View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by CERN Document Server

CERN-TH/99-153
ULB-TH/99-10
hep-th/9906147
June 1999

Effective field theory approach to $\mathcal{N} = 4$ supersymmetric Yang-Mills at finite temperature

Agustin Nieto [1]
CERN – Theory Division, CH-1211 Geneva 23, Switzerland
and
Michel H.G. Tytgat [2]
Service de Physique Théorique, CP225, Université Libre de Bruxelles, Bld du Triomphe, 1050 Brussels, Belgium

Abstract

We study the perturbation expansion of the free energy of $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills at finite temperature in powers of 't Hooft's coupling g^2N in the large N limit. Infrared divergences are controlled by constructing a hierarchy of two 3 dimensional effective field theories. This procedure is applied to the calculation of the free energy to order $(g^2N)^{3/2}$, but it can be extended to higher order corrections.

[1] Agustin.Nieto@cern.ch
[2] mtytgat@ulb.ac.be

In recent years some light has been shed on understanding the relation between string theory and gauge theories [1]. A particularly interesting system is N coincident, parallel D3-branes. This system realizes $\mathcal{N}=4$ supersymmetric $U(N)$ Yang-Mills in its world volume (4 dimensions). In the large N limit, the 3-brane system becomes a black brane whose Bekenstein-Hawking entropy can be obtained by considering Ramond-Ramond charged 3-brane classical solutions [2]. It is therefore interesting to compare the 3-brane thermodynamics with that of $\mathcal{N}=4$ supersymmetric $U(N)$ Yang-Mills. The free energy density of $\mathcal{N}=4$ supersymmetric $U(N)$ Yang-Mills in the ideal gas approximation is $\mathcal{F}_{\rm ideal}=-\pi^2N^2T^4/6$. On the other hand, the free energy of a black 3-brane was found to be [2] $\mathcal{F}_{\rm BH}=(3/4)\times\mathcal{F}_{\rm ideal}$. Maldacena's conjecture [3] helps to understand the relative factor 3/4 between $\mathcal{F}_{\rm ideal}$ and $\mathcal{F}_{\rm BH}$. The conjecture relates type IIB superstrings on $\mathrm{AdS}_5\times\mathrm{S}^5$ to $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills in 4 dimensions and allows to compute the next-to-leading order correction to $\mathcal{F}_{\rm BH}$ in the strong coupling limit [4],

$$\frac{\mathcal{F}_{\rm BH}}{\mathcal{F}_{\rm ideal}}=\frac{3}{4}+\frac{45\zeta(3)}{64\sqrt{2}}\frac{1}{\lambda^{3/2}}\,, \tag{1}$$

where $\lambda=g^2N$ is the 't Hooft coupling. $f(\lambda)\equiv\mathcal{F}/\mathcal{F}_{\rm ideal}$ should be interpreted as a function whose strong coupling limit is $f(\infty)=3/4$, while the weak limit is $f(0)=1$. It has been suggested that $f(\lambda)$ is a monotonic function that interpolates between the strong and the weak coupling limits [1, 4].

We will explore the weak coupling limit of $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills (SYM for short) in 4 dimensions at finite temperature (3 space dimensions). In the present note, we will carry out the calculation of $f(\lambda)$ up to order $\lambda^{3/2}$ by using the effective field theory approach at finite temperature [5, 6]. Related work can be found in [7] and [8, 9], where the terms of order λ and $\lambda^{3/2}$ have also been computed, respectively, but using approaches different from ours.

Feynman rules at finite T are the same as those at $T=0$ except that loops involve infinite sums over Fourier modes [10]. The thermodynamic properties of $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills in 4 dimensions, in thermal equilibrium are described by the free energy density $\mathcal{F}=-T\log\mathcal{Z}/V$, where $\mathcal{Z}$ is the partition function and V is the 3d volume of the system, which is taken to infinity in the thermodynamic limit. The partition function can be written as a path integral over the fields of the Lagrangian $\mathcal{L}$ that describes SYM. Fields are in the adjoint representation of the gauge group, the gauge coupling is g. An expansion for $-T\log\mathcal{Z}/V$ in powers of the coupling constant g can be obtained by summing up the vacuum Feynman diagrams that contribute at a given order in g.

The infrared (IR) behavior of field theories at finite T however prevents us from computing the free energy density in the form just described. There are two sources of IR divergences. First, some fields that are massless at $T=0$, become massive at $T\neq 0$ with a mass of order gT. Diagrams with self-energy insertions have IR divergences that do not cancel order by order in the coupling constant. A second source of IR divergences, which is specific to non-abelian gauge theories, has to do with the gauge field self-interaction terms.

The effective field theory approach allows to remove systematically the IR divergences associated to thermal mass insertions. Also, IR divergences related with the gauge field self-coupling terms are isolated and interpreted as a source of non-perturbative effects [11].

In the effective field theory approach for gauge theories, a hierarchy of two effective field theories in 3 dimensions which only contain static modes are constructed. Since, in addition, these effective field theories do not contain fermionic fields, they are very convenient for non-perturbative studies on the lattice [5].[3]

If non-static modes are integrated out in the partition function $\mathcal{Z}$, we can write

$$\mathcal{Z} = \int \mathcal{D}A_0(\mathbf{x})\mathcal{D}A_i(\mathbf{x})\mathcal{D}\phi_I(\mathbf{x})\; e^{-\int d^3x \mathcal{L}_{\rm eff}}\,, \tag{2}$$

where $\mathcal{L}_{\rm eff}$ is an effective Lagrangian compatible with the internal symmetries of $\mathcal{L}$. $\mathcal{L}_{\rm eff}$ describes a 3 dimensional effective field theory [17]. $\mathcal{L}_{\rm eff}$ is made out of an electrostatic field $A_0(\mathrm{x})$, a magnetostatic gauge field $A_i(\mathrm{x})$, and 6 scalars $\phi_I(\mathrm{x})$, where $I = 1, \ldots, 6$. These fields can be identified, up to field redefinitions, with the static modes of their 4 dimensional counterparts. Note that fermion fields have been integrated out completely because they do not have a static mode. We can write $\mathcal{L}_{\rm eff} = f_E + \mathcal{L}_{\rm ESYM}$, where f_E is a constant that cannot be ignored when computing the free energy. $\mathcal{L}_{\rm ESYM}$ contains the remaining terms and has the form

$$\begin{aligned}\mathcal{L}_{\rm ESYM} \;=\; & \frac{1}{4}\,G^a_{ij}G^a_{ij} \;+\; \frac{1}{2}\,(D_iA_0)^a(D_iA_0)^a \;+\; \frac{1}{2}\,(D_i\phi_I)^a(D_i\phi_I)^a \\ & + \frac{1}{2}m_E^2\,A^a_0A^a_0 \;+\; \frac{1}{2}m_S^2\,\phi^a_I\phi^a_I \;+\; \delta\mathcal{L}_{\rm ESYM},\end{aligned} \tag{3}$$

where $G^a_{ij} = \partial_iA^a_j - \partial_jA^a_i + g_Ef^{abc}A^b_iA^c_j$ is the magnetostatic field strength with gauge coupling constant g_E. The term $\delta\mathcal{L}_{\rm ESYM}$ in (3) represents an infinite number of terms constructed out of $A_0(\mathbf{x})$, $A_i(\mathbf{x})$, and $\phi_I(\mathbf{x})$ that contribute only at order higher than $\lambda^{3/2}$. $\delta\mathcal{L}_{\rm ESYM}$ contains non-renormalizable interaction terms. We call the effective theory constructed by integrating out non-static modes *Electrostatic* SYM (ESYM). The free energy can be written as $\mathcal{F} = T\,(f_E - \log \mathcal{Z}_{\rm ESYM}/V)$, where

$$\mathcal{Z}_{\rm ESYM} = \int \mathcal{D}A_0(\mathbf{x})\mathcal{D}A_i(\mathbf{x})\mathcal{D}\phi_I(\mathbf{x})\; e^{-\int d^3x\, \mathcal{L}_{\rm ESYM}}\,. \tag{4}$$

We can go a step further by integrating out $A_0(\mathbf{x})$, and $\phi_I(\mathbf{x})$ in (4). We obtain

$$\mathcal{Z}_{\rm ESYM} = \int \mathcal{D}A_i(\mathbf{x})\; e^{-\int d^3x\, \widetilde{\mathcal{L}_{\rm eff}}}\,, \tag{5}$$

where $\widetilde{\mathcal{L}_{\rm eff}}$ is an effective Lagrangian compatible with the internal symmetries of the theory which is made out of a magnetostatic gauge field $A_i(\mathbf{x})$. We can write $\widetilde{\mathcal{L}_{\rm eff}} = f_M + \mathcal{L}_{\rm MSYM}$, where f_M is a constant. $\mathcal{L}_{\rm MSYM}$ is of the form

$$\mathcal{L}_{\rm MSYM} = \frac{1}{4}\,H^a_{ij}H^a_{ij} \;+\; \delta\mathcal{L}_{\rm MSYM}, \tag{6}$$

[3]Several theories at finite temperature have been studied perturbatively and non-perturbatively using the effective field theory approach. The ϕ^4-theory is studied in [6], QCD in [12, 11], QED in [13], and scalar QED in [14]. The electroweak phase transition has been studied in the Standard Model [5, 15] and in some of the Minimal Supersymmetric extensions of the Standard Model [16].

where $H_{ij} = \partial_i A_j^a - \partial_j A_i^a + g_M f^{abc} A_i^b A_j^c$ is the magnetostatic field strength with coupling constant g_M. The term $\delta\mathcal{L}_{\rm MSYM}$ in (6) represents an infinite number of terms constructed out of $A_i(\mathbf{x})$. We call this theory *Magnetostatic* SYM (MSYM) because it is made out of magnetostatic fields only. The free energy can therefore be written as

$$\mathcal{F} = T\left(f_E + f_M - \frac{\log \mathcal{Z}_{\rm MSYM}}{V}\right) , \tag{7}$$

where

$$\mathcal{Z}_{\rm MSYM} = \int \mathcal{D}A_i(\mathbf{x})\; e^{-\int d^3x\, \mathcal{L}_{\rm MSYM}} . \tag{8}$$

We may think of using (7) to compute the free energy density of SYM. We need to determine f_E, the effective parameters of $\mathcal{L}_{\rm ESYM}$, f_M, and the effective parameters of $\mathcal{L}_{\rm MSYM}$. The term f_E and the effective parameters of $\mathcal{L}_{\rm ESYM}$ may be determined by computing the same physical quantities in both the full theory (SYM) and in the effective field theory described by $\mathcal{L}_{\rm eff}$ (ESYM) and comparing the result. At first sight this procedure may seem useless if we consider that to compute physical quantities in the full theory we have to remove the IR divergences. It is like having to face the very same problem we wanted to solve at the beginning. However, to compare results, it is actually enough to compute them in a region where we know that both theories describe the same physics.

As mentioned above, self-energy insertions of order gT in the full theory give rise to IR divergences. If we introduce an IR momentum cutoff Λ_E, we will obtain physical quantities as expansions in powers of gT/Λ_E. The expansion diverges when Λ_E goes to zero. However, if we demand $\Lambda_E \gg gT$, the expansion makes sense. Of course, the result depends on Λ_E and one cannot get rid of it unless an infinite number of diagrams are summed up. In the effective theory, the thermal masses (m_E and m_S) are also of order gT (we will explicitly see it below). Therefore, an IR momentum cutoff $\Lambda_E \gg gT$ is equivalent to treating the mass terms in $\mathcal{L}_{\rm eff}$ as interaction terms. By computing physical quantities in this way and comparing the results, we are able to determine the parameters of the effective theory (ESYM) as functions of T, g, and $\Lambda_E \gg gT$.

Once we have determined the parameters of ESYM by matching, we can use ESYM as an ordinary field theory. In general, when computing diagrams using ESYM, we will encounter ultraviolet (UV) divergences. These UV divergences can be regulated by introducing a UV momentum cutoff Λ. Moreover, ESYM is not fully free of IR divergences. In order to regulate the IR divergences originated by the gauge boson self-interaction terms in ESYM, we have to introduce an IR momentum cutoff Λ_M. Analogously to what happened to the full theory with IR cutoff Λ_E, we can make sense of the perturbative expansion of ESYM only if $\Lambda_M \gg g^2T$ [10, 18]. Note that the UV cutoff of ESYM has to satisfy $\Lambda \gg \Lambda_M$. Since $\Lambda_E \gg gT$ and $\Lambda_M \gg g^2T$, we can think of Λ_E as $\Lambda_E \sim T$ and Λ_M as $\Lambda_M \sim gT$. Therefore, we can take Λ_E as the ultraviolet cutoff of ESYM: $\Lambda = \Lambda_E$. The Λ_E dependence of the parameters is canceled by the Λ_E dependence of the loop integrals in ESYM.

We see from (6) that MSYM is a pure gauge theory in 3 dimensions. It is a confining theory that cannot be studied perturbatively. Its Feynman diagrams only contain gauge field self-interaction terms and one should expect the same IR problems that we have already pointed out for the full theory and ESYM. However, if we introduce an IR momentum cutoff

$\Lambda_M \gg g^2 T$, we can still use a diagrammatic expansion to determine the effective parameters of MSYM by matching physical quantities with ESYM. Of course, although we may be able to determine the effective parameters of MSYM, $-\log \mathcal{Z}_{\rm MSYM}/V$ in (7) has still to be calculated non-perturbatively.

We write "$X \approx \cdots$" to denote that the physical quantity X has been computed using the IR cutoffs Λ_E or Λ_M. An expression of this type for X is suitable for matching but it should not be confused with a *correct* perturbative expansion (which should have the IR divergences removed). The perturbative expansions performed with IR cutoffs are called *strict* perturbation expansions.

So far, we have used momentum cutoffs to learn how to organize the perturbative expansion of $\mathcal{N}=4$ supersymmetric Yang-Mills at finite T. However, it turns out to be more convenient to use a regulator that automatically gets rid of power divergences. Such a regulator is dimensional regularization. We therefore consider the full theory in $(3-2\epsilon)+1$ dimensions. Since, in the present note, we are interested in computing the free energy density of $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills to order $\lambda^{3/2}$, we limit ourselves to the effective parameters and corrections that contribute to such an order.

The strict perturbation expansions in the full theory can be greatly simplified if we consider that $\mathcal{N}=4$ supersymmetric Yang-Mills in 4 dimensions (SYM_4) can be obtained by dimensional reduction of $\mathcal{N}=1$ supersymmetric Yang-Mills in 10 dimensions [19]. Physical quantities in SYM_4 can be obtained perturbatively by using SYM_{10} Feynman rules while loop integrals are performed in 4 dimensions ($\mathrm{SYM}_{10\to 4}$) [8]. In this formulation, the 10d A_μ $(\mu=0,1,2,3)$ can be identified, up to scale redefinitions, with the 4d gauge field A_μ. The remaining A_{I+3} $(I=1,\ldots,6)$ are identified with the 6 scalars ϕ_I of $\mathcal{N}=4$ supersymmetric Yang-Mills in 4d. While leading order contributions to the *strict* perturbation expansions in the full theory can be easily computed using either SYM_4 or $\mathrm{SYM}_{10\to 4}$, higher than leading order contributions are much less arduous in $\mathrm{SYM}_{10\to 4}$.

The gauge coupling constant of ESYM, g_E, can be read off from the Lagrangian of the full theory. By substituting $A_0(\mathbf{x},\tau)\to\sqrt{T}A_0(\mathbf{x})$ in the Lagrangian of SYM_4 and comparing $\int_0^\beta d\tau \mathcal{L}_{\rm SYM}$ with $\mathcal{L}_{\rm ESYM}$, we find out, to leading order in g^2,

$$g_E^2 = g^2 T\,. \tag{9}$$

In QCD, the gauge invariant electric screening mass is given by the pole location of the $\mu=\nu=0$ component of the complete gluon propagator [20]. We will use a similar definition in SYM. Using $\mathrm{SYM}_{10\to 4}$, the inverse gauge field propagator has the form $\delta^{ab}\left[\delta_{\mu\nu}k^2+\Pi_{\mu\nu}(0,\mathbf{k})\right]$. In order to compute the static masses in $\mathcal{L}_{\rm ESYM}$, it is convenient to define $\Pi_{\rm el}(k^2)$ and $\Pi_{\rm sc}(k^2)$ such that $\Pi_{00}(0,\mathbf{k})\equiv\Pi_{\rm el}(k^2)$ and $\Pi_{I+3,J+3}(0,\mathbf{k})\equiv\delta_{IJ}\Pi_{\rm sc}(k^2)$ $(I,J=1,\ldots,6)$, respectively. Calculations are performed in Feynman gauge.

The electrostatic mass m_E is determined by computing the electric screening mass in the full theory and matching it with the result obtained in ESYM. The electric screening in SYM is the solution to the equation

$$k^2 \;+\; \Pi_{\rm el}(k^2) \;=\; 0 \qquad \text{at } k^2=-m_{\rm el}^2\,. \tag{10}$$

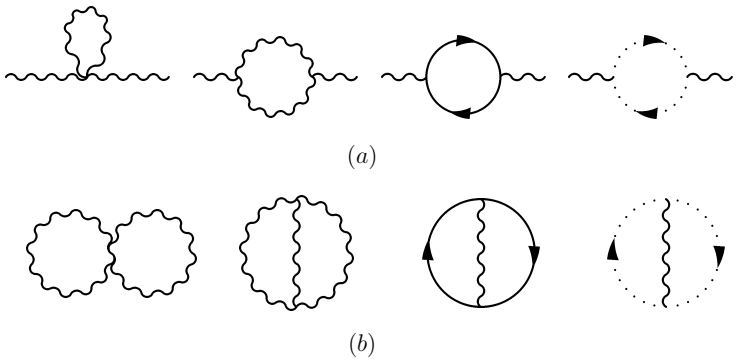

Figure 1: (a) One-loop Feynman diagrams for the gauge field self-energy. (b) Two-loop Feynman diagrams for the free energy. Wavy lines, solid lines, and dotted lines represent the propagators of gauge fields, fermions, and ghosts, respectively using $\mathrm{SYM}_{10\to 4}$.

Note that (10) gives rise to a double expansion in powers of g (or equivalently, in number of loops) and in powers of $m_{\rm el}^2 \sim g^2T^2$. In the calculation of the free energy to order $\lambda^{3/2}$, it is enough to compute $\Pi_{\rm el}$ at one-loop order. Then, the expression for the electric screening mass to leading order in g^2 is $m_{\rm el}^2 \approx \Pi_{\rm el}^{(1)}(0)$. Diagrammatically, $\Pi_{\rm el}^{(1)}(0)$ is given by the graphs shown in Fig. 1(a). These diagrams give rise to sum-integrals that can easily be evaluated by using standard methods [10]. We find that, in the full theory, the strict perturbation expansion for $m_{\rm el}$ is

$$m_{\rm el}^2 \approx 2\, C_A g^2 T^2\,. \tag{11}$$

In ESYM, the electric screening mass $m_{\rm el}$ gives the location of the pole in the propagator for the field $A_0^a(\mathbf{x})$. Denoting the self-energy function by $\Pi_E(k^2)\delta^{ab}$, $m_{\rm el}$ is the solution to

$$k^2 \,+\, m_E^2 \,+\, \Pi_E(k^2) \,=\, 0 \qquad \text{at } k^2 = -m_{\rm el}^2. \tag{12}$$

Similarly to (10), (12) also gives rise to a double expansion in number of loops and in powers of $m_{\rm el}^2 \sim g^2T^2$. The strict perturbation expansion of m_E is obtained as an expansion involving Feynman diagrams evaluated at zero external momentum. Since m_E, m_S, and g_E are treated as perturbation parameters, there is no energy scale in the dimensionally regularized integrals and they all vanish. The solution to the equation (12) for the screening mass is therefore trivial: $m_{\rm el}^2 \approx m_E^2$. Comparing this result with (11), we find that, to leading order in g^2

$$m_E^2 = 2\, C_A g^2 T^2\,. \tag{13}$$

The calculation of the scalar static mass m_S is similar to the calculation of m_E. We define a scalar screening mass $m_{\rm sc}$ which satisfies $k^2 + \Pi_{\rm sc}(k^2) = 0$ at $k^2 = -m_{\rm sc}^2$ in the full theory and an equation analogous to (12) in the effective theory. After comparing the result in SYM with the result in ESYM, we find that, to leading order in g^2

$$m_S^2 = C_A g^2 T^2\,. \tag{14}$$

The effective parameter f_E is evaluated by matching the calculation of the free energy density in the full theory and in the effective theory. The leading order contribution to the

free energy density in the full theory is given by a familiar result from blackbody radiation $\mathcal{F}^{(1)} \approx -\pi^2 T^4/90\,(\delta_B + 7\delta_F/8)$, where δ_B (δ_F) is the number of bosonic (fermionic) degrees of freedom of the theory. In the case of $\mathcal{N} = 4$ SYM$_4$, $\delta_B = \delta_F = 8d_A$, where d_A is the dimension of the adjoint representation of $SU(N)$: $d_A = N^2 - 1$. Alternatively, $\mathcal{F}^{(1)}$ may be obtained by computing the one-loop vacuum diagrams of SYM$_4$ using SYM$_{10\to 4}$. The next-to-leading order contribution to the free energy density is given by the 2-loop Feynman diagrams shown in Fig. 1(b) using SYM$_{10\to 4}$. The evaluation of these diagrams involves elementary sum-integrals that can easily be evaluated by standard methods [10]. Adding the 2-loop order contribution to $\mathcal{F}^{(1)}$, we obtain that the strict perturbation expansion of the free energy density in the full theory is

$$\mathcal{F} \approx -\frac{\pi^2 d_A}{6} T^4 \left(1 - \frac{3}{2\pi^2}\, C_A g^2\right) . \tag{15}$$

In ESYM the effective parameters m_E, m_S, and g_E are treated as perturbation parameters. Then, as there is no scale in the dimensionally regularized integrals, they all vanish. The strict perturbation expansion of the free energy density is therefore trivial: $\mathcal{F} \approx T f_E$. Comparing this result with (15), f_E to order g^2 is

$$f_E = -\frac{\pi^2 d_A}{6} T^3 \left(1 - \frac{3}{2\pi^2}\, C_A g^2\right) . \tag{16}$$

As a check we have also computed the effective parameters of ESYM m_E, m_S, and f_E using SYM$_4$ instead of SYM$_{10\to 4}$ and found the same results given in Eqs. (13), (14), and (16).

To determine f_M, we have to compute the strict perturbation expansion of $-\log \mathcal{Z}_{\rm ESYM}/V$ and compare it with that of $f_M - \log \mathcal{Z}_{\rm MSYM}/V$. Computing the one-loop vacuum diagrams of ESYM, we find $-\log \mathcal{Z}_{\rm ESYM}/V \approx -d_A/(12\pi)\ (m_E^3 + 6\ m_S^3)$. The effective parameters of $\mathcal{L}_{\rm MSYM}$ are treated as perturbation parameters. Feynman diagrams give rise to integrals which have no scale dependence. Therefore, using dimensional regularization all the contributions to $-\log \mathcal{Z}_{\rm MSYM}/V$ vanish. We have $f_M - \log \mathcal{Z}_{\rm MSYM}/V \approx f_M$. Comparing this result with $-\log \mathcal{Z}_{\rm ESYM}/V$, we obtain f_M:

$$f_M = -\frac{d_A}{12\pi}\ (m_E^3 + 6\ m_S^3)\,. \tag{17}$$

The gauge coupling constant g_M can be read off from the Lagrangian of ESYM $\mathcal{L}_{\rm ESYM}$. To leading order in g_E^2 and g^2, we obtain $g_M^2 = g_E^2 = g^2 T$.

The free energy of $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills is given by (7). The term $-\log \mathcal{Z}_{\rm MSYM}/V$ is non-perturbative. It is easy to find out at which order non-perturbative corrections become relevant. The only quantity with dimensions in $\mathcal{L}_{\rm MSYM}$ is g_M^2 (neglecting $\delta\mathcal{L}_{\rm MSYM}$ whose contribution is subleading). Since $-\log \mathcal{Z}_{\rm MSYM}/V$ has dimension 3 in units of energy, its lowest order contribution to the free energy is therefore of order $g_M^6 = (g^2 T)^3$. We conclude that the non-perturbative corrections generated by $\log \mathcal{Z}_{\rm MSYM}/V$ only contribute to the free energy of SYM at order λ^3 or higher.

Using (7), the free energy of $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills in 4 dimensions to order $\lambda^{3/2}$ is given by $\mathcal{F} = T\,[f_E - d_A/(12\pi)\ (m_E^3 + 6\ m_S^3)]$. Substituting (13), (14),

and (16), we finally find that, in the large N limit,

$$f(\lambda) \equiv \frac{\mathcal{F}}{\mathcal{F}_{\rm ideal}} = 1 - \frac{3}{2\pi^2}\,\lambda + \frac{3+\sqrt{2}}{\pi^3}\,\lambda^{3/2}\,. \tag{18}$$

This result is in agreement with previously reported calculations. In Ref. [7], the free energy was evaluated to order λ. In [8] and [9], the term of order $\lambda^{3/2}$ has been computed by regulating the IR divergences of SYM_4 by adding and subtracting out thermal masses to the Lagrangian. Vazquez-Mozo [8] used $\mathrm{SYM}_{10\to 4}$ to compute the first two terms in (18).

Since the term of order $\lambda^{3/2}$ in (18) is positive, it seems that our result does not favor a monotonically decreasing function interpolating between the weak and strong coupling limits. However, such a behavior may be an effect of the zero convergence radius of the weak-coupling expansion (18). It has been suggested [21] that there could be a phase transition in λ at large N. Even though perturbative calculations cannot settle the debate on this matter, it may be possible to find an indication of a phase transition by using Pade approximants. An analysis of (18) using Pade approximants has been carried out in [9]. There, an indication has been found of a smooth interpolation between the weak and strong coupling regimes.

The effective field theory approach provides a systematic way to compute perturbative contributions to the free energy of SYM while taking advantage of dimensional reduction ($\mathrm{SYM}_{10\to 4}$) to determine the effective parameters of ESYM. This is a major simplification for computing $f(\lambda)$ to order less than λ^3 [23]. Starting at order λ^3, there are non-perturbative contributions as well as perturbative contributions. Non-perturbative effects are described by MSYM, a pure gauge theory in 3 dimensions. As we have already pointed out, the effective field theory approach gives rise to effective field theories in 3 dimensions, without fermions, that are suitable for non-perturbative studies on the lattice. Lattice simulations of the effective field theories could therefore be used to explore the non-perturbative sector of $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills.

Acknowledgements

We are grateful to E. Braaten and E. Alvarez for valuable comments on a draft of this paper. M.T. thanks A. Wilch for useful discussions.

References

[1] I.R. Klebanov, hep-th/9901018.

[2] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. **D54**, 3915 (1996) hep-th/9602135.

[3] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) hep-th/9711200.

[4] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B534**, 202 (1998) hep-th/9805156.

[5] K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. **B425**, 67 (1994) hep-ph/9404201.

[6] E. Braaten and A. Nieto, Phys. Rev. **D51**, 6990 (1995) hep-ph/9501375.

[7] A. Fotopoulos and T.R. Taylor, Phys. Rev. **D59**, 061701 (1999) hep-th/9811224.

[8] M.A. Vazquez-Mozo, hep-th/9905030.

[9] C. Kim and S. Rey, hep-th/9905205.

[10] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, 1996).

[11] E. Braaten and A. Nieto, Phys. Rev. **D53**, 3421 (1996) hep-ph/9510408.

[12] E. Braaten, Phys. Rev. Lett. **74**, 2164 (1995) hep-ph/9409434.

[13] J.O. Andersen, Phys. Rev. **D53**, 7286 (1996) hep-ph/9509409.

[14] Nucl. Phys. **B520**, 345 (1998) hep-lat/9711048. K. Kajantie, M. Laine, T. Neuhaus, J. Peisa, A. Rajantie and K. Rummukainen, *ibid* **B546**, 351 (1999) hep-ph/9809334. K. Kajantie, M. Karjalainen, M. Laine, J. Peisa and A. Rajantie, Phys. Lett. **B428**, 334 (1998) hep-ph/9803367. K. Kajantie, M. Karjalainen, M. Laine and J. Peisa, Phys. Rev. **B57**, 3011 (1998) cond-mat/9704056. J.O. Andersen, Z. Phys. **C75**, 147 (1997) hep-ph/9606357.

[15] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Phys. Rev. Lett. **77**, 2887 (1996) hep-ph/9605288. K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. **B458**, 90 (1996) hep-ph/9508379. K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. **B442**, 317 (1995) hep-lat/9412091.

[16] J.M. Cline and K. Kainulainen, Nucl. Phys. **B482**, 73 (1996) hep-ph/9605235. M. Losada, Phys. Rev. **D56**, 2893 (1997) hep-ph/9605266. M. Laine, Nucl. Phys. **B481**, 43 (1996) hep-ph/9605283. D. Bodeker, P. John, M. Laine and M.G. Schmidt, *ibid* **B497**, 387 (1997) hep-ph/9612364. A. Rajantie, *ibid* **B501**, 521 (1997) hep-ph/9702255. J.M. Cline and K. Kainulainen, *ibid* **B510**, 88 (1998) hep-ph/9705201.

[17] T. Appelquist and R.D. Pisarski, *Phys. Rev.* D **23**, 2305 (1981). S. Nadkarni, *ibid* D **27**, 917 (1983). D.J. Gross, R.D. Pisarski, and L.G. Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981).

[18] A. Nieto, Int. J. Mod. Phys. **A12**, 1431 (1997) hep-ph/9612291.

[19] L. Brink, J Schwarz, and J. Scherk, *Nucl. Phys.* **B121**, 77 (1977); F. Gliozzi, J. Scherk, and D. Olive, *ibid* **B122**, 253 (1977);

[20] A.K. Rebhan, Phys. Rev. **D48**, 3967 (1993) hep-ph/9308232. E. Braaten and A. Nieto, Phys. Rev. Lett. **73**, 2402 (1994) hep-ph/9408273.

[21] M. Li, JHEP **03**, 004 (1999) hep-th/9807196.

[22] P. Arnold and O. Espinosa, Phys. Rev. **D47**, 3546 (1993); P. Arnold and C. Zhai, *ibid* **D50**, 7603 (1994); *ibid* **D51**, 1906 (1995); C. Zhai and B. Kastening, *ibid* **D52**, 7232 (1995).

[23] A. Nieto and M. Tytgat, work in progress.