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Leptogenesis in Theories with Large Extra Dimensions

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ABSTRACT

We study the scenario of baryogenesis through leptogenesis in higher-dimensional theories, in which the scale of quantum gravity may be as low as few TeV. The minimal realization of these theories includes an isosinglet neutrino which is considered to propagate in large compact dimensions, whereas all the SM particles are localized on a $(1 + 3)$ -dimensional wall. In the formulation of minimal leptogenesis models, we pay particular attention to the existence of Majorana spinors in higher dimensions. After compactification of the extra dimensions, we obtain a tower of Majorana Kaluza-Klein excitations which act as an infinite series of CP-violating resonators, and derive the necessary conditions for their constructive interference. Based on this CP-violating mechanism, we find that out-of-equilibrium decays of heavy Majorana excitations having masses above the critical temperature of the electroweak phase transition can produce a leptonic asymmetry which is reprocessed into the observed baryonic asymmetry of the Universe via unsuppressed sphaleron interactions.

1 Introduction

Superstring theories have been advocated to provide a consistent theoretical framework that could lead to quantization of gravity, including its possible unification with all other fundamental forces in nature. The quantum nature of gravity is expected to play a central role at energy scales close to the Planck mass, $M_{\text{P}} = 1.2 \times 10^{19}$ GeV. The formulation of superstring theories requires the embedding of our well-established $(1 + 3)$ -dimensional world into a higher dimensional space, in which the new spatial dimensions must be highly curved for both phenomenological and theoretical reasons. In typical string theories, the fundamental string scale is generically of order M_{P} . However, Witten [1], and Hořava and Witten [2] presented an interesting alternative, in which the string scale may be considerably lowered to $\sim 10^{16}$ GeV, thereby enabling the unification of all interactions within the minimal supersymmetric model. An analogous scenario was subsequently discussed by Lykken [3], in which the string scale was further lowered to the TeV range,* but the fundamental Planck scale was kept intact to M_{P} .

Recently, Arkani-Hamed, Dimopoulos and Dvali [5] have considered a more radical scenario, in which the fundamental scale of quantum gravity, M_F , may be as low as few TeV, thereby proposing an appealing solution to the known gauge hierarchy problem [6]. The observed weakness of gravity may then be attributed to the presence of a number δ of large extra spatial dimensions, within which only gravity can propagate and, most probably, fields that are singlets under the Standard Model (SM) gauge group, such as isosinglet neutrinos [7,8]. This higher $[1 + (3 + \delta)]$ -dimensional space is usually termed bulk. On the other hand, all the ordinary SM particles live in the conventional $(1 + 3)$ -dimensional Minkowski subspace, which is called wall. In such a theoretical framework, the ordinary Planck mass M_{P} must be viewed as an effective parameter, which is related to the genuinely fundamental scale M_F via a kind of generalized Gauss's law

$$M_{\text{P}} = M_F (2\pi R M_F)^{\delta/2}, \quad (1.1)$$

where we have assumed, for simplicity, that the additional δ -dimensional volume has the configuration of a torus, with all of its radii being equal. Many astrophysical [9–16] and phenomenological [17] analyses have already appeared in the recent literature for such low string-scale theories.

As has been mentioned already, it is conceivable to assume that isosinglet neutrinos exist in addition to gravitons, and that also feel the presence of large extra space dimensions.

*In a different context, Antoniadis [4] had made an earlier suggestion of a low compactification scale of order TeV in the context of string theories.

In particular, we wish to study novel scenarios, in which the existence of higher-dimensional singlet fields may account for the observed baryonic asymmetry of the Universe (BAU) by means of the Fukugita–Yanagida mechanism of leptogenesis [18]. According to this mechanism, an excess of the lepton number (L) is first generated by out-of-equilibrium L -violating decays of heavy Majorana neutrinos, which is then converted into an asymmetry of the baryon number (B) through $(B + L)$ -violating sphaleron interactions [19]. Such an L -to- B conversion of asymmetries stays unsuppressed, as long as the heavy Majorana-neutrino masses lie above the critical temperature T_c of the electroweak phase transition where sphalerons are supposed to be in thermal equilibrium. Such a scenario of explaining the BAU is often called baryogenesis through leptogenesis.

The presence of extra large dimensions introduces a number of alternatives for leptogenesis which may even have no analog in the conventional 4-dimensional theories. We shall focus our attention on minimal realizations of higher-dimensional leptogenesis models which, after compactification of the extra dimensions, lead to scenarios that admit renormalization assuming a finite number of Kaluza–Klein (KK) excitations. Such models of leptogenesis are therefore endowed with enhanced predictive power. For definiteness, we will consider minimal 4-dimensional extensions of the SM, augmented by one singlet Dirac neutrino, which propagates in the bulk. Parenthetically, we should notice that massive Majorana neutrinos are not defined for spaces with any space-time dimensions but only for those with $2, 3$ and $4 \bmod 8$ dimensions [20,21]. For instance, unlike in 4 dimensions, true Majorana spinors cannot be defined in 5 dimensions. This topic will be discussed in detail in Section 2.

After compactification of the extra dimensions, the kinetic term of the bulk neutrino gives rise to an infinite series of massive KK excitations, with equally spaced Dirac masses, i.e. the mass difference between two neighbouring KK states is of order $1/R$. In order to make the mechanism of leptogenesis work, it is necessary that the model under consideration violates both the lepton number L and the product of symmetries of charge conjugation (C) and (parity) space reflection (P), also known as CP symmetry. The violation of L can be introduced into the theory by simultaneously coupling the different spinorial states of the higher-dimensional Dirac neutrino to the lepton doublets of the SM and to their C-conjugate counterparts. As we will see in Section 3 however, this is not sufficient for the theory to be CP violating. CP nonconservation can be minimally realized in two different ways: one has to either (i) include additional higher-dimensional fermionic bilinears or (ii) extend the Higgs sector of the SM. Obviously, one may also consider more involved models based on combinations of these two minimal scenarios. The first scenario may be regarded as a higher-dimensional extension of the ordinary leptogenesis model [18]. Of most interest

is, however, the second alternative which has no analog in 4 dimensions, as it does not require the inclusion of any heavy Majorana or isosinglet mass scale in the Lagrangian. The characteristic feature of these extensions is that each of the Dirac KK neutrino states splits into two nearly degenerate Majorana neutrinos either at the tree level in the first scenario or at one loop in the second one.

The observed BAU can only arise from those Majorana KK states that are heavier than the critical temperature T_c , namely the temperature at which the electroweak phase transition occurs. The reason is that the $(B+L)$ -violating sphalerons are in thermal equilibrium above T_c and the excess in the lepton number, produced by out-of-equilibrium decays of KK states, can rapidly be reprocessed into the desired baryonic asymmetry. Thus, the KK states that are lighter than $T_c \approx 100\text{--}200$ GeV are irrelevant for the BAU. Furthermore, there are generically two distinct mechanisms that give rise to CP nonconservation in the decays of heavy Majorana KK states. In the first mechanism, CP violation is induced by the interference of the tree-level decay graph with the absorptive part of a one-loop vertex diagram [18,22]; we call the latter ε' -type CP violation in connection with the established terminology of the kaon system. In the second mechanism, which we call it ε -type CP violation, the tree-level diagram interferes with the absorptive part of the one-loop self-energy transition between two heavy Majorana neutrinos [23–25], i.e. between heavy Majorana KK states. If the mass difference of two heavy Majorana states is of the order of their respective widths, the description of ε -type CP violation becomes more subtle field-theoretically [24]. In this case, finite-order perturbation theory does no longer apply, and one is therefore compelled to resort to a resummation approach which consistently takes the instability of the mixed heavy Majorana states into account. This issue has extensively been discussed in [24].

Furthermore, it was shown [24] that ε -type CP violation induced by the mixing of two nearly degenerate heavy Majorana states can be resonantly enhanced up to order of unity. As we shall discuss in more detail in Section 4, an analogous dynamics exhibits the system of the Majorana KK excitations. In fact, each KK pair of the two nearly degenerate Majorana states behaves as an individual CP-violating resonator. In this way, we shall characterize a two-level system that satisfies the resonant conditions of order-unity CP violation. We find that the spacing in mass for two adjacent KK pairs of Majorana states governs the dynamics for constructive or destructive interference of the complete tower of the CP-violating resonators. Owing to cancellations among the different CP-violating vertex contributions, we can explicitly demonstrate that ε' -type CP violation is almost vanishing.

A crucial requirement for successful baryogenesis through leptogenesis is that the

temperature of reheating T_r [26] due to the late decays of gravitons into photons be not much smaller than the critical temperature T_c . Otherwise, the final baryon asymmetry will be dramatically diluted by the entropy released by the decaying gravitons. In particular, it has been argued [12] that it may be difficult to obtain a reheat temperature above T_c in theories with a low scale of quantum gravity M_F , such that sphalerons can effectively reprocess an excess in L into B . For 2 extra large dimensions, the authors [12] derive the mass limit $M_F \gtrsim 100$ TeV, assuming that the reheat temperature is larger than few MeV, so as to ensure that primordial nucleosynthesis proceeds as usual. This bound is also in qualitative agreement with recent constraints derived from considerations of rapid supernovae cooling due to graviton emission [13] and of the cosmic diffuse gamma radiation [14]. Nevertheless, several possibilities have already been reported in the literature that one might think of to avoid possible difficulties associated with a low T_r . For example, one could imagine that the bulk singlet neutrino only resides in a subspace of a multidimensional space spanned by a number $\delta = 6$ of extra dimensions and higher, in which gravity propagates [7]. This could lead to rather suppressed production rates of gravitons, thus allowing much larger reheat temperatures [12]. Another way of resolving the problem of a low T_r is to assume that the compactification radii [15,16] of gravity are not all equal but possess a large hierarchy. Such a possibility would completely change the usual cosmological picture of the previous analyses. In this context, it has been further advocated that gravitons might decay faster on a hidden wall than on the observable wall we live [9] or even a novel type of rapid asymmetric inflation could take place [11]. Because of the variety of the solutions suggested in the literature, in our analysis we shall not put forward a specific mechanism of increasing the reheat temperature but will simply assume that $T_r \gtrsim T_c$. Especially, we shall see how the resonantly enhanced CP asymmetries in the decays of the KK states may help to overcome part of the low- T_r problem.

The paper is organized as follows: Section 2 reviews the topic related to the ability of defining true Majorana spinors in higher-dimensional theories. In Section 3, we formulate minimal renormalizable higher-dimensional models that can lead to successful scenarios of leptogenesis. In Section 4, we derive the necessary conditions for order-unity CP asymmetries due to the constructive interference of the tower of the KK CP-violating resonators, and show the vanishing of all the ε' -type CP-violating contributions. In Section 5, we give an estimate of the baryonic asymmetry which arises from a sphaleron-converted leptonic asymmetry. Finally, Section 6 presents our conclusions.

2 Majorana spinors in higher dimensions

The violation of the lepton-number in leptogenesis models or supersymmetric theories is naturally mediated by Majorana fields, e.g. heavy Majorana neutrinos, neutralinos etc. The KK formulation of these theories necessitates an analogous extension of the notion of the Majorana spinor to higher dimensions [20,21]. The ability of defining true Majorana neutrinos in any dimensions plays a key role in the construction of higher-dimensional leptogenesis models. Here we shall review this topic from a more practical, for our purposes, point of view.

We shall consider d -dimensional theories with one time component and $d - 1$ spatial ones. We assume that the Lagrangian describing these theories is invariant under the generalized Lorentz transformations of the $SO(1, d - 1)$ group. In such an extended d -dimensional Minkowski space, the corresponding Clifford algebra reads

$$\{\gamma_\mu^{(d)}, \gamma_\nu^{(d)}\} = 2g_{\mu\nu}^{(d)} \mathbf{1}, \quad (2.1)$$

where $g_{\mu\nu}^{(d)} = \text{diag}(+1, -1, \dots, -1)$, for $\mu, \nu = 0, 1, \dots, d - 1$, and $\gamma_\mu^{(d)}$ are the generalized Dirac's gamma matrices. The construction of these matrices to any number of dimensions may be found recursively. Our starting point is the representation of gamma matrices for $d = 2$ and $d = 3$, i.e.

$$\gamma_0^{(2,3)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma_1^{(2,3)} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \gamma_2^{(3)} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}. \quad (2.2)$$

The procedure for constructing gamma matrices to higher dimensions is then as follows. If $d = 2m$ ($m = 1, 2, \dots$), we may then define

$$\gamma_0^{(d)} = \begin{bmatrix} 0 & \mathbf{1}_m \\ \mathbf{1}_m & 0 \end{bmatrix}, \quad \gamma_k^{(d)} = \begin{bmatrix} 0 & \gamma_0^{(d-1)} \gamma_k^{(d-1)} \\ -\gamma_0^{(d-1)} \gamma_k^{(d-1)} & 0 \end{bmatrix}; \quad k = 1, \dots, d - 2, \quad (2.3)$$

$$\gamma_{d-1}^{(d)} = \begin{bmatrix} 0 & \gamma_0^{(d-1)} \\ -\gamma_0^{(d-1)} & 0 \end{bmatrix}, \quad \text{and} \quad \gamma_P^{(d)} = \begin{bmatrix} \mathbf{1}_m & 0 \\ 0 & -\mathbf{1}_m \end{bmatrix}, \quad (2.4)$$

where $\mathbf{1}_m$ is the unity matrix in m dimensions. Note that the dimensionality of the representation of the gamma matrices for $d = 2m + 1$ coincides with that of $d = 2m$. The matrix $\gamma_P^{(d)}$ is the generalization of the usual γ_5 matrix in four dimensions, i.e. $\gamma_P^{(d)} = c \prod_{\mu=0}^{d-1} \gamma_\mu^{(d)}$, where the constant c is defined such that $\gamma_P^{(d)2} = \mathbf{1}$. The matrix $\gamma_P^{(d)}$ anticommutes with all $\gamma_\mu^{(d)}$ for $d = 2m$, whereas it commutes with all $\gamma_\mu^{(d)}$ for $d = 2m + 1$, i.e. it is proportional

to the unity matrix. If we know the representation of gamma matrices for $d = 2m$, we can easily construct the respective one for $d = 2m + 1$, just by including

$$\gamma_d^{(d+1)} = i\gamma_P^{(d)}. \quad (2.5)$$

In fact, Eqs. (2.3)–(2.5) are sufficient to construct all $\gamma_\mu^{(d)}$ in any number d of dimensions, starting from the known expressions (2.2) for $d = 2, 3$. In addition, we should notice that the adopted representations of $\gamma_\mu^{(d)}$ are of the Weyl type, having the properties

$$\gamma_0^{(d)} = \gamma_0^{(d)\dagger}, \quad \gamma_k^{(d)} = -\gamma_k^{(d)\dagger}; \quad k = 1, \dots, d-1. \quad (2.6)$$

Finally, a useful property of the above construction is the fact that $\gamma_\mu^{(d)}$ are self-adjoint under the known bar operation, i.e.

$$\overline{\gamma_\mu^{(d)}} \equiv \gamma_0^{(d)} \gamma_\mu^{(d)\dagger} \gamma_0^{(d)} = \gamma_\mu^{(d)}, \quad (2.7)$$

and $\overline{i\gamma_P^{(d)}} = i\gamma_P^{(d)}$.

Let us now define by $\psi(x)$ a massive fermionic free field in a multidimensional Minkowski space, which satisfies the free Dirac equation of motion, i.e. $(i\gamma^\mu\partial_\mu - m)\psi = 0$. Here and henceforth, we shall drop the superscript ‘ (d) ’ on the gamma matrices to simplify notation. The Lorentz adjoint of ψ is then given by $\bar{\psi} = \psi^\dagger\gamma_0$, while invariance of the Dirac equation under generalized Lorentz transformations requires

$$\bar{S} = \gamma_0 S^\dagger \gamma_0 = S^{-1}, \quad (2.8)$$

where

$$S = \exp\left(-\frac{i}{4} \sum_n \omega_n \sigma_{\mu\nu} I_n^{\mu\nu}\right), \quad (2.9)$$

with $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, is the d -dimensional spinorial representation of an arbitrary Lorentz rotation with angles ω_n , and $I_n^{\mu\nu}$ are the generators of $\text{SO}(1, d-1)$. It is easy to see that Eq. (2.8) is equivalent to $\gamma_0 \sigma_{\mu\nu}^\dagger \gamma_0 = \sigma_{\mu\nu}$. The last equality is true by virtue of Eq. (2.7).

To define charge-conjugate fermionic fields in theories with many dimensions, we proceed as follows. We start with the classical Dirac equation by including a background electromagnetic field A_μ coupled to ψ , i.e. $[i\gamma^\mu(\partial_\mu + eA_\mu) - m]\psi = 0$, and then seek for a solution of the respective Dirac equation for the antiparticle field, denoted as ψ^C , which is of the form $[i\gamma^\mu(\partial_\mu - eA_\mu) - m]\psi^C = 0$. In case ψ is a neutral field, e.g. a neutrino, one should initially assume that $e \neq 0$ and then take the limit $e \rightarrow 0$ at the very end of the consideration. In this way, we find that ψ^C may be determined in terms of ψ as follows:

$$\psi^C = C\bar{\psi}^T = C\gamma_0\psi^*, \quad (2.10)$$

where C is the charge-conjugation operator that satisfies the property

$$C^{-1} \gamma_\mu C = -\gamma_\mu^T, \quad (2.11)$$

for massive fermionic fields. For massless fermions, we may also allow the equality

$$C^{-1} \gamma_\mu C = \gamma_\mu^T. \quad (2.12)$$

Furthermore, consistency of charge conjugation with Lorentz invariance implies that

$$C^{-1} S C = (S^{-1})^T, \quad (2.13)$$

or equivalently that $C^{-1} \sigma_{\mu\nu} C = -\sigma_{\mu\nu}^T$, which holds true because of Eqs. (2.11) or (2.12). At this point, we should remark that the transformations

$$\gamma'_\mu = U \gamma_\mu U^{-1}, \quad C' = U C U^T \quad (2.14)$$

preserve all the relations of gamma matrices given above, including Eqs. (2.8) and (2.13).

The necessary and sufficient condition for the existence of a Majorana spinor in any number of dimensions reads

$$\psi = \psi^C, \quad (2.15)$$

which amounts to

$$C \gamma_0^T (C \gamma_0^T)^* = 1. \quad (2.16)$$

This last equality may be rewritten as

$$C^{-1} \gamma_0 C = (C^* C) \gamma_0^T. \quad (2.17)$$

Consequently, massive (massless) Majorana spinors in d dimensions are admitted, if both the construction of a C matrix satisfying Eq. (2.11) (Eq. (2.12)) and $C^* C = -1$ ($C^* C = +1$) is possible. As we will see below, this is not always the case.

For this purpose, it is important to be able to construct a matrix that obeys the identity (2.11) or (2.12). There are only two candidates that could be of interest:

$$C_A = \prod_i^p \gamma_i; \quad \text{with} \quad \gamma_i = -\gamma_i^T = -\gamma_i^\dagger, \quad (2.18)$$

$$C_S = \prod_r^s \gamma_r; \quad \text{with} \quad \gamma_r = \gamma_r^T, \quad \gamma_0 = \gamma_0^\dagger, \quad \gamma_r = -\gamma_r^\dagger \quad (r \neq 0). \quad (2.19)$$

Specifically, C_A (C_S) is formed by the product of all p (s) in number gamma matrices that are pure antisymmetric (symmetric). Employing the identity: $\gamma_\mu \gamma_\mu^\dagger = \mathbf{1}$, we can easily find the following relations for the two C -conjugation matrices:

$$C_A^{-1} = C_A^\dagger = (-1)^p \varepsilon(p) C_A, \quad C_A^T = (C_A^\dagger)^* = (-1)^p \varepsilon(p) C_A^*, \quad C_A = C_A^*, \quad (2.20)$$

$$C_S^{-1} = C_S^\dagger = (-1)^{s-1} \varepsilon(s) C_S, \quad C_S^T = (C_S^\dagger)^* = (-1)^{s-1} \varepsilon(s) C_S^*, \quad C_S^* = (-1)^{s-1} C_S, \quad (2.21)$$

d	s even	$\varepsilon(s) = -1$	p odd	$\varepsilon(p) = 1$	Existence of massive Majorana spinor
2	1	1	1	1	yes
3	2	-1	1	1	yes
4	2	-1	2	-1	yes
5	3	-1	2	-1	no
6	3	-1	3	-1	no
7	4	1	3	-1	no
8	4	1	4	1	no
9	5	1	4	1	no
10	5	1	5	1	yes
11	6	-1	5	1	yes
12	6	-1	6	-1	yes
13	7	-1	6	-1	no

Table 1: Existence of massive Majorana spinors in d dimensions.

with $\varepsilon(z) = (-1)^{z(z-1)/2}$. As advertised, it can be shown that the two C -conjugation matrices satisfy the relations

$$C_A^{-1} \gamma_\mu C_A = (-1)^p \gamma_\mu^T, \quad C_S^{-1} \gamma_\mu C_S = (-1)^{s+1} \gamma_\mu^T. \quad (2.22)$$

On the other hand, the Majorana condition given by Eq. (2.17) may now be translated into

$$C_A^{-1} \gamma_0 C_A = (-1)^p \varepsilon(p) \gamma_0^T, \quad C_S^{-1} \gamma_0 C_S = \varepsilon(s) \gamma_0^T. \quad (2.23)$$

As a consequence, the existence of a massive Majorana spinor in any number of dimensions is ensured, if

$$\varepsilon(p) = 1 \quad \text{and} \quad p \text{ is odd}, \quad (2.24)$$

or if

$$\varepsilon(s) = -1 \quad \text{and} \quad s \text{ is even}. \quad (2.25)$$

Based on Eqs. (2.24) and Eqs. (2.25), we can generate Table 1. As can be seen from this Table, true *massive* Majorana neutrinos exist only in 2, 3 and 4 mod 8 dimensions [20].

If we also allow the possibility of massless Majorana-Weyl spinors, we only need to impose the restrictions:

$$\varepsilon(p) = +1 \quad \text{or} \quad \varepsilon(s) = (-1)^{s+1}. \quad (2.26)$$

d	$\varepsilon(p) = 1$	$\varepsilon(s) = (-1)^{s+1}$		Existence of massless Majorana spinor
5	-1	-1	1	no
6	-1	-1	1	no
7	-1	1	-1	no
8	1	1	-1	yes
9	1	1	1	yes

Table 2: Existence of massless Majorana-Weyl spinors in d dimensions.

The so-generated Table 2 shows that in addition to the result found in the massive case, the definition of massless Majorana fields can be extended to 8 and 9 mod 8 dimensions [21]. For example, our analysis explicitly demonstrates that, as opposed to 4 dimensions, true Majorana neutrinos cannot be defined in theories with 5, 6, and 7 dimensions. In fact, in the latter theories, C loses its very same meaning of being a genuine charge-conjugation matrix. We shall pay special attention to this issue in the next section, while formulating different minimal models of leptogenesis.

3 Higher-dimensional models of leptogenesis

If the SM contains a singlet neutrino that feels extra large dimensions, this additional volume factor of the new spatial dimensions introduces a new possibility to naturally suppress the Higgs Yukawa coupling to neutrinos [7,8]. After spontaneous symmetry breaking (SSB) of the SM Higgs potential, the resulting neutrino masses may naturally be of the order of 10^{-2} eV, which turns out to be in the right ball park for explaining the solar and atmospheric neutrino data [16]. Here we shall formulate minimal models of leptogenesis which, after compactification of the extra dimensions, give rise to theories containing 4-dimensional operators only, and are therefore renormalizable for finitely many KK states. Even though the number of KK excitations is formally infinite, on theoretical grounds, however, one expects the presence of an ultra-violet (UV) cutoff close to the string scale where gravity is supposed to set in. The issue of renormalization will become more clear while describing the leptogenesis models.

For simplicity, we shall consider a 5-dimensional model. The generalization of the results to higher dimensions is then straightforward. Following [7,8], we assume that all particles with non-zero SM charges live in a subspace of $(1+3)$ dimensions. Also, we intro-

duce one Dirac isosinglet neutrino $N(x_\mu, y)$ that propagates in the bulk of all 5 dimensions. We denote by $x_\mu = (x_0, x_1, x_2, x_3)$ the one time and the three spatial coordinates of our observable world and by $y \equiv x_4$ the new spatial dimension. The y -coordinate is to be compactified on a circle of radius R by applying the periodic identification: $y \equiv y + 2\pi R$. Specifically, the minimal field content of a one-generation model of leptogenesis is

$$L(x) = \begin{pmatrix} \nu_L(x) \\ l_L(x) \end{pmatrix}, \quad l_R(x), \quad N(x, y) = \begin{pmatrix} \xi(x, y) \\ \bar{\eta}(x, y) \end{pmatrix}, \quad (3.1)$$

where ν_L , l_L and l_R are 4-dimensional Weyl spinors, and ξ and η are two-component spinors in 5 dimensions. Depending on the model, we shall also assume that ξ (η) is symmetric (antisymmetric) under a y reflection: $\xi(x, y) = \xi(x, -y)$ and $\eta(x, y) = -\eta(x, -y)$.[†] Following the procedure outlined in Section 2, the gamma matrices in 5 dimensions may be represented by

$$\gamma_\mu = \begin{pmatrix} 0 & \bar{\sigma}_\mu \\ \sigma_\mu & 0 \end{pmatrix}, \quad \text{and} \quad \gamma_4 = \begin{pmatrix} i\mathbf{1}_2 & 0 \\ 0 & -i\mathbf{1}_2 \end{pmatrix}, \quad (3.2)$$

where $\sigma^\mu = (\mathbf{1}_2, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbf{1}_2, -\vec{\sigma})$, with $\vec{\sigma}_{1,2,3}$ being the usual Pauli matrices.

As we have mentioned in the introduction, there are two representative minimal scenarios of leptogenesis:

- (i) The first scenario may be viewed as a higher-dimensional generalization of the usual leptogenesis model of Ref. [18], in which the Lorentz- and gauge- invariant fermionic bilinears $\bar{N}N$ and $N^T C^{(5)-1}N$ are included. As we will see, however, if a Z_2 discrete symmetry is imposed on $N(x, y)$, the former bilinear mass term $\bar{N}N$ does not contribute to the effective action. According to Eqs. (2.18) and (2.19), the matrix $C^{(5)}$ satisfies Eqs. (2.12) and (2.13), but not Eq. (2.11) that defines the true C -conjugation matrix for a massive Dirac field. Despite its close analogy to 4 dimensions, the operator $N^T C^{(5)-1}N$ does not represent a genuine bare Majorana mass in 5 dimensions. Nevertheless, after KK compactification, the effective Lagrangian of this scenario displays a dynamics rather analogous to the known scenario of leptogenesis due to Fukugita and Yanagida [18].
- (ii) The second scenario of leptogenesis requires, in addition to the bulk Dirac singlet field $N(x, y)$, that the Higgs sector of the SM be extended by two more Higgs doublets. The first Higgs doublet, denoted as Φ_1 , couples to the lepton isodoublet L , the second

[†]With the imposition of such a symmetry which might be justified within the context of a Z_2 orbifold compactification [8], one may avoid a twofold mass degeneracy in the spectrum of the KK states for the leptogenesis models under study.

Higgs doublet Φ_2 couples to its charge-conjugate counterpart, $C\bar{L}^T$, while the last one Φ_3 has no coupling to matter. The so-extended Higgs potential admits CP nonconservation which originates from the bilinear mixing of the three Higgs doublets. In fact, in this model, both the Majorana masses of the KK excitations and CP violation are generated via loop effects. Most interestingly, as we will detail below, this scenario of leptogenesis has no analog in 4 dimensions.

Of course, one may consider more involved models of leptogenesis that are based on combinations of the basic scenarios (i) and (ii), including their possible supersymmetric extensions. Therefore, it is very instructive to analyze in more detail these two representative models of leptogenesis, as well as an hybrid scenario that includes both the extensions mentioned above, i.e. fermionic bilinears and two additional Higgs doublets.

3.1 The leptogenesis model with fermionic bilinears

In this scenario, the SM is augmented by a higher-dimensional Dirac singlet neutrino $N(x, y)$, while the SM particles are considered to be confined to a 4-dimensional hypersurface which describes our world and is often termed as a 3-brane.[‡] In this picture, the bulk Dirac neutrino field $N(x, y)$ intersects the 3-brane at a position $y = a$, which naturally gives rise to small Yukawa couplings suppressed by the volume of the extra dimensions. This suppression mechanism is very analogous to the one that gravity owes its weakness at long distances in theories with a low scale of quantum gravity [5]. The most general effective Lagrangian of the scenario under discussion is given by

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \int_0^{2\pi R} dy \left\{ \bar{N} \left(i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y \right) N - m\bar{N}N - \frac{1}{2} \left(MN^T C^{(5)-1} N + \text{H.c.} \right) \right. \\ & \left. + \delta(y - a) \left[\bar{h}_1 \bar{L}\Phi\xi + \bar{h}_2 \bar{L}\Phi\eta + \text{H.c.} \right] + \delta(y - a) \mathcal{L}_{\text{SM}} \right\}, \end{aligned} \quad (3.3)$$

where \mathcal{L}_{SM} denotes the SM Lagrangian and

$$C^{(5)} = -\gamma_1\gamma_3 = \gamma_0\gamma_2\gamma_4 = \begin{bmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{bmatrix}. \quad (3.4)$$

In Eq. (3.3), Φ is the SM Higgs doublet and the higher-dimensional two-component spinors ξ and η are defined in Eq. (3.1). Note that \bar{h}_1 and \bar{h}_2 are dimensionful kinematic parameters,

[‡]In a field-theoretic context, Rubakov and Shaposhnikov [27] presented the possibility of dynamically localizing 4-dimensional fermions on a solitonic brane embedded in a higher-dimensional space, by employing the index theorem in a solitonic background [28]

which may be related to the usual 4-dimensional Yukawa couplings h_1 and h_2 through

$$\bar{h}_{1,2} = \frac{h_{1,2}}{(M_F)^\delta}, \quad (3.5)$$

with $\delta = 1$. Here, one must remark that the fundamental scale of quantum gravity M_F occurs naturally in Eq. (3.5), as it is the only available energy scale of the effective Lagrangian to normalize these higher-dimensional Yukawa couplings.

Given that $N(x, y)$ is a periodic function of y , with a period $2\pi R$ and with its two-component spinorial modes being constrained by the afore-mentioned Z_2 discrete symmetry [8], we may expand the two-component spinors ξ and η in a Fourier series as follows:

$$\xi(x, y) = \frac{1}{\sqrt{2\pi R}} \xi_0(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \xi_n(x) \cos\left(\frac{ny}{R}\right), \quad (3.6)$$

$$\eta(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \eta_n(x) \sin\left(\frac{ny}{R}\right), \quad (3.7)$$

where the chiral spinors $\xi_n(x)$ and $\eta_n(x)$ form an infinite tower of KK modes. After integrating out the y coordinate, the effective Lagrangian takes on the form

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{SM}} + \bar{\xi}_0(i\bar{\sigma}^\mu\partial_\mu)\xi_0 + \frac{1}{2}M\xi_0\xi_0 + \left(\bar{h}_1^{(0)}\bar{L}\Phi\xi_0 + \text{H.c.}\right) + \sum_{n=1}^{\infty} \left[\bar{\xi}_n(i\bar{\sigma}^\mu\partial_\mu)\xi_n \right. \\ & + \bar{\eta}_n(i\bar{\sigma}^\mu\partial_\mu)\eta_n + \frac{n}{R}(\xi_n\eta_n + \bar{\xi}_n\bar{\eta}_n) - \frac{1}{2}M(\xi_n\xi_n + \bar{\eta}_n\bar{\eta}_n + \text{H.c.}) \\ & \left. + \sqrt{2}\left(\bar{h}_1^{(n)}\bar{L}\Phi\xi_n + \bar{h}_2^{(n)}\bar{L}\Phi\eta_n + \text{H.c.}\right) \right], \end{aligned} \quad (3.8)$$

where we have chosen the weak basis in which M is positive, and

$$\bar{h}_1^{(n)} = \frac{h_1}{(2\pi M_F R)^{\delta/2}} \cos\left(\frac{na}{R}\right) = \frac{M_F}{M_P} h_1 \cos\left(\frac{na}{R}\right) \quad (n \geq 0), \quad (3.9)$$

$$\bar{h}_2^{(n)} = \frac{h_2}{(2\pi M_F R)^{\delta/2}} \sin\left(\frac{na}{R}\right) = \frac{M_F}{M_P} h_2 \sin\left(\frac{na}{R}\right) \quad (n \geq 1). \quad (3.10)$$

In deriving the last equalities on the RHS's of Eqs. (3.9) and (3.10), we have employed the basic relation given in Eq. (1.1). In agreement with [7,8], we find that independently of the number δ of the extra dimensions, the 4-dimensional Yukawa couplings $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ are naturally suppressed by an extra volume factor $M_F/M_P \lesssim 10^{-10}$. We also observe that the mass term $m\bar{N}N$ drops out from the effective Lagrangian, as a result of the Z_2 discrete symmetry.

In the symmetric (unbroken) phase of the theory, the part of the Lagrangian describing the KK masses is given by

$$-\mathcal{L}_{\text{mass}}^{\text{KK}} = \frac{1}{2}M\xi_0\xi_0 + \frac{1}{2}\sum_{n=1}^{\infty}(\xi_n, \eta_n) \begin{pmatrix} M & -n/R \\ -n/R & M \end{pmatrix} \begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} + \text{H.c.} \quad (3.11)$$

$$= \frac{1}{2} \mu \chi_1^{(0)} \chi_1^{(0)} + \frac{1}{2} \sum_{n=1}^{\infty} (\chi_1^{(n)}, \chi_2^{(n)}) \begin{pmatrix} n/R - \mu & 0 \\ 0 & n/R + \mu \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix} + \text{H.c.},$$

where $\chi_{1(2)}^{(n)} = \frac{1}{\sqrt{2}} \exp(i\phi_{1(2)}^n) [\xi_n + (-)\eta_n]$. As in Ref. [8], we have defined that $\mu = \min(|M - \frac{k}{R}|)$ be the smallest mass eigenvalue for some given value of k , and have re-labelled the remaining KK mass eigenstates $\chi_1^{(n)}$ and $\chi_2^{(n)}$ with respect to k . Thus, after compactification, we see how the heavy isosinglet mass M gets replaced by the small Majorana mass μ , with $\mu \leq 1/R$. Further technical details and discussion may be found in [8]. After expressing the effective Lagrangian in Eq. (3.8) in the newly introduced Majorana-mass basis, we obtain

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{mass}}^{\text{KK}} + \bar{\chi}_1^{(0)} (i\bar{\sigma}^\mu \partial_\mu) \chi_1^{(0)} + (h_1^{(0)} \bar{L} \Phi \chi_1^{(0)} + \text{H.c.}) + \sum_{n=1}^{\infty} \left[\bar{\chi}_1^{(n)} (i\bar{\sigma}^\mu \partial_\mu) \chi_1^{(n)} \right. \\ & \left. + \bar{\chi}_2^{(n)} (i\bar{\sigma}^\mu \partial_\mu) \chi_2^{(n)} + (h_1^{(n)} \bar{L} \Phi \chi_1^{(n)} + h_2^{(n)} \bar{L} \Phi \chi_2^{(n)} + \text{H.c.}) \right], \end{aligned} \quad (3.12)$$

where

$$h_{1(2)}^{(n)} = e^{i\phi_1^n} \bar{h}_1^{(n)} + (-) e^{i\phi_2^n} \bar{h}_2^{(n)}, \quad (3.13)$$

and the Yukawa couplings $\bar{h}_{1,2}^{(n)}$ are given in Eqs. (3.9) and (3.10).

It is now easy to recognize that the Lagrangian in Eq. (3.12) is the known 4-dimensional model of leptogenesis with an infinite number of pairs of Majorana neutrinos $\chi_1^{(n)}$ and $\chi_2^{(n)}$ [18]. With the help of a method based on generalized CP transformations [29], we can derive the sufficient and necessary condition for the theory to be CP invariant. Adapting the result found in [24] to the model under discussion, we find the condition

$$\text{Im Tr} \left(h^\dagger h \widehat{M}_\chi^\dagger \widehat{M}_\chi \widehat{M}_\chi^\dagger h^T h^* \widehat{M}_\chi \right) = 0, \quad (3.14)$$

where $h = (h_1^{(0)}, h_1^{(1)}, h_2^{(1)}, \dots, h_1^{(n)}, h_2^{(n)}, \dots)$ and $\widehat{M}_\chi = \text{diag}(\mu, \frac{1}{R} - \mu, \frac{1}{R} + \mu, \dots, \frac{n}{R} - \mu, \frac{n}{R} + \mu, \dots)$ are formally infinite-dimensional matrices that contain the Higgs Yukawa couplings and the KK mass-eigenvalues, respectively. It is a formidable task to analytically calculate the LHS of Eq. (3.14). Instead, we notice that if one of the following equalities holds true:

$$\mu = 0, \quad a = (0 \text{ or } \pi R), \quad \text{Im}(h_1 h_2^*)^2 = 0, \quad (3.15)$$

the theory is then invariant under CP transformations. Consequently, CP violation requires to have $\mu \neq 0$ and a non-zero shifting of the brane, $a \neq 0$, apart from a relative CP-violating phase between the original Yukawa couplings h_1 and h_2 . Finally, we should remark that if the Z_2 discrete symmetry were not imposed on $N(x, y)$, the resulting Lagrangian would predict a dangerous twofold mass degeneracy in the spectrum of the would-be Majorana KK states that would effectively correspond to the $\mu = 0$ case, and hence would lead to the absence of CP violation as well.

3.2 The leptogenesis model with extended Higgs sector

The second scenario that we will be discussing does not involve the inclusion of any heavy isosinglet mass scale. Instead, in addition to the higher-dimensional Dirac field $N(x, y)$ supplemented by the Z_2 discrete symmetry, we shall extend the Higgs sector by two more Higgs doublets that carry the same hypercharge as the SM Higgs doublet. As we will demonstrate below, such an extension of the Higgs potential by three Higgs doublets, denoted hereafter as Φ_1 , Φ_2 and Φ_3 , is dictated by the necessity of introducing sufficient L and CP violation into the theory. Specifically, this scenario is governed by the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \int_0^{2\pi R} dy \left\{ \bar{N} \left(i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y \right) N + \delta(y-a) \left[\bar{h}_1 \bar{L} \Phi_1 \xi + \bar{h}_2 \bar{L} \Phi_2 \eta + \text{H.c.} \right] \right. \\ & \left. + \delta(y-a) \left[\mathcal{L}'_{\text{SM}}(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3) \right] \right\}, \end{aligned} \quad (3.16)$$

where $\mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3)$ and $\mathcal{L}'_{\text{SM}}(\Phi_1)$ describe the Higgs potential and the residual standard part of the model, respectively. Furthermore, the model is invariant under the transformations:

$$\begin{aligned} N & \rightarrow iN, & \Phi_1 & \rightarrow -i\Phi_1, & \Phi_2 & \rightarrow i\Phi_2, & \Phi_3 & \rightarrow \Phi_3, \\ l_R & \rightarrow -il_R, & u_R & \rightarrow iu_R, & d_R & \rightarrow -id_R, \end{aligned} \quad (3.17)$$

where l_R , u_R and d_R denote the right-handed charged-leptons, the up- and down-type quarks, respectively. Obviously, only Φ_1 couples to the observed SM particles, whereas Φ_3 does not couple to matter at all. The discrete symmetry in Eq. (3.17) is very crucial, as it ensures the renormalizability of the model; the discrete symmetry is only broken softly by operators of dimension two:

$$\mathcal{L}_V^{\text{soft}} = \sum_{i<j=1}^3 m_{ij}^2 \Phi_i^\dagger \Phi_j + \text{H.c.} \subset \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3). \quad (3.18)$$

Notice that the Higgs potential of this scenario is very similar to that of Weinberg's three Higgs-doublet model [30].

One might now naively argue that the third Higgs doublet Φ_3 is not compelling for introducing CP violation into the theory, e.g. $\text{Im}(\bar{h}_1 \bar{h}_2^* m_{12}^2) \neq 0$. However, this is not true. Notwithstanding \bar{h}_1 and \bar{h}_2 might initially be complex in the basis in which m_{12}^2 is real, one can always rephase $L \rightarrow e^{i\phi_l} L$ and $N \rightarrow e^{i\phi} N$ to make both real. If ϕ_{h_1} and ϕ_{h_2} denote the phases of the two Higgs Yukawa couplings, these phases can be eliminated by choosing $\phi_l = (\phi_{h_1} + \phi_{h_2})/2$ and $\phi = (\phi_{h_2} - \phi_{h_1})/2$. In this scenario, CP violation gets



Figure 1: Feynman graphs giving rise to UV-finite KK kinetic terms.

communicated radiatively to the neutrino sector via bilinear Higgs-mixing effects. To be precise, CP nonconservation in the symmetric phase of the Higgs potential \mathcal{L}_V is manifested by the non-vanishing of the following rephasing-invariant quantity [31]:

$$\text{Im} \left(m_{12}^2 m_{23}^2 m_{13}^{*2} \right) \neq 0. \quad (3.19)$$

In addition, CP violation can only occur on a shifted brane, i.e. $a \neq 0$. The latter amounts to non-zero values for both compactified Higgs Yukawa couplings $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$.

Proceeding as in Section 3.1, we integrate out the compact coordinate y in Eq. (3.16) to eventually arrive at

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}'_{\text{SM}}(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3) + \bar{\xi}_0(i\bar{\sigma}^\mu \partial_\mu)\xi_0 + \left(\bar{h}_1^{(0)} \bar{L}\Phi_1\xi_0 + \text{H.c.} \right) \\ & + \sum_{n=1}^{\infty} \left[\bar{\xi}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_n + \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\eta_n - \frac{n}{R} \left(\xi_n\eta_n + \bar{\xi}_n\bar{\eta}_n \right) \right. \\ & \left. + \sqrt{2} \left(i\bar{h}_1^{(n)} \bar{L}\Phi_1\xi_n + i\bar{h}_2^{(n)} \bar{L}\Phi_2\eta_n + \text{H.c.} \right) \right], \end{aligned} \quad (3.20)$$

where $\bar{h}_1^{(n)}$ and $\bar{h}_2^{(n)}$ are given by Eqs. (3.9) and (3.10), respectively. Observe that the effective Lagrangian in Eq. (3.20) still preserves the original discrete symmetry in Eq. (3.17), where the KK components of $N(x, y)$ transform as: $\xi_n \rightarrow i\xi_n$ and $\eta_n \rightarrow -i\eta_n$. At the tree level, the model predicts an infinite number of KK Dirac states that have masses which are equally spaced by an interval $1/R$. Once radiative corrections are included, however, as shown in Fig. 1, each KK Dirac state splits into a pair of nearly degenerate Majorana neutrinos. In fact, radiative effects induce new UV-finite kinetic terms involving the KK states. The new KK kinetic terms are given by

$$\mathcal{L}_{\text{rad}} = \sum_{n,m=1}^{\infty} \kappa_{nm} \bar{\eta}_n(i\bar{\sigma}^\mu \partial_\mu)\xi_m + \text{H.c.}, \quad (3.21)$$

where simple dimensional analysis of the Feynman graphs displayed in Fig. 1 suggests[§]

$$\kappa_{nm} \sim \frac{\bar{h}_2^{(n)*} \bar{h}_1^{(m)}}{8\pi^2} \left[\frac{m_{12}^2}{m_{11}^2 + m_{22}^2} + \frac{m_{13}^2 m_{23}^{*2}}{m_{33}^2 (m_{11}^2 + m_{22}^2)} \right]. \quad (3.22)$$

Since $\kappa_{nm} \ll 1/(RM_F)$, we find that to a good approximation, only the diagonal kinetic transitions $\xi_n \rightarrow \eta_n$ contribute predominantly to the splitting of a KK Dirac state into a pair of Majorana states. After canonically normalizing the KK kinetic terms, the KK mass spectrum is determined by

$$-\mathcal{L}_{\text{mass}}^{\text{KK}} = \sum_{n=1}^{\infty} \frac{n}{2R} (\chi_1^{(n)}, \chi_2^{(n)}) \begin{pmatrix} \frac{1}{1+|\epsilon_n|} & 0 \\ 0 & \frac{1}{1-|\epsilon_n|} \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix} + \text{H.c.}, \quad (3.23)$$

where $\epsilon_n \approx \kappa_{nn}$ and

$$\begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi_\epsilon^n/2} & -e^{-i\phi_\epsilon^n/2} \\ e^{i\phi_\epsilon^n/2} & e^{i\phi_\epsilon^n/2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{1+|\epsilon_n|}} & 0 \\ 0 & \frac{1}{\sqrt{1-|\epsilon_n|}} \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \end{pmatrix}, \quad (3.24)$$

with $\phi_\epsilon^n = \arg(\epsilon_n)$. From Eq. (3.24), we readily see that the radiatively-induced KK Majorana states, $\chi_1^{(n)}$ and $\chi_2^{(n)}$, mix strongly with one another, and so form a two-level CP-violating system, namely a CP-violating resonator. The striking feature of the present scenario is that both the lifting of the dangerous mass degeneracy of the KK Majorana states and CP violation occurs through loop effects. This model of leptogenesis has no analog in 4 dimensions, since the inclusion of a heavy Majorana mass is theoretically not necessary.

3.3 The hybrid leptogenesis model

We shall now consider a model based on the two scenarios discussed in Sections 3.1 and 3.2, in which we include the fermionic bilinears $m\bar{N}N$ and $MN^T C^{(5)-1}N$, as well as the three Higgs doublets Φ_1 , Φ_2 and Φ_3 . As opposed to the previous two cases, we shall not impose the Z_2 discrete symmetry on the bulk Dirac neutrino $N(x, y)$. As we will see, the absence of the Z_2 symmetry yields a distinct prediction for the mass spectrum of the KK states. In

[§]We should remark that our renormalization procedure consists of two steps. In the first step, all UV infinities are absorbed by off-diagonal wave-function and mixing renormalizations of the KK states in the on-shell scheme [32]. To leading order, such a rescaling does not generally affect the original form of the tree-level effective action. The second step, which is of our interest here, consists of a finite renormalization of the kinetic terms.

particular, we find that the heavy mass scales m and M neither decouple completely from the KK mass spectrum nor get replaced by other small quantities of order $1/R$.

The effective Lagrangian of the hybrid model reads

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \int_0^{2\pi R} dy \left\{ \bar{N} (i\gamma^\mu \partial_\mu + i\gamma_4 \partial_y) N - m \bar{N} N - \frac{1}{2} (MN^T C^{(5)-1} N + \text{H.c.}) \right. \\ & + \delta(y-a) [\bar{h}_1 \bar{L} \Phi_1 \xi + \bar{h}_2 \bar{L} \Phi_2 \eta + \text{H.c.}] + \delta(y-a) [\mathcal{L}'_{\text{SM}}(\Phi_1) \\ & \left. + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3)] \right\}. \end{aligned} \quad (3.25)$$

The above Lagrangian possesses a global symmetry given by Eq. (3.17) which is only broken softly by the Higgs mass terms in Eq. (3.18) and by $MN^T C^{(5)-1} N$. This is a crucial fact that ensures the renormalizability of the model.

Since periodicity is the only constraint that applies to $N(x, y)$, the 5-dimensional two-component spinors ξ and η may then be expressed in terms of a Fourier series expansion as follows:

$$\xi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \xi_n(x) \exp\left(\frac{iny}{R}\right), \quad (3.26)$$

$$\eta(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \eta_n(x) \exp\left(\frac{iny}{R}\right). \quad (3.27)$$

Substituting Eqs. (3.26) and (3.27) into the effective Lagrangian (3.25), we find after y integration

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}'_{\text{SM}}(\Phi_1) + \mathcal{L}_V(\Phi_1, \Phi_2, \Phi_3) + \mathcal{L}_{\text{rad}} + \sum_{n=-\infty}^{\infty} \left\{ \bar{\xi}_n (i\bar{\sigma}^\mu \partial_\mu) \xi_n + \bar{\eta}_n (i\bar{\sigma}^\mu \partial_\mu) \eta_n \right. \\ & - \left[\left(m + \frac{in}{R}\right) \xi_n \eta_{-n} + \text{H.c.} \right] - \frac{1}{2} M (\xi_{-n} \xi_n + \bar{\eta}_{-n} \bar{\eta}_n + \text{H.c.}) \\ & \left. + \left(\bar{h}_1^{(n)} \bar{L} \Phi_1 \xi_n + \bar{h}_2^{(n)} \bar{L} \Phi_2 \eta_n + \text{H.c.}\right) \right\}, \end{aligned} \quad (3.28)$$

where

$$\bar{h}_1^{(n)} = \frac{M_F}{M_{\text{P}}} h_1 \exp\left(\frac{ina}{R}\right), \quad \bar{h}_2^{(n)} = \frac{M_F}{M_{\text{P}}} h_2 \exp\left(\frac{ina}{R}\right). \quad (3.29)$$

In Eq. (3.28), \mathcal{L}_{rad} indicates the UV-finite radiative contributions to the KK kinetic terms, i.e.

$$\mathcal{L}_{\text{rad}} = \sum_{n,m=-\infty}^{\infty} \kappa_{n,m} \bar{\eta}_m (i\bar{\sigma}^\mu \partial_\mu) \xi_m + \text{H.c.}, \quad (3.30)$$

where $\kappa_{n,m}$ is given by a formula very analogous to Eq. (3.22). To avoid excessive complication in the calculation, we consider only those radiative terms $\kappa_{n,m}$ that are expected to

have a dominant effect on the KK mass spectrum. More explicitly, we have

$$\begin{aligned} \mathcal{L}_{\text{rad}} \approx & \kappa_{0,0} \bar{\eta}_0 (i\bar{\sigma}^\mu \partial_\mu) \xi_0 + \sum_{n=1}^{\infty} \left[\kappa_{n,n} \bar{\eta}_n (i\bar{\sigma}^\mu \partial_\mu) \xi_n + \kappa_{-n,-n} \bar{\eta}_{-n} (i\bar{\sigma}^\mu \partial_\mu) \xi_{-n} \right. \\ & \left. + \kappa_{n,-n} \bar{\eta}_n (i\bar{\sigma}^\mu \partial_\mu) \xi_{-n} + \kappa_{-n,n} \bar{\eta}_{-n} (i\bar{\sigma}^\mu \partial_\mu) \xi_n \right] + \text{H.c.} \end{aligned} \quad (3.31)$$

Notice that all $|\kappa_{n,m}|$ have the same absolute value and do not depend on the indices n and m .

To evaluate the masses of the KK neutrino states, it is convenient to write the kinetic part of the KK sector as a sum of two terms:

$$\mathcal{L}_{\text{kin}}^{\text{KK}} = \mathcal{L}_{n=0}^{\text{KK}} + \mathcal{L}_{n \geq 1}^{\text{KK}}, \quad (3.32)$$

where

$$\begin{aligned} \mathcal{L}_{n=0}^{\text{KK}} &= (\bar{\xi}_0, \bar{\eta}_0) (i\bar{\sigma}^\mu \partial_\mu) \begin{pmatrix} 1 & \kappa_{0,0} \\ \kappa_{0,0} & 1 \end{pmatrix} \begin{pmatrix} \xi_0 \\ \eta_0 \end{pmatrix} - \frac{1}{2} (\xi_0, \eta_0) \begin{pmatrix} M & m \\ m & M \end{pmatrix} \begin{pmatrix} \xi_0 \\ \eta_0 \end{pmatrix} \\ &+ \text{H.c.}, \quad (3.33) \\ \mathcal{L}_{n \geq 1}^{\text{KK}} &= \sum_{n=1}^{\infty} \left[(\bar{\xi}_n, \bar{\eta}_n, \bar{\xi}_{-n}, \bar{\eta}_{-n}) (i\bar{\sigma}^\mu \partial_\mu) \begin{pmatrix} 1 & \kappa_{n,n}^* & 0 & \kappa_{n,-n}^* \\ \kappa_{n,n} & 1 & \kappa_{n,-n} & 0 \\ 0 & \kappa_{n,-n}^* & 1 & \kappa_{-n,-n}^* \\ \kappa_{n,-n} & 0 & \kappa_{-n,-n} & 1 \end{pmatrix} \begin{pmatrix} \xi_n \\ \eta_n \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} \right. \\ &\left. - \frac{1}{2} (\xi_n, \eta_n, \xi_{-n}, \eta_{-n}) \begin{pmatrix} 0 & 0 & M & \tilde{m}_n \\ 0 & 0 & \tilde{m}_n^* & M \\ M & \tilde{m}_n^* & 0 & 0 \\ \tilde{m}_n & M & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi_n \\ \eta_n \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} + \text{H.c.} \right], \quad (3.34) \end{aligned}$$

with $\tilde{m}_n = m + (in/R)$ (i.e. $\tilde{m}_0 = m$). From the Lagrangian $\mathcal{L}_{n=0}^{\text{KK}}$ in Eq. (3.33), one obtains two Majorana neutrinos, $\chi_1^{(0)}$ and $\chi_2^{(0)}$, with masses

$$m_{\chi_1}^{(0)} = \frac{|M - m|}{|1 - \kappa_{0,0}|} \approx |M - m|, \quad m_{\chi_2}^{(0)} = \frac{M + m}{|1 + \kappa_{0,0}|} \approx M + m. \quad (3.35)$$

We now turn to the evaluation of the KK neutrino masses, for the more involved case with $n \geq 1$. To this end, we first go to a weak basis in which the mass matrix is real by rephasing the KK fields:

$$\xi_n \rightarrow e^{-i\phi_n/2} \xi_n, \quad \eta_n \rightarrow e^{i\phi_n/2} \eta_n, \quad \xi_{-n} \rightarrow e^{i\phi_n/2} \xi_{-n}, \quad \eta_{-n} \rightarrow e^{-i\phi_n/2} \eta_{-n}, \quad (3.36)$$

with $\phi_n = \arg \tilde{m}_n$. Even though one could always work out the most general case, it is, however, very illuminating to make a further assumption that leads to much simpler

analytic results. We assume that all radiative kinetic terms are predominantly real in the new weak basis in Eq. (3.36), i.e. $\text{Im } \kappa_{n,m} \ll \text{Re } \kappa_{n,m} \approx \epsilon$. Then, considering $m \geq M$ for definiteness, we can diagonalize the Lagrangian in Eq. (3.34) through the canonical transformation

$$\begin{pmatrix} \xi_n \\ \eta_m \\ \xi_{-n} \\ \eta_{-n} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} i & 1 & -i & 1 \\ -i & -1 & -i & 1 \\ -i & 1 & i & 1 \\ i & -1 & i & 1 \end{pmatrix} \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{1-2\epsilon}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{1+2\epsilon}} \end{pmatrix} \begin{pmatrix} \chi_1^{(n)} \\ \chi_2^{(n)} \\ \chi_1^{(-n)} \\ \chi_2^{(-n)} \end{pmatrix}, \quad (3.37)$$

which leads to the KK Majorana masses

$$\begin{aligned} m_{\chi_1}^{(n)} &= \sqrt{m^2 + \frac{n^2}{R^2}} - M, & m_{\chi_2}^{(n)} &= \frac{1}{1-2\epsilon} \left(\sqrt{m^2 + \frac{n^2}{R^2}} - M \right), \\ m_{\chi_1}^{(-n)} &= M + \sqrt{m^2 + \frac{n^2}{R^2}}, & m_{\chi_2}^{(-n)} &= \frac{1}{1+2\epsilon} \left(M + \sqrt{m^2 + \frac{n^2}{R^2}} \right). \end{aligned} \quad (3.38)$$

Evidently, Eq. (3.38) shows that the immediate effect of radiative corrections is to lift the dangerous twofold mass degeneracy among the KK states $\chi_1^{(\pm n)}$ and $\chi_2^{(\pm n)}$, thus rendering the theory CP violating. If one considers that $m > M$, the mass of the lowest lying KK state is $m_{\chi_1}^{(0)} \approx m - M$, which can naturally be much bigger than the compactification scale $1/R$. This is a distinctive feature of the present model, since, unlike the previous two scenarios, the heavy mass scale $m - M$ neither decouples from the complete KK mass spectrum nor gets replaced by quantities of order $1/R$.

4 Resonant CP violation

In addition to lepton-number violation, CP nonconservation constitutes another important ingredient for leptogenesis. These two necessary conditions satisfy, by construction, the three models of leptogenesis, discussed in the previous section. However, these conditions may not be sufficient to guarantee an appreciable leptonic asymmetry that results from decays of KK Majorana states according to the standard scenario of leptogenesis [18]. In particular, in theories with low scale of quantum gravity, we have to ensure that the total net effect of the individual CP-violating contributions coming from the tower of the nearly degenerate KK Majorana states does not vanish because of some kind of a GIM [33] cancellation mechanism. In fact, by making use of such a GIM-type mechanism, we can show that all the CP-violating vertex (ϵ' -type) terms almost cancel pairwise. On the other

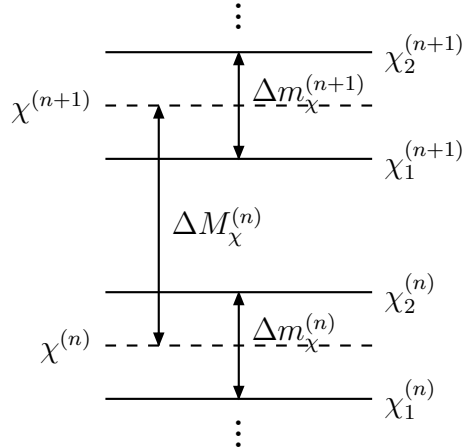


Figure 2: Schematic representation of the mass spectrum of the KK states.

hand, we find that the interference of the CP-violating self-energy (ε -type) contributions is constructive or destructive, depending on the mass spacing of the KK Majorana states.

The mass spectrum of the higher-dimensional models of leptogenesis under discussion consists of an infinite series of pairs of nearly degenerate Majorana neutrinos, which we denote by $\chi_1^{(n)}$ and $\chi_2^{(n)}$. The generic pattern of the KK mass spectrum may be represented by Fig. 2. As we have discussed in Section 3, the mass difference between $\chi_1^{(n)}$ and $\chi_2^{(n)}$ may be induced either at the tree level:

$$\Delta m_\chi^{(n)} \equiv m_{\chi_2}^{(n)} - m_{\chi_1}^{(n)} = 2\mu, \quad (4.1)$$

or through radiative kinetic terms:

$$\Delta m_\chi^{(n)} \sim \kappa_{nm} m_\chi^{(n)} \sim \frac{h_2^* h_1}{8\pi^2} \frac{M_F^2}{M_P^2} m_\chi^{(n)}, \quad (4.2)$$

with

$$m_\chi^{(n)} \equiv \frac{1}{2} (m_{\chi_1}^{(n)} + m_{\chi_2}^{(n)}). \quad (4.3)$$

Furthermore, the mass difference between two adjacent KK pairs is determined by

$$\Delta M_\chi^{(n)} \equiv m_\chi^{(n+1)} - m_\chi^{(n)} \approx \frac{1}{R} = \left(\frac{M_F}{M_P} \right)^{2/\delta} M_F. \quad (4.4)$$

In deriving the approximate equality in Eq. (4.4), we have implicitly assumed that $m \gtrsim n/R \gtrsim T_c$, for the hybrid scenario outlined in Section 3.3. As we will discuss in the next section, this is a reasonable assumption, since only the KK states $\chi_{1,2}^{(n)}$ with masses above the critical temperature T_c of the electroweak phase transition are relevant for baryogenesis. Clearly, if the origin of a non-zero $\Delta m_\chi^{(n)}$ is due to loop effects, one naturally has

$\Delta M_\chi^{(n)} \gg \Delta m_\chi^{(n)}$ for any number δ of extra dimensions. However, if $\Delta m_\chi^{(n)}$ occurs in the Born approximation, then $\Delta m_\chi^{(n)}$ and $\Delta M_\chi^{(n)}$ could be of equal order. As we will see below, the last two quantities determine the magnitude of CP violation that originates from the interference of the tower of the KK states.

Let us first consider ε' -type CP violation in the decays of KK states. For our illustrations, it is sufficient to assume that the KK states decay predominantly to the SM Higgs doublet Φ or to the Higgs doublet with the smallest (thermal) mass in the model with the extended Higgs sector. Since $\Delta m_\chi^{(n)} < \Delta M_\chi^{(n)} \ll m_\chi^{(n)}$, the CP-violating parameter of our interest is

$$\varepsilon_\chi^{\prime(n)} = \frac{|\mathcal{T}_{\chi_1}^{(n)\varepsilon'}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon'}|^2 - |\overline{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'}|^2 - |\overline{\mathcal{T}}_{\chi_2}^{(n)\varepsilon'}|^2}{|\mathcal{T}_{\chi_1}^{(n)\varepsilon'}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon'}|^2 + |\overline{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'}|^2 + |\overline{\mathcal{T}}_{\chi_2}^{(n)\varepsilon'}|^2}, \quad (4.5)$$

where we used the short-hand notations for the transition amplitudes: $\mathcal{T}_{\chi_1}^{(n)\varepsilon'} = \mathcal{T}^{\varepsilon'}(\chi_1^{(n)} \rightarrow L\Phi^\dagger)$, $\overline{\mathcal{T}}_{\chi_1}^{(n)\varepsilon'} = \mathcal{T}^{\varepsilon'}(\chi_1^{(n)} \rightarrow L^C\Phi)$, and likewise for $\chi_2^{(n)}$. In all these amplitudes, only vertex diagrams are included. The parameter $\varepsilon_\chi^{\prime(n)}$ is then found to be

$$\varepsilon_\chi^{\prime(n)} = \frac{1}{8\pi (|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \sum_k \text{Im} (h_1^{(n)*} h_2^{(k)})^2 \left[f\left(\frac{m_{\chi_2}^{(k)2}}{m_{\chi_1}^{(n)2}}\right) - f\left(\frac{m_{\chi_1}^{(k)2}}{m_{\chi_2}^{(n)2}}\right) \right], \quad (4.6)$$

with

$$f(x) = \sqrt{x} \left[1 - \left(1 + x\right) \ln \left(1 + \frac{1}{x}\right) \right]. \quad (4.7)$$

Note that the range of summation over the KK states explicitly depends on the model. Equation (4.6) may further be approximated as

$$\varepsilon_\chi^{\prime(n)} \approx \frac{1}{4\pi (|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \sum_k \text{Im} (h_1^{(n)*} h_2^{(k)})^2 \left(\frac{\Delta m_\chi^{(n)}}{m_\chi^{(n)}} + \frac{\Delta m_\chi^{(k)}}{m_\chi^{(k)}} \right) \frac{m_\chi^{(k)2}}{m_\chi^{(n)2}} f'\left(\frac{m_\chi^{(k)2}}{m_\chi^{(n)2}}\right), \quad (4.8)$$

where $f'(x)$ is the derivative of the function $f(x)$, i.e.

$$f'(x) = \frac{3}{2\sqrt{x}} \left[1 - \left(\frac{2}{3} + x\right) \ln \left(1 + \frac{1}{x}\right) \right]. \quad (4.9)$$

It is obvious that each individual KK term in Eq. (4.8) is suppressed by a factor $\Delta m_\chi^{(k)}/m_\chi^{(k)}$ as a result of a GIM-type cancellation mechanism.

To explicitly demonstrate that ε' -type contributions to CP violation are indeed small, it is instructive to offer an estimate for the sum over the KK states in Eq. (4.8), after making few plausible assumptions. For simplicity, we consider a theory with one additional spatial dimension ($\delta = 1$), and further assume that all the Yukawa couplings $h_1^{(k)}$ and $h_2^{(k)}$ as well as the mass differences, $\Delta m_\chi^{(k)}$ (or $\Delta m_\chi^{(k)}/m_\chi^{(k)}$) and $\Delta M_\chi^{(k)}$, are independent of k . In addition,

we convert the sum over the KK states ‘ k ’ to an energy integral which has an UV cutoff at the fundamental scale of quantum gravity M_F . With these considerations, we obtain

$$\varepsilon'_\chi^{(n)} \approx -\frac{\text{Im}(h_1^{(n)*} h_2^{(n)})^2}{16\pi(|h_1^{(n)}|^2 + |h_2^{(n)}|^2)} \frac{\Delta m_\chi^{(n)}}{\Delta M_\chi^{(n)}} \approx -\frac{\text{Im}(h_1^* h_2)^2}{16\pi(|h_1|^2 + |h_2|^2)} \frac{M_F^2}{M_P^2} \frac{\Delta m_\chi^{(n)}}{\Delta M_\chi^{(n)}}, \quad (4.10)$$

where we have used that $f'(x) \approx -1/(2x^{3/2})$, for $x \gg 1$. The above exercise shows that $\varepsilon'_\chi^{(n)}$ is extremely suppressed by the extra-dimensional volume factor M_F^2/M_P^2 and, in certain scenarios, by the ratio $\Delta m_\chi^{(n)}/\Delta M_\chi^{(n)} \ll 1$. Consequently, we can safely neglect ε' -type CP violation in the decays of the KK Majorana states.

In the following, we shall focus our attention on the self-energy (ε -type) contribution to CP violation. In analogy to Eq. (4.5), the relevant measure of ε -type CP violation may be defined by

$$\varepsilon_\chi^{(n)} = \frac{|\mathcal{T}_{\chi_1}^{(n)\varepsilon}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon}|^2 - |\overline{\mathcal{T}}_{\chi_1}^{(n)\varepsilon}|^2 - |\overline{\mathcal{T}}_{\chi_2}^{(n)\varepsilon}|^2}{|\mathcal{T}_{\chi_1}^{(n)\varepsilon}|^2 + |\mathcal{T}_{\chi_2}^{(n)\varepsilon}|^2 + |\overline{\mathcal{T}}_{\chi_1}^{(n)\varepsilon}|^2 + |\overline{\mathcal{T}}_{\chi_2}^{(n)\varepsilon}|^2}. \quad (4.11)$$

Correspondingly, $\mathcal{T}_{\chi_{1,2}}^{(n)\varepsilon}$ ($\overline{\mathcal{T}}_{\chi_{1,2}}^{(n)\varepsilon}$) indicate the decays $\chi_{1,2}^{(n)} \rightarrow L\Phi^\dagger$ ($\chi_{1,2}^{(n)} \rightarrow L^C\Phi$), where only the self-energy graph has been taken into account. Since each ε -type term is proportional to the mass difference of the KK states involved, one has to avoid that self-energy transitions $\chi_1^{(n+1)} \rightarrow \chi_2^{(n+1)}$ cancel against the transitions $\chi_1^{(n+1)} \rightarrow \chi_2^{(n)}$. From Fig. 2, we may schematically deduce the condition for destructive interference among adjacent KK states, which translates into the relation

$$\Delta m_\chi^{(n)} \sim \frac{1}{2} \Delta M_\chi^{(n)}. \quad (4.12)$$

Instead, if

$$\Delta M_\chi^{(n)} \gg \Delta m_\chi^{(n)}, \quad (4.13)$$

the interference of two neighbouring KK states is constructive. Employing the resummation approach to the mixing of unstable particles which was developed in [24,34], we find that

$$\varepsilon_\chi^{(n)} = \frac{2 \text{Im}(h_1^{(n)} h_2^{(n)*})^2}{(|h_1^{(n)}|^2 + |h_2^{(n)}|^2)^2} \frac{\Delta m_\chi^{(n)} (\Gamma_{\chi_1}^{(n)} + \Gamma_{\chi_2}^{(n)})}{(\Delta m_\chi^{(n)})^2 + \frac{1}{4} \Gamma_{\chi_2}^{(n)2}} \left[1 + \frac{(\Delta m_\chi^{(n)})^2 + \frac{1}{4} \Gamma_{\chi_2}^{(n)2}}{(\Delta m_\chi^{(n)})^2 + \frac{1}{4} \Gamma_{\chi_1}^{(n)2}} \right], \quad (4.14)$$

where

$$\Gamma_{\chi_1}^{(n)} = \frac{1}{8\pi} |h_1^{(n)}|^2 m_{\chi_1}^{(n)} \quad \text{and} \quad \Gamma_{\chi_2}^{(n)} = \frac{1}{8\pi} |h_2^{(n)}|^2 m_{\chi_2}^{(n)} \quad (4.15)$$

are the decay widths of $\chi_1^{(n)}$ and $\chi_2^{(n)}$, respectively. In Eq. (4.14), we have neglected contributions to $\varepsilon_\chi^{(n)}$ of order $\Delta m_\chi^{(n)}/\Delta M_\chi^{(n)}$ (cf. Eq. (4.13)) and $\Delta m_\chi^{(n)}/m_\chi^{(n)}$.

In agreement with Ref. [24], we observe that the CP-violating parameter $\varepsilon_\chi^{(n)}$ given by Eq. (4.14) can be of order one, if the two conditions:

$$(i) \delta_{\text{CP}}^{(n)} \equiv \frac{|\text{Im}(h_1^{(n)} h_2^{(n)*})^2|}{|h_1^{(n)}|^2 |h_2^{(n)}|^2} \sim 1 \quad \text{and} \quad (ii) \Delta m_\chi^{(n)} \sim \frac{\Gamma_{\chi_1}^{(n)}}{2} \text{ or } \frac{\Gamma_{\chi_2}^{(n)}}{2} \quad (4.16)$$

are satisfied. The first condition is rather model-dependent. A priori there is no reason to believe that the phases of the original Yukawa couplings h_1 and h_2 should somehow be aligned and, as a result of this, the parameter $\delta_{\text{CP}}^{(n)}$ must be suppressed. Thus, we consider $\delta_{\text{CP}}^{(n)} \sim 1$.

In a general model, more difficult is, however, to theoretically justify the second condition, as the mass splitting of the mixed particles involved must be of the order of their widths. For the scenarios discussed in Sections 3.2 and 3.3, the mass splitting $\Delta m_\chi^{(n)}$ is radiatively induced by integrating out the Higgs interactions and then canonically normalizing the resulting kinetic terms. Thus, the width of the KK Majorana states and their respective mass difference formally occur at the same electroweak loop order. Therefore, the second condition is naturally implemented for these two models. For the leptogenesis model described in Section 3.1, one has to assume that $\mu \sim \Gamma_{\chi_1}^{(n)}$ or $\Gamma_{\chi_2}^{(n)}$. As a consequence of compactification of the extra large dimensions, the mass parameter μ always comes out to be smaller than $\Delta M_\chi^{(n)}$, so some degree of tuning μ to even smaller values is required in this case.

The models of leptogenesis we have been studying share the generic feature that each KK Dirac state decomposes into a pair of nearly degenerate Majorana neutrinos. Such a pair of KK Majorana neutrinos forms a strongly mixed two-level system that exhibits CP violation of order unity; such a system was called a CP-violating resonator. In fact, it was shown in [24] that the resonant enhancement of CP violation is driven by the non-diagonalizable (Jordan-like) form of the effective Hamiltonian (or equivalently the resummed propagator) of the two-level system, which satisfies conditions very analogous to those of Eq. (4.16). Finally, we should stress that the constructive interference of all the individual KK CP-violating resonators is assured on the basis of the requirement given by Eq. (4.13). This last requirement is more naturally implemented in the models of leptogenesis with extended Higgs sector (see also discussion in Sections 3.2 and 3.3).

5 Baryonic asymmetry of the Universe

Astronomical observations give strong evidence that the present Universe mainly consists of matter rather antimatter, viz. the Universe possesses an excess in the B number. The

observed B asymmetry may be quantified by the nonzero baryon-number-to-entropy ratio of densities [26]

$$Y_{\Delta B} = \frac{n_{\Delta B}}{s} = (0.6 - 1) \times 10^{-10} , \quad (5.1)$$

where $n_{\Delta B} = n_B - n_{\bar{B}} \approx n_B$ and s is the entropy density. As we mentioned in the introduction, an attractive solution that could account for the nonzero value of $Y_{\Delta B}$ by making use of all the necessary conditions imposed by Sakharov [35] may be given by means of the scenario of baryogenesis through leptogenesis [18]. Based on an analysis of chemical potentials [36], one may derive that

$$Y_B(T > T_c) = \frac{8N_F + 4N_H}{22N_F + 13N_H} Y_{(B-L)} . \quad (5.2)$$

Almost independently of the numbers N_F and N_H of flavours and Higgs doublets, one finds that approximately one-third of the initial $B - L$ and/or L asymmetry will be reprocessed by sphalerons into an asymmetry in B . This B asymmetry is not affected by the tower of the right-handed KK states and so persists even after the electroweak phase transition.

Let us first consider the constraints on the parameter space of the leptogenesis models coming from Sakharov's requirement that L or $B - L$ -violating processes, such as decays of KK Majorana modes, must be out-of-thermal equilibrium in an expanding Universe. As was discussed by Abel and Sarkar [37], the presence of low-lying KK states drastically influences the evolution of the Universe, as the number of the relativistic degrees of freedom increases with temperature T . To be more precise, if $m_{\chi_1}^{(0)} \equiv m_{\min}$ represents the mass of the lowest KK state in a given model of leptogenesis, the number of relativistic KK states below T is then roughly given by $[(T - m_{\min})R]^\delta$, where δ is the number of large compact dimensions. Thus, the number of the active degrees of freedom at a given temperature T is determined by

$$\begin{aligned} g(T) &\approx g_* + S_\delta \theta(T - m_{\min}) [(T - m_{\min})R]^\delta \\ &\approx g_* + S_\delta \theta(T - m_{\min}) \left(\frac{M_P}{M_F}\right)^2 \left(\frac{T - m_{\min}}{M_F}\right)^\delta , \end{aligned} \quad (5.3)$$

where $g_* \approx 100$ is the number of active degrees of freedom in usual 4-dimensional extensions of the SM, and $S_\delta = 2\pi^{\delta/2}/\Gamma(\frac{\delta}{2})$ is the surface area of a $(\delta + 1)$ -dimensional sphere of unit radius. From Eq. (5.3), we see that the part of $g(T)$ modified by the presence of KK states, $g_{\text{KK}}(T)$, can generally be much larger than g_* , unless $m_{\min} \sim T_c$, or M_F and δ are sufficiently high for some specific model. For instance, in the hybrid leptogenesis model, one may have $m_{\min} \approx m - M > T_c$, and $g_{\text{KK}}(T)$ is of order g_* for $T \gtrsim T_c$.

Sakharov's requirement that all B - and, because of equilibrated sphaleron interactions, L -violating processes should be out-of-thermal equilibrium translates into the ap-

proximate inequality for the total T -dependent decay rate of the KK states

$$\Gamma_\chi(T) \equiv \sum_{n=\text{int}(m_{\min}R)}^{\text{int}(TR)} \Gamma_\chi^{(i)} \lesssim 2H(T), \quad (5.4)$$

where $n = (n_1, n_2, \dots, n_\delta)$, $\Gamma_\chi^{(n)} = \frac{1}{2}(\Gamma_{\chi_1}^{(n)} + \Gamma_{\chi_2}^{(n)})$ is the average decay width of the n th CP-violating resonator, and

$$H(T) = 1.73 g^{1/2}(T) \frac{T^2}{M_{\text{P}}} \approx 1.73 S_\delta^{1/2} \left(\frac{T - m_{\min}}{M_F} \right)^{\delta/2} \frac{T^2}{M_F}. \quad (5.5)$$

The last approximate equality holds true, provided $g_{\text{KK}}(T) \gg g_*$. Converting the multidimensional sum over the KK modes in Eq. (5.4) into an integral, we find that

$$\Gamma_\chi(T) \approx \frac{|h_1|^2 + |h_2|^2}{16\pi^2} \frac{S_\delta}{\delta} \left(\frac{T - m_{\min}}{M_F} \right)^{\delta+1} M_F. \quad (5.6)$$

An immediate result of the out-of-equilibrium condition in Eq. (5.4) is the constraint

$$\frac{1}{2} (|h_1|^2 + |h_2|^2) \lesssim 32\pi^2 S_\delta^{-1/2} \frac{T^2}{M_F^2} \left(\frac{T - m_{\min}}{M_F} \right)^{-1-\delta/2}, \quad (5.7)$$

which does no longer depend on M_{P} . From Eq. (5.7), it is interesting to see that no serious arrangement of the parameters is necessary for all $\delta \geq 1$ and $M_F > T > T_c$, even if the original Yukawa couplings h_1 and h_2 in Eq. (3.5) are taken to be of order 1. This should be contrasted with the extremely tight limits on the Yukawa couplings in the conventional 4-dimensional models [24], namely $h_{1,2} \lesssim 10^{-6}$. These limits are obtained if one sets $m_{\min} = \delta = 0$, $T = 0.2\text{--}1$ TeV and $M_F = M_{\text{P}}$, and replaces S_δ by $1/g_*$ in Eq. (5.7).

It is interesting to derive the time evolution of the Universe as a function of its temperature in higher-dimensional theories. We assume that the Friedmann–Robertson–Walker metric governs the expansion of the Universe after inflation [26], and that all active relativistic degrees of freedom are in chemical equilibrium and therefore have the same temperature. Imposing entropy conservation, i.e.

$$sR^3 \propto g(T) T^3 R^3 = \text{const.}, \quad (5.8)$$

and differentiating with respect to time t , we then find that

$$H \equiv \frac{1}{R} \frac{dR}{dt} = -\frac{\delta + 3}{3} \frac{1}{T} \frac{dT}{dt}, \quad (5.9)$$

for $g_{\text{KK}}(T) \gg g_*$. If we differentiate the Hubble variable in Eq. (5.5) with respect to t and employ Eq. (5.9), we arrive at the differential equation

$$\frac{dH}{dt} = -\frac{3}{2} \frac{\delta + 4}{\delta + 3} H^2. \quad (5.10)$$

Considering as initial condition $H(t \rightarrow 0) \rightarrow \infty$, the solution of Eq. (5.10) reads

$$t(T) = \frac{2}{3} \frac{\delta + 3}{\delta + 4} \frac{1}{H(T)} \approx (7.6 \times 10^{-28} \text{ sec}) \frac{1}{S_\delta^{1/2}} \frac{\delta + 3}{\delta + 4} \left(\frac{\text{TeV}}{M_F}\right) \left(\frac{T}{M_F}\right)^{-2-\frac{1}{2}\delta}. \quad (5.11)$$

If $g_{\text{KK}}(T) \sim g_*$, which happens for temperatures $T \lesssim m_{\text{min}} + (M_F/M_{\text{P}})^{2/\delta} g_*^{1/\delta} M_F$, the time-temperature relation (5.11) goes over into the canonical 4-dimensional form

$$t(T) = \frac{1}{2H(T)} \approx (2.3 \text{ sec}) \times g_*^{-1/2} \left(\frac{\text{MeV}}{T}\right)^2. \quad (5.12)$$

Comparing the T -dependence in Eqs. (5.11) and (5.12), one readily sees that the presence of large compact dimensions makes Universe cool more rapidly.

We shall now attempt to give a conservative estimate of the baryonic asymmetry that results from a sphaleron-converted leptonic asymmetry at $T > T_c$, including possible dilution factors due to a low reheat temperature T_r . As a starting point, we assume that $n_\chi^{(n)}(T) \approx n_\gamma(T)$, for $m_\chi^{(n)} \lesssim T \lesssim M_F$, where $n_\chi^{(n)}(T)$ is the number density of the n th KK pair of Majorana states and $n_\gamma(T) \approx 2.4 T^3/\pi^2$ is the respective number density for photons. The dominant contribution to the B asymmetry is expected to be encoded in the n th CP-violating resonator at $T \approx m_\chi^{(n)}$ for $m_\chi^{(n)} > T_c, m_{\text{min}}$, when the equilibrium number density of the n th KK pair is of the order of $n_\chi^{(n)}(T)$ [26]. Thus, the n th CP-violating resonator gives rise to the baryonic excess

$$Y_{\Delta B}^{(n)} \approx -\frac{1}{3} \frac{\varepsilon_\chi^{(n)} n_\chi^{(n)}(m_\chi^{(n)})}{s(m_\chi^{(n)})} \approx -\frac{\varepsilon_\chi^{(n)}}{3 g(m_\chi^{(n)})}. \quad (5.13)$$

In deriving the last step of Eq. (5.13), we have used that $s(T) \approx g(T)n_\gamma$. Because $\Gamma_\chi(T) \ll H(T)$, the neutrino KK states will decay at much later times and reheat the Universe at $T_r \ll T_c$. We shall assume that this happens for $T_r \gtrsim 1 \text{ MeV}$ to avoid problems with nucleosynthesis. In this qualitative picture, the entropy release due to the late decays of the KK neutrinos can usually be taken into account by a multiplicative suppression factor $T_r/m_\chi^{(n)}$ on the RHS of Eq. (5.13) [26]. The total B asymmetry may then be estimated by

$$Y_{\Delta B} \approx \sum_{n=\text{int}(T_{\text{min}}R)}^{\text{int}(M_F R)} \frac{T_r}{m_\chi^{(n)}} Y_{\Delta B}^{(n)}, \quad (5.14)$$

with $T_{\text{min}} = \max(T_c, m_{\text{min}})$. In the leptogenesis models under discussion, all the individual CP-violating asymmetries $\varepsilon_\chi^{(n)}$ are of the same order, i.e. $\varepsilon_\chi^{(n)} = -\varepsilon_\chi$ for all n , and their net effect is constructive, as long as the condition in Eq. (4.13) is satisfied. For generality, let us assume that the interference of the CP-violating resonators is constructive up to an

energy scale $M'_F \leq M_F$. Approximating the sum over the n -KK states in Eq. (5.14) by an integral, we then obtain

$$Y_{\Delta B} \approx \frac{1}{3} \left(1 - \frac{T_{\min}}{M'_F} \right) \frac{T_r}{T_{\min}} \varepsilon_\chi. \quad (5.15)$$

From Eq. (5.15), we see that the generated BAU, $Y_{\Delta B}$, is generally independent of the number δ of extra dimensions and can assume its observed value, even if the parameters involved take rather modest values. For example, if $T_r = 1$ MeV, $T_{\min} = 1$ TeV, and $\varepsilon_\chi \approx 10^{-3}$, one then easily arrives at the observed level for the BAU, namely $Y_{\Delta B} \approx 10^{-10}$.

One difficulty of the leptogenesis models we have been discussing is that the late decays of the low KK neutrinos may distort the abundances related to the light elements ${}^4\text{He}$, D, ${}^7\text{Li}$, etc. Of course, for sufficiently large M_F and/or δ values, the lowest KK state, with mass of order $1/R \sim (M_F/M_P)^{2/\delta} M_F$, will be heavy enough to decay just before nucleosynthesis. This may reintroduce a mild hierarchy problem in the parameters of the theory, in case we wish to identify M_F with the scale of soft-supersymmetry breaking [5]. Therefore, among the leptogenesis models which were discussed in Section 3, the hybrid scenario represents the most attractive solution to this problem, as the lowest KK state, with mass $m_{\min} = m - M$, can be made heavy enough, e.g. of order T_c , in order to decay sufficiently rapidly. Whether such a scenario can be embedded to a more general supersymmetric theory is an issue which we shall not address in the present work.

6 Conclusions

We have studied the scenario of baryogenesis through leptogenesis in theories with large compact dimensions. The formulation of these theories requires the extension of the notion of the Majorana spinor to multidimensional Minkowski spaces. We have reviewed this topic in Section 2. In particular, it was shown that genuine massive Majorana neutrinos exist in 2, 3 and 4 mod 8 dimensions only. This limitation is due to the lack of finding Clifford-algebra representations that satisfy the Majorana properties in any number of dimensions. In Section 3, we have formulated minimal models of leptogenesis which are renormalizable if a finite number of KK states are considered. Such a truncation of the number of the KK states may not be very unrealistic, as the fundamental scale of quantum gravity M_F is expected to play the role of an UV cutoff.

The leptogenesis models that we have been discussing initially furnish the field content of the theory with an infinite series of KK Dirac states. Subsequently, each KK Dirac state splits into pairs of nearly degenerate Majorana neutrinos. Such a mass splitting occurs either at the tree level after compactification of the extra dimensions, or more interestingly,

at the one-loop level by integrating out the Higgs interactions of an extended Higgs sector. As a consequence, each pair of the Majorana neutrinos behaves as a CP-violating resonator, i.e. it becomes a strongly mixed two-level system producing a leptonic CP asymmetry of order unity. Depending on the mass difference between two adjacent pairs of KK Majorana neutrinos, the tower of the CP-violating resonators may have a constructive or destructive interference. In Section 4, we have found that such an interference is constructive, if the level spacing between any two nearby pairs of KK Majorana neutrinos is much bigger than the mass difference of the Majorana neutrinos within each pair (cf. Eq. (4.13)).

At temperatures above T_c , the large leptonic asymmetry resulting from the aforementioned mechanism gets converted, through equilibrated sphaleron interactions, into a corresponding large excess in the B number. As has been described in Section 5, long after the electroweak phase transition, the decay products of the KK Majorana neutrinos and possibly those of gravitons may reheat the Universe and so dilute the B asymmetry to reach the presently observed value. In such a cosmological framework, the upper bound on the reheat temperature compatible with baryogenesis may be dramatically weakened in theories with low scale of quantum gravity. For this reason, we believe that embedding the minimal scenarios of leptogenesis that we have studied here into more realistic models of inflation constitutes an interesting issue for future investigations.

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References

- [1] E. Witten, Nucl. Phys. **B471** (1996) 135.
- [2] P. Hořava and E. Witten, Nucl. Phys. **B460** (1996) 506; Nucl. Phys. **B475** (1996) 94.
- [3] J.D. Lykken, Phys. Rev. **D54** (1996) 3693.
- [4] I. Antoniadis, Phys. Lett. **B246** (1990) 377; I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. **B331** (1994) 313.
- [5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429** (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436** (1998) 257.
- [6] See, also, K.R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. **B436** (1998) 55; Nucl. Phys. **B537** (1999) 47.
- [7] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, hep-ph/9811448.
- [8] K.R. Dienes, E. Dudas and T. Gherghetta, hep-ph/9811428 (v3 16 January 1999).
- [9] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. **D59** (1999) 086004.
- [10] For example, see, N. Kaloper and A. Linde, Phys. Rev. **D59** (1999) 101303; T. Banks, M. Dine and A. Nelson, hep-th/9903019; C. Csaki, M. Graesser and J. Terning, hep-ph/9903319; A. Riotto, hep-ph/9904485; J.M. Cline, hep-ph/9904495.
- [11] M. Arkani-Hamed, S. Dimopoulos, N. Kaloper and J. March-Russell, hep-ph/9903224.
- [12] K. Benakli and S. Davidson, hep-ph/9810280.
- [13] S. Cullen and M. Perelstein, hep-ph/9903422.
- [14] L.J. Hall and D. Smith, hep-ph/9904267.
- [15] R. Sundrum, Phys. Rev. **D59** (1999) 085010; N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124.
- [16] G. Dvali and A. Yu. Smirnov, hep-ph/9904211.

- [17] G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. Phys. **B544** (1999) 3; T. Han, J.D. Jykken and R.-J. Zhang, Phys. Rev. **D59** (1999) 105006; J.L. Hewett, hep-ph/9811356; E.A. Mirabelli, M. Perlstein and M.E. Peskin, Phys. Rev. Lett. **82** (1999) 2236; S. Nussinov and R. Shrock, Phys. Rev. **D59** (1999) 105002; K. Agashe and N.G. Deshpande, hep-ph/9902263; M.L. Graesser, hep-ph/9902310; P. Nath and M. Yamaguchi, hep-ph/9902323; hep-ph/9903298; M. Masip and A. Pomarol, hep-ph/9902467; K. Cheung and W.-Y. Keung, hep-ph/9903294.
- [18] M. Fukugita and T. Yanagida, Phys. Lett. **B174** (1986) 45;
For recent reviews, see, A. Pilaftsis, hep-ph/9812256, Int. J. Mod. Phys. A (in press); A. Riotto, hep-ph/9901362.
- [19] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **B155** (1985) 36.
- [20] J. Scherk, “Recent Developments in Gravitation,” proceedings of a Summer Institute held in Cargèse, France, 1978, eds. M. Lévy and S. Deser (Plenum, New York, 1979) p. 479; S. Weinberg, Phys. Lett. **B143** (1984) 97; C. Wetterich, Nucl. Phys. **B234** (1984) 413; K. Shimizu, Prog. Theor. Phys. **74** (1985) 610; R. Finkelstein and M. Villasante, Phys. Rev. **D31** (1985) 425; S. Chadha and M. Daniel, Rutherford preprint, Oxford, 1985, RAL-85-040 (unpublished).
- [21] P. van Nieuwenhuizen, “Supergravity 1981,” proceedings of a conference held in Trieste, Italy, 1981, eds. S. Ferrara and J. Taylor (Cambridge Univ. Press, Cambridge, England, 1982), p. 151.
- [22] M.A. Luty, Phys. Rev. **D45** (1992) 455; C.E. Vayonakis, Phys. Lett. **B286** (1992) 92; R.N. Mohapatra and X. Zhang, Phys. Rev. **D45** (1992) 5331; K. Enqvist and I. Vilja, Phys. Lett. **B299** (1993) 281; G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Lett. **B315** (1993) 325; H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. Lett. **70** (1993) 1912; T. Gherghetta and G. Jungman, Phys. Rev. **D48** (1993) 1546; A. Acker, H. Kikuchi, E. Ma and U. Sarkar, Phys. Rev. **D48** (1993) 5006; M.P. Worah, Phys. Rev. **D53** (1996) 3902; W. Buchmüller and T. Yanagida, Phys. Lett. **B445** (1999) 399.
- [23] J. Liu and G. Segré, Phys. Rev. **D48** (1993) 4609; M. Flanz, E.A. Paschos and U. Sarkar, Phys. Lett. **B345** (1995) 248; L. Covi, E. Roulet and F. Vissani, Phys. Lett. **B384** (1996) 169; M. Flanz, E.A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. **B389** (1996) 693; L. Covi and E. Roulet, Phys. Lett. **B319** (1997) 113; E.Kh. Akhmedov, V.A. Rubakov and A.Yu. Smirnov, Phys. Rev. Lett. **81** (1998) 1359; E. Ma and U.

- Sarkar, Phys. Rev. Lett. **80** (1998) 5716; J. Faridani, S. Lola, P.J. O'Donnell and U. Sarkar, Eur. Phys. J. **C7** (1999) 543; G. Lazarides, Q. Shafi and N. Vlachos, Phys. Lett. **B427** (1998) 53; W. Buchmüller and M. Plümacher, Phys. Lett. **B431** (1998) 354; J. Ellis, S. Lola and D.V. Nanopoulos, Phys. Lett. **B452** (1999) 87; G. Lazarides, Phys. Lett. **B452** (1999) 227; J.-M. Frère, F.-S. Ling, M.H.G. Tytgat and V. Van Elewyck, hep-ph/9901337; E. Ma, M. Raidal and U. Sarkar, hep-ph/9901406; M.S. Berger and B. Brahmachari, hep-ph/9903406; G. Lazarides and N.D. Vlachos, hep-ph/9903511; G.F. Giudice, M. Peloso, A. Riotto and I. Tkachev, hep-ph/9905242.
- [24] A. Pilaftsis, Phys. Rev. **D56** (1997) 5431; Nucl. Phys. **B504** (1997) 61.
- [25] For related phenomena of resonant CP violation at colliders, see A. Pilaftsis, Z. Phys. **C47** (1990) 95; A. Pilaftsis and M. Nowakowski, Phys. Lett. **B245** (1990) 185; Int. J. Mod. Phys. **A9** (1994) 1097; R. Cruz, B. Grzadkowski and J.F. Gunion, Phys. Lett. **B289** (1992) 440; A.S. Joshipura and S.D. Rindani, Phys. Rev. **D46** (1992) 3008; D. Atwood, G. Eilam, A. Soni, R.R. Mendel and R. Migneron, Phys. Rev. Lett. **70** (1993) 1364; T. Arens and L.M. Sehgal, Phys. Rev. **D51** (1995) 3525; A. Skjold and P. Osland, Nucl. Phys. **453** (1995) 3; A. Pilaftsis, Phys. Rev. Lett. **77** (1996) 4996; D. Bowser-Chao and W.-Y. Keung, Phys. Rev. **D56** (1997) 3924; N. Arkani-Hamed, J.L. Feng, L.J. Hall and H.-C. Cheng, Nucl. Phys. **B505** (1997) 3; S.-Y. Choi and M. Drees, Phys. Rev. Lett. **81** (1998) 5509; W. Bernreuther, A. Brandenburg and M. Flesch, hep-ph/9812387.
- [26] E.W. Kolb and M.S. Turner, "The Early Universe," (Addison-Wesley, New York, 1990).
- [27] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **B125** (1983) 136.
- [28] R. Jackiw and C. Rebbi, Phys. Rev. **D13** (1976) 3398; E. Weinberg, Phys. Rev. **D24** (1981) 2669.
- [29] J. Bernabéu, G.C. Branco and M. Gronau, Phys. Lett. **B169** (1986) 243; G.C. Branco, M.N. Rebelo and J.W.F. Valle, Phys. Lett. **B225** (1989) 385.
- [30] S. Weinberg, Phys. Rev. **D42** (1990) 860.
- [31] L. Lavoura and J.P. Silva, Phys. Rev. **D50** (1994) 4619.
- [32] See, e.g. B.A. Kniehl and A. Pilaftsis, Nucl. Phys. **B474** (1996) 286; C. Balzereit, T. Hansmann, T. Mannel and B. Plümper, hep-ph/9810350; P. Gambino, P.A. Grassi and F. Madricardo, hep-ph/9811470.

- [33] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970) 1285.
- [34] For a detailed discussion of the pinch-technique resummation approach to the physics of unstable particles, see, J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75** (1995) 3060; Phys. Rev. **D53** (1996) 2128; Phys. Rev. **D54** (1996) 5315; K. Philippides and A. Sirlin, Nucl. Phys. **B477** (1996) 59; M. Passera and A. Sirlin, Phys. Rev. Lett. **77** (1996) 4146. N.J. Watson, Nucl. Phys. **B494** (1997) 388.
- [35] A.D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5** (1967) 32 (JETP Lett. **5** (1967) 24).
- [36] J.A. Harvey and M.S. Turner, Phys. Rev. **D42** (1990) 3344; B.A. Campbell, S. Davidson, J. Ellis and K.A. Olive, Phys. Lett. **B297** (1992) 118.
- [37] S.A. Abel and S. Sarkar, Phys. Lett. **B342** (1995) 40.