HOW GOOD ARE PRESENT ANALYTICAL QCD-PREDICTIONS ON FLUCTUATIONS IN ANGULAR INTERVALS ?

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Results on two–particle angular correlations in jet cones and on multiplicity fluctuations in one- and two- dimensional angular intervals, delivered by three experiments (DELPHI, L3 and ZEUS, at $\sqrt{s}$ from few to 183 GeV) are compared to present existing analytical QCD calculations, using the LPHD hypothesis. Two different types of functions have been tested. While the differentially normalized correlation functions show substantial deviations from the predictions, a globally normalized correlation function agrees surprisingly well. The role of the QCD parameters α_s, Λ and n_f is discussed. The necessity to include full energy-momentum conservation into the analytical calculations is stressed.

1 Introduction

Phenomenological models which try to describe multihadron production in e^+e^- reactions have to cut off the parton cascade at some scale $Q_0 \geq 1~GeV$. The following non-perturbative hadronisation phase is "handled" with many parameters such that the connection to QCD is "diluted". It has been suggested to extend instead the parton evolution calculated using perturbative QCD down to a lower mass scale. The multihadron final states are then compared directly with the multiparton final states [1] by using the concept of Local Parton Hadron Duality (LPHD) [2]. The obvious difficulty is the overlap of perturbative and non-perturbative QCD. It is unclear what one could expect when entering a region in which nobody knows what to apply. Therefore an experimental input is vital and the only way to learn what on really can apply. The main theoretical effort for evaluating multiparton correlations in the framework of QCD has been based on the Double Log Approximation (DLA) [1,3]. Comparisons with experimental data have to cope with substantial simplifications in the calculations of the perturbative part which are justified only at asymptotic energies, as well as with the question of how far the LPHD hypothesis is valid.

Detailed prescriptions for multiparton angular correlations in cones using DLA have been proposed by three groups [4−−6], which calculated parton correlations produced in gluon cascades radiated off the initial parton. The analytical predictions considered thereby involve only one adjustable param-

eter, namely the QCD scale Λ. The probability for gluon bremsstrahlung in DLA reads as follows:

$$M(k)d^3k = c_a\gamma_0^2\frac{dk}{k}\frac{d\Theta_{pk}}{\Theta_{pk}}\frac{d\Phi_{pk}}{2\pi} \qquad \gamma_0^2 = 6\alpha_S/\pi \qquad (1)$$

where $c_g = 1$ and $c_q = \frac{4}{9}$, p and k are the 3-momenta of the parent parton and the radiated gluon, Θ_{pk} the angle of emission of the gluon, Φ_{pk} the azimuthal angle of the gluon around $\vec{p}$ and γ_0 is dependent on the p_t of the gluon. The inclusive n-particle densities $\rho_n(k_1, k_2, ...k_n)$ (k_i is the 3-momentum of the i-th particle) are obtained by applying the generating functional technique [7] which has been developed for QCD jets first by [8,3]. The calculations have been carried out in DLA where the integrals involved are performed only in phase space regions with dominant contributions given by the singularities of (1). Among the simplifications that had to be used are neglecting energy-momentum conservation and $q\bar{q}$ production, and assuming well developed cascades at very high energies. Angular ordering, however, has been taken into account.

The predictions of [4--6] and the present study use the lowest order QCD relations between the coupling α_s and the QCD scale Λ.

$$\alpha_s = \frac{\pi\beta^2}{6}\frac{1}{\ln(\frac{Q}{\Lambda})} \qquad \beta^2 = 12(\frac{11}{3}n_c - \frac{2}{3}n_f)^{-1} \qquad (2)$$

The aim of this study is to compare experimental measurements of angular correlations in cones around the jet axis as well as multiplicity fluctuations in one- and two- dimensional angular intervals with the available theoretical QCD predictions thus helping the theorists to assess how well their analytical calculation is and what improvements could be done. First experimental measurements [9,10] revealed substantial deviations, but also encouraging agreements.

This study uses results of the LEP experiments DELPHI [11,12] and L3 [13] and the HERA experiment ZEUS [14]. Section 2 contains information about the experimental data used for the comparison. In sections 3 and 4 the theoretical framework is sketched and the analytical calculations are compared with the experimental measurement. Section 5 contains the final discussion and the summary.

2 The data samples

All three experiments considered use specific cuts for hadronic events and track quality (DELPHI [15,16], L3 [13], ZEUS [18]) Various corrections were applied using

events generated from Monte Carlo simulations [11--14]. In the following only corrected data will be shown.

DELPHI: The analysis uses about 600k e^+e^- interactions (after cuts) at $\sqrt{s} = 91$ GeV . A sample of about 1200 high energy events at $\sqrt{s} = 183\ GeV$ collected in 1997 is used to investigate the energy dependence.

L3: About 1 million e^+e^- interactions at $\sqrt{s} = 91$ GeV are used.

The event axis is the sphericity axis both in DELPHI and L3.

ZEUS: The data of this experiment (ep interactions at HERA) are 4 samples with different ranges of Q^2: 85k events ($10 \leq Q^2 \leq 20$ GeV), 28k events ($Q^2 \geq 100$ GeV), 0.9k events ($Q^2 \geq 1000$ GeV) and 0.3k events with ($Q^2 \geq 2000$ GeV). The last sample corresponds to $\sqrt{s} \approx 62\ GeV$, which comes near to LEP1 energies. The event axis is defined using the initial quark momentum of the γ^*q collision in the quark-parton model.

3 Multiplicity fluctuations in 1- and 2- dimensional ring regions around jet cones

The theoretical calculations treat correlations between partons emitted within an angular window defined by two angles ϑ and Θ. The parton and particle density correlations (fluctuations) in this window are described by normalized factorial moments of order n:

$$F^{(n)}(\Theta, \vartheta) = \frac{\int \prod_{k=1}^{n} d\Omega_k \rho^{(n)}(\Omega_1, \ldots, \Omega_n)}{\int \prod_{k=1}^{n} d\Omega_k \rho^{(1)}(\Omega_k)} \qquad (3)$$

The integrals extend over the window chosen. The angular windows considered here are either rings around the jet axis with mean opening angle Θ and half width ϑ in the case of 1 dimension ($D = 1$), or cones with half opening angle ϑ around a direction (Θ, Φ) with respect to the jet axis in the case of 2 dimensions ($D = 2$). At sufficiently large jet energies, the parton flow in these angular windows is dominated by parton avalaches caused by gluon bremsstrahlung off the initial quark.

Ref. [4] derived their predictions explicitly for cumulant moments $C^{(n)}$ [19], whereas [5] and [6] obtained similar expressions for the factorial moments $F^{(n)}$.

For the normalized cumulant moments $C^{(n)}$ [4] and the factorial moments $F^{(n)}$ [5,6], the following prediction has been made:

$$C^{(n)}(\Theta, \vartheta) \text{ or } F^{(n)}(\Theta, \vartheta) \sim \left(\frac{\Theta}{\vartheta}\right)^{\phi_n} \qquad (4)$$

All 3 references [4--6] give in the high energy limit and for large $\vartheta < \Theta$ the

same linear approximation for the exponents ϕ_n:

$$\phi_n \approx (n-1)D - \left(n - \frac{1}{n}\right)\gamma_0 \qquad (5)$$

For fixed α_s (along the parton shower) eq. (5) is asymptotically valid for all angles ϑ.

When the running of α_S with ϑ in the parton cascade is taken into account, ref.[4] obtained

$$\phi_n = (n-1)D - 2\gamma_0(n - \omega(\epsilon,n))/\epsilon \qquad (6)$$

where $D = 1$ for ring regions and $D = 2$ for cones,

$$\omega(\epsilon,n) = n\sqrt{1-\epsilon}(1 - \frac{1}{2n^2}\ln(1-\epsilon)) \qquad (7)$$

and

$$\epsilon = \frac{\ln(\Theta/\vartheta)}{\ln(P\Theta/\Lambda)} \qquad (8)$$

where $P \approx \frac{\sqrt{s}}{2}$ is the momentum of the initial parton.

The dependence on the QCD parameters α_s or Λ enters in the above equations via γ_0 and ϵ that are determined by the scale $Q \approx P\Theta$. In the present study it is, for $\Theta = 25^o$, about 20 GeV for $\sqrt{s}$=91.1 GeV and about 38 GeV for $\sqrt{s} \approx 183\ GeV$.

The corresponding predictions of refs.[5] (eq. 9) and[6] (eq. 10) are analytically different, but numerically similar:

$$\phi_n = (n-1)D - \frac{2\gamma_0}{\epsilon} \cdot \frac{n^2-1}{n}\left(1 - \sqrt{1-\epsilon}\right) \qquad (9)$$

$$\phi_n = (n-1)D - \frac{n^2-1}{n}\gamma_0\left(1 + \frac{n^2+1}{4n^2}\epsilon\right) \qquad (10)$$

These relations depend also on the number of flavours (n_f). Since equ. 2 emerges only from "one loop" calculations, the parameter Λ is not the universal $\Lambda_{\overline{MS}}$, but only an effective parameter Λ_{eff}. But also in this approximation α_s is running having a scale dependence $1/\ln(Q^2/\Lambda^2)$. The running of α_s during the process of jet cascading is implicitly taken into account in (6), (9) and (10) by the dependence of ϕ_n on ϵ (or ϑ). In theory this causes a bending of $F^{(n)}$ when approaching smaller values of ϑ (larger ϵ).
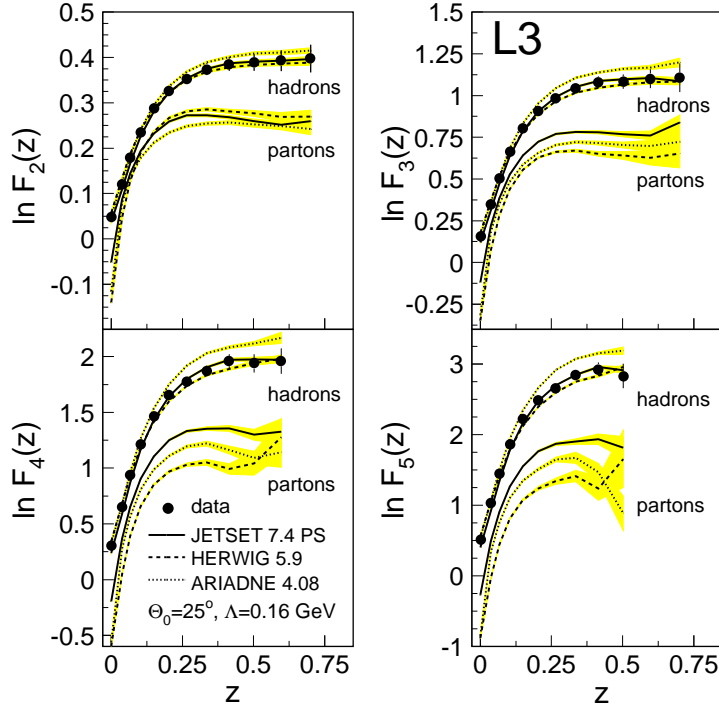
Figure 1: Factorial moments in 1-dim rings are compared with three Monte Carlo models, both on hadronic and partonic level. Please note that the variable ϵ (equ. 8) is here denoted by z.

In fig. 1 (L3) the factorial moments of orders 2,3,4 and 5 are compared to various Monte Carlo simulations at hadronic and partonic level. *For all orders n the hadronic levels agree very well with the data (this is generally the case), while the partonic level disagrees with that of the hadronic level*, so that LPHD seems to be not valid in this case. This is somewhat contradicted by ZEUS [14], which claim an approximate validity for LPHD for $Q^2 \geq 100 \ GeV^2$. When normalizing these moments by the respective value at $\epsilon = 0$ *the agreement between parton and hadron levels is improved* [13]. It is pointed to the fact that the shape of the correlation functions on the partonic level depends very much on the cut-off parameter Q_0 [10] (in fig. 1 Q_0=1 GeV, a rather high value compared to the original understanding of LPHD [2]).
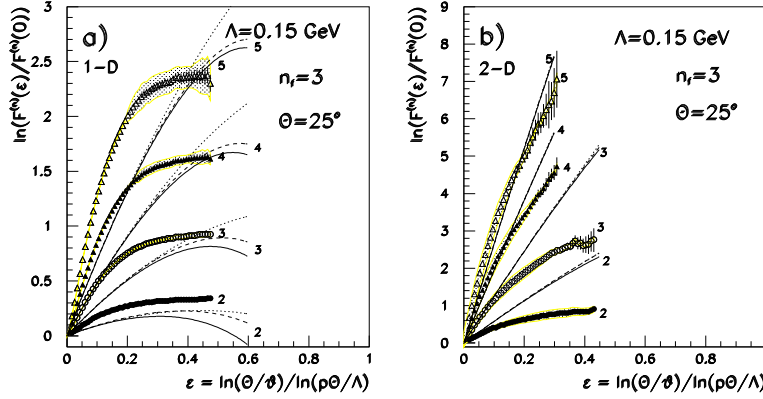
Figure 2: Factorial moments in **a)** 1-dim rings and **b)** 2-dim side cones are compared with analytical calculations of refs[4−−6], eqs. (6) (solid lines), (9) (dashed lines), and (10) (dotted lines), for a cone opening angle of $\Theta = 25^o$. As a consistency test, 1-dim and 2-dim factorial moments are compared with same values of QCD parameters: note the different vertical scales. The values for orders 2 to 5 are indicated in all figures. The statistical errors are shown by the error bars, the shaded regions indicate the systematic errors.

Fig. 2 (DELPHI) shows the normalized factorial moments of orders 2, 3, 4 and 5 together with the predictions of refs.[4−−6], in one- and two- dimensional angular intervals (i.e. rings and side cones) for Λ= 0.15 GeV and n_f=3. *They are not described well by the theoretical predictions*[4−−6]

For the 1-dim case (fig. 1a) the predictions lie below the data for not too large ϵ, differing also in shape. Choosing $n_f = 5$ the discrepances will increase [11], *choosing smaller values of Λ (e.g. = 0.04 GeV) will reduce the discrepancies for small ϵ* [11,13], especially for lower orders n. The angular correlations in 2 dimensions (fig. 2b) *show a different (disagreeing) behaviour* for the lower order moments $n < 4$, where the predictions lie above the data. The higher moments $F^{(4)}$ and $F^{(5)}$ have similar features in the 1-dim and 2-dim case. In both cases the data lie above the predictions at small ϵ and bend below them at larger ϵ.

Similar conclusions for $\sqrt{s}$=91 GeV, for the 1-dim ring regions, are drawn by the L3 collaboration.

It is not possible to find <u>one</u> set of QCD parameters Λ and n_f which simultaneously minimize the discrepancies between data and predictions for moments of all orders 2,3,4 and 5 in both the 1-dim. and 2-dim. cases.

Fig. 3 (DELPHI) shows a comparison with high energy data at $\sqrt{s}$=183 GeV (with a mean energy of $\sqrt{s}$=175 GeV and the corresponding predictions according to equ. (6), where the energy dependence enters via the parameter γ_0. For larger values of ϵ there is better agreement, the statistical errors of the high energy data, however, are substantial. The relative increase of the predicted moments agrees qualitatively with that of the JETSET model.
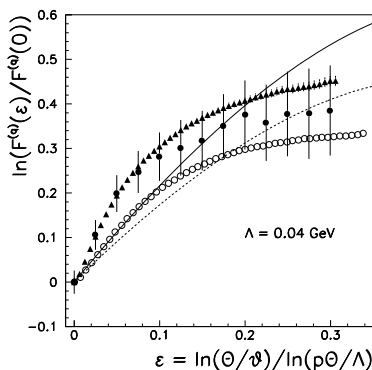
Figure 3: The energy dependence of the normalized factorial moment of order 2 $\sqrt{s}$=91.1 GeV: data (open circles), in full agreement with JETSET, and prediction [4] dashed line and $\sqrt{s}$=175 GeV: data (full circles) and prediction [4] solid line, for $n_f = 3$ and Λ=0.04 GeV. The full triangles denote the high energy JETSET simulation.
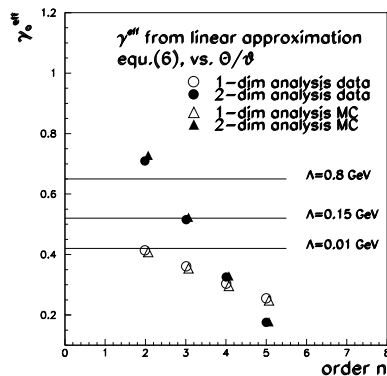
Figure 4: Values of γ_0^{eff} obtained from fitting the linear approximation eq. (5) vs. $\frac{\Theta}{\vartheta}$ the 1-dim case (open circles) and the 2-dim case (full circles), for orders $n = 2, 3, 4, 5$. The measured values of γ_0^{eff} are also compared with those obtained from JETSET at generator level: open triangles (1-dim) and full triangles (2-dim).

The QCD parameter γ_0 is discussed in fig. 4 (DELPHI), where the numerical values of γ_0^{eff} derived from the measured slopes ϕ_n (by fitting equ. (5) for $\epsilon \leq 0.1$) are given for the orders $n = 2, 3, 4, 5$. From the present theoretical understanding, γ_0 is expected to be independent of n. This is indicated by horizontal lines in fig. 4. It has to be pointed out that the average measured values of γ_0^{eff} are not too far from the expectation. *The n-dependence observed, however, is not described by the calculations.* The measured values of γ_0^{eff} agree, however, extremely well with the corresponding values obtained from JETSET, as can be seen in fig. 4.

4 Predictions on 2-particle angular correlations in jet cones

Theoretical predictions[4] concerning the emission of two partons with a relative angle ϑ_{12} - within a cone with half opening angle Θ around the jet axis - have been evaluated using two correlation functions defined as follows :

$$r(\vartheta_{12}) = \frac{\rho_2(\vartheta_{12})}{\rho_1 \otimes \rho_1(\vartheta_{12})} \tag{11}$$

$$\tilde{r}(\vartheta_{12}) = \frac{\rho_2(\vartheta_{12})}{\bar{n}^2(\Theta)} \tag{12}$$

with the correlation integrals[20] $\rho_2(\vartheta_{12}) = \int_\Theta d^3k_1 d^3k_2 \rho_2(k_1, k_2)\delta(\vartheta_{12}-\vartheta(k_1, k_2))$ and
$\rho_1 \otimes \rho_1(\vartheta_{12}) = \int_\Theta d^3k_1 d^3k_2 \rho_1(k_1)\rho_1(k_2)\delta(\vartheta_{12}-\vartheta(k_1, k_2))$ where $\rho_1(k)$ is the single particle distribution and $\bar{n}(\Theta)$ is the mean multiplicity of partons emitted

into the Θ-cone. The quantities in eqns. (11) and (12) exhibit very different structures. $\rho_2(\vartheta_{12})$ consists of 2 terms $\rho_2(\vartheta_{12}) = C_2(\vartheta_{12}) + \rho_1 \otimes \rho_1(\vartheta_{12})$ where only $C_2(\vartheta_{12})$ describes the genuine correlations, $\rho_1 \otimes \rho_1(\vartheta_{12})$, on the other side, is obtained from the single particle spectra. Consequently $r(\vartheta_{12})$ is given essentially by the normalized C_2-term whereas it turns out that the dominant term of $\tilde{r}(\vartheta_{12})$ is given by $\rho_1 \otimes \rho_1(\vartheta_{12})$.

Distinct predictions for $r(\vartheta_{12})$ and $\tilde{r}(\vartheta_{12})$ have been evaluated which depend essentially only on the QCD parameters Λ and n_f [4].

At high energy and for sufficiently large angles $\vartheta_{12} \leq \Theta$ the following power law is expected:

$$r(\vartheta_{12}) = \left(\frac{\Theta}{\vartheta_{12}} \right)^{0.5\gamma_0} \tag{13}$$

and the scale determining γ_0 is again given by $Q \approx P\Theta$ (see section 3).

For asymptotically high energies the quantity

$$\frac{\ln(r(\vartheta_{12}))}{\sqrt{\ln \frac{P\Theta}{\Lambda}}} \approx 2\beta(\omega(\epsilon, 2) - 2\sqrt{1 - \epsilon}) \tag{14}$$

with

$$\epsilon = \frac{\ln \frac{\Theta}{\vartheta_{12}}}{\ln \frac{P\Theta}{\Lambda}} \quad , \qquad \beta^2 = 12(11 - \frac{2}{3}n_f)^{-1} \tag{15}$$

and ω given by equ. 7, is expected to be independent of the cone opening angle Θ and primary momentum P, meaning that it is a scaling function.

The expected scaling properties of the quantity $\frac{\ln(r(\vartheta_{12}))}{\sqrt{\ln P\Theta/\Lambda}}$ (eqn. (14)) are tested in figs. 5 and 6.

The dependence on the cone opening angle Θ is shown in figs. 5b (ZEUS) and 6a (DELPHI) (left side of figs 5, 6). It can be seen that for $15° \leq \Theta \leq 60°$ the *dependence on Θ is very weak already at $\sqrt{s} = 91\ GeV$*, in agreement with the predictions of eqn. (14). For small energies (fig 5b) there is no scaling for $\epsilon \leq 0.3$. The *shape predicted* by eqn. (14) *differs appreciably from the measurement* at LEP (fig. 6). The shape of the data is only similar to that predicted in the sense that it is rising and levelling off; the data are much smaller and flatter. There is a "hook" in data at small ϵ which is a reflection of momentum conservation. It is almost absent, however, in the ZEUS data. Thus at LEP energies the analytic QCD calculations do not describe quantitatively the 2-particle angular correlations r(ϑ_{12}). These differences are much less pronounced for the low energies (solid lines in fig. 5), but this could be also due to the different way to define the jet axis. The dependence of (14) on energy is shown in figs. 5a (ZEUS) and 6b (DELPHI) for a fixed value of Θ (right hand
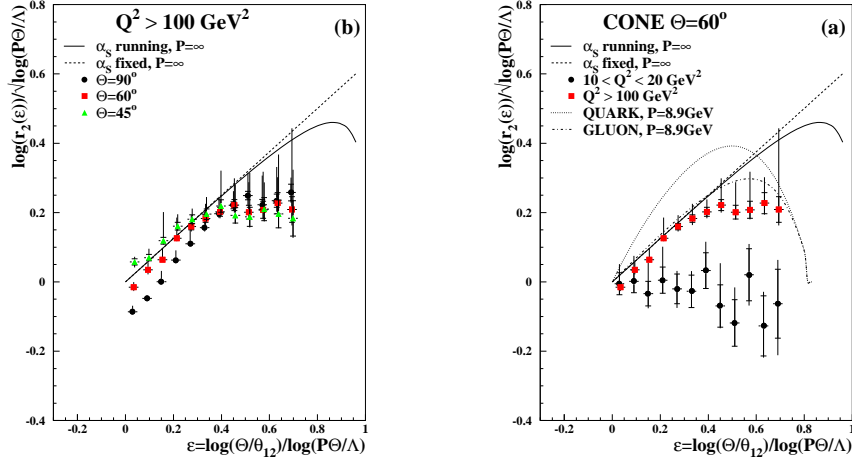
Figure 5: The r_2 correlation function (14) dependence on **b)** cone size Θ and **a)** Q^2 in a lower energy range - and comparison with QCD predictions [4].
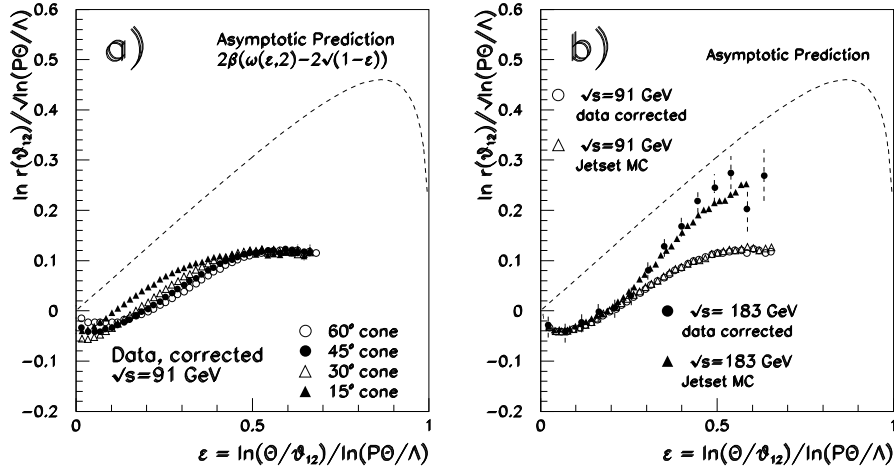

Figure 6: **a)** The rescaled function r_2 (14) is plotted against the scaling variable ϵ in order to test prediction [4], dashed line. **b)** The corrected data for $\Theta = 45°$, at 91 GeV (open circles) and at 183 GeV (full circles), using $\Lambda = 0.15\ GeV$, are shown together with JETSET Monte Carlo calculations (open resp. full triangles).

sides of figs. 5,6). The ZEUS data indicate an energy dependence only at lowest energies [14]. This is in disagreement with the the observations of DELPHI which observes *a strong energy dependence also at the highest energies.*

In equs. (16), (17) a scaling function $Y(\epsilon)$ is defined, which is predicted to be independent of Θ and the primary momentum P.

$$\tilde{r}(\epsilon) = \vartheta_{12}\tilde{r}(\vartheta_{12})\ln\frac{P\Theta}{\Lambda} \qquad (16)$$

$$Y(\epsilon) = -\frac{\ln(\tilde{r}(\epsilon)/b)}{2\sqrt{\ln(\frac{P\Theta}{\Lambda})}} = 2\beta(1 - 0.5\omega(\epsilon, 2)) \quad , \quad b = 2\beta\sqrt{\ln(\frac{P\Theta}{\Lambda})} \qquad (17)$$

The expected scaling properties of $Y(\epsilon)$ are tested in figs. 7 (ZEUS) and 8 (DELPHI). From $\sqrt{s} \approx 18 \; GeV$ upwards the *distributions agree very well with the prediction* in the region $\epsilon \geq 0.2$, therefore exhibiting scaling both in angle Θ and energy. There is also good agreement with the coresponding JETSET simulations on both the partonic and hadronic level, which *supports parton hadron duality* - for the special function $Y(\epsilon)$. Note that no arbitrary normalization has been applied in the above figures.

Similar to the case of factorial moments (fig. 4), values of γ_0^{eff} have been extracted from $r(\vartheta_{12})$ by fitting the DELPHI data to equ. (13) [12]. The corresponding values of α_s depending on the cone opening angle Θ are shown in fig. 9a. Similar to the case of factorial moments the data agree better with lower values of Λ. This is at variance with the function $Y(\epsilon)$ where the data agree best with the value $\Lambda \approx 0.3 \; GeV$ (fig. 9b). It has to be noted that DLA takes only the leading singularities in all cases considered which could lead to different redefinitions of the effective QCD parameters (e.g. Λ_{eff}) [12].

5 Summary and conclusions

Present available QCD predictions on angular correlations, based on the DLA approximation, with first order relationship between α_s and Λ, are checked, comparing them to relevant results of three collaborations (DELPHI, L3 and ZEUS). The experimental measurements, which agree generally well with Monte Carlo simulations on hadronic level and not so well with the partonic level cover 2-particle angular correlations in cones and 1-dim and 2-dim multiplicity fluctuations in angular intervals. It turns out that functions which contain mainly single particle terms ($Y(\epsilon)$) are predicted well in every respect. Also the extraction of QCD parameters (i.e. Λ_{eff}) out of these measurements leads to reasonable results. Differentially normalised correlations ($r(\vartheta_{12})$, $F(\frac{\Theta}{\vartheta})$) that contain 2 or more particles, however, show often strong disagreements in shapes. The
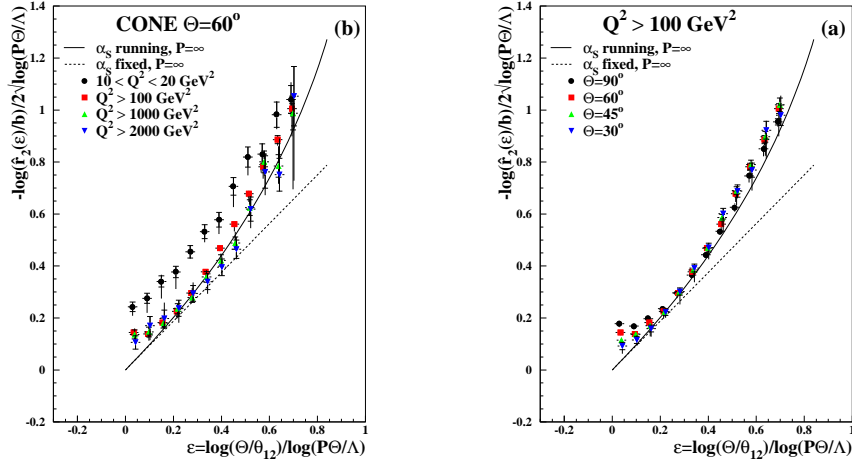
Figure 7: The function $Y(\epsilon)$ (equ. 17) evolving with **b)** Q^2 and **a)** cone opening angle Θ.
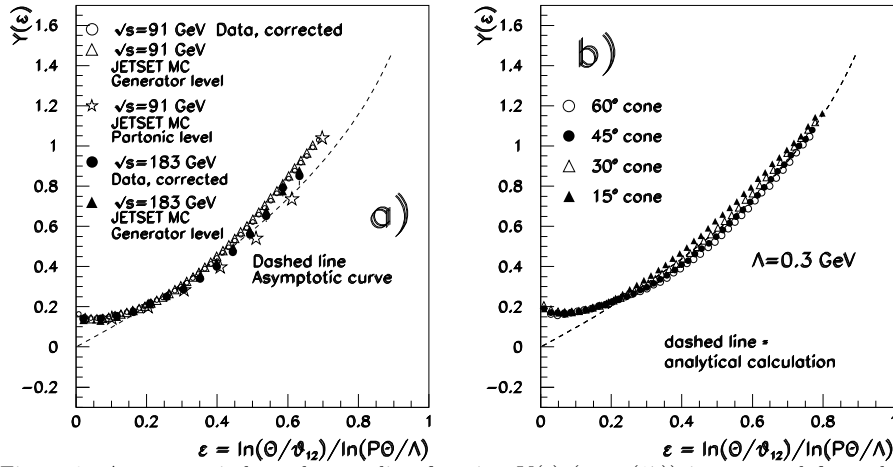

Figure 8: An energy independent scaling function $Y(\epsilon)$ (eqn. (17)) is extracted from the 2-body angular correlation function defined by eqn. (12). The dashed lines represent the asymptotic prediction eqn. (17). Statistical and systematic errors are smaller than 0.01 for the 91 GeV data. **a)** The corrected data for $\Theta = 45°$, at 91 GeV (open circles) and at 183 GeV (full circles), using $\Lambda = 0.15\ GeV$, are shown together with Monte Carlo calculations (open resp. full triangles). **b)** Test of the Θ-scaling behaviour of the data as predicted by eqn. (17), using $\Lambda = 0.3\ GeV$.
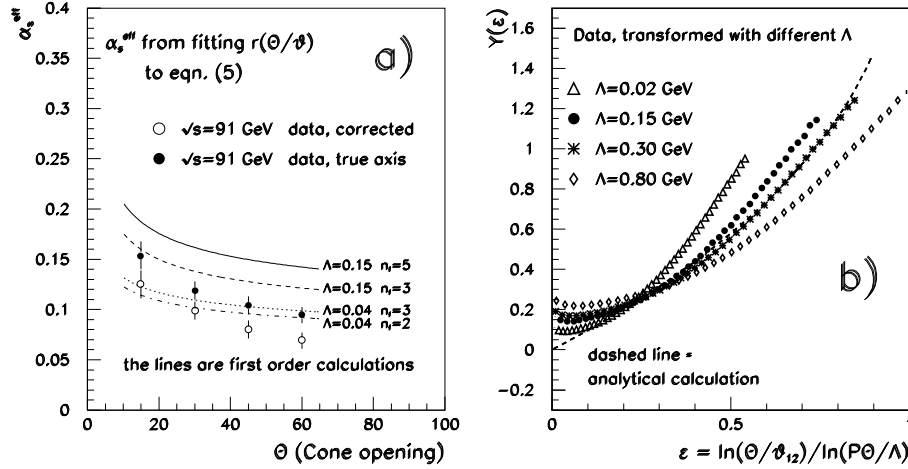
Figure 9: **a)** The measured values α_S^{eff} ($\circ$) from eqn. (13) for different values of Θ are compared with lowest order QCD relations equ. (2), with $Q \approx P\Theta$, for different values of Λ and n_f . Applying Monte Carlo corrections for choosing the true axis (of the initial parton) increases the value for α_S^{eff} ($\bullet$). The errors shown are systematic ones only, since the statistical errors are much smaller. **b)** Variation of the measured $Y(\epsilon)$ by choosing different values of Λ.

basic qualitative features, on the other hand, are fulfilled. Here smaller values of Λ_{eff} (resp. γ_0^{eff} or α_s^{eff}) are favoured as well as $n_f=3$ rather than $n_f=5$. Up to now no set of QCD parameters will minimise simultaneously the various discrepancies.

The main shortcoming of the analytical calculations is the missing energy-momentum conservation adopted. Some "partial" introduction of NLO corrections [11,13] did not improve matters significantly.

Considering, however, that the analytical calculation does not use any free parameter besides Λ or α_s, that they treat all kinds of correlations functions in an universal manner, that the asymptotic energies might be still far away for some subtle functions and that, nevertheless, the basic features of the predictions are always seen in the experimental measurement, further theoretical efforts are encouraged.

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