Elliptic and hexadecupole flow from AGS to LHC energies

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Using a hydrodynamic model we study the effects of the initial spatial anisotropy in non-central heavy-ion collisions on the momentum distributions of the emitted hadrons. We show that the elliptic flow measured at midrapidity in 158 A GeV/c Pb+Pb collisions can be quantitatively reproduced by hydrodynamic expansion, indicating early thermalization in the collision. We predict the excitation function of the elliptic and hexadecupole flow from AGS to LHC energies and discuss its sensitivity to the quark-hadron phase transition.

The recent observation of transverse collective flow phenomena in non-central heavy-ion collisions at ultrarelativistic beam energies [1–4] has led to renewed intense theoretical interest in this topic (see [5] for a review). Collective flow is the consequence of pressure in the system and thereby provides access to the equation of state of the hot and dense matter ("fireball") formed in the reaction zone. This access is indirect since the flow in the final state represents a time integral over the pressure history of the fireball. Sorge [6] has argued that different types of transverse flow (radial, directed, elliptic, see [5]) show different sensitivities to the early and late stages of the collision such that a combination of flow observables may allow for a more differential investigation of the equation of state. In particular, he pointed out that the elliptic flow (which develops in non-central collisions predominantly at midrapidity and manifests itself as a quadrupole deformation of the hadronic momentum distributions around the beam axis) is a signature for the early stage of the collision: its driving force is the spatial anisotropy of the dense nuclear overlap region which, if thermalized quickly enough, leads to an anisotropy of the pressure gradients which cause the expansion. Since the developing anisotropic flow reduces the elliptic spatial deformation of the fireball, it acts against its own cause and thus shuts itself off after some time. Radial flow, on the other hand, requires just pressure, but no pressure anisotropies to develop; it therefore exists also in central collisions, and in non-central collisions it continues to grow even after the initial elliptic spatial deformation of the fireball has disappeared.

A phase transition from a hadron gas to a colordeconfined quark-gluon plasma causes a softening of the

equation of state: as the temperature crosses the critical value for the phase transition, the energy and entropy densities increase rapidly while the pressure rises slowly. The resulting small ratio of p/e at the upper end of the transition region ("the softest point" [7]) weakens the build-up of flow as the system passes through it. Shuryak [8] and van Hove [9] therefore suggested that a plot of the mean transverse momentum against the central multiplicity density should show a plateau. Later hydrodynamic calculations did not confirm the existence of a plateau, showing only a slight flattening in a strictly monotonic curve [10]. While the acceleration of the matter is weak in the transition region, the system also takes a long time to cross it, thereby allowing for the flow to build up over a longer time. This considerably reduces the sensitivity of the final radial flow to the existence of a soft region in the equation of state.

Recently Sorge [11] revived van Hove's idea in connection with elliptic flow: using a modified version of RQMD which allows to simulate an equation of state with a "softest point", he found that the response of the final elliptic flow to the initial spatial deformation of the fireball was weakened for initial conditions in the phase transition region. Using a hydrodynamic model, Teaney and Shuryak [12] argued that the existence of the phase transition should, at higher energies, also lead to other dramatic effects in the transverse expansion pattern of non-central collisions, in particular to the formation of two well-separated shells moving into the reaction plane. In the present Letter we follow up on these ideas, trying to understand in more detail the transverse dynamics in non-central collisions and what experimental data can tell us about it. We use a similar hydrodynamic approach as in [12,13], adjust its free parameters to data from central Pb+Pb collisions at the SPS, demonstrate that it correctly reproduces the measured elliptic flow of pions and protons at midrapidity [2,14], and then use it to make predictions at other beam energies. In particular we discuss the sensitivity of the excitation functions of v_2 and v_4 , the elliptic and hexadecupole flow coefficients, to the existence of a deconfining phase transition.

In the hydrodynamical model one assumes that shortly after the impact the produced strongly interacting matter reaches a state of local thermal equilibrium and subsequently expands adiabatically. In the conservation laws for energy-momentum and baryon number

$$\partial_{\mu}T^{\mu\nu}(x) = 0, \quad \partial_{\mu}j^{\mu}(x) = 0 \tag{1}$$

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one can then use the ideal fluid decompositions $T^{\mu\nu} = (e+p)u^{\mu}u^{\nu} - g^{\mu\nu}p$, $j^{\mu} = n u^{\mu}$ in terms of the energy density e, the pressure p, the (net) baryon number density n, and the fluid four-velocity u^{μ} . One thus obtains the equations of ideal (non-dissipative) relativistic hydrodynamics. An equation of state p(e, n) is needed to close the set of equations; its direct connection with the developing flow pattern makes hydrodynamics the most appropriate framework for an investigation of the equation of state.

We are here mostly interested in the transverse expansion dynamics in non-central $(b \neq 0)$ heavy-ion collisions. The lack of azimuthal symmetry leads to a non-trivial 3+1 dimensional problem, requiring considerable numerical resources. As noted in [13], the complexity of the task is significantly reduced if one focusses on the transverse plane at midrapidity and assumes that the longitudinal expansion can be described analytically by Bjorken's scaling solution [15] $v_z = z/t$. The latter is known to correctly reproduce the longitudinal expansion dynamics at asymptotically high beam energies, and it works phenomenologically very well even at SPS and AGS energies [16]. This assumption reduces the numerical problem to 2+1 dimensions. While it should be harmless at midrapidity, it forbids to make reliable predictions at forward and backward rapidities. Hence we cannot describe the rapidity dependence of the transverse flow pattern.

We investigated three different equations of state: (i) an ideal gas of massless particles, $p = \frac{e}{3}$ (EOS I); (ii) a hadron resonance gas including all known resonances [17] with masses below 2 GeV and a repulsive mean field potential $\mathcal{V}(n) = \frac{1}{2}Kn^2$, with K = 0.45 GeV fm³ [18] (EOS H; for small *n* this equation of state can be well characterized by the simple relation p = 0.15 e); (iii) an equation of state with a first order phase transition at $T_c(n=0) = 164$ MeV, constructed by matching EOS H and EOS I using a bag constant $B^{1/4} = 230$ MeV (EOS Q) [18]. EOS Q features at n = 0 a latent heat of 1.15 GeV/fm³: the mixed phase ranges from $e_{\rm H} = 0.45$ GeV/fm³ to $e_{\rm Q} = 1.6$ GeV/fm³. We show results only for the semi-realistic cases EOS H and EOS Q.

For $b \neq 0$ the initial energy density distribution in the transverse plane has an almond shape, characterized by a spatial deformation $\alpha_x = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} > 0$. (x denotes the transverse direction parallel to the impact parameter **b**, y the one orthogonal to it.) This results in larger pressure gradients and thus in larger flow velocities in x than in y direction. Hence the final $p_{\rm T}$ -distribution is anisotropic. Its azimuthal angular dependence can be characterized by (even) Fourier coefficients [13] v_2, v_4, \ldots (at midrapidity the odd ones, in particular the "directed flow" v_1 , vanish by symmetry):

$$\frac{dN}{d\tilde{y}d\varphi} = \frac{dN}{2\pi d\tilde{y}} \left(1 + 2v_2\cos(2\varphi) + 2v_4\cos(4\varphi) + \ldots\right), \quad (2)$$
$$\frac{dN}{d\tilde{y}p_{\rm T}dp_{\rm T}d\varphi} = \frac{dN}{2\pi d\tilde{y}p_{\rm T}dp_{\rm T}} \left(1 + 2v_2(p_{\rm T})\cos(2\varphi)\right)$$

$$+2 v_4(p_{\rm T}) \cos(4\varphi) + \ldots)$$
. (3)

 $(\tilde{y} = \operatorname{Artanh}(p_z/E)$ is the longitudinal rapidity of the particles, and \tilde{y}_{cm} below denotes the midrapidity point.)

For each impact parameter b, we parametrize the initial transverse energy density $e(\mathbf{r})$ by a Glauber-inspired formula [13,19] which assumes that the deposited energy is proportional to the number of participating nucleons $N_{\text{part}}(\mathbf{b})$ (more exactly: to the sum of the nuclear transmission functions $T_{A,B}(\boldsymbol{b},\boldsymbol{r})$). The corresponding initial baryon density $n(\mathbf{r})$ is taken proportional to $e(\mathbf{r})$. At SPS energies a slightly nonlinear dependence of $dN/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}$ on $N_{\rm part}$ was observed [20]. At RHIC energies and above this nonlinearity is expected to become stronger since the energy deposition will be dominated by semi-hard processes (minijets) which are proportional to the number of NN collisions, not to N_{part} . However, this is not expected to strongly change the relation between the initial energy density and the final multiplicity density; it will mostly affect its dependence on the impact parameter b and on \sqrt{s} . The uncertainty in the \sqrt{s} -dependence prompts us to present excitation functions as functions of $dN/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}$; the "energy calibration" $dN/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}(\sqrt{s})$ will be provided by experiment.

The final particle distribution $dN/(d\tilde{y}p_{\rm T}dp_{\rm T}d\varphi)$ is calculated using the Cooper-Frye formula [21] with a freezeout hypersurface of constant temperature. We adjust the model parameters, i.e. the initial central energy and baryon densities e_0 and n_0 at b=0, the equilibration time τ_0 , and the decoupling temperature $T_{\rm dec}$, by fitting [19] the measured [22] negative hadron and proton spectra from central 158 $A\,{\rm GeV}$ Pb+Pb collisions. For EOS Q we find $T_{\rm dec} = 120 \text{ MeV}, e_0 = 9.0 \text{ GeV}/\text{fm}^3, n_0 = 0.95 \text{ fm}^{-3}$ and $\tau_0 = 0.8 \text{ fm}/c$. The freeze-out temperature and the average radial flow resulting from these initial conditions agree well with previous studies [23–25]. In calculating the negative hadron spectrum we included [26] decays of all resonances up to the mass of the $\Delta(1232)$; resonance decays are found to reduce the momentum anisotropies $v_{2,4}$ for pions by 10-15%.

Having adjusted the model parameters in b=0 collisions, we can calculate the initial density distributions also for $b \neq 0$ collisions, using the same Glauber formula. The equilibration time τ_0 and decoupling temperature T_{dec} are left unchanged. We have tested this procedure on $p_{\rm T}$ -spectra from non-central Pb+Pb(Au) collisions for pions [3] and protons [4,27] and found very good agreement between data and hydrodynamical simulations up to impact parameters of about 10 fm [19]. We then proceed to compute the elliptic and hexadecupole flow coefficients v_2 and v_4 in Eq. (2) as functions of the number of participating nucleons, $N_{\rm part}$ (or, equivalently, as functions of the impact parameters given above, are shown in Fig. 1.



FIG. 1. The coefficients of elliptic and hexadecupole flow for pions as functions of the number of participating nucleons. Also shown is the momentum-space anisotropy α_p . For details see text.

In addition to $v_2 = \langle \cos(2\varphi) \rangle$ (where the average is taken with the particle momentum distribution) we also show the momentum-space analogue α_p of the spatial anisotropy α_x introduced above:

$$\alpha_p = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{\langle p_{\rm T}^2 \cos(2\varphi) \rangle}{\langle p_{\rm T}^2 \rangle} \,. \tag{4}$$

This quantity was denoted by $\bar{\alpha}$ in [13] and gives the $p_{\rm T}^2$ -weighted elliptic flow. Fig. 1 shows that for pions v_2 and α_p differ by an overall factor of about 2, but otherwise have the same impact parameter dependence.



FIG. 2. Elliptic flow v_2 for pions and protons, as a function of rapidity, as measured by NA49 in 158 A GeV Pb+Pb collisions [14]. The dashed lines are to guide the eye. The circled crosses at midrapidity show our hydrodynamical results, with the same cuts in b and p_T as the data.

This factor 2 is important: previously α_p and v_2 have often been used synonymously and, based on Ollitrault's results for α_p [13], one concluded that hydrodynamic calculations overpredict the elliptic flow at the SPS by about a factor of 2. Fig. 2 shows that this is not the case: a correct comparison of the data with the calculated v_2 (instead of α_p) shows good quantitative agreement. The data [14] were obtained from Pb+Pb collisions at the SPS, with a cut on the collision centrality and on the particle $p_{\rm T}$ as given in the figure. Our calculation was done for b = 7 fm and, using Eq. (3), the same $p_{\rm T}$ -cut as in the data was applied to the calculated spectra. The thus calculated values for v_2 at midrapidity are 2.9% for pions and 11.7% for protons.

The good agreement of the data (both the shape of the $p_{\rm T}$ -spectra as a function of b and the absolute values of v_2) with hydrodynamical calculations strongly suggests very early thermalization and pressure buildup in these collisions. In the calculation we can follow the time history of the elliptic flow: we found that α_p for pions is nearly identical to

$$\tilde{\alpha}_p = \frac{\langle\!\langle T_{xx} - T_{yy} \rangle\!\rangle}{\langle\!\langle T_{xx} + T_{yy} \rangle\!\rangle},\tag{5}$$

if the spatial average $\langle \langle \ldots \rangle \rangle$ is performed at the time when the fireball center freezes out. The quantity $\tilde{\alpha}_p$ does not require knowledge of the particle spectra and can be evaluated also at other times from the solution of the hydrodynamic equations. As expected we find that $\tilde{\alpha}_p$ saturates as soon as the spatial anisotropy α_x goes to zero, and that for Pb+Pb collisions at the SPS $\frac{1}{6}$ of the final elliptic flow is created while the fireball center is in a pure QGP phase, $\frac{1}{2}$ of it is created in the mixed phase, and the final $\frac{1}{3}$ is generated during the hadronic stage. This agrees with Sorge's conclusion [11] that the elliptic flow at the SPS indeed probes the deconfining phase transition and the existence of a QGP phase.

However, is it also *sensitive* to the existence of a phase transition? To answer this question we recalculated v_2 and v_4 with EOS I and EOS H, readjusting the initial conditions to the measured h^- and $p - \bar{p}$ spectra from central Pb+Pb collisions [19]. (While for EOS H an acceptable fit is possible, the fit for EOS I is quite bad, as found before by several other authors.) Whereas EOS I (which can already be excluded from the b = 0 spectra) gives about 30-40% larger values for v_2 , the elliptic flow developed by EOS H is quite similar to that of EOS Q. v_4 is about 60% larger with EOS H than with EOS Q. The time history of $\tilde{\alpha}_n$ reveals that the softening of the EOS near the phase transition delays the buildup of elliptic flow by about 1.5-2 fm/c but that in the end it reaches the same value. The mechanism is the same as discussed in the context of van Hove's plateau: the phase transition weakens the elliptic flow, but since the system also spends more time in the transition region, its net effect on v_2 is much less than naively expected.



FIG. 3. Hydrodynamic excitation functions of the elliptic (v_2) and hexadecupole flow (v_4) , for pions from A + A collisions $(A \approx 200)$ at impact parameter b = 7 fm. The vertical dashed lines indicate the produced total pion multiplicity densities at midrapidity for 11.6 and 158 A GeV/c beam momentum, respectively (upper ends of the AGS and SPS ranges). The dash-dotted vertical line indicates the threshold above which, at b = 7 fm, the fireball center is initially in a pure QGP phase.

Sorge [11] suggested that, for beam energies near the "softest point", the response function $A_2 = \frac{v_2}{\alpha_r} \approx 0.5 \frac{\alpha_p}{\alpha_r}$ should show a plateau-like structure as a function of impact parameter: If, as the collision centrality is decreased, the initial energy density in the fireball center drops from supercritical to subcritical values, this should show up by a weaker elliptic flow for impact parameters corresponding to energy densities in the transition region. Adjusting our initial parameters such that these conditions were satisfied, we could not confirm this finding: α_x and α_p stay strictly proportional to each other (see Fig. 2) even if the phase transition is crossed as one varies N_{part} or b. We found $A_2=0.16$ for pions and $A_2=0.27$ for protons, independent of N_{part} . (The results given in [13]) also yield constant A_2 -values.) The plateau in A_2 found by Sorge within RQMD must thus be entirely a nonequilibrium effect. Sorge himself stressed [11] that within his approach non-equilibrium mechanisms are responsible for the saturation of v_2 as a function of time. The experimental confirmation of a plateau-like structure in $A_2(N_{\text{part}})$ would thus be a crucial test for (the lack of) local thermalization.

Given the apparent insensitivity of elliptic flow to the phase transition at a fixed beam energy, one may still hope for distinctive features in the excitation function of anisotropic flow [28]. In Fig. 3 we show the excitation functions for v_2 and v_4 for the hadron resonance gas and QGP equations of state, for Pb+Pb collisions at impact parameter b = 7 fm. (The initial equilibration time $\tau_0 =$ 0.8 fm/c, the ratio e_0/n_0 , and the freeze-out temperature $T_{\text{dec}} = 120$ MeV are held fixed.) Above SPS energies, both v_2 and v_4 are seen to rapidly approach constant asymptotic values. For the equation of state with a phase transition (EOS Q) these asymptotic values are smaller than for EOS H: for pions v_2 drops from 6 % to 5 %, while v_4 is reduced by about 40 %. (v_4 is of the order of a few permille and thus hard to measure.) This is the main effect of the softness of EOS Q near T_c . Larger differences would be seen in comparison with the (unrealistic) ideal gas EOS I. Altogether, at high energies the anisotropic flow coefficients show a qualitatively similar dependence on the EOS as the radial flow [10].

Interestingly, in the phase transition region v_2 and v_4 show very little sensitivity to the EOS; only above the critical energy density for creation of a pure QGP the two excitation functions are different. The most prominent qualitative pattern is a maximum and subsequent decrease in the excitation function of v_2 for EOS Q which is absent for EOS H. v_4 features monotonic excitation functions for both equations of state although it is considerably *reduced* for EOS Q.

Fig. 3 covers the range of initial energy densities $1 \leq$ $e_0 \leq 25 \text{ GeV/fm}^3$. We caution, however, that towards the left of Fig. 3 our results become unreliable: we kept $T_{\rm dec}$ fixed although at lower beam energies freeze-out is known to occur at lower temperatures [29], giving more time for flow buildup and yielding larger values $v_2 > 0$. On the other hand, below 1-2 A GeV/c the elliptic flow at midrapidity builds up before the projectile and target spectators have moved out of the way (as assumed in our model); this "inertial confinement" causes the elliptic flow to develop perpendicular to the reaction plane ("squeeze-out" [30], $v_2 < 0$) instead of in-plane as in our calculations. Experimentally this sign change of v_2 occurs near $E_{\text{beam}} = 4 A \text{ GeV} [31]$. None of these phenomena is, however, directly related to the existence of a phase transition; where we see sensitivity in the excitation function to the phase transition, our calculations are not affected by these issues. Our results do not confirm the expectation of strong structures in the excitation function of v_2 between AGS and SPS energies [32].

As a last point we discuss why the seemingly so dramatic phenomenon of the "cracked nut", recently advocated by Teaney and Shurvak [12] as a hydrodynamic signature for the existence of a QGP phase transition, doesn't leave stronger traces in the anisotropic flow pattern. These authors argued that at high energies (RHIC or LHC) a soft region in the EOS leads to the development of a "shell" at the edge of the almond-like initial fireball which is then cracked by the high pressure inside, with two separating half-shells expanding into the reaction plane. While we confirm their numerical results [12], we tend to interpret them more cautiously. To illustrate our point of view we show in Fig. 4 the freeze-out surface $\tau(x, y)$ for a Pb+Pb collision at b=8 fm, initiated with a central temperature $T_0=870$ MeV at $\tau_0=0.2$ fm/c and yielding a pion midrapidity density $dN_{\pi}/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}\approx 530$ (corresponding to $dN_{\pi}/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}\approx 1600$ for central collisions). Note its mushroom-like structure: While at the SPS hydrodynamics predicts freeze-out surfaces which shrink with time, the present surface features dramatic transverse growth [33] before freezing out *nearly instantaneously* after 13 fm/c. One thus expects a very small emission time duration signal in two-particle Bose-Einstein correlations [34], in spite of the phase transition.



FIG. 4. Freeze-out hypersurface $\tau(x, y, z=0)$ for b=8 fm Pb+Pb collisions, for an initial temperature $T_0=1.14$ GeV $(dN_{\pi}/d\tilde{y}|_{\tilde{y}=\tilde{y}_{\rm cm}}\approx 1600)$ in central collisions. Note the dramatic transverse growths, followed by sudden freeze-out.

Already before freeze-out the initial elliptic spatial deformation has vanished. The ripple on the top of the mushroom near its outer edge in x-direction is the "nut shell" [12]: in a cut through the surface at $\tau \approx 13$ fm/c it shows up as a crescent-shaped half shell at $x \sim 7$ fm. However, a mere 0.5 fm/c later, the matter in this shell has frozen out, too. For EOS H one finds a very similar mushroom, but without the ripple at the edge. Since there is no qualitative difference in the momentum-space structure of the "shell" compared to the rest of the matter, this explains why it is impossible to uniquely identify this particular structure by an anisotropic flow analysis. As suggested in [12], two-particle correlations may be more promising, but require extensive studies.

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- J. Barrette et al. (E877 Coll.), Phys. Rev. Lett. 73 (1994) 2532.
- [2] H. Appelshäuser et al. (NA49 Coll.), Phys. Rev. Lett. 80 (1998) 4136.
- [3] M.M. Aggarwal et al. (WA98 Coll.), Phys. Rev. Lett. 81 (1998) 4087.
- [4] F. Ceretto et al. (CERES Coll.), Nucl. Phys. A 638 (1998) 467c.
- [5] J.Y. Ollitrault, Nucl. Phys. A 638 (1998) 195c.
- [6] H. Sorge, Phys. Lett. B 402 (1997) 251.
- [7] C.M. Hung, E.V. Shuryak, Phys. Rev. Lett. 75 (1995) 4003.
- [8] E.V. Shuryak, O.V. Zhirov, Sov. J. Nucl. Phys. 28 (1978) 247; Phys. Lett. B 89 (1980) 253.
- [9] L. van Hove, Phys. Lett. B 118 (1982) 138.
- [10] M. Kataja, P.V. Ruuskanen, L.D. McLerran, H. von Gersdorff, Phys. Rev. D 34 (1986) 2755.
- [11] H. Sorge, Phys. Rev. Lett. 82 (1999) 2048.
- [12] E.V. Shuryak, hep-ph/9903297; D. Teaney, E.V. Shuryak, nucl-th/9904006.
- [13] J.Y. Ollitrault, Phys. Rev. D 46 (1992) 229.
- [14] The data were taken from an update of those published in [2] and can be found at the URL: http://na49info.cern.ch/na49/Archives/Images/Publications/Phys.Rev.Lett.80:4136-4140,1998
- [15] J.D. Bjorken, Phys. Rev. D 27 (1983) 140.
- see, e.g., H. Dobler, J. Sollfrank, U. Heinz, nuclth/9904018, Phys. Lett. B, in press; E. Schnedermann, J. Sollfrank, U. Heinz, Phys. Rev. C 48 (1993) 2462.
- [17] Particle Data Group, Eur. Phys. J. C 3 (1998) 1.
- [18] J. Sollfrank et al., Phys. Rev. C 55 (1997) 392.
- [19] P.F. Kolb, Diploma Thesis, Univ. Regensburg, 1999;P.F. Kolb, J. Sollfrank, U. Heinz, in preparation.
- [20] R. Lietava et al. (WA97 Coll.), J. Phys. G 25 (1999) 181;
 F. Sikler et al. (NA49 Coll.), talk given at Quark Matter '99, Nucl. Phys. A, in press.
- [21] F. Cooper, G. Frye, Phys. Rev. D 10 (1974) 186.
- [22] H. Appelshäuser et al., (NA49 Coll.), Phys. Rev. Lett. 82 (1999) 2471.
- [23] B. Kämpfer et al., J. Phys. G 23 (1997) 2001.
- [24] H. Appelshäuser et al. (NA49 Coll.), Eur. Phys. J. C 2 (1998) 661.
- [25] B. Tomášik, PhD Thesis, Univ. Regensburg, 1999;
 B. Tomášik, U.A. Wiedemann, U. Heinz, in preparation.
- [26] J. Sollfrank, P. Koch, U. Heinz, Phys. Lett. B 252 (1990) 256; Z. Phys. C 52 (1991) 593.
- [27] P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 638 (1998) 3c.
- [28] P. Danielewicz et al., Phys. Rev. Lett. 81 (1998) 2438.
- [29] N. Herrmann, Nucl. Phys. A 610 (1996) 49c.
- [30] H. Stöcker et al., Phys. Ref. C 25 (1982) 1873.
- [31] C. Pinkenburg et al. (E895 Coll.), nucl-ex/9903010, Phys. Rev. Lett., in press.
- [32] E.V. Shuryak, talk at Quark Matter '99, Nucl. Phys. A, in press.
- [33] M. Kataja, J. Letessier, P.V. Ruuskanen, A. Tounsi, Z. Phys. C 55 (1992) 153.
- [34] U. Heinz, B.V. Jacak, nucl-th/9902020, Ann. Rev. Nucl. Part. Sci. 49 (1999), in press.







