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Mixing Effects in the Finite-Temperature Effective Potential of the MSSM with a Light Stop

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Abstract: We incorporate the effects of mixing arising from the trilinear terms in the MSSM potential to the effective three dimensional theory for the MSSM at high temperature in the limit of large m_A. There are relevant one-loop effects that modify the 3D parameters of the effective theory. We calculate the two-loop effective potential of the 3D theory for the Higgs and the right handed stop to analyse the possible phase transitions and to determine the precise region in the m_h-$m_{\tilde{t}_2}$ plane for which the sphaleron constraint for preservation of the baryon asymmetry is satisfied. There is an upper bound on the value of the mixing parameter coming from stop searches. We also compare with previous results obtained using 4D calculations of the effective potential for the regime of large m_Q. A two-stage phase transition persists for a small range of values of $m_{\tilde{t}_2}$ for given values of the mixing parameter and $\tan\beta$. This can further constrain the allowed region of parameter space. Electroweak baryogenesis requires a value of $m_{\tilde{t}_2} \lesssim 170$ and $m_h \lesssim 105$ GeV for $m_Q = 300$ GeV.

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1 Introduction

The region of parameter space for which the electroweak baryogenesis scenario is viable in the context of the Minimal Supersymmetric Standard Model (MSSM) is currently being tested at existing accelerators. Electroweak baryogenesis requires a strong first-order electroweak phase transition. This ensures that sphaleron transitions are switched off in the broken phase, permitting the preservation of a baryon asymmetry [1, 2] (for reviews, see [3]-[6]). In the context of the Standard Model the only free parameter relevant to the phase transition is the Higgs mass. Non-perturbative results of the phase transition show that for the Standard Model there is no experimentally allowed value of the Higgs mass for which the phase transition is sufficiently first order [7, 8][2]. Non-perturbative studies of the phase transition are necessary at finite temperature due to the existence of infrared divergences in gauge theories. However, recent lattice simulations have shown that for the case of the MSSM the perturbative results are conservative in the constraints they impose on the parameters [9]. Perturbative analyses of the phase transition have pointed out different mechanisms that can enhance the strength of the phase transition in the MSSM with respect to the Standard Model case. The fact that there are additional scalar particles in the spectrum, which strongly couple to Standard Model fields, thus contributing significantly to the finite-temperature effective potential opens some room for electroweak baryogenesis. The main conclusion from the perturbative analysis in the 4D theory [10]-[15] was that low values of the ratio of the vacuum expectation values of the two Higgs doublets $\tan\beta = \frac{v_2}{v_1}$, large values of the pseudoscalar mass m_A and a very light right-handed stop were favoured. Given the current experimental limits on the masses in this model incorporating two-loop corrections QCD corrections is necessary in order to determine the allowed parameter space for which electroweak baryogenesis can take place [16]- [19]. In addition the presence of the trilinear terms in the potential is necessary in order to provide the required CP-violating sources for the production mechanism of electroweak baryogenesis. We recall that the ratio of the vacuum expectation value of the scalar field to the temperatyre, $\frac{\phi}{T_c}$, determines the rate of the sphaleron transitions in the broken phase and non-zero values for the trilinear couplings tend to reduce the strength of the phase transition in the Higgs doublet direction if the other parameters are kept fixed [14, 20]. This is because the mixing in the squark sector reduces the light stop

[2]Non-perturbative analyses also show good agreement with perturbation theory for Higgs mass values $m_h \lesssim 70$GeV [7].

contribution to the cubic term in the potential, it increases the critical temperature of the phase transition and radiative corrections make the Higgs heavier, thus weakening the strength of the phase transition. However, ref. [20] showed that the upper bound on the Higgs mass was arising from a zero-temperature effect and not from the requirement of having out-of-equilibrium sphaleron transitions. A lighter stop can compensate for the three effects that weaken the phase transition.

Alternatively the analysis of the phase transition can be performed in the context of a bosonic effective 3D theory obtained using dimensional reduction [7], [21]- [25]. Perturbative calculations within the effective theory reproduce in a relatively simple way the results obtained with the 4D perturbative analysis. The additional benefit of the construction of the purely bosonic effective theory is that it provides a link between the parameters employed in 3D lattice simulations with the underlying 4D parameters of the theory. The parameters in the 3D Lagrangian are obtained using dimensional reduction at high temperature by matching the static Green's functions in the two theories, to a given order in the perturbative expansion, by integrating out the non-zero Matsubara modes with masses of the order of πT, where T is the temperature. A further reduction can also be performed noting that some of the static modes in the theory have acquired thermal masses proportional to a gauge coupling multiplied by the temperature, $\sim g_w T, g_s T$. These so-called heavy particles can then be integrated out as well. References [7], [9], [26], [27]-[30], give more details concerning the construction of effective theories for both the Standard Model and the MSSM.

Interestingly the analysis of the phase transition with a light stop shows that for some region of parameter space a possible two-stage phase transition, first into a colour and and charge breaking (CCB) minimum and subsequently to the broken $SU(2)$ minimum occurs. Initial lattice analysis suggest that the second transition is too strong and might not have taken place on cosmological time scales [9]. Furthermore, in a recent paper Cline et al. [31] showed that although tunneling into the CCB minimum is viable the next step of getting out of this minimum appears not to be possible. Given this result, an accurate determination of the critical temperatures of the phase transitions can exclude an additional region of parameter space. Here we closely follow the procedure of refs. [7, 30] in order to determine very precisely the critical temperature of the phase transitions. In particular, the 3D scalar masses require ultraviolet renormalization and a two-loop calculation (in 4D) must be performed to fix the scales entering the mass parameters. This requires a full 4D effective-potential calculation incorporating mixing effects arising from the trilinear terms in the

MSSM potential. In this paper we extend the analysis of ref. [30] to identify the range of parameter space for which a sufficiently strong first-order phase transition can occur and when a two-stage phase transition can exist.

The perturbative component of the analysis relies on the validity of the high temperature expansion. The value of the masses of the particles which are integrated out will define the regime of validity of the approach. It is interesting to consider the effects on the parameters of the effective theory as the mass of the third-generation left-handed squark doublet is varied. We present the formulae that allow us to continously pass from the low-$m_Q \sim 300$ GeV to large-$m_Q \sim 1$ TeV regime at one-loop with squark mixing and, for the latter regime, we compare our with results presented in the literature. In fact as we will discuss below for values the most relevant effects from the trilinear couplings are one-loop effects. For large values of m_Q, the high temperature expansion is no longer valid; in this case we estimate the two-loop effects of the trilinear couplings.

The paper is organized as follows: in section 2 we present the dimensional reduction to the effective bosonic theory at one-loop valid for values of $m_Q \sim 300$ GeV. Section 2.2 presents a further one-loop reduction in the 3D theory, eliminating the heavy fields. In section 3 we present our results for the critical temperatures and the strength of the phase transition. The allowed region for electroweak baryogenesis to occur is also given here. Finally, in section4 we conclude. Appendix A presents the one-loop formulae needed when the trilinear terms are included and shows how the contributions to the 3D masses and couplings are modified for large values of third-generation left-handed squark doublet mass. In appendix B we give the expression for the two-loop unresummed effective potential in 4D, fully incorporating mixing effects, which is necessary for evaluating Λ_{H_3}, Λ_{U_3}. The contribution from the "heavy" particles that were integrated out at the second stageis given in this appendix. We discuss in appendix C the relevant zero-temeperature effects that must be included in our analysis.

2 Dimensional Reduction

We will now perform dimensional reduction by matching, as was previously done in refs. [7],[9],[26]-[30] for different models. Our initial 4D Lagrangian corresponds to the MSSM in the large-m_A limit. The particles that contribute to the thermal bath are the Standard Model particles plus third-generation squarks: $\tilde{t}_L, \tilde{b}_L, \tilde{t}_R, \tilde{b}_R$. There are two stages of reduction. The first one corresponds to the integration out of all non-zero Matsubara modes, that is with a thermal mass of the order of $\sim \pi T$. We calculate all one-loop contributions to mass terms and coupling constants of the static fields to order g^4, where g denotes a gauge or top Yukawa coupling. The second stage of reduction corresponds to the integration of heavy particles with masses of the order of $g_w T$, $g_s T$.

2.1 First stage

The potential in the 3D effective theory after integration over non-zero Matsubara modes is of the form

$$
\begin{aligned}
V &= m_{H_3}^2 H^\dagger H + \lambda_{H_3}(H^\dagger H)^2 + m_{U_3}^2 U^\dagger U + \lambda_{U_3}(U^\dagger U)^2 + \gamma_3 (H^\dagger H)(U^\dagger U) \\
&+ m_{Q_3}^2 Q^\dagger Q + m_{D_3}^2 D^\dagger D + \Lambda_3^Q (H^\dagger H)(Q^\dagger Q) + \Lambda_4^c (H^\dagger Q)(Q^\dagger H) \\
&+ (\Lambda_4^s + h_t^L)|\epsilon_{ij} H^i Q^j|^2 + (h_t^{QU} + g_{s_1}^{QU}) Q_{i\alpha}^* U_\alpha^* Q_{i\beta} U_\beta \\
&+ g_{s_2}^{QU} U_\alpha U_\alpha^* Q_{j\gamma}^* Q_{j\gamma} + g_{s_1}^{QD} D_\alpha D_\beta^* Q_{j\beta}^* Q_{j\alpha} \\
&+ g_{s_2}^{QD} D_\alpha D_\alpha^* Q_{j\gamma}^* Q_{j\gamma} + g_{s_1}^{UD} U_\alpha U_\gamma^* D_\gamma^* D_\alpha \\
&+ g_{s_2}^{UD} U_\alpha U_\alpha^* D_\gamma^* D_\gamma + \Lambda_1 (Q^\dagger Q)^2 + \lambda_{D_3}(D^\dagger D)^2 + \lambda_{Q_3}(Q_i^\dagger Q_i)^2 \\
&+ g_{s_1}^{QQ} Q_{i\alpha}^* Q_{j\alpha}^* Q_{i\gamma} Q_{j\gamma} + g_{s_2}^{QQ} Q_{i\alpha} Q_{i\alpha}^* Q_{j\gamma}^* Q_{j\gamma} \\
&+ \frac{1}{2} m_{A_0}^2 A_0^a A_0^a + \frac{1}{2} m_{C_0}^2 C_0^A C_0^A + \frac{1}{4} g_{w_3}^2 (H^\dagger H)(A_0^a A_0^a) \\
&+ \frac{1}{4} g_{s_3}^2 C_0^A C_0^B (U^*)^\dagger \lambda^A \lambda^B U^* - \epsilon_{ij} \overline{X}_t h_t H^i Q^j U^* . \qquad (1)
\end{aligned}
$$

Here H is the Higgs doublet field, $U(D)$ is the right-handed stop (sbottom) field, and Q is the third generation left-handed squark doublet field. The longitudinal components of the SU(2) and SU(3) gauge fields are denoted by A_o and C_o, respectively. The Latin (Greek) indices indicate SU(2) (SU(3)) components. As usual, the fields in eq. (1) are the static components of the scalar fields properly renormalized, the dimension of the fields in 3D is $[\mathrm{GeV}]^{1/2}$. Quartic couplings are of order $g_i^2(h_t^2)T$, having dimensions of [GeV]; here $g_i(h_t)$ denotes a gauge (top Yukawa) coupling. The zero-temperature trilinear coupling is $X_t = A \sin\beta - \mu \cos\beta = \tilde{A}_t \sin\beta$. We work throughout in Landau gauge. In the following we have not included the correction to the quartic coupling between the doublet Higgs field and the triplet scalar field A_0, or the corresponding correction for the SU(3) counterparts. We work throughout in the Landau gauge. We do not rewrite the expressions for the 3D masses (m_{A_o}, m_{C_o}) and all of the quartic couplings which are not modified by the presence of the trilinear couplings, they are given in ref. [30].

2.1.1 Mass terms

For the Higgs doublet we have[3]

$$
\begin{aligned}
m_{H_3}^2 &= m_H^2\Big(1+\frac{9}{4}g_w^2\frac{L_b}{16\pi^2}-3h_t^2\sin^2\beta\frac{L_f}{16\pi^2}\Big) \\
&+ T^2\Big(\frac{\lambda}{2}+\frac{3}{16}g_w^2+\frac{1}{16}g'^2+\frac{1}{4}h_t^2\sin^2\beta+\frac{1}{4}(2h_t^2\sin^2\beta+2\lambda_3+\lambda_4)\Big) \\
&- \frac{L_b}{16\pi^2}\Big(6\lambda m_H^2+3(m_Q^2+m_U^2)h_t^2\sin^2\beta+N_c h_t^2 X_t^2\Big),
\end{aligned}
\tag{2}
$$

where the Higgs mass parameter is denoted by m_H, and $\lambda = \frac{(g_w^2+g'^2)}{8}\cos^2 2\beta, \lambda_3 = \frac{g_w^2}{4}, \lambda_4 = -\frac{g_w^2}{2}$. Similarly, for the third-generation squark mass terms we have

$$
\begin{aligned}
m_{U_3}^2 &= m_U^2\Big(1+4g_s^2\frac{L_b}{16\pi^2}\Big)+T^2\Big(\frac{1}{3}g_s^2+\frac{2}{3}\lambda_U+\frac{1}{6}h_t^2\sin^2\beta+\frac{1}{6}h_t^2\Big) \\
&- \frac{L_b}{16\pi^2}\Big(\frac{4}{3}g_s^2 m_U^2+2h_t^2\sin^2\beta(m_H^2+m_Q^2)+2h_t^2X_t^2\Big),
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
m_{Q_3}^2 &= m_Q^2\Big(1+(\frac{9}{4}g_w^2+4g_s^2)\frac{L_b}{16\pi^2}\Big)+T^2\Big(\frac{3}{16}g_w^2+\frac{\lambda_1}{2}+\frac{4}{9}g_s^2+\frac{1}{12}h_t^2(1+\sin^2\beta)\Big) \\
&- \frac{L_b}{16\pi^2}\Big(\frac{4}{3}g_s^2m_Q^2+6\lambda_1 m_Q^2+h_t^2m_U^2+h_t^2\sin^2\beta m_H^2+h_t^2X_t^2\Big),
\end{aligned}
\tag{4}
$$

where the soft SUSY-breaking mass for the third generation left-handed squark doublet is denoted by m_Q, and $\lambda_U = \frac{g_s^2}{6}$, $\lambda_1 = \frac{g_w^2}{8}$.

[3]Throughout the paper we mostly neglect the hypercharge coupling g'. The only exception is in the contribution to the tree-level expression of the Higgs self-coupling λ, as this latter quantity is fundamental in determining the strength of the phase transition.

2.1.2 Couplings

We present here the expressions for the trilinear couplings in 3D, which were absent for the case of zero-squark mixing. The modifications of the scalar quartic couplings are suppressed at this stage for the values of the mixing parameters which we will consider, and we do not include the corrections from diagrams of those in fig. 1. However, at the next-stage of integration-out we will include them:

$$\begin{aligned}\overline{X}_t h_t &= X_t h_t T^{1/2}\Big(1+\frac{9}{4}g^2\frac{L_b}{16\pi^2}-\frac{N_c}{2}h_t^2\sin^2\beta\frac{L_f}{16\pi^2} \\ &+ 4g_s^2\frac{L_b}{16\pi^2}\Big)-X_t h_t T^{1/2}\Big(\lambda_3+2\lambda_4\sin^2\beta+3h_t^2\sin^2\beta+N_c h_t^2 \\ &- \frac{4}{3}g_s^2\Big)\frac{L_b}{16\pi^2}. \end{aligned} \tag{5}$$

2.2 Second stage

Another simplification of the effective theory can be obtained by integrating out the scalar fields, which are massive at the transition point. As we have seen, the static modes corresponding to the scalar fields Q, D, A_o, C_o acquired thermal masses proportional to $\sim g_{w(s)}T$, as a consequence of the integration out of the non-zero Matsubara modes. The second stage proceeds in exactly the same way as in refs. [18, 30]. We include the additional corrections arising from the couplings we have considered.

2.2.1 Couplings

The final expression for the tree-level 3D potential is given by

$$V_{3D}=\overline{m}_{H_3}^2 H^\dagger H+\overline{\lambda}_{H_3}(H^\dagger H)^2+\overline{m}_{U_3}^2 U^\dagger U+\overline{\lambda}_{U_3}(U^\dagger U)^2+\overline{\gamma}_3 H^\dagger H U^\dagger U, \tag{6}$$

where the scalar couplings are now

$$\overline{\lambda}_{H_3} = \lambda_{H_3}-\frac{3}{16}\frac{g_{w_3}^4}{8\pi m_{A_0}}-\frac{3}{8\pi m_{Q_3}}\Big(\Lambda_3^2+\Lambda_3(\Lambda_4^c+\Lambda_4^s)$$

$$
\begin{aligned}
&+ \quad \frac{1}{2}\Big((\Lambda_4^c)^2 + (\Lambda_4^s)^2\Big) + h_t^L \Lambda_3^Q + h_t^L \Lambda_4^s + \frac{1}{2}(h_t^L)^2\Big) \\
&+ \quad 3\overline{X}_t^2 h_t^2 (\Lambda_3^Q + \Lambda_4^s + h_t^L) \frac{1}{8\pi m_{Q_3}} \frac{1}{(m_{Q_3} + m_{U_3})^2} \\
&+ \quad 3\overline{X}_t^2 h_t^2 \gamma_3 \frac{1}{8\pi m_{U_3}} \frac{1}{(m_{Q_3} + m_{U_3})^2} - \frac{3}{2}\overline{X}_t^4 h_t^4 f(m_{Q_3}, m_{U_3}, m_{U_3}),
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\overline{\lambda}_{U_3} &= \lambda_{U_3} - \frac{13}{36}\frac{g_{s_3}^4}{8\pi m_{C_0}} - \frac{1}{8\pi m_{D_3}}\Big(\frac{1}{2}(g_{s_1}^{UD} + g_{s_2}^{UD})^2 + (g_{s_2}^{UD})^2\Big) \\
&- \frac{1}{8\pi m_{Q_3}}\Big((h_t^{QU})^2 - 2h_t^{QU} g_{s_1}^{QU} + 2h_t^{QU} g_{s_2}^{QU} + (g_{s_1}^{UD} + g_{s_2}^{UD})^2 + 2(g_{s_2}^{UD})^2\Big) \\
&+ \overline{X}_t^2 h_t^2 (h_t^{QU} + g_{s_1}^{QU} + g_{s_2}^{QU}) \frac{1}{8\pi m_{Q_3}} \frac{1}{(m_{Q_3} + m_{H_3})^2} + \overline{X}_t^2 h_t^2 \gamma_3 \frac{1}{8\pi m_{H_3}} \frac{1}{(m_{Q_3} + m_{H_3})^2} \\
&- \overline{X}_t^4 h_t^4 f(m_{Q_3}, m_{H_3}, m_{H_3}),
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\overline{\gamma_3} &= \gamma_3 - \frac{1}{8\pi m_{Q_3}}(h_t^{QU} + g_{s_1}^{QU} + 3g_{s_2}^{QU})(2\Lambda_3^Q + \Lambda_4^c + \Lambda_4^s + h_t^L) \\
&+ \overline{X}_t^2 h_t^2 (\Lambda_3^Q + \frac{1}{2}(\Lambda_4^s + h_t^L)) \frac{1}{8\pi m_{Q_3}} \frac{1}{(m_{Q_3} + m_{H_3})^2} \\
&+ \overline{X}_t^2 h_t^2 \frac{1}{2}(h_t^{QU} + g_{s_1}^{QU} + 3g_{s_2}^{QU}) \frac{1}{8\pi m_{Q_3}} \frac{1}{(m_{Q_3} + m_{U_3})^2} \\
&+ 3\overline{X}_t^2 h_t^2 \Lambda_{H_3} \frac{1}{8\pi m_{H_3}} \frac{1}{(m_{Q_3} + m_{H_3})^2} + 4\overline{X}_t^2 h_t^2 \Lambda_{U_3} \frac{1}{8\pi m_{U_3}} \frac{1}{(m_{Q_3} + m_{U_3})^2} \\
&+ \overline{X}_t^2 h_t^2 \gamma_3 \frac{1}{4\pi} \frac{1}{(m_{Q_3} + m_{U_3})} \frac{1}{(m_{Q_3} + m_{H_3})} \frac{1}{(m_{U_3} + m_{H_3})} - \overline{X}_t^4 h_t^4 f(m_{Q_3}, m_{H_3}, m_{U_3}).
\end{aligned} \tag{9}
$$

and

$$
f(m_1, m_2, m_3) = \frac{1}{8\pi} \frac{2m_1 + m_2 + m_3}{m_1(m_1 + m_3)^2(m_1 + m_2)^2(m_2 + m_3)}. \tag{10}
$$

2.2.2 Mass terms

The one-loop contribution to the mass terms can be obtained directly as shown in ref. [7]:

$$\overline{m}^2_{H_3} = m^2_{H_3}\left(1 - \frac{N_c h_t^2 \overline{X}_t^2}{12\pi(m_{Q_3}+m_{U_3})^3}\right) - \frac{3}{16\pi} g_{w_3} m_{A_0} - \frac{3}{4\pi}(2\Lambda_3^Q + \Lambda_4^c + \Lambda_4^s + h_t^L) m_{Q_3}$$
$$- 3h_t^2 \overline{X}_t^2 \frac{1}{4\pi(m_{Q_3}+m_{U_3})}, \quad (11)$$

$$\overline{m}^2_{U_3} = m^2_{U_3}\left(1 - \frac{2 h_t^2 \overline{X}_t^2}{12\pi(m_{Q_3}+m_{H_3})^3}\right) - \frac{1}{3\pi} g_{s_3} m_{C_0} - \frac{1}{4\pi}(2h_t^{QU} + 2g_{s_1}^{QU} + 6g_{s_2}^{QU}) m_{Q_3}$$
$$- \frac{1}{4\pi}(2g_{s_1}^{UD} + 6g_{s_2}^{UD}) m_{D_3} - 2h_t^2 \overline{X}_t^2 \frac{1}{4\pi(m_{Q_3}+m_{H_3})}. \quad (12)$$

In order to precisely fix the scales of the couplings that appear in the thermal polarizations of eqs. (2) and (3), one needs to perform a 2-loop evaluation of the effective potential. In addition, the mass parameters are renormalized in the 3D theory:

$$\overline{m}^2_{H_3}(\mu) = \overline{m}^2_{H_3} + \frac{1}{(16\pi^2)} f_{2m_H} \log\frac{\Lambda_{H_3}}{\mu}, \quad (13)$$

$$\overline{m}^2_{U_3}(\mu) = \overline{m}^2_{U_3} + \frac{1}{(16\pi^2)} f_{2m_U} \log\frac{\Lambda_{U_3}}{\mu}. \quad (14)$$

The expressions for the two-loop beta functions f_{2m_H}, f_{2m_U} for the mass parameters have been given in ref. [18]. In appendix B we perform a two-loop calculation of the effective potential for the H (ϕ-direction) and U(χ-direction) fields, including the effects of mixing with the third generation left-handed squark doublet[4]. We incorporate all of the corrections to the 3D couplings obtained in the previous sections, so as to determine the exact values of Λ_{H_3} and Λ_{U_3}. Using the results of appendix B it can be checked that the trilinear couplings do not produce scale dependent logarithmic contributions to the mass terms. In other words when both stop fields are light, the residual dependence on the mixing term in the divergent parts of the contributions to the mass terms appears only through the couplings which enter the expressions for the beta functions f_{2m_H}, f_{2m_U}. These couplings are given by the one-loop expressions in equations (7) -(9) . We will analyse the effect of including these corrections on the critical temperatures [5].

[4] Some of the results given previously in the literature are corrected.

[5] The necessary modifications of f_{2m_H}, f_{2m_U} in the case of non-zero mixing for large values of m_Q appear in ref.

3 Results

With the results of the previous sections and those in appendices B and C we can analyse the phase transition for values of $m_Q = 300$ GeV. In fig. 2 we show the critical temperatures for the transitions in the ϕ (solid)- and χ (dotted)-directions as a function of the lightest stop mass $m_{\tilde{t}_2}$, for $\tan\beta = 5$ and for values of the parameter $\tilde{A}_t = \frac{X_t}{\sin\beta} = 100, 200$ GeV. We find that, for $m_Q \sim 300$ GeV, there still is a region in which a two-stage phase transition can occur. This region is to the left of the crossing point of the curves. As shown in figs. 2 and 3, the range of values of the lighest stop mass for which the phase transition is sufficiently strong and a two-stage phase transition occurs decreases as X_t increases. Thus the experimental bound on the stop mass puts an upper bound on the value of the mixing parameter. The figures display the effects produced by the non-zero trilinear couplings as described in the introduction. Keeping all other parameters fixed, lowering the stop mass will decrease the critical temperature in the ϕ-direction, slightly reduce the Higgs mass and strengthens the transition. The light-stop mechanism of enhancing the phase transition also eliminates the restriction of having low values of $\tan\beta$.

The allowed region in parameter space is shown in fig. 4, as a function of the Higgs and stop masses, for $\tilde{A}_t = 200$ GeV, the region on the left of the solid line indicates when a sufficiently strong first-order phase transition occurs. The dotted line gives the condition for absolute stability of the physical vacuum. As explained in appendix C, to the left of this line the colour-breaking minimum is lower than the physical one at zero temperature. The dashed line is obtained when the critical temperatures of the transitions in the ϕ- and χ-directions are the same. A two-stage phase transition occurs to the left of the dashed line. If the second transition from the CCB minimum to the electroweak minimum does not occur, as indicated by the analysis of ref. [31], there are stronger constraints on the allowed region. For each value of the mixing parameter $\tilde{A}_t$ the available region is restricted to the band within the dashed and the solid lines. Note that, for low values of m_Q and non-zero squark mixing, there is no cross-over between the dashed and dotted lines. The end-points of the lines correspond to the maximum value of the Higgs mass which is reached by the effect of the zero-temperature radiative corrections for a given value of m_Q and $\tilde{A}_t$. In fig. 5 we display the full allowed region in parameter space coming from the requirement of having a strong

[20]. The results of appendix B are no longer applicable as the high temperature expansion expressions for the D functions are not valid for the third generation left-handed squark doublet, see below.

enough phase transition. The solid vertical line is determined t varying $\tan\beta$ for $\tilde{A}_t = 0$ and the solid diagonal line corresponds to the variation of $\tilde{A}_t$ for $\tan\beta = 12$. The region to the left and below these solid lines provides a sufficiently strong phase transition for values of $2 \lesssim \tan\beta \lesssim 12$ and $0 \lesssim \tilde{A}_t \lesssim 280$ GeV. The dashed line shown defines when the critical temperatures in the ϕ- and χ-directions are equal for the same variations of $\tan\beta$ and $\tilde{A}_t$. For this value of m_Q the maximum allowed masses are $m_h \lesssim 105$GeV and $m_{\tilde{t}_2} \lesssim 170$GeV.

We now turn to the analysis of the phase transition for $m_Q = 1$ TeV. The expressions for the 3D parameters are given in appendix A. The one-loop calculation is exact, the two-loop contributions are only estimated, using the corresponding two-loop beta functions for the mass parameters with non-zero mixing at large m_Q given in ref. [32]. Here the scales Λ_{H_3} and Λ_{U_3} have been fixed to agree with the numerical results obtained in the case with zero-mixing in ref. [30]. We point out that the exact contributions from the two-loop sunset and figure eight diagrams when the high temperature expansion is no longer valid are not known [6]. These contributions are necessary for terms involving the third generation left-handed squark doublet for large values of m_Q. Figure 6 shows that the phase-diagram structure is maintained for $m_Q = 1$ TeV for several values of $\tilde{A}_t$. Again, a possible two-stage phase transition persists, in agreement with previous results [20, 33]. We remark that the qualitative effects of increasing $\tilde{A}_t$ remain unchanged with respect to the case of $m_Q = 300$GeV. In figure 7 the solid line determines the constant ratio $\frac{\phi}{T_c} = 1$, as a function of the lightest stop mass for $\tan\beta = 5$, varying the mixing parameter in the range $0 \lesssim \tilde{A}_t \lesssim 650$ GeV. The dashed line is obtained when the critical temperatures in the ϕ- and χ- directions are the same. The dotted line gives the zero-temperature condition for absolute stability of the physical minimum. Imposing the constraint which eliminatesthe region where tunneling into a CCB minima at finite temperature is possible reduces the allowed region to the band between the dashed and solid line. The experimental constraints on the stop mass $m_{\tilde{t}_2} \gtrsim 80$ GeV, will also restrict the value of the mixing parameter. Finally, in fig. 8 we present the contours of $\frac{\phi}{T_c} = 1$ in the m_h-$m_{\tilde{t}_2}$ plane varying $\tan\beta$ for $\tilde{A}_t = 0, 100, 200, 300, 400, 600$. The regions of possible two-stage phase transitions defined by each set of values of A_t and $\tan\beta$ are not shown. The maximum allowed Higgs mass is now $m_h \lesssim 120$ GeV.

[6]We hope to return to this point in the future.

4 Conclusions

We have performed a full two-loop dimensional reduction of 4D MSSM parameters to the 3D couplings and masses of the effective theory, including the effects from the trilinear terms in the potential. The preservation of the baryon asymmetry can be ensured tuning the stop mass in order to compensate the negative effects of the mixing term on the strength of the phase transition. We conclude that the direct stop searches restrict the amount of mixing in the stop sector. The allowed range of masses is $m_h \lesssim 107$ GeV and $m_{\tilde{t}_1} \lesssim 170$ GeV for $m_Q = 300$ GeV. Our calculation allows us to determine the critical temperatures of the transitions precisely. We find that the phase-diagram still allows a possible two-stage phase transition for a small range of values of $m_{\tilde{t}_2}$ and any value of $\tilde{A}_t$ for values of m_Q for which the high-temperature expansion can be applied. This range of values is shifted for different values of the stop mixing parameter. If the second phase transition does not occur there is a further restriction of the allowed regions of parameter space for electroweak baryogenesis for each value of the mixing parameter and $\tan\beta$. At large values of m_Q the qualitative dependence on the parameters in the theory remains unchanged.

A One-loop contributions from trilinear terms and large-m_Q

The first diagram that contributes to the two-point functions corresponds to fig. 1a [34, 35]:

$$I(m) = A(m) + f_1(a) \tag{15}$$

where

$$\begin{aligned} A(m) &= -\frac{m^2}{16\pi^2}\Big(\Delta + \ln\frac{\mu^2}{m^2} + 1\Big) \\ f_1(a) &= \frac{T^2}{2\pi^2}\int_o^\infty dx \frac{x^2}{(x^2+a^2)^{\frac{1}{2}}} \frac{1}{e^{(x^2+a^2)^{\frac{1}{2}}} - 1} \end{aligned} \tag{16}$$

and $a = \frac{m}{T}$. The expression that is valid for a high-temperature expansion is

$$I(m) = \frac{T^2}{12}(1+\epsilon i_\epsilon) - \frac{m^2}{16\pi^2}\Big(\frac{1}{\epsilon} + L_b\Big) + O(\frac{m^2}{T^2}), \tag{17}$$

where $L_b = 2\log\frac{\overline{\mu}e^{\gamma}}{4\pi T} \approx 2\log\frac{\mu}{7.055T}$, $L_f = L_b + 4\log 2$. Here $\overline{\mu}$ is the mass scale defined by the modified minimal subtraction ($\overline{MS}$) scheme. For a low-temperature expansion that is $m \gg T$ only the zero-temperature contribution remains as the temperature-dependent part is suppressed. Trilinear coupling induce an additional contribution to the two-point function shown in fig. 1b:

$$I_2(m_1,m_2) = B_o(p,m_1,m_2) + \frac{1}{2\pi^2}f_2(a,b), \tag{18}$$

where $B_o(p,m_1,m_2)$ is the usual Veltman-Passarino scalar function and in particular,

$$\begin{aligned} B_o(0,m_1,m_2) &= \frac{1}{16\pi^2}\Big(\Delta + 1 + \ln\frac{\mu^2}{m_2^2} + \Big(\frac{m_1^2}{m_1^2-m_2^2}\Big)\ln\frac{m_2^2}{m_1^2}\Big) \\ f_2(a,b) &= -\frac{1}{a^2-b^2}\int_o^\infty dx\Big(\frac{x^2}{(x^2+a^2)^{\frac{1}{2}}}\frac{1}{e^{(x^2+a^2)^{\frac{1}{2}}}-1} - \frac{x^2}{(x^2+b^2)^{\frac{1}{2}}}\frac{1}{e^{(x^2+b^2)^{\frac{1}{2}}}-1}\Big) \end{aligned} \tag{19}$$

where $a = m_1/T, b = m_2/T$.

For a high-temperature expansion this becomes

$$I_2(m_1,m_2) = \frac{1}{16\pi^2}(\frac{1}{\epsilon} + L_b) + O(\frac{m^2}{T^2}) + O(\frac{p^2}{T^2}). \tag{20}$$

For $m_1 \ll T$ and $m_2 \gg 2\pi T$ is

$$I_2(m_1,m_2) = B_o(p,m_1,m_2) + \frac{1}{m_1^2}f_1(m_2) \tag{21}$$

and for the case in which $m_1 = m_2 \gg T$ this reduces to the zero-temperature contribution

$$I_2(m,m) = B_o(0,m,m) = \frac{1}{16\pi^2}\ln\frac{\mu^2}{m^2} \tag{22}$$

plus wave-function renormalization effects. We remind the reader that this can be obtained from $I_2(m,m) = -\frac{\partial}{\partial m^2}$ I(m). Similarly, we can obtain the corresponding results for other diagrams, in fig. 1 taking the appropiate derivatives. We now give only the low-temperature expressions valid for $m_1 \gg T$, the corresponding high-temperature expressions can be found in the literature:

$$I_3(m_1,m_2,m_2) = C_o(0,m_1,m_2,m_2) + \frac{1}{16\pi^2}\frac{1}{m_1^2}\ln\frac{m_2^2}{\mu_T^2} \tag{23}$$

$$I_3(m_1,m_1,m_2) = C_o(0,m_1,m_1,m_2) + \frac{1}{m_1^4} f_1(m_2) \tag{24}$$

$$I_4(m_1,m_1,m_2,m_2) = D_o(0,m_1,m_1,m_2,m_2) + \frac{1}{m_1^4}\frac{1}{16\pi^2}\ln\frac{m_2^2}{\mu_T^2}. \tag{25}$$

where the functions C_o and D_o are the scalar one-loop functions of Veltman-Passarino.

The 3D couplings in equation 6 for the case of large m_Q are given by:

$$\begin{aligned}
\overline{\lambda}_{H_3} &= \lambda T\Big(1 + \frac{9}{2}g_w^2\frac{L_b}{16\pi^2} - 6h_t^2\sin^2\beta\frac{L_f}{16\pi^2}\Big) \\
&- \frac{T}{16\pi^2}\Big[L_b\Big(\frac{9}{16}g_w^4 + 12\lambda^2 + \frac{1}{2}h_t^4\sin^4\beta\Big) + 3\Big(\lambda_3^2 + \lambda_3\lambda_4 + \lambda_4^2\cos^4\beta + \lambda_4^2\sin^4\beta \\
&+ h_t^2\sin^2\beta(\lambda_3 + \lambda_4\sin^2\beta) + \frac{1}{2}h_t^4\sin^4\beta\Big)\ln\frac{\mu^2}{m_Q^2} + \frac{3}{8}g_w^4 + 3h_t^4\sin^4\beta L_f \\
&+ 3X_t^2h_t^2(\lambda_3 + \lambda_4\sin^2\beta + h_t^2\sin^2\beta)\frac{1}{m_Q^2} \\
&+ 3X_t^2h_t^4\sin^2\beta(-\frac{1}{m_Q^2}(1 - \ln\frac{m_Q^2}{\mu_T^2})) - \frac{3}{2}X_t^4h_t^4(-\frac{1}{m_Q^4}(2 + \ln\frac{\mu_T^2}{m_Q^2})) - N_c\lambda h_t^2\frac{X_t^2}{m_Q^2}\Big],
\end{aligned} \tag{26}$$

$$\begin{aligned}
\overline{\lambda}_{U_3} &= \frac{g_s^2}{6}T\Big(1 + 8g_s^2\frac{L_b}{16\pi^2}\Big) - \frac{T}{16\pi^2}\Big[L_b\Big(\frac{17}{36}g_s^4 + \frac{13}{12}g_s^4 + h_t^4\Big) - \Big(\frac{2}{3}h_t^2g_s^4 \\
&+ \frac{g_s^2}{6} + h_t^4\sin^4\beta\Big)\ln\frac{\mu^2}{m_Q^2} + \frac{13}{18}g_s^4 + 2X_t^2h_t^2(h_t^2 - \frac{1}{3}g_s^2)\frac{1}{m_Q^2}) \\
&+ 2X_t^2h_t^4\sin^2\beta(-\frac{1}{m_Q^2}(1 - \ln\frac{m_Q^2}{\mu_T^2}) - X_t^4h_t^4(-\frac{1}{m_Q^4}(2 - \ln\frac{m_Q^2}{\mu_T^2})) - \frac{g_s^2}{3}h_t^2\frac{X_t^2}{m_Q^2}\Big],
\end{aligned} \tag{27}$$

$$\begin{aligned}
\overline{\gamma}_3 &= h_t^2\sin^2\beta T\Big(1 + \frac{9}{4}g_w^2\frac{L_b}{16\pi^2} - 3h_t^2\sin^2\frac{L_f}{16\pi^2} + 4g_s^2\frac{L_b}{16\pi^2}\Big) \\
&- \frac{T}{16\pi^2}\Big[L_b\Big(\frac{4}{3}h_t^2\sin^2\beta g_s^2 + 2h_t^4\sin^4\beta + 6\lambda h_t^2\sin^2\beta\Big) \\
&+ \Big(h_t^2(2\lambda_3 + \lambda_4 + h_t^2\sin^2\beta\Big)\ln\frac{\mu^2}{m_Q^2} + X_t^2h_t^2\Big(2(\lambda_3 + \frac{1}{2}\lambda_4\sin^2\beta + h_t^2\sin^2\beta + h_t^2)\frac{1}{m_Q^2} + (6\lambda + 4\frac{g_s^2}{3} \\
&+ 2h_t^2\sin^2\beta)(-\frac{1}{m_Q^2}(1 - \ln\frac{m_Q^2}{\mu_T^2})) - X_t^2h_t^2(-\frac{1}{m_Q^4}(2 + \ln\frac{\mu_T}{m_Q^2})) - h_t^2\sin^2\beta(\frac{1}{m_Q^2} + \frac{3}{m_Q^2})\Big],
\end{aligned} \tag{28}$$

and the 3D masses are given by

$$
\begin{aligned}
m_{H_3}^2 &= m_H^2\Big(1+\frac{9}{4}g_w^2\frac{L_b}{16\pi^2}-3h_t^2\sin^2\beta\frac{L_f}{16\pi^2}\Big) \\
&+ T^2\Big(\frac{\lambda}{2}+\frac{3}{16}g_w^2+\frac{1}{16}g'^2+\frac{1}{4}h_t^2\sin^2\beta+\frac{1}{4}h_t^2\sin^2\beta\Big)-3(h_t^2\sin^2\beta+2\lambda_3+\lambda_4)\frac{m_Q^2}{16\pi^2}\Big(\ln\frac{\mu^2}{m_Q^2}+1\Big) \\
&- \frac{L_b}{16\pi^2}\Big(6\lambda m_H^2+3m_U^2)h_t^2\sin^2\beta\Big)-3\frac{h_t^2X_t^2}{16\pi^2}\Big(-\frac{m_H^2}{2m_Q^2}+\frac{m_U^2}{m_Q^2}\Big(\ln\frac{\mu_T^2}{m_Q^2}+1\Big) \\
&+ \ln\frac{\mu^2}{m_Q^2}+1\Big)-3h_t^2\frac{X_t^2}{m_Q^2}\frac{T^2}{12}-\frac{3}{16\pi}g_{w_3}m_{A_0}+\frac{1}{(16\pi^2)}f_{2m_H}\log\frac{\Lambda_{H_3}}{\mu},
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
m_{U_3}^2 &= m_U^2\Big(1+4g_s^2\frac{L_b}{16\pi^2}\Big)+T^2\Big(\frac{1}{3}g_s^2+\frac{2}{3}\lambda_U+\frac{1}{6}h_t^2\sin^2\beta\Big)-2h_t^2\frac{m_Q^2}{16\pi^2}\Big(\ln\frac{\mu^2}{m_Q^2}+1\Big) \\
&- \frac{L_b}{16\pi^2}\Big(\frac{4}{3}g_s^2m_U^2+2h_t^2\sin^2\beta(m_H^2+m_Q^2)\Big)-2\frac{h_t^2X_t^2}{16\pi^2}\Big(-\frac{m_U^2}{2m_Q^2} \\
&+ \ln\frac{\mu^2}{m_Q^2}+1+\frac{m_H^2}{m_Q^2}\Big(\ln\frac{\mu_T^2}{m_Q^2}+1\Big)\Big)-2h_t^2\frac{X_t^2}{m_Q^2}\frac{T^2}{12}-\frac{1}{3\pi}g_{s_3}m_{C_0}+\frac{1}{(16\pi^2)}f_{2m_U}\ln\frac{\Lambda_{U_3}}{\mu},
\end{aligned}
\tag{30}
$$

where $\mu_T = 4\pi e^{-\gamma}T$.

B Two-loop contributions with non-zero squark mixing

The strategy we employ follows that of refs. [7, 30]. The idea is to use the 4D two-loop effective potential in order to fix the scales in the 3D theory, and to use the 3D effective potential expressions for the Higgs and stop fields given in ref. [18] to analyse the phase transition. We calculate the unresummed two-loop effective potential in order to include all 4D corrections to the mass parameters; resummation is automatically included in the calculation of the two-loop effective potential in the 3D theory. We must also include the contributions to the two-loop effective potential of the static modes, which have been integrated out at the second stage (includes the effects of resummation of the heavy fields).

There are several effects that must be considered in order to obtain all of the contributions (constant and logarithmic) to the mass parameters. From the 4D effective potential, one finds the two-loop contributions from the gauge bosons, Higgs, right-handed stop, left-handed squark doublet, right-handed sbottom, and top quark. The main difference is that the D functions appearing below correspond to the unresummed expressions. Additionally we must include the effects arising at the second stage of reduction from the left-handed squark doublet, the right-handed sbottom, the scalar triplet and the scalar octet[7].

We now derive the effective potential at finite temperature using the background fields ϕ and $\chi = \tilde{t}_{R\alpha} u^\alpha$, where we have chosen the unit vector in colour space $u^\alpha = (1,0,0)$. We first write the expressions in the shifted theory of the mass spectrum after the first stage of integration.

B.1 ϕ-direction

The gauge-boson masses are

$$m_{W,Z}^2 = \frac{1}{4} g_w^2 \phi^2. \tag{31}$$

The stop mass matrix elements are given by

$$m_{\tilde{t}_L}^2(\phi) = m_Q^2 + m_t^2(\phi) + \frac{1}{4}(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)(g_w^2 + g'^2)\phi^2 \cos 2\beta \tag{32}$$

$$m_{\tilde{t}_R}^2(\phi) = m_U^2 + m_t^2(\phi) + \frac{1}{4}(\frac{2}{3}\sin^2\theta_W)(g_w^2 + g'^2)\phi^2 \cos 2\beta \tag{33}$$

$$m_{\tilde{t}_{LR}}^2(\phi) = \frac{h_t}{\sqrt{2}}(A_t \sin\beta - \mu\cos\beta)\phi \equiv \frac{h_t}{\sqrt{2}} X_t \phi; \tag{34}$$

the corresponding eigenvalues are $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, and the eigenstates are given by

$$\begin{aligned} \tilde{t}_1 &= \cos\alpha_t \tilde{t}_L + \sin\alpha_t \tilde{t}_R \\ \tilde{t}_2 &= -\sin\alpha_t \tilde{t}_L + \cos\alpha_t \tilde{t}_R, \end{aligned} \tag{35}$$

where $\sin 2\alpha_t = \frac{2m_{\tilde{t}_{LR}}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}$. Below we will use the abbreviations $c_t = \cos\alpha_t$, $s_t = \sin\alpha_t$, etc.

Neglecting mixing effects in the sbottom sector, we have

$$m_{\tilde{b}_L}^2(\phi) = m_Q^2 + \frac{1}{4}(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W)(g_w^2 + g'^2)\phi^2 \cos 2\beta \tag{36}$$

[7] The expression given for the 4D effective potential would correspond to the usual resummed two-loop 4D effective potential if we used the resummed expressions in the D functions appearing below.

$$m^2_{\tilde{b}_R}(\phi) = m^2_D + \frac{1}{4}(-\frac{1}{3}\sin^2\theta_W)(g_w^2 + g'^2)\phi^2\cos 2\beta. \tag{37}$$

For the Higgs sector, the Goldstone bosons and Higgs masses are

$$\begin{aligned} m^2_\pi &= m^2_H + \lambda\phi^2, \\ m^2_h &= m^2_H + 3\lambda\phi^2. \end{aligned} \tag{38}$$

The additional corrections that arise from supersymmetric particles can be calculated using the two-loop unresummed potential. Our notation for the D-functions corresponds to that of ref. [7]. The contributions from the two-loop graphs containing supersymmetric particles are given below. For the ϕ-direction, we can drop the colour index of the squark masses:

$$\begin{aligned} (SSV) &= -\frac{g_w^2}{8}N_c[c_t^4 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_W) + s_t^4 D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_W) + 2s_t^2c_t^2 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_W) \\ &+ D_{SSV}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, m_W) + 4(c_t^2 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{b}_L}, m_W) + s_t^2 D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{b}_L}, m_W))] \\ &- \frac{g_s^2}{4}(N_c^2 - 1)[D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, 0) + D_{SSV}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, 0) \\ &+ D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, 0) + D_{SSV}(m_{\tilde{b}_R}, m_{\tilde{b}_R}, 0)], \end{aligned} \tag{39}$$

$$\begin{aligned} (SSS) &= -\frac{N_c}{2}\Big\{\Big[\Big(h_t^2\sin^2\beta + \frac{g_w^2}{4}c_t^2\cos 2\beta\Big)\phi + \sqrt{2}X_t h_t c_t s_t\Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_h) \\ &+ \Big[\Big(h_t^2\sin^2\beta + \frac{g_w^2}{4}s_t^2\cos 2\beta\Big)\phi - \sqrt{2}X_t h_t c_t s_t\Big]^2 D_{SSS}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_h) \\ &+ 2\Big[-\Big(\frac{g_w^2}{4}c_t s_t\cos 2\beta\Big)\phi + \frac{1}{\sqrt{2}}X_t h_t c_{2t}\Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_h) \\ &+ \Big(\frac{g_w^2}{4}\cos 2\beta\phi\Big)^2 D_{SSS}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, m_h) \\ &+ h_t^2 X_t^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_\pi) + \Big[\phi\Big(h_t^2\sin^2\beta + \frac{g_w^2}{2}\cos 2\beta\Big)c_t + \sqrt{2}h_t X_t s_t\Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{b}_L}, m_\pi) \\ &+ \Big[\phi\Big(h_t^2\sin^2\beta + \frac{g_w^2}{2}\cos 2\beta\Big)s_t - \sqrt{2}h_t X_t c_t\Big]^2 D_{SSS}(m_{\tilde{t}_2}, m_{\tilde{b}_L}, m_\pi)\Big\}, \end{aligned} \tag{40}$$

$$
\begin{aligned}
(SV) &= -\frac{1}{4}g_s^2(N_c^2-1)[D_{SV}(m_{\tilde{t}_1},0)+D_{SV}(m_{\tilde{b}_L},0)+D_{SV}(m_{\tilde{t}_2},0)+D_{SV}(m_{\tilde{b}_R},0)] \\
&- \frac{3}{8}g_w^2 N_c[c_t^2 D_{SV}(m_{\tilde{t}_1},m_W)+s_t^2 D_{SV}(m_{\tilde{t}_2},m_W)+D_{SV}(m_{\tilde{b}_L},m_W)], \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
(SS) &= \frac{g_s^2}{6}N_c(N_c+1)[c_{2t}^2 D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_1})+D_{SS}(m_{\tilde{b}_R},m_{\tilde{b}_R})+c_{2t}^2 D_{SS}(m_{\tilde{t}_2},m_{\tilde{t}_2})+D_{SS}(m_{\tilde{b}_L},m_{\tilde{b}_L})] \\
&+ \frac{g_s^2}{6}N_c^2(c_t^4+s_t^4+10c_t^2 s_t^2)D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_2})-\frac{g_s^2}{2}N_c(c_t^4+s_t^4-\frac{2}{3}s_t^2 c_t^2)D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_2}) \\
&+ \frac{g_w^2}{4}N_c(2-N_c)[c_t^2 D_{SS}(m_{\tilde{t}_1},m_{\tilde{b}_L})+s_t^2 D_{SS}(m_{\tilde{t}_2},m_{b_L})] \\
&+ h_t^2 N_c[(c_t^4+s_t^4)D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_2})+s_t^2 D_{SS}(m_{\tilde{b}_L},m_{\tilde{t}_1}) \\
&+ c_t^2 D_{SS}(m_{\tilde{b}_L},m_{\tilde{t}_2})+(N_c+1)c_t^2 s_t^2[D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_1})+D_{SS}(m_{\tilde{t}_2},m_{\tilde{t}_2})]-2N_c s_t^2 c_t^2 D_{SS}(m_{\tilde{t}_2},m_{\tilde{t}_1})] \\
&+ \left(\frac{g_w^2}{8}\right)N_c(N_c+1)[c_t^4 D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_1})+2c_t^2 s_t^2 D_{SS}(m_{\tilde{t}_1},m_{\tilde{t}_2}) \\
&+ s_t^4 D_{SS}(m_{\tilde{t}_2},m_{\tilde{t}_2})+D_{SS}(m_{\tilde{b}_L},m_{\tilde{b}_L})] \\
&+ N_c\Big[\left(\frac{1}{2}h_t^2\sin^2\beta+\frac{1}{8}g_w^2\cos 2\beta c_t^2\right)[D_{SS}(m_{\tilde{t}_1},m_h)+D_{SS}(m_{\tilde{t}_1},m_\pi)] \\
&+ \left(\frac{1}{2}h_t^2\sin^2\beta+\frac{1}{8}g_w^2\cos 2\beta s_t^2\right)[D_{SS}(m_{\tilde{t}_2},m_h)+D_{SS}(m_{\tilde{t}_2},m_\pi)]\Big] \\
&- \frac{1}{8}N_c g_w^2\cos 2\beta[D_{SS}(m_{\tilde{b}_L},m_h)-D_{SS}(m_{\tilde{b}_L},m_\pi)+2c_t^2 D_{SS}(m_{\tilde{t}_1},m_\pi)+2s_t^2 D_{SS}(m_{\tilde{t}_2},m_\pi)] \\
&+ N_c h_t^2\sin^2\beta[s_t^2 D_{SS}(m_{\tilde{t}_1},m_\pi)+c_t^2 D_{SS}(m_{\tilde{t}_2},m_\pi)+D_{SS}(m_{b_L},m_\pi)]. \qquad (42)
\end{aligned}
$$

There are no additional finite contributions apart from those given in ref. [30], from counterterms when non-zero mixing is included in the stop sector. One can check that when all contributions are added there is no dependence on the mixing angle in the divergent part of the potential.

B.2 χ-direction

The gauge-boson masses are

$$
m_G^2=\frac{1}{4}g_s^2\chi^2, \quad \overline{m}_G^2=\frac{4}{3}m_G^2. \qquad (43)
$$

For the Higgs sector, the Goldstone bosons and Higgs masses are

$$\begin{aligned} m_\omega^2 &= m_{\overline{\omega}}^2 = m_U^2 + \lambda_U \chi^2 + h_t^2 \sin^2\beta \frac{\phi^2}{2}, \\ m_u^2 &= m_U^2 + 3\lambda\chi^2 + h_t^2 \sin^2\beta \frac{\phi^2}{2}. \end{aligned} \tag{44}$$

The masses of the rest of the scalars contributing to the effective potential are given by

$$m_{h_1}^2 = m_H^2 + h_t^2 \sin^2\beta \frac{\chi^2}{2}, \tag{45}$$

$$m_{\tilde{b}_{L_1}}^2 = m_Q^2 + \left(h_t^2 - \frac{g_s^2}{3}\right)\frac{\chi^2}{2}, \tag{46}$$

$$m_{h_1\tilde{b}}^2 = -\frac{h_t}{\sqrt{2}} X_t \chi \tag{47}$$

$$m_{h_2}^2 = m_H^2 + h_t^2 \sin^2\beta \frac{\chi^2}{2}, \tag{48}$$

$$m_{\tilde{t}_{L_1}}^2 = m_Q^2 + \left(h_t^2 - \frac{g_s^2}{3}\right)\frac{\chi^2}{2}, \tag{49}$$

$$m_{h_2\tilde{t}}^2 = \frac{h_t}{\sqrt{2}} X_t \chi \tag{50}$$

so the mixing angles are related by

$$\sin q_1 = -\sin q_2 \equiv s_q \tag{51}$$

with eigenstates given by

$$\begin{aligned} H_1 &= c_q h_1 - s_q \tilde{b}_{L1} \\ \tilde{q}_2 &= s_q h_1 + c_q b_{L1} \\ H_2 &= c_q h_2 + s_q \tilde{t}_{L1} \\ \tilde{q}_2 &= -s_q h_2 + c_q t_{L1} \end{aligned} \tag{52}$$

$$m^2_{\tilde{t}_{L_{2,3}}} = m^2_Q + \left(\frac{g_s^2}{6}\right)\frac{\chi^2}{2}, \tag{53}$$

$$m^2_{\tilde{b}_{L_{2,3}}} = m^2_Q + \left(\frac{g_s^2}{6}\right)\frac{\chi^2}{2}, \tag{54}$$

$$m^2_{\tilde{b}_{R_1}} = m^2_D + \left(\frac{g_s^2}{6}\right)\frac{\chi^2}{2}, \tag{55}$$

$$m^2_{\tilde{b}_{R_{2,3}}} = m^2_D - \left(\frac{g_s^2}{3}\right)\frac{\chi^2}{2}. \tag{56}$$

In the expressions below, we also include the contributions from the Higgs doublet. The two-loop unresummed effective potential in the χ-direction is given by the following contributions[8]:

$$\begin{aligned}
(SSV) &= -\frac{g_w^2}{8}[D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_1}, 0) + D_{SSV}(m_{H_1}, m_{H_1}, 0) + D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, 0) \\
&+ D_{SSV}(m_{\tilde{t}_{L3}}, m_{\tilde{t}_{L3}}, 0) + D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{q}_2}, 0) + D_{SSV}(m_{H_2}, m_{H_2}, 0) \\
&+ D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, 0) + D_{SSV}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{L3}}, 0) \\
&+ 4(D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_2}, 0) + D_{SSV}(m_{H_1}, m_{H_2}, 0) \\
&+ D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}, 0) + D_{SSV}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}}, 0))] \\
&- g_s^2\frac{1}{4}\Big[(N_c-1)D_{SSV}(m_\omega, \overline{m}_\omega, m_G) + (N_c-1)D_{SSV}(m_\omega, m_u, m_G) \\
&+ \frac{N_c-1}{N_c}D_{SSV}(\overline{m}_\omega, m_u, m_G) + \frac{1}{N_c}D_{SSV}(m_\omega, m_\omega, \overline{m}_G) + N_c(N_c-2)D_{SSV}(m_\omega, m_\omega, 0) \\
&+ 2(N_c-1)(c_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{t}_{L2}}, m_G) + s_q^2 D_{SSV}(m_{H_2}, m_{\tilde{t}_{L2}}, m_G)) \\
&+ \frac{N_c-1}{N_c}(c_q^4 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_1}, \overline{m}_G) + s_q^4 D_{SSV}(m_{H_2}, m_{H_2}, \overline{m}_G) \\
&+ 2s_q^2 c_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{H_2}, \overline{m}_G)) + \frac{1}{N_c}D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, \overline{m}_G) \\
&+ N_c(N_c-2)D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, 0) \\
&+ 2(N_c-1)(c_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{b}_{L2}}, m_G) + s_q^2 D_{SSV}(m_{H_1}, m_{\tilde{b}_{L2}}, m_G))
\end{aligned}$$

[8] As $m_{\tilde{t}_{L2}} = m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L2}} = m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R2}} = m_{\tilde{b}_{R3}}$ in the χ-direction, we just multiply by a factor of 2 the contributions from these fields in some of the following expressions.

$$
\begin{aligned}
&+ \quad \frac{N_c-1}{N_c}\Big(c_q^4 D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{q}_2}, \overline{m}_G) + s_q^4 D_{SSV}(m_{H_1}, m_{H_1}, \overline{m}_G) + 2s_q^2 c_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{H_1}, \overline{m}_G)\Big) \\
&+ \quad \frac{1}{N_c} D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, \overline{m}_G) + N_c(N_c-2) D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, 0) \\
&+ \quad 2(N_c-1) D_{SSV}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}}, m_G) + \frac{N_c-1}{N_c} D_{SSV}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}, \overline{m}_G) \\
&+ \quad \frac{1}{N_c} D_{SSV}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}, \overline{m}_G) + N_c(N_c-2) D_{SSV}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}, 0)\Big]. \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
(SV) \quad = \quad & -\frac{g_s^2}{8}[2N_c(N_c-2) D_{SV}(m_\omega, 0) + (N_c-1)[3D_{SV}(m_\omega, m_G) + D_{SV}(m_u, m_G)] \\
&+ \quad \frac{1}{N_c}[(N_c+1) D_{SV}(m_\omega, \overline{m}_G) + (N_c-1) D_{SV}(m_u, \overline{m}_G)] \\
&+ \quad 2N_c(N_c-2) D_{SV}(m_{\tilde{t}_{L2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{t}_{L2}}, m_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_1}, m_G) \\
&+ \quad s_q^2 D_{SV}(m_{H_2}, m_G))] + \frac{1}{N_c}[2D_{SV}(m_{\tilde{t}_{L2}}, \overline{m}_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_1}, \overline{m}_G) + s_q^2 D_{SV}(m_{H_2}, \overline{m}_G))] \\
&+ \quad 2N_c(N_c-2) D_{SV}(m_{\tilde{b}_{L2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{b}_{L2}}, m_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_2}, m_G) \\
&+ \quad s_q^2 D_{SV}(m_{H_1}, m_G))] + \frac{1}{N_c}[2D_{SV}(m_{\tilde{b}_{L2}}, \overline{m}_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_2}, \overline{m}_G) + s_q^2 D_{SV}(m_{H_1}, \overline{m}_G))] \\
&+ \quad 2N_c(N_c-2) D_{SV}(m_{\tilde{b}_{R2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{b}_{R2}}, m_G) + 2D_{SV}(m_{\tilde{b}_{R1}}, m_G)] \\
&+ \quad \frac{1}{N_c}[2D_{SV}(m_{\tilde{b}_{R2}}, \overline{m}_G) + 2D_{SV}(m_{\tilde{b}_{R1}}, \overline{m}_G)] \\
&- \quad \frac{3}{8} g_w^2 [D_{SV}(m_{\tilde{q}_1}, 0) + D_{SV}(m_{\tilde{t}_{L2}}, 0) + D_{SV}(m_{\tilde{t}_{L3}}, 0) \\
&+ \quad D_{SV}(m_{\tilde{q}_2}, 0) + D_{SV}(m_{\tilde{b}_{L2}}, 0) + D_{SV}(m_{\tilde{b}_{L3}}, 0) + D_{SV}(m_{H_1}, 0) + D_{SV}(m_{H_2}, 0)], \qquad (58)
\end{aligned}
$$

$$
\begin{aligned}
(SSS) \quad = \quad & -\lambda_U^2 \chi^2 [3D_{SSS}(m_u, m_u, m_u) + (2N_c-1) D_{SSS}(m_u, m_\omega, m_\omega)] \\
&- \quad \frac{1}{2}\Big(h_t^2 \sin^2\beta c_q^2 \chi + (h_t^2 - \frac{1}{3} g_s^2) s_q^2 \chi + \sqrt{2} X_t h_t c_q s_q\Big)^2 [D_{SSS}(m_u, m_{H_1}, m_{H_1}) \\
&+ \quad D_{SSS}(m_u, m_{H_2}, m_{H_2})] \\
&- \quad \frac{1}{2}\Big(h_t^2 \sin^2\beta s_q^2 \chi + (h_t^2 - \frac{1}{3} g_s^2) c_q^2 \chi - \sqrt{2} X_t h_t c_q s_q\Big)^2 [D_{SSS}(m_u, m_{\tilde{q}_1}, m_{\tilde{q}_1}) \\
&+ \quad D_{SSS}(m_u, m_{\tilde{q}_2}, m_{\tilde{q}_2})]
\end{aligned}
$$

$$
\begin{aligned}
&- \quad 2\Big((h_t^2 - \frac{1}{2}g_s^2)\frac{\chi}{\sqrt{2}}c_q - s_q X_t h_t\Big)^2 [D_{SSS}(m_\omega, m_{\tilde{q}_1}, m_{\tilde{t}_{L2}}) + D_{SSS}(m_\omega, m_{\tilde{q}_2}, m_{\tilde{b}_{L2}}] \\
&- \quad 2[(h_t^2 - \frac{1}{2}g_s^2)\frac{\chi}{\sqrt{2}}s_q + c_q X_t h_t]^2 [D_{SSS}(m_\omega, m_{H_2}, m_{\tilde{t}_{L2}})) + D_{SSS}(m_\omega, m_{H_1}, m_{\tilde{b}_{L2}})] \\
&- \quad (-h_t^2 \sin^2\beta s_q c_q \chi + c_{2q}\frac{X_t}{\sqrt{2}} + s_q c_q (h_t^2 - \frac{1}{3}g_s^2)\chi)^2 D_{SSS}(m_u, m_{H_2}, m_{q_1}) \\
&- \quad (h_t^2 \sin^2\beta s_q c_q \chi + c_{2q}\frac{X_t}{\sqrt{2}} - s_q c_q (h_t^2 - \frac{1}{3}g_s^2)\chi)^2 D_{SSS}(m_u, m_{H_1}, m_{q_2}) \\
&- \quad [c_{2q}\frac{X_t}{\sqrt{2}}h_t]^2 (D_{SSS}(m_\omega, m_{H_1}, m_{q_2}) + D_{SSS}(m_\omega, m_{H_2}, m_{q_1})) \\
&- \quad \frac{\chi^2}{2}\Big(2(\frac{g_s^2}{6})^2 D_{SSS}(m_u, m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}) + 2(\frac{g_s^2}{6})^2 D_{SSS}(m_u, m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}) \\
&+ \quad (\frac{1}{3}g_s^2)^2 D_{SSS}(m_u, m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}) + 2(\frac{1}{2}g_s^2)^2 D_{SSS}(m_\omega, m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}}) \\
&+ \quad 2(\frac{g_s^2}{6})^2 D_{SSS}(m_u, m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}})\Big),
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
(SS) \quad = \quad & 2\lambda\Big[c_q^4 (D_{SS}(m_{h_1}, m_{h_1}) + D_{SS}(m_{h_2}, m_{h_2}) + D_{SS}(m_{h_1}, m_{h_2})) \\
+ \quad & s_q^4 (D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_1}) + D_{SS}(m_{\tilde{q}_2}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2})) \\
+ \quad & c_q^2 s_q^2 (2D_{SS}(m_{h_1}, m_{\tilde{q}_2}) + 2D_{SS}(m_{h_2}, m_{\tilde{q}_1}) + D_{SS}(m_{h_1}, m_{\tilde{q}_1}) + D_{SS}(m_{h_2}, m_{\tilde{q}_2}))\Big] \\
+ \quad & \Big(\frac{g_w^2}{8} + \frac{g_s^2}{6}\Big)[2c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_1}) + 2s_q^4 D_{SS}(m_{h_2}, m_{h_2}) + 4s_q^2 c_q^2 D_{SS}(m_{h_2}, m_{\tilde{q}_1}) \\
+ \quad & 4(s_q^2 D_{SS}(m_{h_2}, m_{\tilde{t}_{L2}}) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{t}_{L2}})) + 6D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}) \\
+ \quad & 2c_q^4 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{q}_2}) + 2s_q^4 D_{SS}(m_{h_1}, m_{h_1}) + 4s_q^2 c_q^2 D_{SS}(m_{h_1}, m_{\tilde{q}_2}) \\
+ \quad & 4(s_q^2 D_{SS}(m_{h_1}, m_{\tilde{t}_{L2}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{t}_{L2}})) + 6D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}})] \\
+ \quad & \frac{1}{4}(\frac{g_s^2}{6})[3D_{SS}(m_u, m_u) + 10D_{SS}(m_u, m_\omega) + 35D_{SS}(m_\omega, m_\omega) \\
+ \quad & 8D_{SS}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}) + 24D_{SS}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}) + 16D_{SS}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}})] \\
+ \quad & \frac{1}{2}(h_t^2 - \frac{1}{2}g_s^2)[c_q^2 D_{SS}(m_{\tilde{q}_1}, m_u) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_\omega) + s_q^2 D_{SS}(m_{h_2}, m_u) + s_q^2 D_{SS}(m_{h_2}, m_\omega) \\
+ \quad & 2D_{SS}(m_{\tilde{t}_{L2}}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L3}}, m_\omega)] \\
+ \quad & \frac{1}{12}g_s^2[c_q^2 D_{SS}(m_u, m_{\tilde{q}_1}) + 5c_q^2 D_{SS}(m_\omega, m_{\tilde{q}_1}) + s_q^2 D_{SS}(m_u, m_{h_2})
\end{aligned}
$$

$$
\begin{aligned}
+ & \quad 5s_q^2 D_{SS}(m_\omega, m_{h_2}) + D_{SS}(m_u, m_{\tilde{t}_{L2}}) \\
+ & \quad 5D_{SS}(m_\omega, m_{\tilde{t}_{L2}}) + D_{SS}(m_u, m_{\tilde{t}_{L3}}) + 5D_{SS}(m_\omega, m_{\tilde{t}_{L3}})] \\
+ & \quad \frac{1}{2}(h_t^2 - \frac{1}{2}g_s^2)[c_q^2 D_{SS}(m_{\tilde{q}_2}, m_u) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_\omega) \\
+ & \quad s_q^2 D_{SS}(m_{h_1}, m_u) + s_q^2 D_{SS}(m_{h_1}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L2}}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L3}}, m_\omega)] \\
+ & \quad \frac{1}{12}g_s^2[c_q^2 D_{SS}(m_u, m_{\tilde{q}_2}) + 5c_q^2 D_{SS}(m_\omega, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_u, m_{h_1}) \\
+ & \quad 5s_q^2 D_{SS}(m_\omega, m_{h_1}) + D_{SS}(m_u, m_{\tilde{b}_{L2}}) \\
+ & \quad 5D_{SS}(m_\omega, m_{\tilde{b}_{L2}}) + D_{SS}(m_u, m_{\tilde{b}_{L3}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{L3}})] \\
+ & \quad \left(h_t^2 \sin^2\beta + \frac{1}{4}g_w^2 \cos 2\beta\right)\Big[2c_q^2 s_q^2[D_{SS}(m_{h_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{h_2}, m_{h_2}) + D_{SS}(m_{h_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{\tilde{q}_2})] + c_q^2[D_{SS}(m_{\tilde{t}_{L2}}, m_{h_2}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{h_2}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{h_1})] + s_q^2[D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{q}_2})] + c_{2q}^2[D_{SS}(m_{\tilde{q}_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_2}, m_{h_1})]\Big] \\
+ & \quad \left(\frac{1}{4}g_w^2(c_q^4 + s_q^4) - (c_q^4 + s_q^4)\frac{1}{2}g_w^2 \cos^2\beta - 2s_q^2 c_q^2(-\frac{1}{2}g_w^2 \sin^2\beta + h_t^2 \sin^2\beta)\right)[D_{SS}(m_{\tilde{q}_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{h_2})] - \frac{g_w^2}{2}s_q^2 c_q^2[D_{SS}(m_{h_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2})] \\
- & \quad \left(\frac{1}{4}g_w^2 \cos 2\beta\right)[c_q^2(D_{SS}(m_{\tilde{b}_{L2}}, m_{h_2}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{h_2}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{h_1})) + s_q^2(D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{q}_1}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_2}))] \\
+ & \quad \frac{1}{2}h_t^2 \sin^2\beta[c_q^2 D_{SS}(m_u, m_{h_1}) + c_q^2 D_{SS}(m_u, m_{h_2}) + s_q^2 D_{SS}(m_u, m_{\tilde{q}_1}) + s_q^2 D_{SS}(m_u, m_{\tilde{q}_2}) \\
+ & \quad 5(c_q^2 D_{SS}(m_\omega, m_{h_1}) + c_q^2 D_{SS}(m_\omega, m_{h_2}) + s_q^2 D_{SS}(m_\omega, m_{\tilde{q}_1}) + s_q^2 D_{SS}(m_\omega, m_{\tilde{q}_2}))] \\
+ & \quad \frac{g_w^2}{4}(2 - N_c)[c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) \\
+ & \quad s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2 D_{SS}(m_{h_2}, m_{\tilde{q}_2}) + s_q^2 c_q^2 D_{SS}(m_{h_1}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}})] \\
+ & \quad \frac{1}{2}g_s^2[c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2(D_{SS}(m_{\tilde{q}_1}, m_{h_1})
\end{aligned}
$$

$$
\begin{aligned}
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{h_2})) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{t}_{L3}})] \\
- & \quad \frac{1}{6} g_s^2 [c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2 (D_{SS}(m_{\tilde{q}_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{h_2})) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{L2}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{L2}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{L3}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{L3}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_{\tilde{t}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L3}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_{\tilde{t}_{L3}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L2}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}}) \\
- & \quad \frac{1}{2} g_s^2 [c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R2}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R3}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R1}}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R3}})] \\
+ & \quad \frac{1}{6} g_s^2 (c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R1}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R2}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R2}}) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R3}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R3}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R3}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R2}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R3}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R1}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R2}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R2}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R3}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R3}}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R3}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R1}}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R3}})] \\
+ & \quad \frac{1}{4} g_s^2 [D_{SS}(m_u, m_{\tilde{b}_{R1}}) + D_{SS}(m_\omega, m_{\tilde{b}_{R1}}) + 2D_{SS}(m_\omega, m_{\tilde{b}_{R2}}) + 2D_{SS}(m_\omega, m_{\tilde{b}_{R3}})] \\
- & \quad \frac{1}{12} g_s^2 [D_{SS}(m_u, m_{\tilde{b}_{R1}}) + D_{SS}(m_u, m_{\tilde{b}_{R2}}) + D_{SS}(m_u, m_{\tilde{b}_{R3}}) \\
+ & \quad 5D_{SS}(m_\omega, m_{\tilde{b}_{R1}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{R2}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{R3}})]. \qquad (60)
\end{aligned}
$$

B.3 Integration over the heavy scale

The second part of the calculation arises, as noticed in the paper by Kajantie et al. [7]: when the "heavy" particles have been integrated out their contributions to the 3D mass parameters should also be included, as they can substantially vary the value of the parameters $\Lambda_{H_3}, \Lambda_{U_3}$. In order to do this we must calculate the two-loop contributions to the effective potential in the ϕ- and χ-directions from the heavy fields: Q, D, C_o, A_o. In the following rotation to eigenstates the angle is temperature-dependent; however, if we verify that the eigenstates are always well separated close to the transition point, there is no ambiguity about which is the field that is being integrated out at the second stage.

The masses in the shifted theory are now given by

$$m^2_{\tilde{t}_{L_1}} = \overline{m}^2_{Q_3} + (h^L_t + \Lambda_3 + \Lambda^s_4)\frac{\phi^2}{2} + (h^{QU}_t + g^{QU}_{s_1} + g^{QU}_{s_2})\frac{\chi^2}{2}, \tag{61}$$

$$m^2_{\tilde{t}_{L_{2,3}}} = \overline{m}^2_{Q_3} + (h^L_t + \Lambda_3 + \Lambda^s_4)\frac{\phi^2}{2} + (g^{QU}_{s_2})\frac{\chi^2}{2}, \tag{62}$$

$$m^2_{\tilde{b}_{L_1}} = \overline{m}^2_{Q_3} + (\Lambda_3 + \Lambda^c_4)\frac{\phi^2}{2} + (h^{QU}_t + g^{QU}_{s_1} + g^{QU}_{s_2})\frac{\chi^2}{2}, \tag{63}$$

$$m^2_{\tilde{b}_{L_{2,3}}} = \overline{m}^2_{Q_3} + (\Lambda_3 + \Lambda^c_4)\frac{\phi^2}{2} + (g^{QU}_{s_2})\frac{\chi^2}{2}, \tag{64}$$

$$m^2_{\tilde{b}_{R_1}} = m^2_{D_3} + (g^{UD}_{s_1} + g^{UD}_{s_2})\frac{\chi^2}{2}, \tag{65}$$

$$m^2_{\tilde{b}_{R_{2,3}}} = \overline{m}^2_{D_3} + (g^{QU}_{s_2})\frac{\chi^2}{2}. \tag{66}$$

and the relevant mixing terms are:

$$m^2_{\tilde{t}_{LR}}(\phi) = \frac{h_t}{\sqrt{2}}(\overline{A}_t \sin\beta - \overline{\mu}\cos\beta)\phi \equiv \frac{h_t}{\sqrt{2}}\overline{X}_t\phi \tag{67}$$

for the ϕ-direction, and

$$m^2_{\tilde{t}_{\pm}}(\phi) = \pm\frac{h_t}{\sqrt{2}}\overline{X}_t\chi \tag{68}$$

for the χ-direction.

The expressions for the rest of the fields are given in [18]. The two-loop contributions from the heavy scale are given below. We stress that the D-integrals in eqs. (69) and (70) are just $3D$ integrals, our notation follows that of refs. [18, 36, 7][9]. We do not write the contributions arising from the longitudinal components of the gauge fields A_o and C_o, since the only modification that is necessary is to substitute the gauge couplings $g_{w(s)} \rightarrow g_{w(s)_3}$, which are given in ref. [30] .

B.3.1 ϕ-direction

$$
\begin{aligned}
V_2^{heavy}(\phi) &= -\frac{g_{w_3}^2}{8} N_c [c_t^4 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_W) + s_t^4 D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_W) + 2 s_t^2 c_t^2 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_W) \\
&+ D_{SSV}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, m_W) + 4(c_t^2 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{b}_L}, m_W) + s_t^2 D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{b}_L}, m_W))] \\
&- \frac{g_{s_3}^2}{4}(N_c^2 - 1)[c_t^4 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, 0) + s_t^4 D_{SSV}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, 0) + 2 c_t^2 s_t^2 D_{SSV}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, 0) \\
&+ D_{SSV}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, 0) + D_{SSV}(m_{\tilde{b}_R}, m_{\tilde{b}_R}, 0)] \\
&- \frac{N_c}{2}\Big\{ \Big[\Big((h_t^L + \Lambda_3 + \Lambda_4^s) c_t^2 \Big) \phi + \sqrt{2} \overline{X}_t h_t c_t s_t \Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_1}, m_h) \\
&+ \Big[(h_t^L + \Lambda_3 + \Lambda_4^s) s_t^2 \Big) \phi - \sqrt{2} X_t h_t c_t s_t \Big]^2 D_{SSS}(m_{\tilde{t}_2}, m_{\tilde{t}_2}, m_h) \\
&+ 2\Big[-\Big((h_t^L + \Lambda_3 + \Lambda_4^s) c_t s_t \Big) \phi + \frac{1}{\sqrt{2}} \overline{X}_t h_t c_{2t} \Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_h) \\
&+ \Big(\Lambda_3 + \Lambda_4^c \Big)^2 \phi D_{SSS}(m_{\tilde{b}_L}, m_{\tilde{b}_L}, m_h) \\
&+ h_t^2 \overline{X}_t^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_\pi) + \Big[\phi \Big(h_t^L - \Lambda_4^c + \Lambda_4^s \Big) c_t + \sqrt{2} h_t \overline{X}_t s_t \Big]^2 D_{SSS}(m_{\tilde{t}_1}, m_{\tilde{b}_L}, m_\pi) \\
&+ \Big[\phi \Big(h_t^L - \Lambda_4^c + \Lambda_4^s \Big) s_t - \sqrt{2} h_t \overline{X}_t c_t \Big]^2 D_{SSS}(m_{\tilde{t}_2}, m_{\tilde{b}_L}, m_\pi) \Big\} \\
&- \frac{1}{4} g_{s_3}^2 (N_c^2 - 1)[c_t^2 D_{SV}(m_{\tilde{t}_1}, 0) + s_t^2 D_{SV}(m_{\tilde{t}_2}, 0) + D_{SV}(m_{\tilde{b}_L}, 0) + D_{SV}(m_{\tilde{b}_R}, 0)] \\
&- \frac{3}{8} g_{w_3}^2 N_c [c_t^2 D_{SV}(m_{\tilde{t}_1}, m_W) + + s_t^2 D_{SV}(m_{\tilde{t}_2}, m_W) + D_{SV}(m_{\tilde{b}_L}, m_W)] \\
&+ N_c(N_c+1)[\lambda_{Q_3} c_t^4 + g_{s_1}^{QU} s_t^2 c_t^2 + g_{s_2}^{QU} c_t^2 s_t^2] D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_1}) \\
&+ N_c(N_c+1)[\lambda_{Q_3} s_t^4 + g_{s_1}^{QU} s_t^2 c_t^2 + g_{s_2}^{QU} c_t^2 s_t^2] D_{SS}(m_{\tilde{t}_2}, m_{\tilde{t}_2}) + \lambda_{Q_3} N_c (N_c+1) D_{SS}(m_{\tilde{b}_L}, m_{\tilde{b}_L})
\end{aligned}
$$

[9]Our convention for the functions D_{VVV}, D_{VVS} is that of [18].

$$
\begin{aligned}
&+ \quad N_c^2(2c_t^2 s_t^2 \lambda_{Q_3} - 2c_t^2 s_t^2 g_{s_1}^{QU} + (c_t^4 + s_t^4) g_{s_2}^{QU}) D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
&+ \quad N_c(2c_t^2 s_t^2 \lambda_{Q_3} - 2c_t^2 s_t^2 g_{s_2}^{QU} + (c_t^4 + s_t^4) g_{s_1}^{QU}) D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
&+ \quad \lambda_{D_3} N_c(N_c+1) D_{SS}(m_{\tilde{b}_R}, m_{\tilde{b}_R}) + (2\Lambda_1) N_c(2-N_c)[c_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{b}_L}) \\
&+ \quad s_t^2 D_{SS}(m_{\tilde{t}_2}, m_{b_L})] \\
&+ \quad (N_c g_{s_1}^{QQ} + N_c^2 g_{s_2}^{QQ})[c_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{b}_L}) \\
&+ \quad s_t^2 D_{SS}(m_{\tilde{t}_2}, m_{b_L})] \\
&+ \quad N_c(g_{s_1}^{QU} + N_c g_{s_2}^{QU})[s_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{b}_L}) + c_t^2 D_{SS}(m_{\tilde{t}_2}, m_{\tilde{b}_L})] \\
&+ \quad h_t^{QU} N_c[(c_t^4 + s_t^4) D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}) + s_t^2 D_{SS}(m_{\tilde{b}_L}, m_{\tilde{t}_1}) \\
&+ \quad c_t^2 D_{SS}(m_{\tilde{b}_L}, m_{\tilde{t}_2}) + (N_c+1) c_t^2 s_t^2 [D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_1}) + D_{SS}(m_{\tilde{t}_2}, m_{\tilde{t}_2})] - 2D_{SS}(m_{\tilde{t}_2}, m_{\tilde{t}_1})] \\
&+ \quad \Lambda_1 N_c(N_c+1)[c_t^4 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_1}) + 2c_t^2 s_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{t}_2}) \\
&+ \quad s_t^4 D_{SS}(m_{\tilde{t}_2}, m_{\tilde{t}_2}) + D_{SS}(m_{\tilde{b}_L}, m_{\tilde{b}_L})] \\
&+ \quad \frac{N_c}{2}\Big[\Big((h_t^L + \Lambda_3 + \Lambda_4^s) c_t^2\Big)[D_{SS}(m_{\tilde{t}_1}, m_h) + D_{SS}(m_{\tilde{t}_1}, m_\pi)] \\
&+ \quad \Big((h_t^L + \Lambda_3 + \Lambda_4^s) s_t^2\Big)[D_{SS}(m_{\tilde{t}_2}, m_h) + D_{SS}(m_{\tilde{t}_2}, m_\pi)]\Big] \\
&- \quad \frac{1}{2} N_c(\Lambda_3 + \Lambda_4^c)[D_{SS}(m_{\tilde{b}_L}, m_h) - D_{SS}(m_{\tilde{b}_L}, m_\pi) + 2c_t^2 D_{SS}(m_{\tilde{t}_1}, m_\pi) + 2s_t^2 D_{SS}(m_{\tilde{t}_2}, m_\pi)] \\
&+ \quad N_c h_t^L [D_{SS}(m_{b_L}, m_\pi)] \\
&+ \quad (g_{s_1}^{QD} + 3g_{s_2}^{QD}) N_c[c_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{b}_R}) + s_t^2 D_{SS}(m_{\tilde{t}_2}, m_{\tilde{b}_R}) + D_{SS}(m_{\tilde{b}_L}, m_{\tilde{b}_R})] \\
&+ \quad (g_{s_1}^{UD} + 3g_{s_2}^{UD}) N_c[s_t^2 D_{SS}(m_{\tilde{t}_1}, m_{\tilde{b}_R}) + c_t^2 D_{SS}(m_{\tilde{t}_2}, m_{\tilde{b}_R})]. \qquad (69)
\end{aligned}
$$

B.3.2 χ-direction

$$
\begin{aligned}
V_2^{heavy}(\chi) \quad &= \quad -\frac{g_{w_3}^2}{8}[c_q^4 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_1}, 0) + s_q^4 D_{SSV}(m_{H_1}, m_{H_1}, 0) + 2c_q^2 s_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{H_2}, 0) \\
&+ \quad D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, 0) + D_{SSV}(m_{\tilde{t}_{L3}}, m_{\tilde{t}_{L3}}, 0) \\
&+ \quad c_q^4 D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{q}_2}, 0) + s_q^4 D_{SSV}(m_{H_2}, m_{H_2}, 0) + 2c_q^2 s_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{H_1}, 0) \\
&+ \quad D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, 0) + D_{SSV}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{L3}}, 0) \\
&+ \quad 4(c_q^4 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_2}, 0) + s_q^4 D_{SSV}(m_{H_1}, m_{H_2}, 0) + 2c_q^2 s_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{H_1}, 0) \\
&+ \quad 2c_q^2 s_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{H_2}, 0) + D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}, 0) + D_{SSV}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}}, 0))]
\end{aligned}
$$

$$
\begin{aligned}
- & \; g_{s_3}^2 \frac{1}{4}[2(N_c-1)(c_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{t}_{L2}}, m_G) + s_q^2 D_{SSV}(m_{H_2}, m_{\tilde{t}_{L2}}, m_G)) \\
+ & \; \frac{N_c-1}{N_c}(c_q^4 D_{SSV}(m_{\tilde{q}_1}, m_{\tilde{q}_1}, \overline{m}_G) + s_q^4 D_{SSV}(m_{H_2}, m_{H_2}, \overline{m}_G) \\
+ & \; 2s_q^2 c_q^2 D_{SSV}(m_{\tilde{q}_1}, m_{H_2}, \overline{m}_G)) + \frac{1}{N_c} D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, \overline{m}_G) \\
+ & \; N_c(N_c-2) D_{SSV}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}, 0) \\
+ & \; 2(N_c-1)(c_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{b}_{L2}}, m_G) + s_q^2 D_{SSV}(m_{H_1}, m_{\tilde{b}_{L2}}, m_G)) \\
+ & \; \frac{N_c-1}{N_c}(c_q^4 D_{SSV}(m_{\tilde{q}_2}, m_{\tilde{q}_2}, \overline{m}_G) + s_q^4 D_{SSV}(m_{H_1}, m_{H_1}, \overline{m}_G) + 2s_q^2 c_q^2 D_{SSV}(m_{\tilde{q}_2}, m_{H_1}, \overline{m}_G) \\
+ & \; \frac{1}{N_c} D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, \overline{m}_G) + N_c(N_c-2) D_{SSV}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}, 0) \\
+ & \; 2(N_c-1) D_{SSV}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}}, m_G) + \frac{N_c-1}{N_c} D_{SSV}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}, \overline{m}_G) \\
+ & \; \frac{1}{N_c} D_{SSV}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}, \overline{m}_G) + N_c(N_c-2) D_{SSV}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}, 0)] \\
- & \; \frac{1}{2}((h_t^{QU} + g_{s_1}^{QU} + g_{s_2}^{QU}) s_q^2 \chi + \sqrt{2}\overline{X}_t h_t c_q s_q)^2 [D_{SSS}(m_u, m_{H_1}, m_{H_1}) + D_{SSS}(m_u, m_{H_2}, m_{H_2})] \\
- & \; \frac{1}{2}((h_t^{QU} + g_{s_1}^{QU} + g_{s_2}^{QU}) c_q^2 \chi - \sqrt{2}\overline{X}_t h_t c_q s_q)^2 [D_{SSS}(m_u, m_{\tilde{q}_1}, m_{\tilde{q}_1}) \\
+ & \; D_{SSS}(m_u, m_{\tilde{q}_2}, m_{\tilde{q}_2})] \\
- & \; 2[(h_t^{QU} + g_{s_1}^{QU}) \frac{\chi}{\sqrt{2}} c_q - s_q \overline{X}_t h_t]^2 [D_{SSS}(m_\omega, m_{\tilde{q}_1}, m_{\tilde{t}_{L2}}) + D_{SSS}(m_\omega, m_{\tilde{q}_2}, m_{\tilde{b}_{L2}}] \\
- & \; 2[(h_t^{QU} + g_{s_1}^{QU}) \frac{\chi}{\sqrt{2}} s_q + c_q \overline{X}_t h_t]^2 [D_{SSS}(m_\omega, m_{H_2}, m_{\tilde{t}_{L2}})) + D_{SSS}(m_\omega, m_{H_1}, m_{\tilde{b}_{L2}})] \\
- & \; (c_{2q} \frac{\overline{X}_t}{\sqrt{2}} + s_q c_q (h_t^{QU} + g_{s_1}^{QU} + g_{s_2}^{QU}) \chi)^2 D_{SSS}(m_u, m_{H_2}, m_{q_1}) \\
- & \; (c_{2q} \frac{\overline{X}_t}{\sqrt{2}} - s_q c_q (h_t^{QU} + g_{s_1}^{QU} + g_{s_2}^{QU}) \chi)^2 D_{SSS}(m_u, m_{H_1}, m_{q_2}) \\
- & \; [c_{2q} \frac{\overline{X}_t}{\sqrt{2}} h_t]^2 (D_{SSS}(m_\omega, m_{H_1}, m_{q_2}) + D_{SSS}(m_\omega, m_{H_2}, m_{q_1})) \\
- & \; \frac{\chi^2}{2}[2(g_{s_2}^{QU})^2 D_{SSS}(m_u, m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}) \\
+ & \; 2(g_{s_2}^{QU})^2 D_{SSS}(m_u, m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}}) \\
+ & \; (g_{s_1}^{UD} + g_{s_2}^{UD})^2 D_{SSS}(m_u, m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}) + 2(g_{s_1}^{UD})^2 D_{SSS}(m_\omega, m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}})
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 2(g_{s_2}^{UD})^2 D_{SSS}(m_u, m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}})] \\
&- \quad \frac{g_{s_3}^2}{8}[2N_c(N_c-2)D_{SV}(m_{\tilde{t}_{L2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{t}_{L2}}, m_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_1}, m_G) \\
&+ \quad s_q^2 D_{SV}(m_{H_2}, m_G))] + \frac{1}{N_c}[2D_{SV}(m_{\tilde{t}_{L2}}, \overline{m}_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_1}, \overline{m}_G) + s_q^2 D_{SV}(m_{H_2}, \overline{m}_G))] \\
&+ \quad 2N_c(N_c-2)D_{SV}(m_{\tilde{b}_{L2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{b}_{L2}}, m_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_2}, m_G) \\
&+ \quad s_q^2 D_{SV}(m_{H_1}, m_G))] + \frac{1}{N_c}[2D_{SV}(m_{\tilde{b}_{L2}}, \overline{m}_G) + 2(c_q^2 D_{SV}(m_{\tilde{q}_2}, \overline{m}_G) + s_q^2 D_{SV}(m_{H_1}, \overline{m}_G))] \\
&+ \quad 2N_c(N_c-2)D_{SV}(m_{\tilde{b}_{R2}}, 0) + (N_c-1)[2D_{SV}(m_{\tilde{b}_{R2}}, m_G) + 2D_{SV}(m_{\tilde{b}_{R1}}, m_G)] \\
&+ \quad \frac{1}{N_c}[2D_{SV}(m_{\tilde{b}_{R2}}, \overline{m}_G) + 2D_{SV}(m_{\tilde{b}_{R1}}, \overline{m}_G)] \\
&- \quad \frac{3}{8}g_{w_3}^2[c_q^2 D_{SV}(m_{\tilde{q}_1}, 0) + D_{SV}(m_{\tilde{t}_{L2}}, 0) + D_{SV}(m_{\tilde{t}_{L3}}, 0) \\
&+ \quad c_q^2 D_{SV}(m_{\tilde{q}_2}, 0) + D_{SV}(m_{\tilde{b}_{L2}}, 0) + D_{SV}(m_{\tilde{b}_{L3}}, 0) + s_q^2 D_{SV}(m_{H_1}, 0) + s_q^2 D_{SV}(m_{H_2}, 0)] \\
&+ \quad \Big(\Lambda_1 + \lambda_{Q_3}\Big)[2c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_1}) + 2s_q^4 D_{SS}(m_{h_2}, m_{h_2}) + 6s_q^2 c_q^2 D_{SS}(m_{h_2}, m_{\tilde{q}_1}) \\
&+ \quad 4(s_q^2 D_{SS}(m_{h_2}, m_{\tilde{t}_{L2}}) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{t}_{L2}}) + 6D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{t}_{L2}}) \\
&+ \quad 2c_q^4 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{q}_2}) + 2s_q^4 D_{SS}(m_{h_1}, m_{h_1}) + 6s_q^2 c_q^2 D_{SS}(m_{h_1}, m_{\tilde{q}_2}) \\
&+ \quad 4(s_q^2 D_{SS}(m_{h_1}, m_{\tilde{t}_{L2}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{t}_{L2}}) + 6D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{L2}})] \\
&+ \quad \lambda_{D_3}[8D_{SS}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R1}}) + 24D_{SS}(m_{\tilde{b}_{R2}}, m_{\tilde{b}_{R2}}) + 16D_{SS}(m_{\tilde{b}_{R1}}, m_{\tilde{b}_{R2}})] \\
&+ \quad \frac{1}{2}(h_t^{QU} + g_{s_1}^{QU})[c_q^2 D_{SS}(m_{\tilde{q}_1}, m_u) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_\omega) + s_q^2 D_{SS}(m_{h_2}, m_u) + s_q^2 D_{SS}(m_{h_2}, m_\omega) \\
&+ \quad 2D_{SS}(m_{\tilde{t}_{L2}}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L3}}, m_\omega)] \\
&+ \quad \frac{1}{2}g_{s_2}^{QU}[c_q^2 D_{SS}(m_u, m_{\tilde{q}_1}) + 5c_q^2 D_{SS}(m_\omega, m_{\tilde{q}_1}) + s_q^2 D_{SS}(m_u, m_{h_2}) \\
&+ \quad 5s_q^2 D_{SS}(m_\omega, m_{h_2}) + D_{SS}(m_u, m_{\tilde{t}_{L2}}) \\
&+ \quad 5D_{SS}(m_\omega, m_{\tilde{t}_{L2}}) + D_{SS}(m_u, m_{\tilde{t}_{L3}}) + 5D_{SS}(m_\omega, m_{\tilde{t}_{L3}})] \\
&+ \quad \frac{1}{2}(h_t^{QU} + g_{s_1}^{QU})[c_q^2 D_{SS}(m_{\tilde{q}_2}, m_u) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_\omega) \\
&+ \quad s_q^2 D_{SS}(m_{h_1}, m_u) + s_q^2 D_{SS}(m_{h_1}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L2}}, m_\omega) + 2D_{SS}(m_{\tilde{t}_{L3}}, m_\omega)] \\
&+ \quad \frac{1}{2}g_{s_2}^{QU}[c_q^2 D_{SS}(m_u, m_{\tilde{q}_2}) + 5c_q^2 D_{SS}(m_\omega, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_u, m_{h_1}) \\
&+ \quad 5s_q^2 D_{SS}(m_\omega, m_{h_1}) + D_{SS}(m_u, m_{\tilde{b}_{L2}})
\end{aligned}
$$

$$
\begin{aligned}
+ & \quad 5D_{SS}(m_\omega, m_{\tilde{b}_{L2}}) + D_{SS}(m_u, m_{\tilde{b}_{L3}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{L3}})] \\
+ & \quad \Big(h_t^L + \Lambda_3 + \Lambda_4^s\Big)\Big[2c_q^2 s_q^2 [D_{SS}(m_{h_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{h_2}, m_{h_2}) + D_{SS}(m_{h_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{\tilde{q}_2})] + c_q^2[D_{SS}(m_{\tilde{t}_{L2}}, m_{h_2}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{h_2}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{h_1})] + s_q^2[D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{q}_2})] + c_{2q}^2[D_{SS}(m_{\tilde{q}_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_2}, m_{h_1})]\Big] \\
+ & \quad \Big((\Lambda_3 + \Lambda_4^c)(c_q^4 + s_q^4) - 2s_q^2 c_q^2(\Lambda_4^s + h_t^L)\Big)[D_{SS}(m_{\tilde{q}_1}, m_{h_1}) + D_{SS}(m_{\tilde{q}_2}, m_{h_2})] \\
+ & \quad +2\Lambda_4^c s_q^2 c_q^2[DSS(m_{h_1}, m_{h_2}) + D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2})] \\
- & \quad \Big(\Lambda_3 + \Lambda_4^c\Big)[c_q^2(D_{SS}(m_{\tilde{b}_{L2}}, m_{h_2}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{h_2}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{h_1})) + s_q^2(D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{q}_1}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_2}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_2}))] \\
+ & \quad 2\Lambda_1(2 - N_c)[c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) \\
+ & \quad s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2 D_{SS}(m_{h_2}, m_{\tilde{q}_2}) + s_q^2 c_q^2 D_{SS}(m_{h_1}, m_{\tilde{q}_1}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}})] \\
+ & \quad g_{s_1}^{QQ}[c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2(D_{SS}(m_{\tilde{q}_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{h_2})) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{t}_{L3}})] \\
+ & \quad g_{s_2}^{QQ}[c_q^4 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{q}_2}) + s_q^4 D_{SS}(m_{h_1}, m_{h_2}) + s_q^2 c_q^2(D_{SS}(m_{\tilde{q}_1}, m_{h_1}) \\
+ & \quad D_{SS}(m_{\tilde{q}_2}, m_{h_2})) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{L2}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{L2}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{L3}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{L3}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_{\tilde{t}_{L2}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L2}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{L3}}) \\
+ & \quad c_q^2 D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{q}_2}) + s_q^2 D_{SS}(m_{\tilde{t}_{L3}}, m_{h_1}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L2}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{L3}}) \\
+ & \quad g_{s_1}^{QD}[c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R2}}) \\
+ & \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R3}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R1}}) \\
+ & \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R3}})]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad g_{s_2}^{QD}(c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R1}}) \\
&+ \quad c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R2}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R2}}) + c_q^2 D_{SS}(m_{\tilde{q}_1}, m_{\tilde{b}_{R3}}) + s_q^2 D_{SS}(m_{h_2}, m_{\tilde{b}_{R3}}) \\
&+ \quad D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{t}_{L2}}, m_{\tilde{b}_{R3}}) \\
&+ \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R2}}) \\
&+ \quad D_{SS}(m_{\tilde{t}_{L3}}, m_{\tilde{b}_{R3}}) \\
&+ \quad c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R1}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R1}}) \\
&+ \quad c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R2}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R2}}) + c_q^2 D_{SS}(m_{\tilde{q}_2}, m_{\tilde{b}_{R3}}) + s_q^2 D_{SS}(m_{h_1}, m_{\tilde{b}_{R3}}) \\
&+ \quad D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R1}}) + D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L2}}, m_{\tilde{b}_{R3}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R1}}) \\
&+ \quad D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R2}}) + D_{SS}(m_{\tilde{b}_{L3}}, m_{\tilde{b}_{R3}})] \\
&+ \quad \frac{1}{2} g_{s_1}^{UD}[D_{SS}(m_u, m_{\tilde{b}_{R1}}) + D_{SS}(m_\omega, m_{\tilde{b}_{R1}}) + 2D_{SS}(m_\omega, m_{\tilde{b}_{R2}}) + 2D_{SS}(m_\omega, m_{\tilde{b}_{R3}})] \\
&+ \quad \frac{1}{2} g_{s_2}^{UD}[D_{SS}(m_u, m_{\tilde{b}_{R1}}) + D_{SS}(m_u, m_{\tilde{b}_{R2}}) + D_{SS}(m_u, m_{\tilde{b}_{R3}}) \\
&+ \quad 5D_{SS}(m_\omega, m_{\tilde{b}_{R1}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{R2}}) + 5D_{SS}(m_\omega, m_{\tilde{b}_{R3}})]. \qquad (70)
\end{aligned}
$$

C Zero-temperature renormalization

The most important zero-temperature renormalization effects with respect to our calculation concern the mass parameters. We will not go into the details of the renormalization, but refer the reader to the literature in which the pole masses for the relevant particles of our calculation have been obtained considering the full particle spectrum of the MSSM [37, 38]. We use the expressions given in ref. [37], keeping only the top Yukawa coupling, in the appropriate (large-m_A) limit. In this limit the relevant expression for the one-loop corrected Higgs mass is given by [39, 32, 40]:

$$
\begin{aligned}
m_h^2 &= m_Z^2 \cos^2 2\beta + \frac{3 g_w^2 m_t^4}{16\pi^2 m_W^2}[(2 \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + (\frac{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}{4 m_t^2} \sin^2 2\alpha_t)^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \\
&+ \quad 2(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)/(2m_t^2)\ \sin^2 2\alpha_t \ln[m_{\tilde{t}_2}/m_{\tilde{t}_1}])] \qquad (71)
\end{aligned}
$$

where $f(x,y) = 2 - \frac{x+y}{(x-y)} \ln \frac{x}{y}$.

In principle, there exists a metastable region in which the colour-breaking minimum is lower than the physical one at zero temperature. The constraint for absolute stability can be obtained by studying the effective potential at zero temperature [14, 20]. This gives the constraint $-m_U^2 \leq (m_U^c)^2$, where

$$m_U^c = \left(\frac{m_h^2 v^2 g_s^2}{12}\right)^{1/4}. \tag{72}$$

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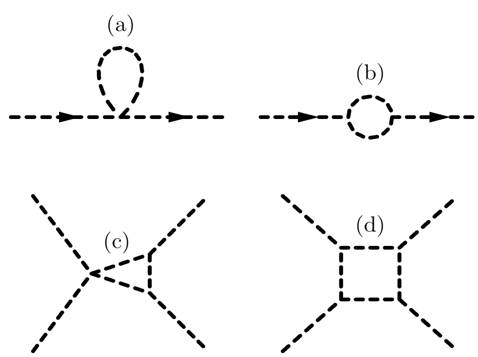

Figure 1: Feynman diagrams contributing to the two- and four-point Green functions.

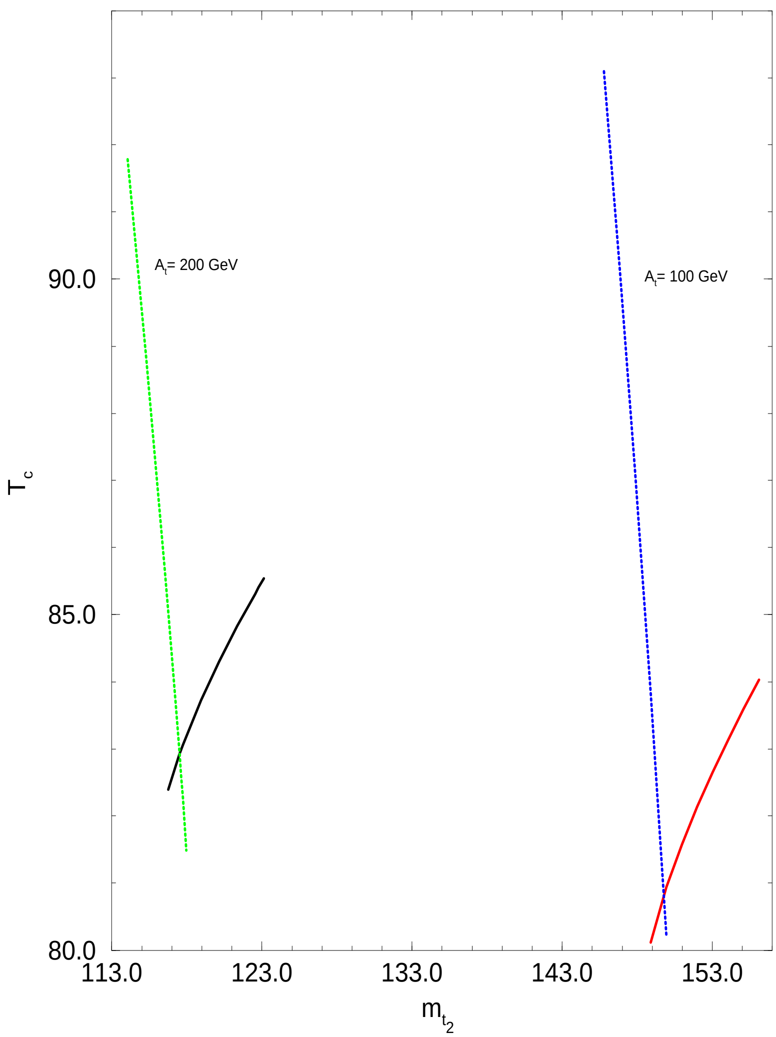

Figure 2: Critical temperatures in the ϕ- (solid) and χ- (dotted) directions as functions of $m_{\tilde{t}_2}$ for $\tan\beta = 5$ and $m_Q = 300$ GeV.

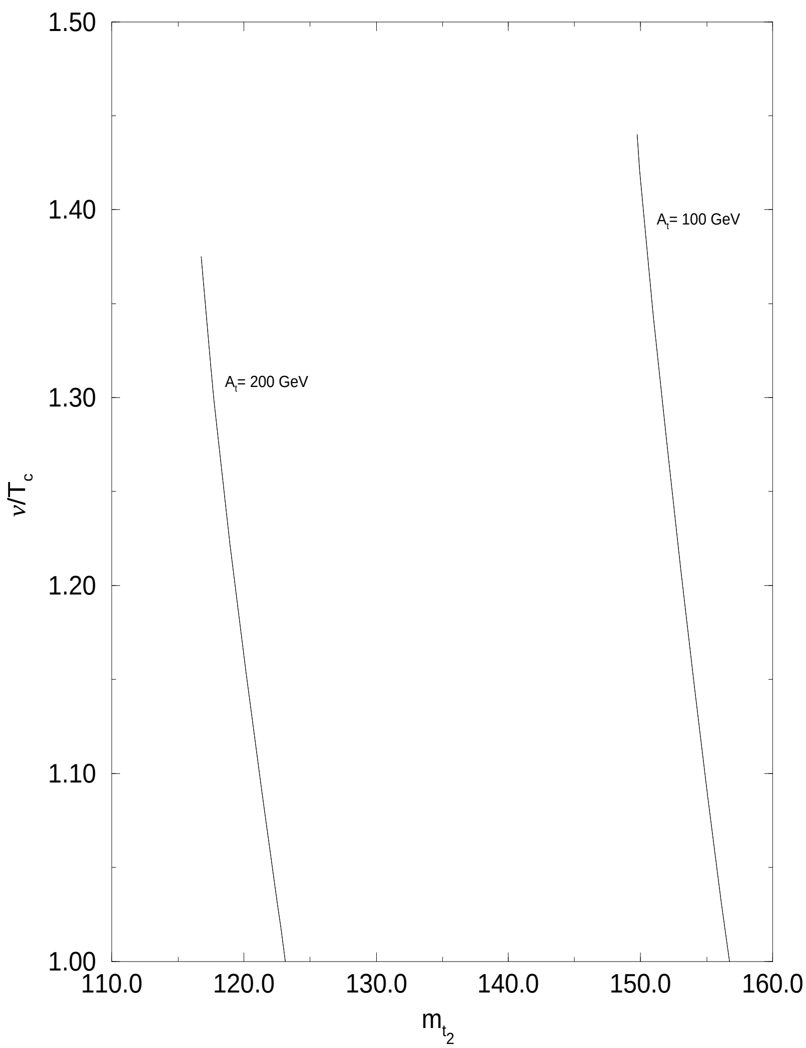

Figure 3: Plot of $\frac{v}{T}$ as a function of $m_{\tilde{t}_1}$ in the ϕ- direction for $\tan\beta = 5$, $m_Q = 300$ GeV and $A_t = 100, 200$ GeV.

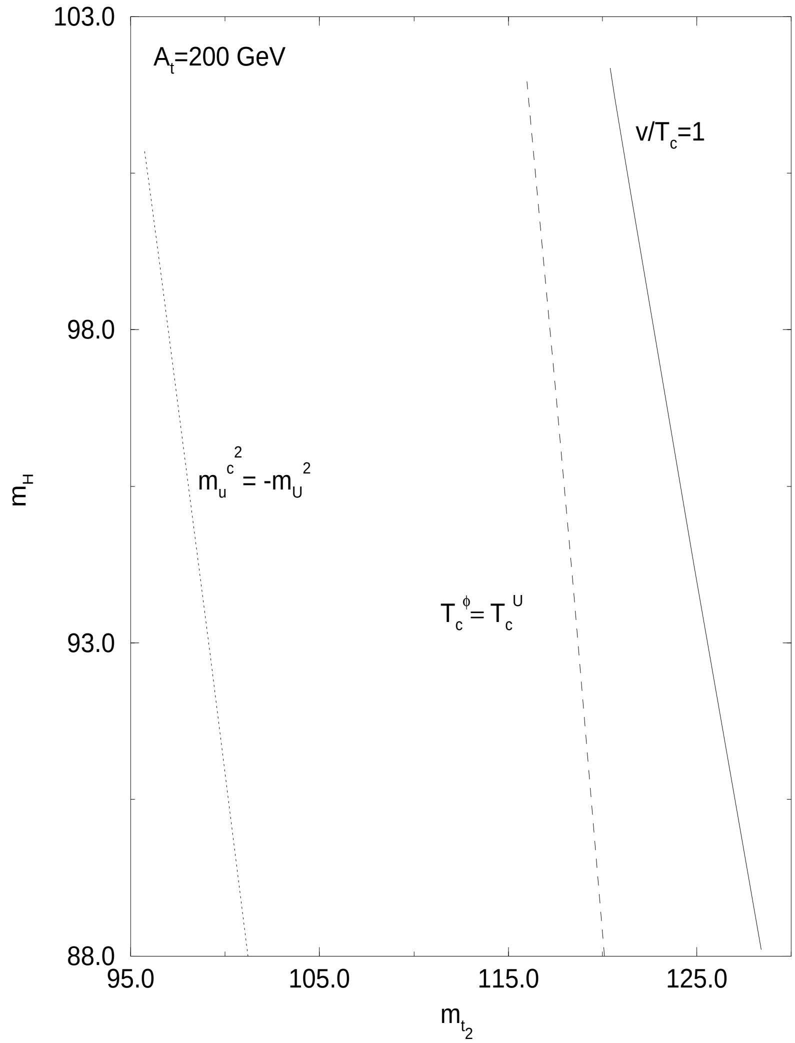

Figure 4: Allowed region in m_h-$m_{\tilde{t}_2}$ plane for $m_Q = 300$ GeV and $\frac{X_t}{\sin\beta} = 200$ GeV. To the left of the solid line there is a sufficiently strong first-order phase transition, to the right of the dotted line the physical vacuum is absolutely stable. The dashed line separates the region for which a two-stage phase transition can occur.

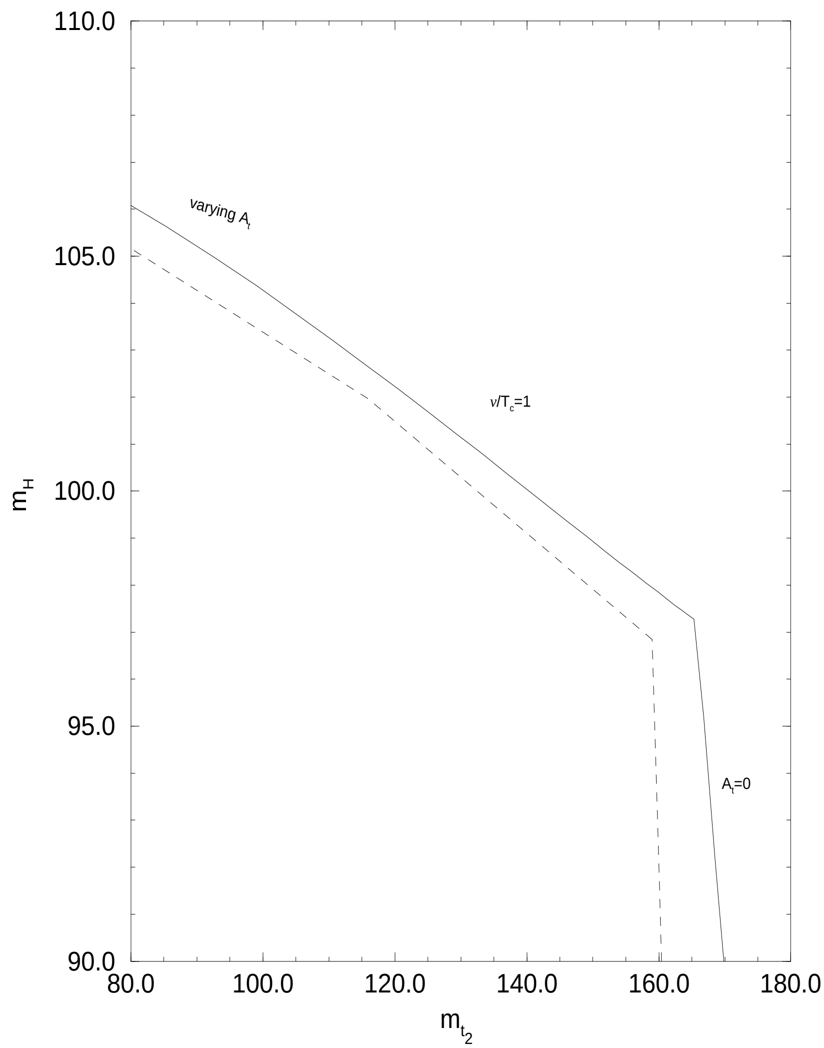

Figure 5: Allowed region of parameter space in the m_h-$m_{\tilde{t}_2}$ plane for $m_Q = 300$ GeV, varying $\tilde{A}_t$ and $\tan\beta$. The dashed line is defined when the critical temperatures in the ϕ- and χ- directions are equal for the same variations of $\tan\beta$ and $\tilde{A}_t$.

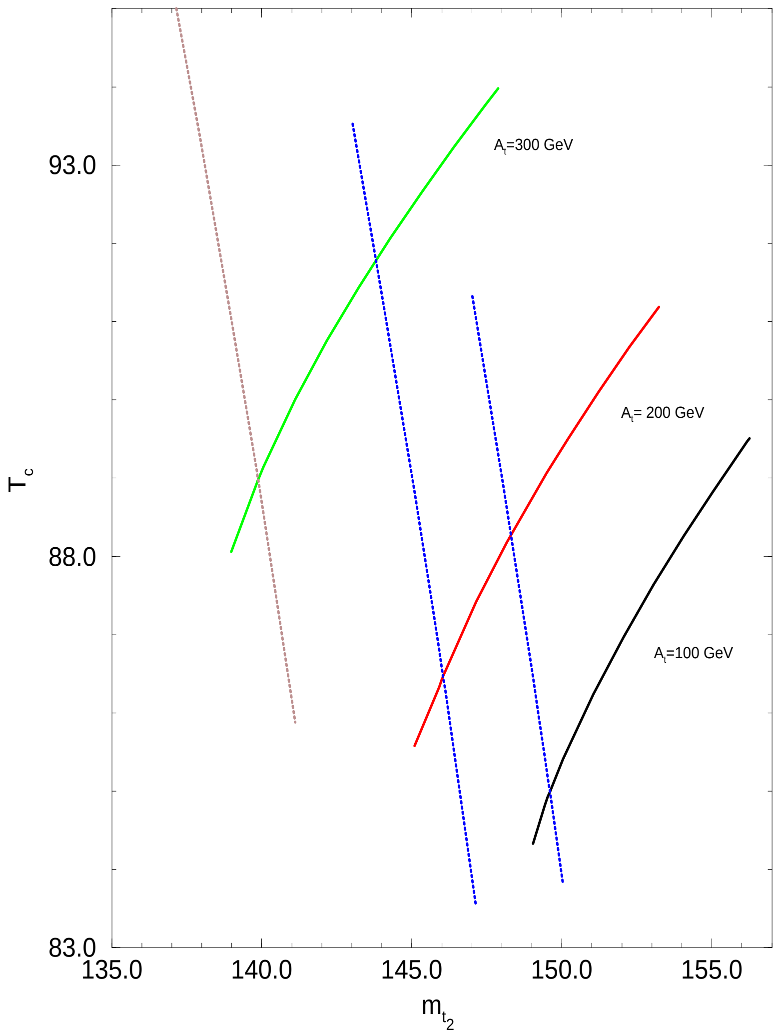

Figure 6: Critical temperatures in the ϕ- (solid) and χ- (dotted) directions as functions of $m_{\tilde{t}_2}$ for $\tan\beta = 5$, $m_Q = 1$ TeV and $\tilde{A}_t = 100, 200, 300$ GeV.

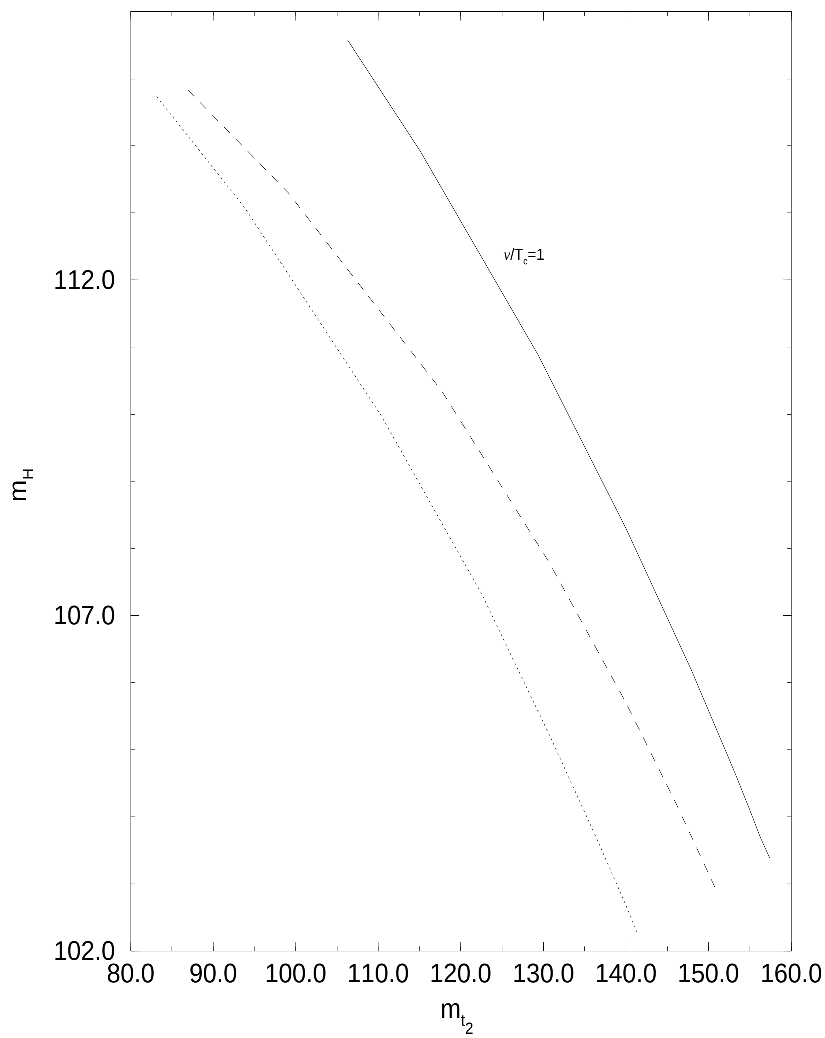

Figure 7: Allowed region of parameter space in the m_h-$m_{\tilde{t}_2}$ plane for $m_Q = 1$ TeV, $0 \leq \tilde{A}_t \leq 650$ GeV and $\tan\beta = 5$. To the left of the solid line there is a sufficiently strong first-order phase transition, to the right of the dotted line the physical vacuum is absolutely stable. The dashed line separates the region for which a two-stage phase transition can occur.

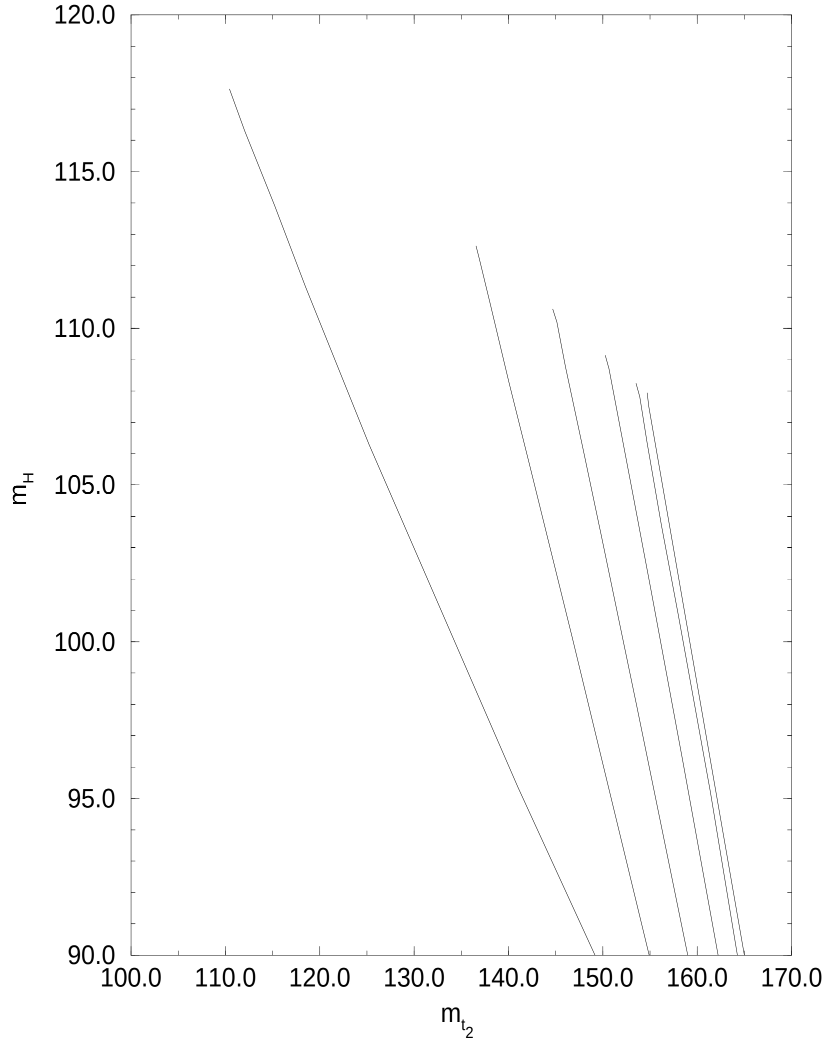

Figure 8: Contours of $\frac{\phi}{T_c} = 1$ in the m_h-$m_{\tilde{t}_2}$ plane for $m_Q = 1$ TeV, for $\tilde{A}_t = 0, 100, 200, 300, 400, 600$ GeV.