The D4-D8 Brane System and Five Dimensional Fixed Points

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Abstract

We construct dual Type I' string descriptions to five dimensional supersymmetric fixed points with E_{N_f+1} global symmetry. The background is obtained as the near horizon geometry of the D4-D8 brane system in massive Type IIA supergravity. We use the dual description to deduce some properties of the fixed points.

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1   Introduction

The consideration of the near horizon geometry of branes on one hand, and the low energy dynamics on their worldvolume on the other hand has lead to conjectured duality relations between field theories and string theory (M theory) on certain backgrounds [1, 2, 3] [1]. The field theories under discussion are in various dimensions, can be conformal or not, and with or without supersymmetry. These properties are reflected by the type of string/M theory backgrounds of the dual description.

In this note we will construct dual Type I' string descriptions to five dimensional supersymmetric fixed points with E_{N_f+1} global symmetry [2]. These fixed points are obtained in the limit of infinite bare coupling of $\mathcal{N} = 2$ supersymmetric gauge theories with gauge group $Sp(Q_4)$, $N_f < 8$ massless hypermultiplets in the fundamental representation and one massless hypermultiplet in the anti-symmetric representation [5, 6]. The dual background is obtained as the near horizon geometry of the D4-D8 brane system in massive Type IIA supergravity. The ten dimensional space is a fibration of AdS_6 over S^4 and has the isometry group $SO(2,5) \times SO(4)$. This space provides the spontaneous compactification of massive Type IIA supergravity in ten dimensions to the $F(4)$ gauged supergravity in six dimensions [7].

The paper is organized as follows. In section 2 we will discuss the D4-D8 brane system and its relation to the five dimensional fixed points. In section 3 we will construct the dual string description and use it to deduce some properties of the fixed points.

2   The D4-D8 Brane System

We start with Type I string theory on $\mathbb{R}^9 \times S^1$ with N coinciding D5 branes wrapping the circle. The six dimensional D5 brane worldvolume theory possesses $\mathcal{N} = 1$ supersymmetry. It has an $Sp(N)$ gauge group, one hypermultiplet in the antisymmetric representation of $Sp(N)$ from the DD sector and 16 hypermultiplets in the fundamental representation from the DN sector. Performing T-duality on the circle results in Type I' theory compactified on the interval $S^1/\mathbb{Z}_2$ with two orientifolds (O8 planes) located at the fixed points. The D5 branes become D4 branes and there are 16 D8 branes located at points on the interval. They cancel the -16 units of D8 brane charge carried by the two O8 planes. The locations of the D8 branes correspond to masses for the hypermultiplets in the funda-

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[1]For a recent review of the subject see [4].

[2]$E_{N_f+1} = (E_8, E_7, E_6, E_5 = Spin(10), E_4 = SU(5), E_3 = SU(3) \times SU(2), E_2 = SU(2) \times U(1), E_1 = SU(2))$.

mental representation arising from the open strings between the D4 branes and the D8 branes. The hypermultiplet in the antisymmetric representation is massless.

Consider first $N = 1$, namely one D4 brane. The worldvolume gauge group is $Sp(1) \simeq SU(2)$. The five-dimensional vector multiplet contains as bosonic fields the gauge field and one real scalar. The scalar parametrizes the location of the D4 branes in the interval, and the gauge group is broken to $U(1)$ unless the D4 brane is located at one of the fixed points. A hypermultiplet contains four real (two complex) scalars. The N_f massless matter hypermultiplets in the fundamental and the antisymmetric hypermultiplet (which is a trivial representation for $Sp(1)$) parametrize the Higgs branch of the theory. It is the moduli space of $SO(2N_f)$ one-instanton. The theory has an $SU(2)_R$ R-symmetry. The two supercharges as well as the scalars in the hypermultiplet transform as a doublet under $SU(2)_R$. In addition, the theory has a global $SU(2) \times SO(2N_f) \times U(1)_I$ symmetry. The $SU(2)$ factor of the global symmetry group is associated with the massless antisymmetric hypermultiplet and is only present if $N > 1$, the $SO(2N_f)$ group is associated with the N_f massless hypermultiplets in the fundamental and the $U(1)_I$ part corresponds to the instanton number conservation.

Consider the D8 brane background metric. It takes the form

$$ds^2 = H_8^{-1/2}(-dt^2 + dx_1^2 + \ldots + dx_8^2) + H_8^{1/2}dz^2 \quad , \quad e^{-\phi} = H_8^{5/4} \ . \tag{2.1}$$

H_8 is a harmonic function on the interval parametrized by z. Therefore H_8 is a piecewise linear function in z where the slope is constant between two D8 branes and decreases by one unit for each D8 brane crossed. Thus,

$$H_8(z) = c + 16\frac{z}{l_s} - \sum_{i=1}^{16} \frac{|z - z_i|}{l_s} - \sum_{i=1}^{16} \frac{|z + z_i|}{l_s} \ , \tag{2.2}$$

where the z_i denote the locations of the 16 D8 branes.

Denote the D4 brane worldvolume coordinates by $t, x_1 \ldots x_4$. The D4 brane is located at some point in $x_5 \ldots x_8$ and z. We can consider it to be a probe of the D8 branes background. The gauge coupling g of the D4 brane worldvolume theory and the harmonic function H_8 are related as can be seen by expanding the DBI action of a D4 brane in the background (2.2). We get

$$g^2 = \frac{l_s}{H_8} \ , \tag{2.3}$$

where $g_{cl}^2 = \frac{c}{l_s}$ corresponds to the classical gauge coupling. In the field theory limit we take $l_s \to 0$ keeping the gauge coupling g fixed, thus,

$$g^2 = \text{fixed}, \quad \Rightarrow \quad \phi = \frac{z}{l_s^2} = \text{fixed}, \quad l_s \to 0 \ . \tag{2.4}$$

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In this limit we have

$$\frac{1}{g^2} = \frac{1}{g_{cl}^2} + 16\phi - \sum_{i=1}^{16} |\phi - m_i| - \sum_{i=1}^{16} |\phi + m_i| \ , \qquad (2.5)$$

where the masses $m_i = \frac{z_i}{l_s^2}$. Note that in the field theory limit we are studying the region near $z = 0$. The coordinate ϕ takes values in $\mathbb{R}^+$ and parametrizes the field theory Coulomb branch.

Seiberg argued [5] that the theory at the origin of the Coulomb branch obtained in the limit $g_{cl} = \infty$ with $N_f < 8$ massless hypermultiplets is a non trivial fixed point. The restriction $N_f < 8$ can be seen in the supergravity description as a requirement for the harmonic function H_8 in equation (2.2) to be positive when $c = 0$ and $z_i = 0$. At the fixed point the global symmetry is enhanced to $SU(2) \times E_{N_f+1}$. The Higgs branch is expected to become the moduli space of E_{N_f+1} one-instanton.

The generalization to $N = Q_4$ D4 branes is straightforward [6]. The gauge group is now $Sp(Q_4)$ and the global symmetry is as before. The Higgs branch is now the moduli space of $SO(2N_f)$ Q_4-instantons. At the fixed point the global symmetry is enhanced as before and the Higgs branch is expected to become the moduli space of E_{N_f+1} Q_4-instanton. Our interest in this paper will be in finding dual string (supergravity) descriptions of these fixed points.

3   The Supergravity (String) Description

The low energy description of our system is given by Type I' supergravity [8]. The region between two D8 branes is, as discussed in [9], described by massive Type IIA supergravity [10]. The configurations that we will study have N_f D8 branes located at one O8 plane and $16 - N_f$ D8 branes at the other O8 plane. Therefore we are always between D8 branes and never encounter the situation where we cross D8 branes, and the massive Type IIA supergravity description is sufficient.

The bosonic part of the massive Type IIA action (in string frame) including a six-form gauge field strength which is the dual of the RR four-form field strength is

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi\right) - \frac{1}{2 \cdot 6!}|F_6|^2 - \frac{1}{2}m^2 \right) \qquad (3.1)$$

where the mass parameter is given by[3].

$$m = \sqrt{2}(8 - N_f)\mu_8\kappa_{10} = \frac{8 - N_f}{2\pi l_s} \ . \tag{3.2}$$

The Einstein equations derived from (3.1) read (in Einstein frame metric)

$$
\begin{aligned}
2R_{ij} &= g_{ij}(R - \frac{1}{2}|\partial\Phi|^2 - \frac{e^{-\Phi/2}}{2 \cdot 6!}|F_6|^2 - \frac{m^2}{2}e^{5\Phi/2}) + \\
&+ \partial_i\Phi\partial_j\Phi + \frac{e^{-\Phi/2}}{5!}F_{im_1...m_5}F_j^{m_1...m_5} \ , \\
0 &= \nabla^j\partial_j\Phi - \frac{5m^2}{4}e^{5\Phi/2} + \frac{e^{-\Phi/2}}{4 \cdot 6!}|F_6|^2 \ , \\
0 &= \nabla^i\left(e^{-\Phi/2}F_{im_1...m_5}\right) \ .
\end{aligned}
\tag{3.3}
$$

Except for these formulas we will be using the string frame only.

For identifying the solutions of D4 branes localized on D8 branes it is more convenient to start with the conformally flat form of the D8 brane supergravity solution. It takes the form [8]

$$ds^2 = \Omega(z)^2\left(-dt^2 + \ldots + dx_4^2 + d\tilde{r}^2 + \tilde{r}^2 d\Omega_4^2 + dz^2\right) \ , \tag{3.4}$$

$$e^{\Phi} = C\left(\frac{3}{2}Cmz\right)^{-\frac{5}{6}} \ , \quad \Omega(z) = \left(\frac{3}{2}Cmz\right)^{-\frac{1}{6}} \ .$$

In these coordinates the harmonic function of Q_4 localized D4 branes in the near horizon limit derived from (3.3) reads

$$H_4 = \frac{Q_4}{l_s^{10/3}(\tilde{r}^2 + z^2)^{5/3}} \ . \tag{3.5}$$

This localized D4-D8 brane system solution, in a different coordinate system, has been constructed in [11]. One way to determine the harmonic function (3.5) of the localized D4 branes is to solve the Laplace equation in the background of the D8 branes.

It is useful to make a change of coordinates $z = r\sin\alpha, \tilde{r} = r\cos\alpha, 0 \leq \alpha \leq \pi/2$. We get

$$
\begin{aligned}
ds^2 &= \Omega^2\left(H_4^{-\frac{1}{2}}(-dt^2 + \ldots dx_4^2) + H_4^{\frac{1}{2}}(dr^2 + r^2 d\Omega_4^2)\right) \ , \\
e^{\Phi} &= C\left(\frac{3}{2}Cmr\sin\alpha\right)^{-\frac{5}{6}}H_4^{-\frac{1}{4}} \ , \quad \Omega = \left(\frac{3}{2}Cmr\sin\alpha\right)^{-\frac{1}{6}} \ , \tag{3.6} \\
F_{01234r} &= \frac{1}{C}\partial_r\left(H_4^{-1}\right) \ ,
\end{aligned}
$$

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[3]We use the conventions $\kappa_{10} = 8\pi^{7/2}l_s^4$ and $\mu_8 = (2\pi)^{-9/2}l_s^{-5}$ for the gravitational coupling and the D8 brane charge, respectively.

where C is an arbitrary parameter of the solution [8] and

$$d\Omega_4^2 = d\alpha^2 + (\cos\alpha)^2 d\Omega_3^2 \ . \tag{3.7}$$

The background (3.6) is a solution of the massive Type IIA supergravity equations (3.3).

The metric (3.6) of the D4-D8 system can be simplified to

$$ds^2 = \left(\frac{3}{2}Cm\sin\alpha\right)^{-\frac{1}{3}} \left(Q_4^{-\frac{1}{2}}r^{\frac{4}{3}}dx_\parallel^2 + Q_4^{\frac{1}{2}}\frac{dr^2}{r^2} + Q_4^{\frac{1}{2}}d\Omega_4^2\right) \tag{3.8}$$

where $dx_\parallel^2 \equiv -dt^2 + \ldots + dx_4^2$. Define now the energy coordinate U by $r^2 = l_s^5 U^3$. That this is the energy coordinate can be seen by calculating the energy of a fundamental string stretched in the r direction or by using the DBI action as in the previous section. In the field theory limit, $l_s \rightarrow 0$ with the energy U fixed, we get the metric in the form of a *warped product* [12] of $AdS_6 \times S^4$

$$ds^2 = l_s^2 \left(\frac{3}{4\pi}C(8 - N_f)\sin\alpha\right)^{-\frac{1}{3}} \left(Q_4^{-\frac{1}{2}}U^2 dx_\parallel^2 + Q_4^{\frac{1}{2}}\frac{9dU^2}{4U^2} + Q_4^{\frac{1}{2}}d\Omega_4^2\right) \ , \tag{3.9}$$

and the dilaton is given by

$$e^\Phi = Q_4^{-\frac{1}{4}}C\left(\frac{3}{4\pi}C(8 - N_f)\sin\alpha\right)^{-\frac{5}{6}} \ . \tag{3.10}$$

The ten dimensional space described by (3.9) is a fibration of AdS_6 over S^4. It is the most general form of a metric that has the isometry of an AdS_6 space [13]. The space has a boundary at $\alpha = 0$ which corresponds to the location of the O8 plane ($z = 0$). The boundary is of the form $AdS_6 \times S^3$. In addition to the $SO(2,5)$ AdS_6 isometries, the ten dimensional space has also $SO(4)$ isometries associated with the spherical part of the metric (3.9). In general S^4 has the $SO(5)$ isometry group. However, this is reduced due to the warped product structure. As is easily seen from the form of the spherical part (3.7), only transformations excluding the α coordinate are isometries of (3.9). We are left with an $SO(4) \sim SU(2) \times SU(2)$ isometry group.

The two different viewpoints of the D4-D8 brane system, the near horizon geometry of the brane system on one hand, and the low energy dynamics on the D4 branes worldvolume on the other hand suggest a duality relation. Namely, *Type I' string theory compactified on the background (3.9), (3.10) with a 4-form flux of Q_4 units on S^4 is dual to an $\mathcal{N} = 2$ supersymmetric five dimensional fixed point*[4]. The fixed point is obtained in the limit of infinite coupling of $Sp(Q_4)$ gauge theory with N_f hypermultiplets in the

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[4]The 4-form flux is just the dual of the 6-form in (3.6).

fundamental representation and one hypermultiplet in the antisymmetric representation, where $m \sim (8 - N_f)$ as in (3.2). The $SO(2, 5)$ symmetry of the compactification corresponds to the conformal symmetry of the field theory. The $SU(2) \times SU(2)$ symmetry of the compactification corresponds $SU(2)_R$ R-symmetry and to the $SU(2)$ global symmetry associated with the massless hypermultiplet in the antisymmetric representation.

At the boundary $\alpha = 0$ the dilaton (3.10) blows up and Type I' is strongly coupled. In the weakly coupled dual heterotic string description this is seen as an enhancement of the gauge symmetry to E_{N_f+1}. One can see this enhancement of the gauge symmetry in the Type I' description by analysing the D0 brane dynamics near the orientifold plane [14, 15]. This means that we have E_{N_f+1} vector fields that propagate on the $AdS_6 \times S^3$ boundary, as in the Horava-Witten picture [16]. The scalar curvature of the background (3.9), (3.10)

$$\mathcal{R} l_s^2 \sim (C(8 - N_f))^{\frac{1}{3}} Q_4^{-\frac{1}{2}} (\sin \alpha)^{-\frac{5}{3}} , \tag{3.11}$$

blows up at the boundary as well. In the dual heterotic description the dilaton is small but the curvature is large, too. For large Q_4 there is a region, $\sin \alpha \gg Q_4^{-\frac{3}{10}}$, where both curvature (3.11) and dilaton (3.10) are small and thus we can trust supergravity.

The AdS_6 supergroup is $F(4)$. Its bosonic subgroup is $SO(2, 5) \times SU(2)$. Romans constructed an $\mathcal{N} = 4$ six dimensional gauged supergravity with gauge group $SU(2)$ that realizes $F(4)$ [7]. It was conjectured in [17] that it is related to a compactification of the ten dimensional massive Type IIA supergravity. Indeed, we find that the ten dimensional background space is the warped product of AdS_6 and S^4 (3.9) (with $N_f = 0$). The reduction to six dimensions can be done in two steps. First we can integrate over the coordinate α. This yields a nine dimensional space of the form $AdS_6 \times S^3$. We can then reduce on S^3 to six dimensions, while gauging its isometry group. Roman's construction is based on gauging an $SU(2)$ subgroup of the $SO(4)$ isometry group.

The massive Type IIA supergravity action in the (string frame) goes like

$$l_s^{-8} \int \sqrt{-g} e^{-2\Phi} \mathcal{R} \sim Q_4^{5/2} , \tag{3.12}$$

suggesting that the number of degrees of freedom goes like $Q_4^{5/2}$ in the regime where it is an applicable description. Terms in the Type I' action coming from the D8 brane DBI action turn out to be of the same order. Viewed from M theory point of view, we expect the corrections to the supergravity action to go like $l_p^3 \sim l_s^3 e^\Phi \sim 1/Q_4$, where l_p is the eleven-dimensional Planck length. This seems to suggest that the field theory has a $1/Q_4$ expansion at large Q_4. For example, the one-loop correction of the form

$$l_s^{-8} \int \sqrt{-g} l_s^6 \mathcal{R}^4 \sim Q_4^{1/2} \tag{3.13}$$

is suppressed by Q_4^2 compared to the tree-level action.

According to the AdS/CFT correspondence the spectrum of chiral primary operators of the fixed point theory can be derived from the spectrum of Kaluza-Klein excitations of massive Type IIA supergravity on the background (3.9). We will not carry out the detailed analysis here but make a few comments. The operators fall into representations of the $F(4)$ supergroup. As in the case of the six dimensional $(0,1)$ fixed point with E_8 global symmetry [18] we expect E_{N_f+1} neutral operators to match the Kaluza-Klein reduction of fields in the bulk geometry, and E_{N_f+1} charged operators to match the Kaluza-Klein reduction of fields living on the boundary. Among the E_{N_f+1} neutral operators we expect to have dimension $3k/2$ operators of the type $Tr\phi^k$ where ϕ is a complex scalar in the hypermultiplet, which parametrize the Higgs branch of the theory. Like in [18] we do not expect all these operators to be in short multiplets. We expect that those in long multiplets will generically receive $1/Q_4$ corrections to their anomalous dimensions. Unlike the hypermultiplet, the vector multiplet in five dimensions is not a representation of the superconformal group $F(4)$. Therefore, we do not expect Kaluza-Klein excitations corresponding to neutral operators of the type $Tr\varphi^k$ where φ is a real scalar in the vector multiplet, which parametrizes the Coulomb branch of the theory. Among the E_{N_f+1} charged operators we expect to have the dimension four E_{N_f+1} global symmetry currents that couple to the massless E_{N_f+1} gauge fields on the boundary.

Acknowledgement

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