CLIC Note 400

OPEN-99-009
28/04/99

A trajectory correction based on Multi-step Lining-up for the CLIC main Linac.

E. T. d'Amico, G. Guignard

April 28, 1999

Abstract

In the CLIC main linac it is very important to minimise the trajectory excursion and consequently the emittance dilution in order to obtain the required luminosity. Several algorithms have been proposed and lately the ballistic method has proved to be very effective. The trajectory method described in this note retains the main advantages of the latter while adding some interesting features. It is based on the separation of the unknown variables like the quadrupole misalignments, the offset and slope of the injection straight line and the misalignments of the beam position monitors (BPM). This is achieved by referring the trajectory relatively to the injection line and not to the average pre-alignment line and by using two trajectories each corresponding to slightly different quadrupole strengths. A reference straight line is then derived onto which the beam is bent by a kick obtained by moving the first quadrupole. The other quadrupoles are then aligned on that line. The quality of the correction depends mainly on the BPM's and micro-movers' resolution and on the stability of the quadrupole strengths which should be at least of the order of 0.05 %. Although the beam follows a broken straight line, its offset from the center of the quadrupoles is typically 1.5 μm r.m.s.

1 Introduction

Given the importance of the trajectory correction and the alignment tolerances in linear colliders, various methods of correction and beam-based re-alignment have been proposed and simulated numerically according to the design specificity. Beam based alignment algorithms studied at NLC [1] [2] pointed out the benefits of using the beam to re-align the quadrupoles on a straight line defined over sections of the linac. In NLC, the endpoint beam position monitors (BPM) determine this reference line and the relative off-set of the BPMs with respect to the quadrupole centers are assumed to be known within small tolerances. In CLIC, where the BPMs are not attached to the quadrupoles as in NLC but independently pre-aligned, two studies of trajectory correction are going on, the ballistic method [3] and the multi-step lining-up described in this paper. To find the reference line defined by the endpoint BPMs, the ballistic method steers the beam over a bin of quadrupoles which are all switched off except the first, and measures the off-sets of all the intermediate BPMs relative to this line. The quadrupoles, switched on again, are then re-aligned around the same line via a few-to-few correction. Combining this correction with emittance bumps proved through numerical simulations to be sufficient for achieving the small emittance growth required for the CLIC performance [4].

The Multi-step Lining-up (ML) method has in common with the NLC correction and the ballistic method the idea to align as well as possible the main components of the linac on a straight line defined by the beam. In addition ML is based on the observation that to align the quadrupoles on a reference line is more important that the real choice and straightness of this line, provided the latter is not too far away from the average-alignment line fixed by the positions of the components resulting from the pre-alignment. Instead of switching-off the quadrupoles, the Multi-step Lining-up relies on a small change of their strengths (several %) in a bin or section and on measurements of trajectory differences, like for instance in the dispersion-free correction [5]. This has the advantages to minimize the heat-load variations, to make the remanent-field and hysteresis effects negligible, to keep the beam focused and not too distant from the center of the elements (reducing wakefields effects) and eventually to allow on-line corrections by matching the detuned section to the rest of the linac while the results still remain only dependent on the BPM resolution as in the ballistic method. As a first step, the measured trajectory difference is used in ML to work out the off-sets of the quadrupoles with respect to a virtual (bin-)injection line. In the second step, a least squares fit of the BPM measurements allows a good estimation of the injection parameters. This permits the definition of a reference line onto which the quadrupoles are actually moved. Correcting for the estimated injection angle by moving the first quadrupole of the bin sends the beam on this reference line with an accuracy depending on the various resolutions. As a last step in the correction, the BPMs sitting at the head of each girder are also displaced toward the reference line (by nullifying their measurements). This also aligns all the girders, reducing the cavity misalignments to their scattering with respect to each girder. Because the optics model is perturbed by the wakefields and other imperfections, the correction is an iterative process rapidly converging [6]. Investigations show that with acceptable tolerances on the measurement- and micromover-resolution, acquisition noise and resolution of the quadrupole power supplies, the ML trajectory correction allows a very good control of the beam offsets with respect to the center of the quadrupoles. Numerical investigations of the emittance blow-up after Multi-step Lining-up will be done as soon as the algorithm is entirely implemented in a tracking code.

2 Possible procedure of correction based on ML

In this section, a correction procedure based on the Multi-step Lining-up method is proposed. As the result of the survey all the components of the linac are assumed to be randomly scattered around the so-called averaged-prealignment line (see Fig.5). Numerical modeling indicates that acceptable r.m.s. offset amplitudes are of the order of 50 μm r.m.s. for both the quadrupoles and BPMs. For the accelerating structures (cavities) placed on a single girder, the relevant quantity is their pre-alignment offset with respect to the BPM sitting on the same girder. This has to be within 2 and 10 μm r.m.s. . The relative misalignments between girders are given by those of the BPMs. For the first correction a single bunch is injected. Bins of N quadrupoles are successively dealt with in the linac. N is optimized for an accurate definition of the reference line and a good least squares fit of the measurements over the BPMs used to estimate the injection parameters. Preliminary modeling shows that N can be as large as 50 or more, which opens the way to a correction section which could in principle be as long as a linac sector, defined by the FODO lattice being constant. The beam is injected into the nominal lattice (focal distance f_1) and the beam positions are measured at each BPM. To gain a factor 10 on the resolution and acquisition errors, measurements should be averaged over typically 100 pulses (about 1 s). The beam is then injected into a lattice only slightly detuned in the section considered and betatron-matched to the rest of the linac. The focal distance increment by ~ 5 % has been found to be sufficient because the difference between the two trajectories is enhanced by the phase advance shift (see Fig.1 and Fig.2). The beam positions are again measured and averaged over 100 pulses.

The ML algorithm is now applied following these steps:

1. Isolate the contribution of the quadrupole misalignments by building up the differences between the two trajectories. Solve the obtained triangular system of $N-1$ equations and $N-1$ unknowns which are the quadrupole displacements $d_{q,k}$ with respect to the injection line. Actually the results will be estimations $\hat{d}_{q,k}$ because the trajectories differences are known only up to the resolution of the BPMs. Restore the nominal lattice with focal length f_1 and suppress the betatron-matching.

2. Subtract the estimated contribution of the quadrupole misalignments from the nominal trajectory BPM readings. Assuming that the BPM misalignments are randomly distributed, these measurements scatter around the injection straight line. Estimates of its offset and slope are obtained by a least squares fit.

3. Compute the change of the slope (kick) that would steer the beam towards the average-prealignment line by using the estimated injection parameters. The obtained straight line is the reference line on which the quadrupoles

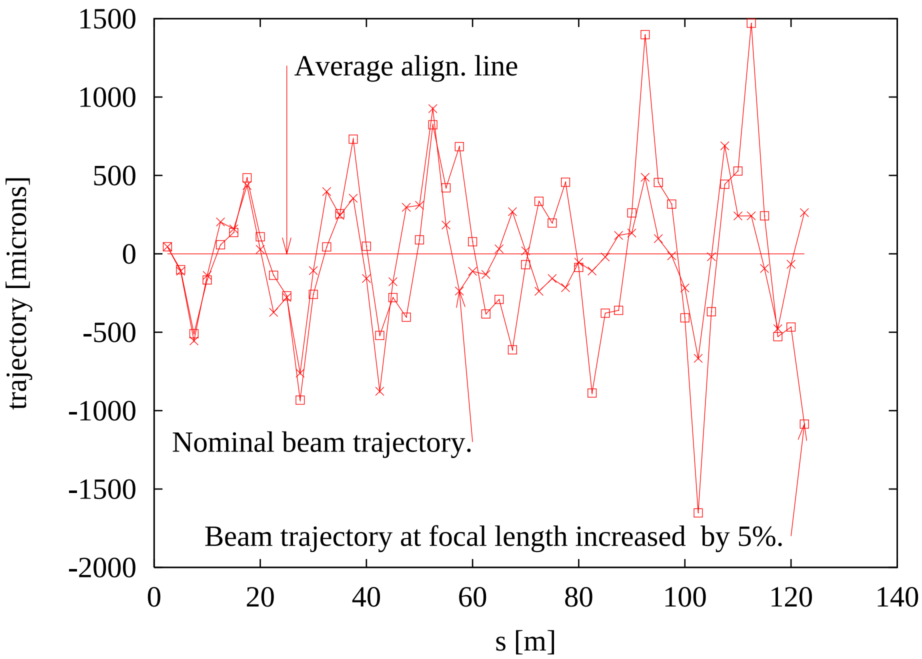

Figure 1: Beam trajectories at nominal strength and at strength decreased by 5 %.

should be aligned. To achieve this, the first quadrupole of the bin should be displaced in order to apply the computed kick. All the other quadrupoles should then be moved in order to align them onto the reference line.

4. Move all the BPMs sitting at the head of each girder to the reference line, by nullifying their reading within their resolution. This is a kind of "calibration" of the measurement system.

Moving the BPMs will also realign the girders. The cavity position scattering is consequently reduced to their starting pre-alignment imperfections on a single girder. The wake-field effects are accordingly decreased limiting the emittance growth. At least one iteration of this process is necessary because the lattice model used in the algorithm does not include the wake-fields. The procedure described here must be repeated section after section over the whole linac, before the full-intensity beam can be injected. Note that the bins can likely coincide with the linac sectors (12 for 3 TeV c.m.) and the matching insertions be used for bin matching when f is varied.
Simulations assume a BPM resolution of 0.1 μm, an acquisition noise of 0.1 μm, a micro-mover resolution of 0.5 μm and a precision of the quadrupole strength of $\Delta f/f = 5 \times 10^{-4}$. Results indicate that the reference line deviation with respect to the average prealignment line is of the order of 30 μm r.m.s. at the end of a section of 125 m. The most important result is that, although the actual line followed by the beam does not exactly coincide with the reference line, the remaining offsets between the beam and the quadrupole-centers (independent of the initial pre-alignment and of the line parameters) is of the order of $\sim$ 1.5 μm

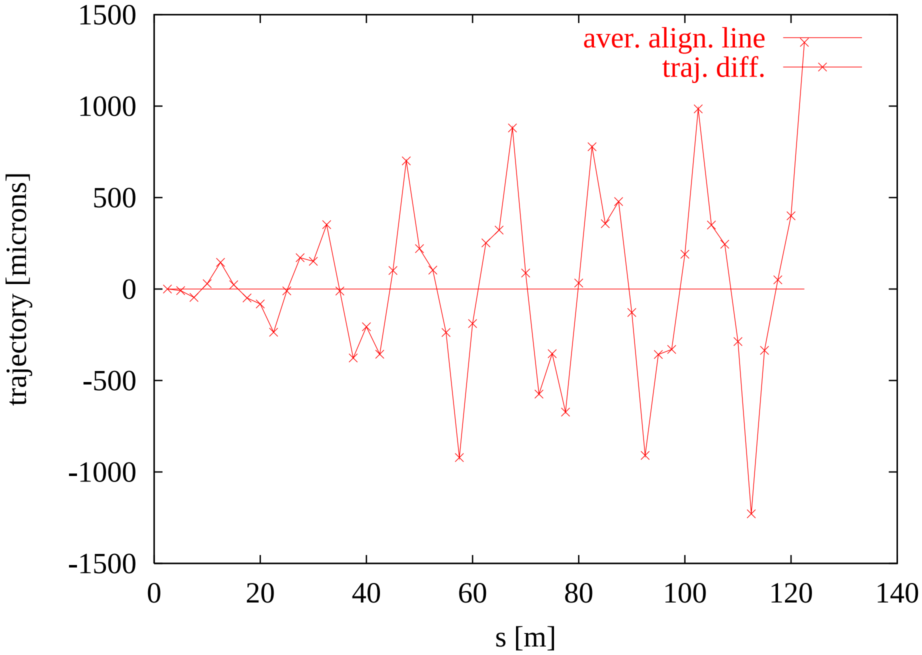

Figure 2: Difference between the two beam trajectories.

r.m.s. (see Fig.3 and the zooming of beam trajectory inside the quadrupoles in Fig.4).

Turning to time-dependent drifts of the components after the first correction has been completed, the BPMs may begin to measure non-zero deviations if the beam does not follow the change of the geometry. An on-line one-to-one feedback can be applied in the case of smooth displacements like in an ATL-model. When BPM measurements indicate short-range position variations or the beam moving away from the linac component centers, the ML correction has to be repeated. It is hoped to apply it on-line with only small focal changes and sector-matching, without interrupting full-beam acceleration. That has to be fully simulated with a tracking program e.g. [7] before drawing a clear conclusion and re-optimizing the proposed procedure according to the results.

Considering now the first bin or linac-sector, the reference line obtained by ML can be used to correct for the injection jitter while maintaining the same trajectory over the rest of the linac. For this, it is necessary to apply two correction kicks (based on the "pantograph" principle), near the first quadrupole and for instance the one in the middle, in order to maintain the beam on a constant trajectory through the BPMs following the second quadrupole (by keeping their reading equal to zero). To use this correction as a feedback may require two fast kickers near the two quadrupoles mentioned while static corrections of the injection can be done by actually moving the two quadrupoles.

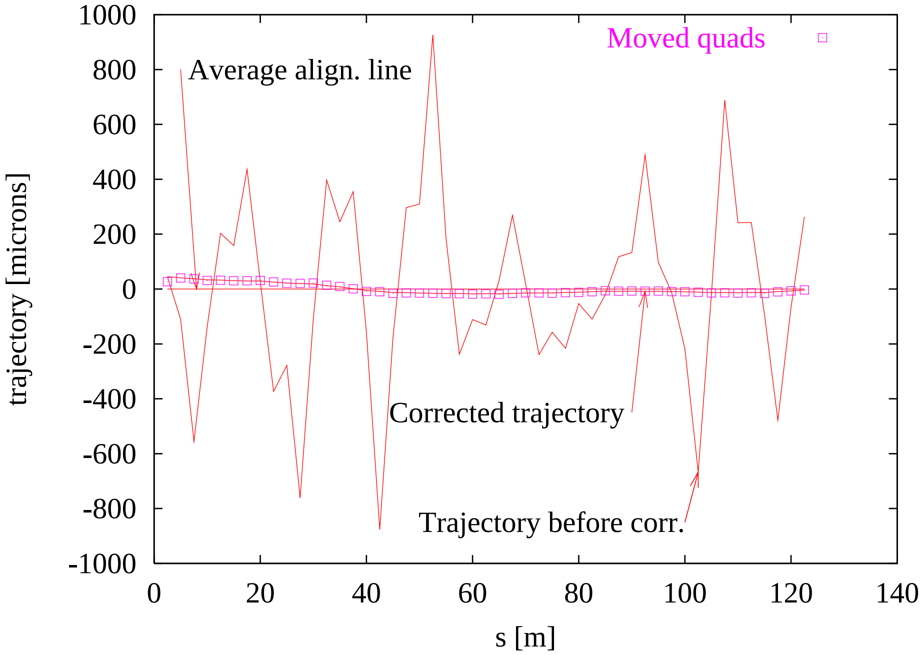

Figure 3: Beam trajectory before and after ML correction.

3 Description of the method

The magnetic quadrupoles and the beam position monitors (BPM) of the CLIC main linac are assumed to be pre-aligned around an ideal line which will be referred to as the average pre-alignment line from now on. For sake of simplicity the two transverse planes are considered independent and the method is developed only in one plane. The optics is a FODO with a focal length f and a distance between quadrupoles L. The BPM is placed at a distance l in front of the quadrupole. Considering a bin of N quadrupoles, let $\delta_{q,i}$ and $\delta_{p,i}$ be the offsets of the i-th quadrupole and i-th BPM from the average pre-alignment line respectively. The beam entering the first quadrupole follows the injection line defined by the injection offset x_{inj} and slope s_{inj} (see Fig.5). It should be noted that the first BPM and last quadrupole are not relevant for the method because the measurement of the former is of no use concerning the following quadrupoles and the effect of the latter can not be detected.

The beam trajectory between quadrupoles is a straight line with slope defined by the expression :

$$s_i = s_{i-1} + \frac{(-1)^i}{f}(t_i + l s_{i-1} - \delta_{q,i}) \tag{3.1}$$

where s_i is the beam slope between the i-th quadrupole and the next and t_i is the offset of the beam from the average pre-alignment line at the i-th BPM. The trajectory value t_i can be expressed iteratively by :

$$t_i = t_{i-1} + l s_{i-2} + (L - l) s_{i-1} \tag{3.2}$$

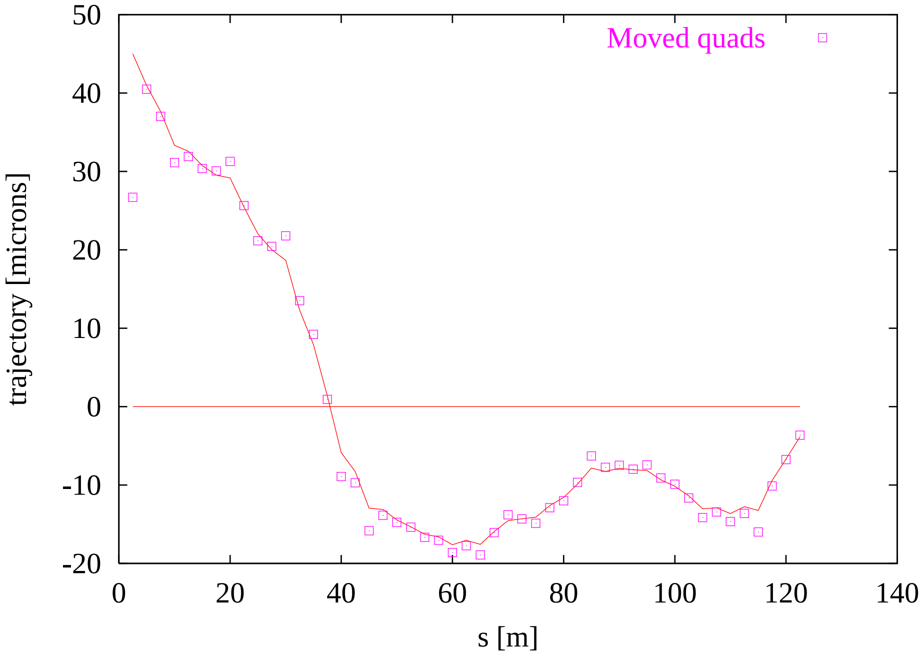

Figure 4: Zoom of the beam trajectory after ML correction.

the first value being $t_1 = x_{inj} + s_{inj}(L - l)$ by definition. Summing up term by term, we get :

$$t_i = x_{inj} + Ls_{inj} + L\sum_{j=1}^{i-1} s_j - ls_{i-1}, \qquad i = 2, \ldots, N \tag{3.3}$$

Replacing (3.3) into (3.1) we obtain :

$$s_i = s_{i-1} + \frac{L}{f}(-1)^i \sum_{j=1}^{i-1} s_j + \frac{(-1)^i}{f}(x_{inj} + Ls_{inj} - \delta_{q,i}), \qquad i = 2, \ldots, N-1 \tag{3.4}$$

The first slope s_1 being given by :

$$s_1 = s_{inj}\left(1 - \frac{L}{f}\right) - \frac{1}{f}(x_{inj} - \delta_{q,1}) \tag{3.5}$$

The iteration (3.4) show that the slope s_i can be written under the form :

$$s_i = x_{inj}a_i + s_{inj}b_i + c_i, \qquad i = 2, \ldots, N-1 \tag{3.6}$$

where a_i and b_i depend only from the optics parameters (L, l, f) while c_i is also a function of the quadrupole displacements $\delta_{q,i}$. This can be readily seen by replacing (3.6) in (3.4) and observing that the obtained equation should be valid for all the values of x_{inj} and s_{inj} :

$$a_i = a_{i-1} + (-1)^i \frac{L}{f} \sum_{j=1}^{i-1} a_j + \frac{(-1)^i}{f}, \qquad a_1 = -\frac{1}{f} \tag{3.7}$$

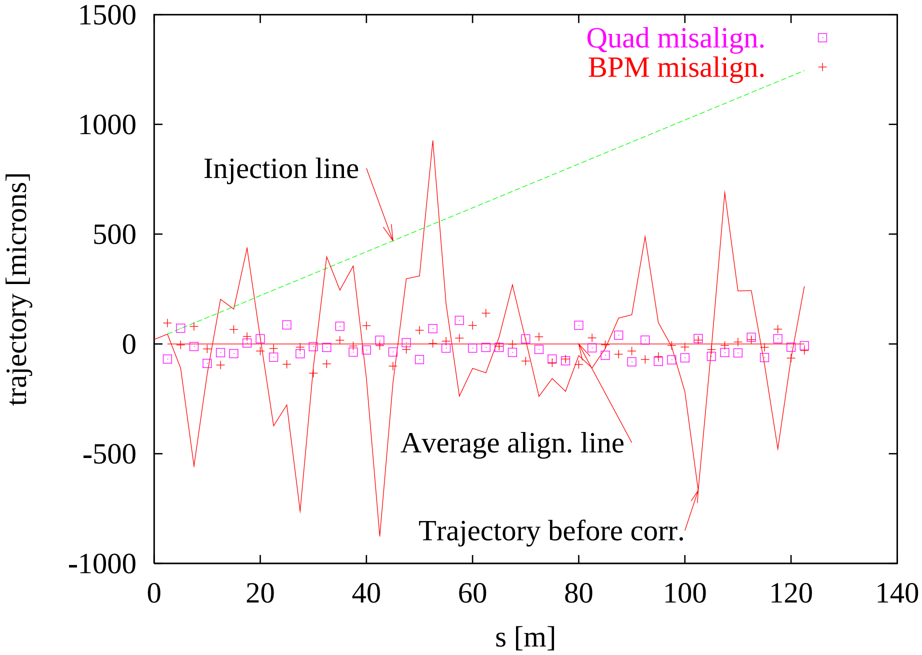

Figure 5: Quadrupoles and BPMs misalignments for a correction bin. Beam trajectory and the injection straight line are also drawn.

$$b_i = b_{i-1} + (-1)^i \frac{L}{f} \sum_{j=1}^{i-1} b_j + (-1)^i \frac{L}{f}, \qquad b_1 = 1 - \frac{L}{f} \tag{3.8}$$

$$c_i = c_{i-1} + (-1)^i \frac{L}{f} \sum_{j=1}^{i-1} c_j - (-1)^i \frac{\delta_{q,i}}{f}, \qquad c_1 = \frac{\delta_{q,1}}{f} \tag{3.9}$$

It is easy to see that c_i can be expressed as a linear sum of the quadrupole displacements :

$$c_i = \sum_{k=1}^{i} p_{i,k} \delta_{q,k}, \qquad p_{1,1} = \frac{1}{f} \tag{3.10}$$

The coefficients $p_{i,k}$ may be obtained by replacing (3.10) in (3.9) :

$$\sum_{k=1}^{i} p_{i,k} \delta_{q,k} = \sum_{k=1}^{i-1} p_{i-1,k} \delta_{q,k} + (-1)^i \frac{L}{f} \sum_{j=1}^{i-1} \left(\sum_{k=1}^{j} p_{j,k} \delta_{q,k} \right) - (-1)^i \frac{\delta_{q,i}}{f} \tag{3.11}$$

The double sum in (3.11) can be written in the following way :

$$\sum_{j=1}^{i-1} \left(\sum_{k=1}^{j} p_{j,k} \delta_{q,k} \right) = \sum_{k=1}^{i-1} \left(\sum_{j=k}^{i-1} p_{j,k} \right) \delta_{q,k} \tag{3.12}$$

Replacing (3.12) in (3.11) and reordering according the quadrupole displacements, we obtain :

$$\sum_{k=1}^{i-1}\left(p_{i,k}-p_{i-1,k}-(-1)^i\frac{L}{f}\sum_{j=k}^{i-1}p_{j,k}\right)\delta_{q,k}+\left(p_{i,i}+\frac{(-1)^i}{f}\right)\delta_{q,i}=0 \qquad (3.13)$$

This equation should be valid for all values of the quadrupole displacements which means that their coefficients should be identically null :

$$\begin{aligned} p_{i,i} &= -\frac{(-1)^i}{f} \\ p_{i,k} &= p_{i-1,k}+(-1)^i\frac{L}{f}\sum_{j=k}^{i-1}p_{j,k}, \quad k=1,...,i-1 \end{aligned} \qquad (3.14)$$

These iterative relations make possible to compute the coefficients $p_{i,j}$ from the given optics parameters (L, l, f). Replacing (3.10) into (3.6) and then the obtained expression of the slope s_j into (3.3) we get :

$$\begin{aligned} t_i =& x_{inj}\left(1+L\sum_{j=1}^{i-1}a_j-la_{i-1}\right)+s_{inj}\left(L+L\sum_{j=1}^{i-1}b_j-lb_{i-1}\right)+ \\ & L\sum_{j=1}^{i-1}\left(\sum_{k=1}^{j}p_{j,k}\delta_{q,k}\right)-l\sum_{k=1}^{i-1}p_{i-1,k}\delta_{q,k}, \qquad i=2,\ldots,N \end{aligned} \qquad (3.15)$$

Replacing the double sum by the expression (3.12) and defining

$$u_{i,k}=L\sum_{j=k}^{i-1}p_{j,k}-lp_{i-1,k}, \qquad i=2,\ldots,N \qquad k=1,\ldots,i-1 \qquad (3.16)$$

the expression (3.15) can be rewritten as

$$t_i=x_{inj}\left(1+L\sum_{j=1}^{i-1}a_j-la_{i-1}\right)+s_{inj}\left(L+L\sum_{j=1}^{i-1}b_j-lb_{i-1}\right)+\sum_{k=1}^{i-1}u_{i,k}\delta_{q,k} \qquad (3.17)$$

The coefficients of the unknown variables x_{inj}, s_{inj} can be further developed by observing that the trajectory is $t_i = x_{inj} + s_{inj}(iL - l)$ if the quadrupole offsets are $\delta_{q,k} = x_{inj} + s_{inj}kL$. Replacing these quadrupole offsets in (3.17) and reordering according to x_{inj} and s_{inj} we get for this special case :

$$\begin{aligned} & x_{inj}\left(L\sum_{j=1}^{i-1}a_j-la_{i-1}+\sum_{k=1}^{i-1}u_{i,k}\right)+ \\ & s_{inj}\left(L+L\sum_{j=1}^{i-1}b_j-lb_{i-1}+L\sum_{k=1}^{i-1}ku_{i,k}-iL+l\right)=0 \end{aligned} \qquad (3.18)$$

which should be valid for all values of the injection parameters. This means that their coefficients should be identically null providing two useful expressions

$$\begin{aligned} L\sum_{j=1}^{i-1} a_j - l a_{i-1} &= -\sum_{k=1}^{i-1} u_{i,k} \\ L + L\sum_{j=1}^{i-1} b_j - l b_{i-1} &= iL - l - L\sum_{k=1}^{i-1} k u_{i,k} \end{aligned} \tag{3.19}$$

Replacing these relations in (3.17) we obtain

$$t_i = x_{inj}\left(1 - \sum_{k=1}^{i-1} u_{i,k}\right) + s_{inj}\left(iL - l - L\sum_{k=1}^{i-1} k u_{i,k}\right) + \sum_{k=1}^{i-1} u_{i,k}\delta_{q,k} \tag{3.20}$$

and separating the terms which do not depend on the focal length :

$$\boxed{t_i = x_{inj} + s_{inj}(iL - l) + \sum_{k=1}^{i-1} u_{i,k}\delta_{q,k} - x_{inj}\sum_{k=1}^{i-1} u_{i,k} - s_{inj}L\sum_{k=1}^{i-1} k u_{i,k}} \tag{3.21}$$

This is the principal equation giving the trajectory amplitude in the i-th BPM due to injection offset x_{inj} and slope s_{inj} and to the quadrupoles offsets $\delta_{q,k}$ from the injection straight line. The coefficients $u_{i,k}$ are the elements of the transfer matrix depending only on the parameters (f, L, l) of the FODO line. The subtraction of the sums from t_i obviously provides the beam injection line identical to the ballistic line in absence of errors and a virtual straight line close to it in the presence of these errors. This virtual line is used to compute the reference line on which the quadrupoles are re-aligned.
Inside each BPM the offset of the trajectory from its center is

$$\begin{aligned} t_{p,i}(f) = & x_{inj} + s_{inj}(iL - l) + \sum_{k=1}^{i-1} u_{i,k}(f)\delta_{q,k} - \\ & x_{inj}\sum_{k=1}^{i-1} u_{i,k}(f) - s_{inj}L\sum_{k=1}^{i-1} k u_{i,k}(f) - \delta_{p,i} \end{aligned} \tag{3.22}$$

where the dependence on the focal length is explicitely shown. To get rid of the BPM displacements $\delta_{p,i}$ another beam trajectory is generated by slightly increasing the quadrupole focal length. The difference between the two trajectories inside each BPM, one at the nominal focal length f_1 and one at the focal length f_2, is

$$\begin{aligned} t_{p,i}(f_1) - t_{p,i}(f_2) = & \sum_{k=1}^{i-1}\left[u_{i,k}(f_1) - u_{i,k}(f_2)\right]\delta_{q,k} - \\ & x_{inj}\sum_{k=1}^{i-1}\left[u_{i,k}(f_1) - u_{i,k}(f_2)\right] - s_{inj}L\sum_{k=1}^{i-1} k\left[u_{i,k}(f_1) - u_{i,k}(f_2)\right] \\ & i = 2, \ldots, N \end{aligned} \tag{3.23}$$

This linear systems consists of $N-1$ equations and $N+1$ unknowns ($N-1$ quadrupole offsets and the two injection parameters x_{inj} and s_{inj}). It is underdetermined but its determinant is null because the coefficients of x_{inj} and s_{inj} are linear combinations of the coefficients of the $\delta_{q,k}$. Thus we can express the $\delta_{q,k}$ as linear combinations of x_{inj}, s_{inj} and of the new unknown variables $d_{q,k}$, defined by :

$$\delta_{q,k} = d_{q,k} + x_{inj} + s_{inj} kL \tag{3.24}$$

This is equivalent to change the reference system of the quadrupole offsets by referring them to the injection line itself. The offset of the trajectory from the center of each BPM becomes :

$$\boxed{t_i = x_{inj} + s_{inj}(iL - l) + \sum_{k=1}^{i-1} u_{i,k} d_{q,k}} \tag{3.25}$$

We obtain $d_{q,k}$ iteratively from the triangular linear system of $N-1$ equations and $N-1$ unknowns $d_{q,k}$:

$$t_{p,i}(f_1) - t_{p,i}(f_2) = \sum_{k=1}^{i-1} \left[u_{i,k}(f_1) - u_{i,k}(f_2)\right] d_{q,k} \qquad i = 2, \ldots, N \tag{3.26}$$

which has the following solution :

$$\begin{aligned} d_{q,1} =& \frac{t_{p,2}(f_1) - t_{p,2}(f_2)}{u_{2,1}(f_1) - u_{2,1}(f_2)} \\ d_{q,i} =& \frac{1}{u_{i+1,i}(f_1) - u_{i+1,i}(f_2)} \Big\{ t_{p,i+1}(f_1) - t_{p,i+1}(f_2) - \\ & \sum_{k=1}^{i-1} \left[u_{i+1,k}(f_1) - u_{i+1,k}(f_2)\right] d_{q,k} \Big\}, \qquad i = 2, \ldots, N-1 \end{aligned} \tag{3.27}$$

The actual BPM's reading is given by :

$$t_{m,i}(f) = t_{p,i}(f) + \varepsilon_{p,i}(f) \tag{3.28}$$

The error $\varepsilon_{p,i}(f)$ is due to the limited BPM's resolution. Thus the distance of each quadrupole centers from the injection line can be estimated by :

$$\hat{d}_{q,i} = d_{q,i} + \varepsilon_{d,i} \tag{3.29}$$

the corresponding error depending only on the BPM resolutions. Then the sum in (3.25) can be estimated by :

$$\sum_{k=1}^{i-1} u_{i,k}(f) \hat{d}_{q,k} = \sum_{k=1}^{i-1} u_{i,k}(f) d_{q,k} + \sum_{k=1}^{i-1} u_{i,k}(f) \varepsilon_{d,k} \tag{3.30}$$

and subtracting it from each BPM's measurement a virtual trajectory is obtained :

$$\begin{aligned} t_{v,i}(f_1) &= t_{m,i}(f_1) - \sum_{k=1}^{i-1} u_{i,k}(f_1)\hat{d}_{q,k} \\ &= t_{p,i}(f_1) + \varepsilon_{p,i}(f_1) - \sum_{k=1}^{i-1} u_{i,k}(f_1) d_{q,k} - \sum_{k=1}^{i-1} u_{i,k}(f_1)\varepsilon_{d,k} \\ &= x_{inj} + s_{inj}(iL - l) - \delta_{p,i} + \varepsilon_{p,i}(f_1) - \sum_{k=1}^{i-1} u_{i,k}(f_1)\varepsilon_{d,k} \end{aligned} \tag{3.31}$$

This virtual trajectory is very close (identical when all $\varepsilon_{d,k} = 0$) to the measured trajectory if all the quadrupoles are switched off as in the ballistic method. Given a group of N quadrupoles a reference line can be defined once the injection offset and slope have been estimated. The simplest way is to obtain them from the first and last BPM readings as in the ballistic method :

$$\begin{aligned} \hat{s}_{inj} &= \frac{t_{v,N}(f_1) - t_{v,1}(f_1)}{L(N-1)} \\ \hat{x}_{inj} &= t_{v,1}(f_1) - (L-l)\hat{s}_{inj} \end{aligned} \tag{3.32}$$

Noting that the virtual trajectory is a straight line modified by small random errors, a better estimation is obtained by minimizing the sum of the squares of these errors. After a little bit of algebra we get :

$$\begin{aligned} \hat{x}_{inj} &= \frac{12}{L(N^2-1)} \left\{ \frac{(N+1)}{2} \left[\frac{L(2N+1)}{3} - l \right] \Sigma_1 - \left[\frac{L(N+1)}{2} - l \right] \Sigma_2 \right\} \\ \hat{s}_{inj} &= \frac{12}{L(N^2-1)} \left[\Sigma_2 - \frac{N+1}{2} \Sigma_1 \right] \end{aligned} \tag{3.33}$$

with :

$$\Sigma_1 = \frac{1}{N} \sum_{i=1}^{N} t_{v,i}(f_1), \qquad \Sigma_2 = \frac{1}{N} \sum_{i=1}^{N} i t_{v,i}(f_1)$$

The reference straight line on which the beam should be directed is the straight line which passes inside the first quadrupole at the estimated offset $\hat{x}_{inj} + \hat{s}_{inj} L$ from the average pre-alignment line and intersects the latter inside the last quadrupole of the bin. Its slope s_{ref} is given by :

$$s_{ref} = -\frac{\hat{x}_{inj} + \hat{s}_{inj} L}{(N-1)L} \tag{3.34}$$

It is possible to bend the beam in the direction of the reference line by moving the first quadrupole of the bin by the displacement :

$$\Delta_{q,1} = -\hat{d}_{q,1} + f\Delta s \tag{3.35}$$

where :

$$\Delta s = s_{ref} - \hat{s}_{inj} = -\frac{\hat{x}_{inj} + \hat{s}_{inj} NL}{(N-1)L}$$

The other $N-2$ quadrupoles should be aligned onto the reference line by displacing them by the quantities :

$$\Delta_{q,i} = -\hat{d}_{q,i} + \Delta s(i-1)L = -\hat{d}_{q,i} - (i-1)\frac{\hat{x}_{inj} + \hat{s}_{inj} NL}{(N-1)}, \qquad i = 2, \dots, N-1 \tag{3.36}$$

It is important to note that the reference line is different from the broken straight line followed by the beam. Actually already at the first quadrupole in the bin the reference line is displaced from the beam line by $\hat{x}_{inj} - x_{inj} + (\hat{s}_{inj} - s_{inj})L$ However it will be shown in the rest of this section that the differences between the beam and the centers of all the quadrupoles in the bin but the first one are quite small because they depend only on the BPMs measurement resolution. Let $\tilde{t}_i$ and $\tilde{\delta}_{q,i}$ be the offsets of the beam in the i-th quadrupole and of its center from the average pre-alignment line respectively once the quadrupoles have been moved.

In the first quadrupole we have :

$$\begin{aligned} \tilde{t}_1 &= x_{inj} + s_{inj} L \\ \tilde{\delta}_{q,1} &= \delta_{q,1} - \hat{d}_{q,1} + f\Delta s \end{aligned} \tag{3.37}$$

By making use of the definitions (3.29) and (3.24), the difference $\tilde{t}_1 - \tilde{\delta}_{q,1}$ can be expressed by :

$$\tilde{t}_1 - \tilde{\delta}_{q,1} = f\Delta s - \varepsilon_{d,1} \tag{3.38}$$

This difference is dominated by the term $f\Delta s$ which shows the effect of the 'kink' at the beginning of each bin. This effect is reduced by taking bins as long as possible (i.e. a full sector) because less 'kinks' will occur and the bin injection offset and slope being better estimated, the reference line is closer to the average pre-alignment line. The offset of the i-th quadrupole ($i = 2, \dots, N-1$) from the average pre-alignment line is given by :

$$\tilde{\delta}_{q,i} = \delta_{q,i} - \hat{d}_{q,i} + \Delta s(i-1)L = x_{inj} + s_{inj} iL + \Delta s(i-1)L - \varepsilon_{d,i} \tag{3.39}$$

where the definitions (3.29) and (3.24) have been used. The position of the beam in the i-th quadrupole ($i = 2, \dots, N-1$) relative to the average pre-alignment line can be obtained from (3.25) where the coefficients $u_{i,k}$ are computed with $l = 0$. Let us call the new coefficients $u'_{i,k}$. It is easy to prove :

$$u'_{i,k} = L \sum_{j=k}^{i-1} p_{j,k}, \qquad i = 2, \dots, N, \qquad k = 1, \dots, i-1 \tag{3.40}$$

Thus the required position is expressed by :

$$\tilde{t}_i = x_{inj} + iLs_{inj} + \sum_{k=1}^{i-1} u'_{i,k} \left[\tilde{\delta}_{q,k} - x_{inj} - s_{inj} kL\right] \qquad i = 2, \dots, N-1 \tag{3.41}$$

It is necessary to distinguish the case $i = 2$ from the rest because $\tilde{\delta}_{q,1}$ has a special expression. Thus using the expression (3.37) :

$$\begin{aligned}\tilde{t}_2 &= x_{inj} + 2Ls_{inj} + u'_{2,1}\left[\tilde{\delta}_{q,1} - x_{inj} - s_{inj}L\right] \\ &= x_{inj} + 2Ls_{inj} + u'_{2,1}\left[f\Delta s - \varepsilon_{d,1}\right]\end{aligned} \tag{3.42}$$

Thus the difference $\tilde{t}_2 - \tilde{\delta}_{q,2}$ is expressed by :

$$\tilde{t}_2 - \tilde{\delta}_{q,2} = \Delta s\left(u'_{2,1}f - L\right) + \varepsilon_{d,2} - u'_{2,1}\varepsilon_{d,1} \tag{3.43}$$

Using the expressions (3.40) and (3.14) we get $u'_{2,1} = Lp_{1,1} = \frac{L}{f}$. Replacing it in (3.43) we obtain :

$$\tilde{t}_2 - \tilde{\delta}_{q,2} = \varepsilon_{d,2} - u'_{2,1}\varepsilon_{d,1} \tag{3.44}$$

which shows the remarquable results that the parameters of the reference straight line have no effect on the position of the beam inside the second quadrupole. This position depend only on the resolution of the first and second BPMs. We will now prove this result for the other quadrupoles. The expression (3.41) can be written :

$$\begin{aligned}\tilde{t}_i =& x_{inj} + iLs_{inj} + u'_{i,1}\left(f\Delta s - \varepsilon_{d,1}\right) + \sum_{k=2}^{i-1} u'_{i,k}\left[\tilde{\delta}_{q,k} - x_{inj} - s_{inj}kL\right] \\ =& x_{inj} + iLs_{inj} + u'_{i,1}\left(f\Delta s - \varepsilon_{d,1}\right) + \sum_{k=2}^{i-1} u'_{i,k}\left[\Delta s(k-1)L - \varepsilon_{d,k}\right] \\ =& x_{inj} + iLs_{inj} + \Delta s\left[u'_{i,1}f + L\sum_{k=2}^{i-1} u'_{i,k}k - L\sum_{k=2}^{i-1} u'_{i,k}\right] - \\ & u'_{i,1}\varepsilon_{d,1} - \sum_{k=2}^{i-1} u'_{i,k}\varepsilon_{d,k}, \qquad i = 3, \ldots, N-1\end{aligned} \tag{3.45}$$

The relations (3.19) with $l = 0$ provide the following expressions :

$$\begin{aligned}\sum_{k=1}^{i-1} u'_{i,k} &= -L\sum_{j=1}^{i-1} a_j \\ \sum_{k=1}^{i-1} u'_{i,k}k &= (i-1) - \sum_{j=1}^{i-1} b_j\end{aligned} \tag{3.46}$$

from which one gets :

$$\begin{aligned}\sum_{k=2}^{i-1} u'_{i,k} &= -L\sum_{j=1}^{i-1} a_j - u'_{i,1} \\ \sum_{k=2}^{i-1} u'_{i,k}k &= (i-1) - \sum_{j=1}^{i-1} b_j - u'_{i,1}\end{aligned} \tag{3.47}$$

Replacing them in (3.45) and using (3.40) we get :

$$\begin{aligned}\tilde{t}_i =& x_{inj} + iLs_{inj} + \Delta sL\left(f\sum_{j=1}^{i-1} p_{j,1} + i - 1 - \sum_{j=1}^{i-1} b_j - u_{i,1}^{'} + L\sum_{j=1}^{i-1} a_j + u_{i,1}^{'}\right) - \\ & u_{i,1}^{'}\varepsilon_{d,1} - \sum_{k=2}^{i-1} u_{i,k}^{'}\varepsilon_{d,k} \\ =& x_{inj} + iLs_{inj} + \Delta sL\left[i - 1 + \sum_{j=1}^{i-1}\left(fp_{j,1} - b_j + La_j\right)\right] - \sum_{k=1}^{i-1} u_{i,k}^{'}\varepsilon_{d,k}\end{aligned} \tag{3.48}$$

Let us buildup $b_j - La_j$ by iteration using the definitions (3.7) and (3.8) :

$$\begin{aligned}b_j - La_j =& b_{j-1} + (-1)^j \frac{L}{f}\sum_{k=1}^{j-1} b_k + (-1)^j \frac{L}{f} - La_{j-1} - (-1)^j \frac{L^2}{f}\sum_{k=1}^{j-1} a_k - (-1)^j \frac{L}{f} \\ =& \left(b_{j-1} - La_{j-1}\right) + (-1)^j \frac{L}{f}\sum_{k=1}^{j-1}\left(b_k - La_k\right) \\ b_1 - La_1 =& 1\end{aligned} \tag{3.49}$$

Similarly we get from (3.14):

$$\begin{aligned}fp_{j,1} &= fp_{j-1,1} + (-1)^j \frac{L}{f}\sum_{k=1}^{j-1} fp_{j,1} \\ fp_{1,1} &= 1\end{aligned} \tag{3.50}$$

Comparing (3.49) and (3.50) one obtains the equation :

$$fp_{j,1} = b_j - La_j \tag{3.51}$$

Replacing it in the expression (3.48) one gets :

$$\tilde{t}_i = x_{inj} + iLs_{inj} + \Delta sL\,(i-1) - \sum_{k=1}^{i-1} u_{i,k}^{'}\varepsilon_{d,k} \tag{3.52}$$

Using the relation (3.39) the difference $\tilde{t}_i - \tilde{\delta}_{q,i}$ becomes :

$$\boxed{\tilde{t}_i - \tilde{\delta}_{q,i} = \varepsilon_{d,i} - \sum_{k=1}^{i-1} u_{i,k}^{'}\varepsilon_{d,k}, \qquad i = 2, \ldots, N-1} \tag{3.53}$$

This proves that the position of the beam inside every quadrupole but the first depends only on the BPMs resolutions and not on the parameters of the straight line on to which the beam has been directed.

4 Conclusions

It was mentioned in the introduction that the first step of the ML correction method was quite similar to the procedure of the dispersion-free correction. Actually it can be easily shown that it corresponds to a "pure dispersion-free (pure DF)" where only the dispersion term is retained and the trajectory term is neglected. The minimisation of the squared trajectory difference $[t_{p,i}(f_1) - t_{p,i}(f_2)]^2$ with respect to the quadrupole offsets gives the expression (3.26). Thus the solution of the "pure DF" is the set of the quadrupole offsets $d_{q,k}$ referred to the injection line. In other words the "pure DF" aligns the quadrupoles along the injection line if the errors are neglected. It is intituively evident because in this configuration the trajectory is not changed by a modification of the quadrupole strengths.
It can also be said that the injection line is detected by the first step of the ML correction. The second step provides its parameters (offset and slope) either by the same mechanism used in the "ballistic method" or by least squares estimation. Finally the beam is steered accordingly. Concluding these remarks, the ML correction can be seen as a kind of combination of a "pure DF" with a "ballistic" steering which does not require to switch off the quadrupoles.

5 Acknowledgements

We would like to thank T. Raubenheimer and D. Schulte for their useful comments.

References

[1] R.W. Assmann and T. O. Raubenheimer, "Operation and Expected Performance of the NLC Main Linacs", Proc. of 1996 DPF/DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, CO, 25 Jun - 12 Jul 1996. SLAC-PUB-7354

[2] R.W. Assmann, C. Adolphsen, K. Bane, T. O. Raubenheimer and K. Thompson, "Emittance and Trajectory Control in the Main Linacs of the NLC", Proc. 18th International Linac Conference (Linac 96), Geneva, Switzerland, 26-30 Aug 1996. SLAC-PUB-7301 (1996).

[3] T. O. Raubenheimer and D. Schulte, "The Ballistic Alignment Method", CERN PS/99-024 and CLIC Note 395.

[4] D. Schulte, "The CLIC Main Linac Lattice at 1 TeV", CLIC Note 356, 1998.

[5] T. O. Raubenheimer, "The Generation and Acceleration of Low Emittance Flat Beams for Future Linear Colliders", SLAC Report 387, 1991.

[6] The NLC Design Group, "Zeroth-Order Design Report for the Next Linear Collider", SLAC report 474, 1996.

[7] G. Guignard and J. Hagel, "MUSTAFA- A Tool for Numerical Simulations of the Beam Behaviour in a Linac", Proc. of the LINAC98 conf., Chicago, 1998.