

THE LHC DYNAMIC APERTURE

J.-P. Koutchouk, *for the LHC Team*,
CERN, Geneva, Switzerland

Abstract

In 1996, the expected field errors in the dipoles and quadrupoles yielded a long-term dynamic aperture of some 8σ at injection. The target was set to 12σ to account for the limitations of our model (imperfections and dynamics). From scaling laws and tracking, a specification for the field imperfections yielding the target dynamic aperture was deduced. The gap between specification and expected errors is being bridged by *i*) an improvement of the dipole field quality, *ii*) a balance between geometric and persistent current errors, *iii*) additional correction circuits (a_3, b_4). With the goal in view, the emphasis has now turned to the sensitivity of the dynamic aperture to the optical parameters. The distortion of the dynamics at the lower amplitudes effectively reached by the particles is minimized by optimizing the distribution of the betatron phase advance. At collision energy, the dynamic aperture is limited by the field imperfections of the low- β triplets, enhanced by the crossing angle. With correction of the most important aberrations, the dynamic aperture reaches the target set to 10σ .

1 INTRODUCTION

With the approval of LHC in 1994, the studies of dynamic aperture (D.A.) became more strictly targeted towards establishing the requirements of the magnetic field quality. For that purpose, the largest stable amplitude was selected as the most reliable indicator of stability. The computing power was increased significantly [1] to a level where it is not a serious limit anymore. The correctness of the model of the imperfections and the understanding of the stability limit became essential as compared to tracking issues. We describe in this paper the methods selected for LHC, the results obtained and the consequences for the machine design (field quality, correction systems and optics).

2 DEFINITION OF THE DYNAMIC APERTURE

In the following, the D.A. is defined as the radius of the largest circle inscribed inside the domain of initial conditions in (x, y) space observed to be stable after 10^5 turns, i.e. about 10 seconds of LHC time. The reduction of the 6D hyper-volume to a circle is practically necessary for tracking and justified by the observed irrelevance of the initial angles [2] and by the choice of a maximum relative momentum deviation which maximizes the chromatic perturbations. Testing the connexity of this area is practically limited by the mesh size ($\sigma/30$). Thanks to the ever increasing power of computers, 60 possible instances of multipoles distributions are tracked. The D.A. is approached

from below. The average is used as a relative indicator, e.g. to compare correction systems. The **minimum** is retained as the absolute D.A. of LHC (95% confidence level) and expressed in units of the unperturbed rms beam size. Out of the thousands of cases analysed, no evidence was found of abnormally small D.A.'s inside the stability domain.

3 SOURCES OF NON-LINEARITIES

The combination of all available results over the last few years, though not always strictly comparable, gives the relative importance of the non-linear perturbations (Table 1). These results assume the baseline multipole correction scheme which includes b_3 and b_5 coils at the end of each dipole. Unlike in low emittance electron machines,

Non-linearity	Injection	Collision
Chromatic sextupoles	28	≈ 70
Multipoles in Dipoles	6.5	> 27
Multipoles in Lattice Quads	≈ 12	> 27
Multipoles in Low- β Quads.	> 23	6.5
Long-range Beam-beam Kicks	6.5	6

Table 1: Computed D.A.s versus non-linear sources in LHC V2 and V4

the sextupolar non-linearities can be neglected (lower beam momentum spread). At injection, the multipoles in the dipoles dominate. At collision energy, the effect of the low- β quadrupole imperfections becomes overwhelming due to the very large β -function and the crossing angle. In both cases, the beam separation on either side of the crossing points sets the maximum D.A.

4 REQUIREMENT FOR THE DYNAMIC APERTURE

4.1 Actual Dynamic Aperture

To protect efficiently the s.c. magnets from quenching (a loss of only 100 ppm of the full beam intensity is sufficient), the collimation system reduces the useful aperture to 6σ , where we require the motion to be stable over long times. This figure can be compared to the 4.5σ D.A. found in HERA [3] to allow good performance of the machine, in the absence of constraints from collimators. The SPS Experiment [4] suggests, on the other hand, that a D.A. of 8σ could be insufficient to ensure a long lifetime in presence of large tune modulation. These few data seem to confirm that a target of 6σ is reasonable, given the larger sensitivity of LHC to modulation.

4.2 Computed Dynamic Aperture

Table 2 summarizes our assumptions on the predictability of the D.A. from the tracking results. Items (1) and (3) are related to the sensitivity to initial conditions. To reduce complexity, the linear imperfections (2) are usually not included in the D.A. calculation. When this is done and after correction, the D.A. is reduced by about 5%. The variation of the multipole errors occurring during the snap-back (5) are approximately taken into account by selecting the worst case for each multipole and then assuming a static situation. Items (1), (3), (4) and (6) are related to the limits in computer power. The initial conditions are mostly chosen on the diagonal in the x, y plane. Tracking is mostly carried out to 10^5 turns although the injection plateau is expected to last about 400 times longer. Relevant studies of ripple require tracking beyond 10^5 turns. Longer tracking with a more complete exploration of initial conditions and ripple are carried out exceptionally to evaluate the impact on the D.A.. Fits of the survival plots [5] show a moderate reduction of the D.A. from 10^5 to 10^7 turns. Finally, the safety

Table 2: Relation between long-term and computed D.A.'s.

Source or Uncertainty	Impact	D.A. in σ
Target D.A. at 10^5 turns		12.0
(1) Finite mesh size	-5%	
(2) Linear Imperfections	-5%	
(3) Amplitude ratio x_i/y_i	-5%	
(4) Extrapolation to $4 \cdot 10^7$ turns	-7%	9.6
(5) Time-dependent multipoles	-10%	
(6) Ripple	-10%	7.8
(7) safety margin	-20%	6.2
Long term D.A.		6.2

margin (7) is estimated from the SPS D.A. Experiment [4] and from Fermilab/E778 [6] to be 20%. This is probably an optimistic view of the predictability of the D.A.: in HERA, the D.A. was predicted [7] [8] using the **measured** field imperfections, without or with the decay of persistent currents; compared to the measured D.A., the discrepancies are in a range of 10% to 100%.

In order to cope with these uncertainties, the impacts are combined linearly and the target for the computed D.A. is fixed to 12σ at injection. Such an approach in a highly non-linear domain is not un-controversial: the large target D.A. might create artificial requirements on the higher-order multipoles. This was not found to be the case for LHC.

The 12σ target is significantly more ambitious than the D.A.'s recorded a few years ago (see table 1). It is now reached with improvements discussed below. At collision energy, the limitations are only in the low- β quadrupoles and thus better defined. We have suppressed the safety margin and set the target to 10σ .

5 EFFECT OF THE MAIN DIPOLES

5.1 The Most Significant Imperfections

The multipole expansion of the main dipole field is listed in Table 3. Expressed in relative field error at 12σ , the

Order	Normal			Skew		
	S	U	R	S	U	R
3 (sext.)	-9.7	1.38	1.47	-.08	.87	.48
4 (oct.)	.23	.34	.51	-	.14	.51
5 (dec.)	.89	.44	.43	.007	.42	.34
6	-.01	.06	.09	-	.06	.17
7	-.16	.05	.22	.017	-	.08
8	-	-	.04	-	-	.08
9	.36	.03	.07	-.006	-	.11
10	-	-	-	-	-	.01
11	.57	-	-	.002	-	-

Table 3: Table 9901: Multipole expansion b_n, a_n (Systematic, Uncertainty, Random) of the dipole field at injection energy at the reference radius of 17mm in units of 10^{-4} .

field error is one order of magnitude above that of the SPS or LEP but only slightly worse after a perfect correction of b_3 and b_5 . The errors can be systematic (finite number of blocks in the coil, 2-in-1 design), uncertain, i.e. predictable in a range (production bias) or random from dipole to dipole. The uncertainties are large compared to the bias of the random components. In the model of the imperfections, we assume eight production lines with different uncertainties; the dipoles in one arc come from the **same** line.

The computation of the D.A. for either all random errors combined **R** or for individual systematic multipole errors (Figure 1) shows that the LHC D.A. is dominated by systematic effects. The effect of b_3 is almost perfectly suppressed by the sextupolar coils in the dipole ends. The effect of b_5 is sufficiently though not completely corrected by the decapole coils in the dipole ends. b_4 is the next limitation. The D.A.'s due to the other multipoles are rather balanced. To avoid a D.A. which would be too sensitive to optics parameters, we require both the effect of individual multipoles and their combined effect to be small enough.

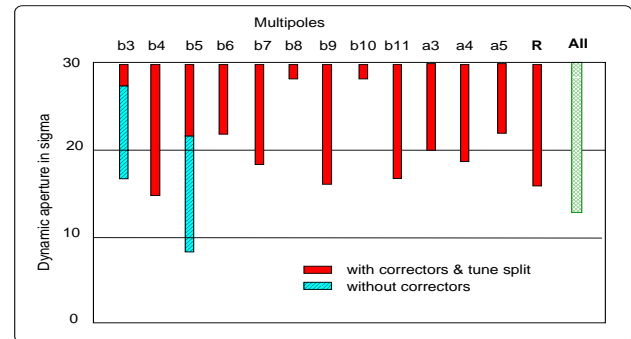


Figure 1: D.A. per source for error table 9901

5.2 The Consequences on the Beam Dynamics

Figure 2 shows the D.A. as a function of Q_x . The nominal fractional betatron tunes are .28 and .31 between 3rd and 4th-order resonances. This position is favourable to avoid high-order resonances in collision mode and was used at the Sp \bar{p} S and HERA. The D.A.'s in this report are computed for this working point. A second point between 4th and 5th-order resonances is explored occasionally in case the 3rd-order resonances would be too strong or in case tuning the two rings differently would be an advantage.

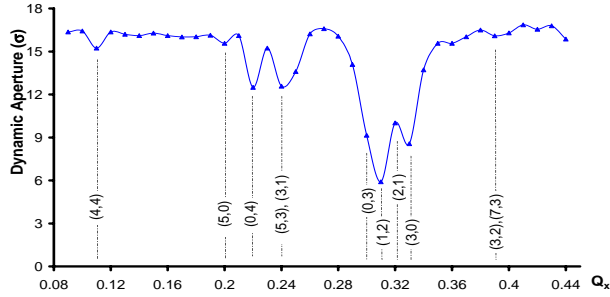


Figure 2: Average short-term D.A. versus tunes ($Q_y = Q_x + .03$) for the target error table [9] (no linear errors)

The chromaticity due to the persistent currents is about 400 units. It is corrected in each dipole with b_3 -coils powered in series per arc. A break-down of the correctors in one arc causes a 8% loss of D.A. together with an increase of the excitation of 3rd order resonances [10], justifying a distributed correction scheme.

The effect of b_5 causes mostly an amplitude detuning larger than 0.01 at 4σ and $\Delta p/p = 1.5\sigma_e$. The correction is efficient but not sufficiently local to allow b_5 to increase too much above its design value: the distance between the dipole center and end (where the b_5 coil is mounted) is significant at this order. On the other hand, corrector break-downs in several arcs are tolerable [10], confirming that the main effect is indeed a detuning.

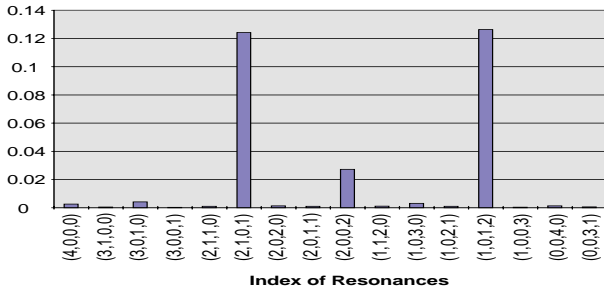


Figure 3: Driving terms of 4th order resonances, from [11]

The amplitude detuning for on-momentum particles is mostly due to the uncertain b_4 in the dipoles and partly to a second-order in lattice sextupole perturbation; in some cases, a second order in the uncertain a_4 appears as well. A significant correlation [12] is found between b_4 , amplitude

detuning (of up to 0.01 at 8σ) and D.A. A full recovery of the loss of D.A. (about 1.5σ) is obtained by correcting the detuning with b_4 correctors in the dipole ends distributed in 50% of the arcs [13].

a_4 excites the ‘sub-resonance’ $Q_x - Q_y = p$. Figure 3 shows that its driving terms (2101, 1012) are by far the largest fourth-order terms; on the frequency map [14] (Figure 4), large amplitude particles appear attracted by the resonance. Its correction by a_4 coils in 50% of the arc dipoles

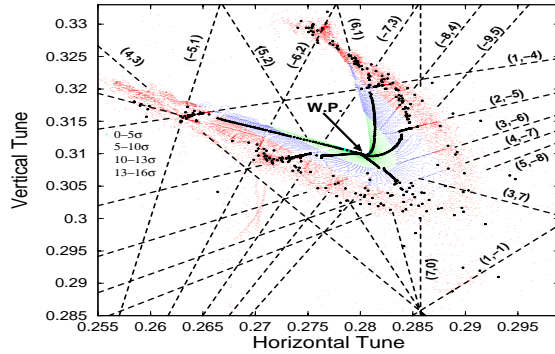


Figure 4: Frequency map showing the attraction by the 4th-order resonance (1,-1) of large amplitude particles.

allows full recovery of the loss of D.A. (about 1.5σ). The higher-order detuning which is resonant with $Q_x - Q_y = p$ is decreased by the same token [13].

a_3 causes a smearing in phase space, a loss of D.A. of about 0.5σ and a large second-order chromaticity. The mechanism is a momentum dependent betatron coupling [15] which modulates the tunes at twice the synchrotron frequency. The modulation depth of 0.01 or more depends on the working point. A resonant correction with skew sextupoles cures these effects.

The LHC D.A. appears to depend mostly on first-order amplitude detunings and resonances driven by field uncertainties. This is different from the former SSC and LHC observations where the detunings were associated with systematic errors, the resonances to random higher-order terms and the D.A. to the interplay of higher-order resonances. This simpler phenomenology stems from a better knowledge of the sources of field errors. It opens possibilities for optimal lattice design as long as the hypotheses on the errors are confirmed.

5.3 The Target Table for Imperfections

Using the scaling law of D.A. for individual multipoles and combining them according to an empirical conjecture [16], we could construct rather rapidly a ‘Target Table’ (Table 4) for field imperfections which yields the target D.A. of 12σ , starting from the error table 9607 which yielded 8 to 9σ . Tracking globally confirmed the prediction. This approach allowed the dipole designers to include the constraint of field quality at the design stage rather than after

Order	Normal	Skew
3	1	0.05
4	.02	1 (.28)
5	0.5 (0.37)	.9
7	.6 (0.15)	-
9	1	-
11	1 (0.38)	-

Table 4: Ratio of multipoles in the Target Error Table over those of the realistic Tables 9901 (or 9607).

and speeded up the improvements.

5.4 Improvements of the Dipole Field Quality

The change from a 5-block to a 6-block coil design [17] permitted a better compensation of persistent currents at injection. The allowed field errors are close to target (Table 4). The uncertainty on a_4 is reduced by alternating the orientation of the dipole collared coils assemblies.

5.5 Multipole Corrector Schemes

The multipoles a_3 and b_4 were however found irreducible, thus requiring new correction schemes.

We foresee for LHC version 6 a non local resonant correction of the resonance $Q_x - Q_y = p$ driven by a_3 . The required integrated strength of the skew sextupoles is comparable to that of all lattice sextupoles at injection. An elegant implementation [18] involves tilting 4 lattice sextupoles per arc, chosen such as to minimize geometric aberrations.

The correction of the amplitude detuning will be carried out by octupole windings lodged into the decapolar corrector of the dipoles. Tracking shows indeed [13] that reducing the amplitude detunings to 0.001 at 8σ provides more robust D.A.'s which can be expected to be less sensitive to tune drifts during operation. The Landau damping octupoles, close to the quadrupoles, cannot reduce simultaneously the three terms of the detuning. With a cell phase advance close to 90° , they may produce other aberrations. We take advantage of the fact that the b_4/b_5 correctors correct mainly detuning terms to equip the dipoles in every second cell only. This upgrade of the correction system is almost cost neutral and provides the required efficiency.

5.6 Optimized Optics

Up to version 4, the integer LHC betatron tunes were identical. A tune split was introduced in LHC version 5 when the linear coupling was identified to be exceptionally strong [19]. It is caused by the systematic part of a_2 , and the feed-downs of the b_3 corrections coils due to alignment tolerances. This was achieved by breaking the optics antisymmetry with a more flexible hardware [20]. The tune split was adjusted to 4 units (2 in the arcs, 2 arising naturally in the dispersion suppressors) to weaken sufficiently the perturbation arising from the systematic and uncertain parts of a_2 (Figure 5). It increased the D.A. of version 5 by

about 1σ . The optimal tune split for coupling cancellation is 1 per arc. The LHC quadrupole gradients are specified to allow this potentially interesting possibility (used in [21]).

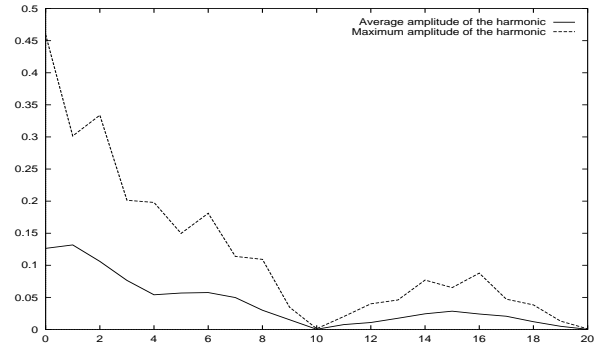


Figure 5: Azimuthal harmonics of a field uncertainty averaged over 1000 seeds

With the present understanding of the consequences of a_3 and a_4 (section 5.2) and the identification of a hidden super-periodicity in $\mu_x - \mu_y$ leading to a possibly constructive build-up of $Q_x - Q_y = p$, the tune split is now increased from 4 to 5 in version 6. The D.A. increases again by about 1σ (Table 5). It is close to target with realistic errors (Figure 1). The onset of chaos is observed at 9 to 10σ .

Besides this pragmatic use of the tune split, first-order perturbation theory [22] was used to compute the largest amplitude allowed in presence of a realistic mechanical aperture at 10σ . In this 4D approach, the normal and skew octupolar imperfections were found to perturb most the dynamics. On Figure 6, the variation of the integer tunes allows a minimization of the perturbation (the size of the ellipse is a measure of the aperture loss). The new tunes of

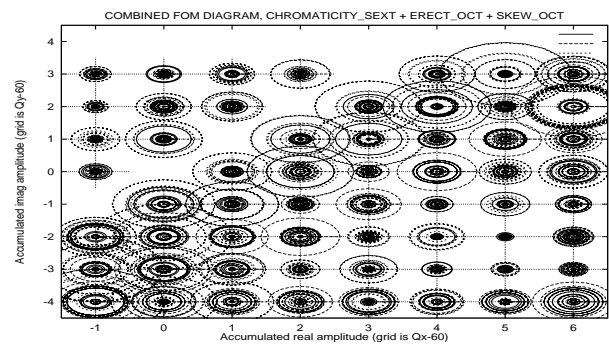


Figure 6: Distortion versus integer tunes (origin is 60/60)

LHC version 6 are 64.28/59.31, i.e. one of the best combinations.

Pushing further the idea of minimizing the driving terms of first-order resonances, it was shown that a tune split of 1 per arc complemented by a condition on the horizontal cell phase advance cancels the contribution of each arc to al-

most all first-order resonances [21], provided the field multipoles are constant in each arc. Tracking not only confirms that the D.A. is increased (Table 5), but shows that this is still true if b_4 is increased significantly [23].

Tunes	Aver. D.A.
63/59 (LHC V5)	12.0
64/59 (LHC V6)	13.4
65/58 (LHC V6)	13.8
68/59 (LHC V6)	14.5

Table 5: D.A. versus tunes (Target Error Table + expected a_4/b_4 from 9712)

6 EFFECT OF THE BEAM-BEAM KICKS

To meet the D.A. requirement, the beam separation in the section common to the two beams was increased to 10σ [24] at injection and 7σ in collision [25]. The corresponding crossing angle is $\pm 150\mu\text{rad}$ at 7 TeV.

7 EFFECT OF THE QUADRUPOLES

At injection, the b_6 due to persistent currents in the lattice quadrupoles (Table 1) contributed to the limitation of the D.A.. This effect was partly compensated by changing the coil geometry, leaving a negligible residue.

At 7 TeV, the field errors in the low- β quadrupoles limit severely the D.A. (Table 1). In the first design, the main effect [26] was that of a too large $b_{10} \approx 0.5$ units, enhanced by the large β -function (4.7km) and the off-axis orbit ($\pm 5\text{mm}$). Since then the field quality was much improved in the US/LHC design [27]. The aberrations are further minimized by choosing the magnet orientation, by body-end compensation and by adding non-linear correctors ($n = 3, 4, 6$). The present performance fully satisfies the requirements with a D.A. of 10σ and an amplitude detuning small compared to the beam-beam detuning [27]. The KEK quadrupole field quality is still being improved.

Case	DA(σ_{xy})	Min. DA	$\Delta\nu$ (10^{-3})
no correction	10.7 ± 1.7	8	1.9 ± 1.1
with correction	13.3 ± 1.6	10	1.0 ± 0.7

Table 6: D.A. with and without correctors at 7 TeV

8 METHODS

The model of the LHC machine and the tracking methods are detailed in [28] [24]. Basically, element by element tracking is performed using thin lenses. The power of the computer cluster has been increased by a factor of 2.5 since 1997. The computation of a D.A. takes 2 days of CPU and much more in preparation and analysis. Fast tracking with symplectified maps was not operational for the design of LHC but could become interesting for optimization.

Modern tools such as map analysis by resonant and non-resonant normal forms and frequency analysis of the betatron motion are used to compute global quantities. The classical resonance theory has remained so far the only approach to design multipolar correction circuits or optimized optics. It retains indeed the azimuthal dependence of the imperfections and corrections and has yielded satisfactory results for LHC. The frequency map analysis seems a useful qualitative help in understanding the dynamics, e.g. to judge on the respective efficiency of correction schemes.

The real issue in improving our knowledge of the D.A., as formally noted by Ritson [29] lies first in the ability to describe in a realistic way the machine with realistic imperfections, achievable corrections, time-dependent effects,...

9 CONCLUSION

The target D.A. of 12σ at injection and 10σ at 7 TeV is now reached. It may be viewed either as ensuring a realistic D.A. of 6σ with a safety factor of 2 or as a guaranty that the onset of chaos occurs beyond the amplitudes practically allowed by the collimation system. The improvement of the field quality of the dipoles and low- β quadrupoles, the addition of correcting circuits and the tune split all contributed to the improvement. The dominant role played by first-order resonances and detuning terms opens the possibility of handling the consequences of unexpected systematic multipole imperfections by optimizing the optics.

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