THE NON-PERTURBATIVE QCD DEBYE MASS
FROM A WILSON LINE OPERATOR

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Abstract

According to a proposal by Arnold and Yaffe, the non-perturbative g^2T-contribution to the Debye mass in the deconfined QCD plasma phase can be determined from a single Wilson line operator in the three-dimensional pure SU(3) gauge theory. We extend a previous SU(2) measurement of this quantity to the physical SU(3) case. We find a numerical coefficient which is more accurate and smaller than that obtained previously with another method, but still very large compared with the naive expectation: the correction is larger than the leading term up to $T \sim 10^7 T_c$, corresponding to $g^2 \sim 0.4$. At moderate temperatures $T \sim 2T_c$, a consistent picture emerges where the Debye mass is $m_D \sim 6T$, the lightest gauge invariant screening mass in the system is $\sim 3T$, and the purely magnetic operators couple dominantly to a scale $\sim 6T$. Electric ($\sim gT$) and magnetic ($\sim g^2T$) scales are therefore strongly overlapping close to the phase transition, and the colour-electric fields play an essential role in the dynamics.

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1 Introduction

An important concept in the description of the deconfined finite temperature QCD plasma is the Debye mass m_D. It is the inverse of the spatial screening length felt by colour-electric excitations, in analogy with Debye screening in abelian plasmas, which is felt by electric fields but not by magnetic fields. The leading order result for m_D is perturbatively computable [1], but the next-to-leading order result is already non-perturbative [2], making the determination of m_D (and, in fact, already its precise definition) a challenging theoretical problem. The numerical value of m_D has a number of applications, in particular in the phenomenology for quark-gluon plasma formation (for a review, see [3]). It is also interesting to note that, even though m_D is originally a strictly static quantity, it might have significance even for some dynamical observables, such as the rates of anomalous processes in the plasma [4].

A gauge invariant, non-perturbative definition for m_D has been given by Arnold and Yaffe [5]. This definition is based on a (Euclidian) time reflection operation, denoted also by $\mathcal{R}_z$, which reverses the sign of the temporal component of the gauge field: $\mathcal{R}_z A_0^a \to -A_0^a$ (for a more detailed discussion, see [5]). The Debye mass is defined as the smallest among the exponential fall-offs that are found for gauge invariant operators odd under this symmetry. A further observation is that due to finite temperature dimensional reduction [6]–[9], the measurement of m_D in numerical simulations can be carried out in a three-dimensional (3d) SU(N) + adjoint Higgs theory. Lattice measurements in this theory were performed, both for SU(2) and SU(3), in [10][3].

However, there is a problem in making precise simulations in the 3d SU(3) + adjoint Higgs theory. The reason is that in the regime of asymptotically small couplings, there is a scale hierarchy between the leading order result $m_D \sim gT$ and the non-perturbative correction $\delta m_D \sim g^2 T$, which one is trying to determine (the "magnetic" degrees of freedom are also associated with the scale $g^2 T$). As always, a scale hierarchy is difficult to accommodate on a single lattice of presently available sizes, and thus makes the approach to the continuum limit quite problematic in practical simulations.

Fortunately, there is a way out: parametrically (i.e., as a power series in g), even the field A_0^a can be integrated out perturbatively, leaving a pure 3d SU(3) gauge theory. The operator odd in A_0^a goes over to a non-local but gauge invariant Wilson line in the pure SU(3) gauge theory [5]. This allows to determine the coefficient of $g^2 T$.

For the SU(2) case, this Wilson line operator was studied with lattice simulations in [14]. We found a result for the $g^2 T$ term which is more precise and smaller than that found in [10]. The purpose of the present paper is to extend the SU(2) measurement to the physical SU(3) case.

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[3]Progress towards a lattice measurement of m_D directly with 4d lattice simulations has been reported in [11] (see below). Another gauge invariant definition within the 3d SU(3)+adjoint Higgs model was suggested in [12]. Other definitions of m_D, based on gauge fixing, have recently been pursued in [13].

2 The operator in the continuum

Let us start by briefly reviewing the basic idea of the approach in [5]. Consider gauge invariant operators $O(x)$ odd under $A_0 \to -A_0$ (i.e., in the notation of [5], with $\mathcal{R}_z = -1$). In practice, we consider $O(x)$ containing only one power of A_0, since otherwise (for $A_0^n, n \geq 3$) the leading behaviour for small g is expected to be of the form $n \cdot m_0$, where $m_0 = (N/3 + N_f/6)^{1/2} gT$ is the leading order Debye mass [1] (N_f is the number of flavours). Imagine now doing the path integral over A_0 before doing that over A_i. Then,

$$\frac{\int \mathcal{D}A_0\, O(x)O(y)\exp(-S_{A_0})}{\int \mathcal{D}A_0\, \exp(-S_{A_0})} \sim O'_a(x)G^{ab}(x,y)O'_b(y), \tag{1}$$

where $O' = dO/dA_0$,

$$S_{A_0} = \int d^3x \left[\frac{1}{2}(D_i A_0)^a (D_i A_0)^a + \frac{1}{2}m_0^2 A_0^a A_0^a\right], \tag{2}$$

and $G^{ab}(x,y)$ is the tree-level Green's function in the background of A_i:

$$G^{ab}(x,y) = \frac{\exp(-m_0|x-y|)}{4\pi|x-y|}W^{ab}(x,y), \tag{3}$$

with $W^{ab}(x,y)$ the adjoint Wilson line,

$$W^{ab}(x,y) = 2\mathrm{Tr}\, T^a W^{\mathrm{fund}}(x,y)T^b[W^{\mathrm{fund}}(x,y)]^\dagger, \tag{4}$$

where $W^{\mathrm{fund}}(x,y) = \mathcal{P}\exp(ig\int_x^y dx_i A_i^a T^a)$.

Thus, the measurement of the Debye mass can be reduced to the measurement of

$$G_F(x,y) \equiv \left\langle O'_a(x)W^{ab}(x,y)O'_b(y)\right\rangle, \tag{5}$$

in the pure SU(N) gauge theory. The measurement of this operator produces an exponential decay with a coefficient m, to which the perturbative part m_0 from Eq. (3) has to be added in order to obtain the physical result.

Including also loop corrections (i.e., allowing A_i to fluctuate at all length scales), maintains the form of the answer but replaces the perturbative m_0 in Eq. (3) by an effective parameter m_0^{eff} [5]:

$$m_0 \to m_0^{\mathrm{eff}} = m_0 + \frac{Ng_3^2}{4\pi}\left(\ln\frac{m_0}{\Lambda} + d_\Lambda\right) + \mathcal{O}\left(g_3^2\frac{m_0}{\Lambda}, \frac{g_3^4}{m_0}, ...\right), \tag{6}$$

where $g_3^2 = g^2 T$, Λ is some ultraviolet cutoff, and d_Λ depends on Λ. In particular, in the case of a lattice regularization, $\Lambda = a^{-1}$ and $d_\Lambda = -(3\ln 2 + 1)/2$ (see [5, 14]). It follows that if the physical Debye mass is written as

$$m_D = m_0 + \frac{Ng_3^2}{4\pi}\ln\frac{m_0}{g_3^2} + c_N g_3^2 + \mathcal{O}(g^3 T), \tag{7}$$

then

$$c_N = \frac{m}{g_3^2} + \frac{N}{8\pi}\left(\ln\frac{N^2}{2\beta_G^2} - 1\right),\tag{8}$$

where m is the mass measured from the operator in Eq. (5), and $\beta_G = 2N/(ag_3^2)$.

We thus need to find operators O' such that we get the lowest possible coefficient for the exponential falloff in Eq. (5). From [10], we expect the operators for Eq. (1) to be $O \sim \epsilon_{3jk}\mathrm{Tr}\, A_0 F_{jk}$ and suitable variants thereof (see below), leading to $O'_a = F_{ij}^a$ in Eq. (5). In principle, one could get rid of the specification of the operators altogether by measuring the coefficient of the perimeter law of the adjoint Wilson loop [5]. However, in practice the perimeter law can only be observed on the lattice by including this very kind of operators in the basis, due to a very small overlap of the lattice Wilson loop with the asymptotic lightest state [15].

3 A lattice formulation

On the lattice, we replace Eq. (5) with operators of the type

$$G_{F,ijkl}(x, y) = \left\langle 4\mathrm{Tr}\left(F_{ij}(x)T^a\right)\Gamma^{ab}(x, y)\mathrm{Tr}\left(F_{kl}(y)T^b\right)\right\rangle,\tag{9}$$

where F_{ij} is the naive lattice analogue of the field strength tensor,

$$F_{ij}(x) = -\frac{i}{2}\Big[P_{ij}(x) - P_{ij}^\dagger(x)\Big],\tag{10}$$

P_{ij} is the plaquette, and Γ^{ab} is the lattice Wilson line:

$$\Gamma^{ab}(x, y) = 2\mathrm{Tr}\left(T^a S(x, y)T^b S^\dagger(x, y)\right),\tag{11}$$

$$S(x, y) = \prod_{n=0}^{N-1} U_3(x + n\hat{3}), \qquad y = x + N\cdot\hat{3}.\tag{12}$$

Here U_i is a link variable and $\hat{3}$ is the unit vector in the x_3-direction. Eq. (9) may be rewritten as

$$G_{F,ijkl}(x, y) = 2\left\langle\mathrm{Tr}\left[F_{ij}(x)SF_{kl}(y)S^\dagger\right] - \frac{1}{N}\mathrm{Tr}\, F_{ij}(x)\mathrm{Tr}\, F_{kl}(y)\right\rangle,\tag{13}$$

where the latter term vanishes fast close to the continuum limit and is not numerically important (for SU(2), it vanishes identically). In our simulations, we use G_F in this last form, with the components $ij = kl = 12$.

In our practical simulations, each plaquette at the end of the Wilson line was replaced by the sum over all four spatial plaquettes of the same orientation sharing the end point of the Wilson line, a configuration called the "clover". This operator is expected to have
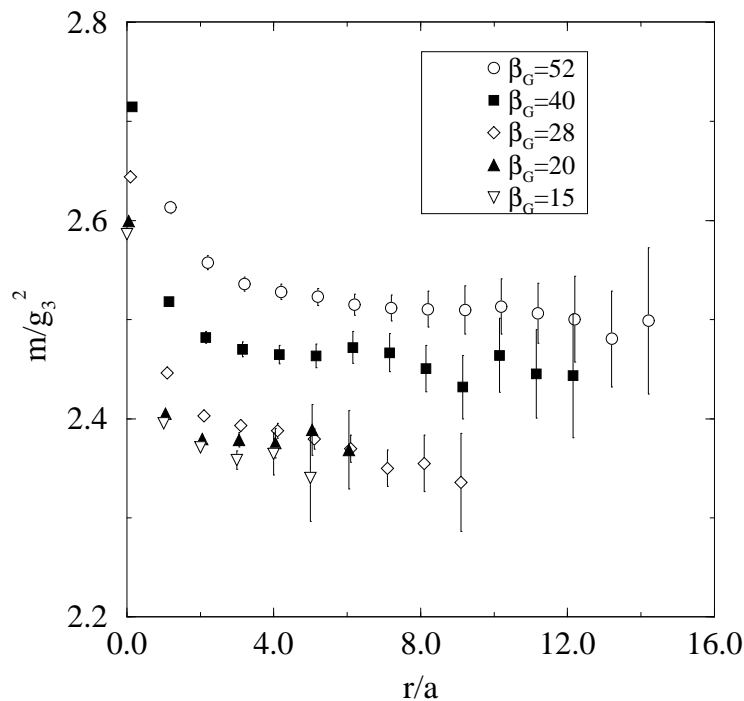
Figure 1: The local mass values, $m\,a = -\ln[G_F(n+1)/G_F(n)]$, from typical diagonalized operators at the different β_G's. The masses have been converted to continuum units by multiplying the lattice values $m\,a$ with $\beta_G/6$.

a better projection onto the ground state (i.e. to be less contaminated by higher spin excitations [16]), as well as to improve statistics. We have carried out measurements using single plaquettes, as well, and checked that the two operator types give consistent results, with the clover indeed providing a much better signal. In order to further increase the projection onto the lightest mass, we used smearing and diagonalization, as in [14]. The idea is to measure a whole cross correlation matrix within a channel of given quantum numbers, and then to search for the optimal operator by a variational calculation. The basis is obtained from gauge invariant "smeared" operators with different numbers of smearing iterations. For details, we refer to [14].

We gathered statistics until the statistical errors of the local masses at their plateaux were well under 10%, which required between 500 and 4000 measurements, depending on the lattice size. The choice of lattice sizes was governed by the experience from SU(3) glueball measurements [16] as well as our previous SU(2) measurements [14]. We have also explicitly checked that finite volume effects are within the statistical errors at single values of β_G (see Tables 1, 2), and then increased the volume with β_G, keeping it fixed in physical units.

β_G	volume	$m\,a$	m/g_3^2	c_2
9	30^3	0.685(8)	1.54(2)	1.17(2)
12	$30^2 \cdot 42$	0.517(7)	1.55(2)	1.13(2)
16	42^3	0.401(4)	1.60(2)	1.14(2)
	54^3	0.397(3)	1.59(2)	1.12(2)
20	52^3	0.327(5)	1.64(3)	1.13(3)
25	64^3	0.268(2)	1.68(2)	1.14(2)
∞		–	–	1.14(4)

Table 1: The results for SU(2). The continuum extrapolation is from a linear fit to $\beta_G \geq 12$, with $\chi^2/\mathrm{d.o.f.} = 0.03$ (see Fig. 2).

4 Numerical results

Typical local mass values from the correlation functions in the case of SU(3) are shown in Fig. 1, exhibiting rather pronounced plateaux. The mass values quoted in this paper have been obtained by fitting an exponential to the optimal diagonalized correlation function, and choosing among various fitting ranges those giving the best values for χ^2/dof. The length of the fitting ranges is between 5 and 11 timeslices, and the χ^2-values are all between $0.5 - 1.5$. We now discuss our results both for SU(2) and SU(3).

SU(2). Previously we carried out measurements for SU(2) with $\beta_G = 9, 12, 16$ [14]. We have now added values closer to the continuum limit, $\beta_G = 20, 25$. The results are shown in Table 1 and in Fig. 2. As a continuum extrapolation we obtain $c_2 = 1.14(4)$. Note that we see the onset of a quadratic $\mathcal{O}(a^2)$ term in c_2 on coarser lattices (the same is true also for SU(3)).

SU(3). The results for SU(3) are shown in Table 2 and in Fig. 2. As for SU(2), the results show could scaling with β_G after the removal of the logarithmic divergence. As a final continuum value we cite $c_3 = 1.65(6)$.

5 Discussion

Comparison with other measurements in 3d. Let us compare our results with those of previous studies. In [10], c_2, c_3 were measured using operators $O \sim \epsilon_{3jk} \mathrm{Tr}\, A_0 F_{jk}$ in the 3d SU(N) + adjoint Higgs theory, with the result $c_2 = 1.58(20)$, $c_3 = 2.46(15)$. In [14], we found the smaller result $c_2 = 1.06(4)$, using the field strength correlator in the pure 3d SU(2) theory as in this work. Adding two more lattice spacings to SU(2) and evaluating also SU(3), our present results are $c_2 = 1.14(4)$, $c_3 = 1.65(6)$. In conclusion, the values are considerably smaller than in [10]. However, there the system had multiple length scales, so it need not be a surprise that the result has some
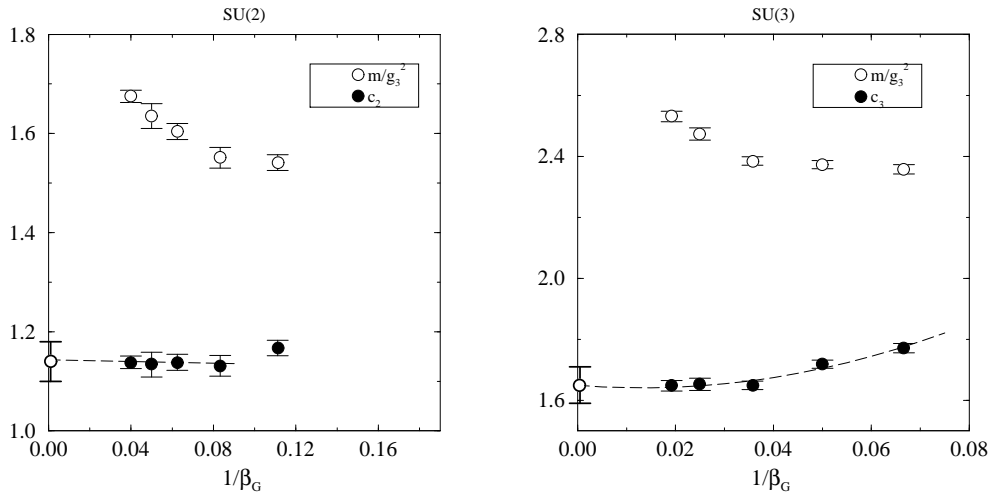
Figure 2: The results for SU(2) (left) and SU(3) (right). The continuum fits from Tables 1, 2 have also been shown. The ranges of $1/\beta_G$ on the x-axes have been chosen such that in lattice units, the masses have comparable values for SU(2) and SU(3).

β_G	volume	$m\,a$	m/g_3^2	c_3
15	18^3	0.943(6)	2.36(2)	1.77(2)
20	24^3	0.712(4)	2.37(2)	1.72(2)
	32^3	0.707(5)	2.36(2)	1.70(2)
28	32^3	0.511(3)	2.38(2)	1.65(2)
40	46^3	0.371(3)	2.47(2)	1.65(2)
52	60^3	0.292(2)	2.53(2)	1.65(2)
∞		$-$	$-$	1.65(6)

Table 2: The results for SU(3). The continuum extrapolation is from a quadratic fit to all the β_G-values, with $\chi^2/\text{d.o.f.} = 1.25$. We use a quadratic fit for SU(3) since we are slightly farther from the continuum limit than for SU(2), see Fig. 2. A linear fit to $\beta_G \geq 28$ gives 1.65(5), with $\chi^2/\text{d.o.f.} = 0.04$.

systematic uncertainties. Our present results scale quite well with N, a behaviour which is also observed for the glueball spectrum in SU(N) theories [16]. Furthermore, the bare numbers m/g_3^2 extracted from the Wilson lines exhibit the logarithmic divergence expected from lattice perturbation theory [5], adding further confidence to them.

The mass scales in units of temperature. We now proceed to estimate the numerical value of the Debye mass m_D in units of T. For definiteness we take $N_f = 0$,

in which case $T_c/\Lambda_{\overline{\text{MS}}} \approx 1.03(19)$ for SU(3) [17]. (Let us stress that the values of c_2, c_3 as such do not depend on the fermionic content of the original 4d theory, and thus c_3 applies also to the physical QCD with $N_f \neq 0$.) At moderate temperatures of, say, $2T_c$, we then estimate that $g_3^2 \approx 2.7T$ [9], and thus $m_D \sim 6T$. This estimate is of course somewhat rough, since terms of order $g^3 T$ etc. have been neglected[4]. However, the results in [10] suggest that these corrections should come with very small coefficients.

An important question for the construction of effective theories concerns the degree to which the Debye mass is perturbative, and the related issue of how well the electric ($\sim gT$) and magnetic ($\sim g^2 T$) sectors are separated, once non-perturbative coefficients are taken into account. Even at very large temperatures, say, $T \sim 10^4 T_c$ with a weaker coupling $g_3^2 \approx 0.64T$, the correction ($\sim 1.1T$) is still larger than the leading term ($\sim 0.8T$). Both are of the same order ($\sim 0.7T$) only at $T \sim 10^7 T_c$.

It is also interesting to compare with the masses in the magnetic sector, especially at low temperatures $T \sim 2T_c$. There the ordering of states according to the naive expectations is reversed. A purely magnetic excitation, the lightest glueball, has a mass $\sim 2.4g_3^2 \sim 6T$ [16], making it as heavy as m_D. However, there is a lighter excitation with the same quantum numbers which then determines, e.g., the decay of the Polyakov loop in the 3d SU(3) + adjoint Higgs theory. It is represented by the mass of the bound state $A_0^a A_0^a$, whose numerical value is about $\sim 3T$ [9, 18].

Comparison with measurements in 4d. All these numbers are in good qualitative agreement with those found with direct 4d simulations in [11]. In Table 5 of [11], $m(A_1^{++}) \approx 2.60(4)T$ corresponds to the lightest mass in the 0^{++} channel [in the language of the 3d theory, to $m(A_0^a A_0^a) \sim 3T$], while the $\mathcal{R}_z = -1$ [5, 11] mass $m(A_1^{-+}) \approx 6.3(2)T$ should correspond to the lightest mass in the electric sector [in the language of the 3d theory, to m_D as defined here, $\sim 6T$]. Since the 4d measurement of m_D contains all orders in g this agreement would indeed suggest corrections of order $g^3 T$ and higher to be small.

Incidentally, we believe that, contrary to the suggestion in [11], the lightest masses measured in [11] do not correspond to the glueball masses measured in [16], but to scalar states involving A_0: the screening masses corresponding to static glueball correlators are heavier. Thus, close to T_c, the naive expectation that A_0 is heavy and decouples is completely unjustified (see also [8, 9, 20]). This should not be a surprise, since the actual transition is assumed to be driven by the Z(N) symmetry related to the Polyakov line and A_0. At the same time, it is interesting to note that the higher lying glueball spectrum itself is expected to be relatively little affected by A_0 [19].

Conclusions. In conclusion, we have determined the non-perturbative contribution of the soft modes $\sim g^2 T$ to the Debye mass in QCD and found it to be larger than the

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[4]Note also that in principle, some other $\mathcal{R}_z = -1$ operator such as Tr A_0^3 could have a lighter mass in this region.

"leading" perturbative contributions for all reasonable temperatures. The approximate agreement between the values for the Debye mass as determined in this simulation and with the 4d measurement, further suggests that the $\mathcal{O}(g^3T)$ and higher corrections in the Debye mass are small even at relatively small temperatures, as already argued in [10]. The consistent picture emerging from our results, as well as from previous analytical work and simulations [8, 9, 10, 20], is that a dimensionally reduced theory gives a reliable description of correlation functions down to quite moderate temperatures $T \sim 2T_c$, when A_0 is kept in the action. However, at such temperatures A_0 may not be integrated out, but is in fact an essential constituent in the lightest physical degrees of freedom.

Acknowledgements

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