EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Determination of the T- and CPT-violation parameters in the neutral-kaon system using the Bell–Steinberger relation and data from CPLEAR.

CPLEAR Collaboration

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Abstract

Data from the CPLEAR experiment, together with the most recent world averages for some of the neutral-kaon parameters, were constrained with the Bell–Steinberger (or unitarity) relation, allowing the T-violation parameter $\mathrm{Re}(\epsilon)$ and the CPT-violation parameter $\mathrm{Im}(\delta)$ of the neutral-kaon mixing matrix to be determined with an increased accuracy: $\mathrm{Re}(\epsilon) = (164.9 \pm 2.5) \times 10^{-5}$, $\mathrm{Im}(\delta) = (2.4 \pm 5.0) \times 10^{-5}$. Moreover, the constraint allows the CPT-violation parameter for the neutral-kaon semileptonic decays, $\mathrm{Re}(y)$, to be determined for the first time. The $\Delta S \neq \Delta Q$ parameters $\mathrm{Re}(x_-)$ and $\mathrm{Im}(x_+)$ are given with an increased accuracy. The quantity $\mathrm{Re}(y + x_-)$, which enters the T-violation CPLEAR asymmetry previously published, is determined to be $(0.2 \pm 0.3) \times 10^{-3}$. The value obtained for $\mathrm{Re}(\delta)$ is in agreement with the one resulting from a previous unconstrained fit and has a slightly smaller error.

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1 Introduction

The CPLEAR experiment has directly measured for the first time the violation of T invariance in the neutral-kaon system [1] and has provided a new, more accurate limit for the CPT-violation parameter $\mathrm{Re}(\delta)$ [2]. These results were obtained by analysing the rate asymmetries between K^0 and $\overline{\mathrm{K}}^0$ for decays to $\mathrm{e}^{\pm}\pi^{\mp}\nu$, as a function of the decay time $t = \tau$.

In continuation of this work we have studied the constraints on our results deriving from the Bell–Steinberger (or unitarity) relation [3–5]. The Bell–Steinberger relation relates all decay channels of neutral kaons to the parameters describing T and CPT non-invariance. With the present precision of the two-pion decay parameters the dominant uncertainties arise from the three-pion and semileptonic decays. Moreover, the semileptonic decays enter the relation with the parameter $\mathrm{Re}(y)$, describing CPT violation in semileptonic decays and as yet not measured. Having improved the precision of three-pion decays [7,8] and measured precisely the semileptonic decay rates, CPLEAR allows the determination of all parameters with an unprecedented accuracy. We stress here that they are obtained free of theoretical assumptions, apart from unitarity, unlike others obtained previously also using the unitarity relation [6].

2 Phenomenology and the Bell–Steinberger relation

For the semileptonic decays of the neutral kaons we may consider four independent decay rates as a function of the decay time, depending on the strangeness of the kaon (K^0 or $\overline{\mathrm{K}}^0$) at the production time $t = 0$ and on the charge of the decay lepton (e^+ or e^-):

$$R_+(\tau) \equiv R\left[\mathrm{K}^0_{\,t=0} \to \mathrm{e}^+\pi^-\nu_{t=\tau}\right], \qquad \overline{R}_-(\tau) \equiv R\left[\overline{\mathrm{K}}^0_{\,t=0} \to \mathrm{e}^-\pi^+\overline{\nu}_{t=\tau}\right],$$
$$R_-(\tau) \equiv R\left[\mathrm{K}^0_{\,t=0} \to \mathrm{e}^-\pi^+\overline{\nu}_{t=\tau}\right], \qquad \overline{R}_+(\tau) \equiv R\left[\overline{\mathrm{K}}^0_{\,t=0} \to \mathrm{e}^+\pi^-\nu_{t=\tau}\right].$$

These four rates are parametrized (see Ref. [2]) as a function of the mixing parameters ϵ (T-violation parameter) and δ (CPT-violation parameter):

$$\epsilon = \frac{\Lambda_{\overline{\mathrm{K}}^0,\mathrm{K}^0} - \Lambda_{\mathrm{K}^0,\overline{\mathrm{K}}^0}}{2(\lambda_{\mathrm{L}} - \lambda_{\mathrm{S}})} \qquad \text{and} \qquad \delta = \frac{\Lambda_{\overline{\mathrm{K}}^0,\overline{\mathrm{K}}^0} - \Lambda_{\mathrm{K}^0,\mathrm{K}^0}}{2(\lambda_{\mathrm{L}} - \lambda_{\mathrm{S}})}.$$

Here, Λ_{ij} are the elements of the matrix Λ, describing the time evolution of K^0 and $\overline{\mathrm{K}}^0$, and $\lambda_{\mathrm{L,S}} = m_{\mathrm{L,S}} - (\mathrm{i}/2)\Gamma_{\mathrm{L,S}}$ its eigenvalues; $m_{\mathrm{L,S}}$ and $\Gamma_{\mathrm{L,S}}$ are the masses and decay widths for the $\mathrm{K_L}$ and $\mathrm{K_S}$ states. We also define $\Delta m = m_{\mathrm{L}} - m_{\mathrm{S}}$ and $\gamma = \Gamma_{\mathrm{S}} + \Gamma_{\mathrm{L}}$. The $\mathrm{K_L}$ mixing parameter is defined as $\epsilon_{\mathrm{L}} = \epsilon - \delta$.

The decay amplitudes corresponding to the four rates can be written as

$$\langle \mathrm{e}^+\pi^-\nu|\Lambda|\mathrm{K}^0\rangle = a + b, \qquad \langle \mathrm{e}^-\pi^+\overline{\nu}|\Lambda|\overline{\mathrm{K}}^0\rangle = a^* - b^*,$$
$$\langle \mathrm{e}^-\pi^+\overline{\nu}|\Lambda|\mathrm{K}^0\rangle = c + d, \qquad \langle \mathrm{e}^+\pi^-\nu|\Lambda|\overline{\mathrm{K}}^0\rangle = c^* - d^*.$$

The amplitudes b and d are CPT violating, c and d describe possible violations of the $\Delta S = \Delta Q$ rule, and the imaginary parts are all T violating. The quantities

$$x = \frac{c^* - d^*}{a + b} \qquad \text{and} \qquad \overline{x} = \frac{c^* + d^*}{a - b}$$

describe the violation of the $\Delta S = \Delta Q$ rule in decays into positive and negative leptons, respectively, while $y = -b/a$ describes CPT violation in semileptonic decays in the case where the $\Delta S = \Delta Q$ rule holds. The parameters $x_+ = (x + \overline{x})/2$ and $x_- = (x - \overline{x})/2$ describe a violation of the $\Delta S = \Delta Q$ rule in CPT-conserving and CPT-violating amplitudes, respectively. We note that the parametrizations used here are equivalent to those of Refs. [9,10] but formulated in a slightly different notation.

In the $\mathrm{K_S} - \mathrm{K_L}$ basis the Bell–Steinberger relation [3] can be written as

$$-i(\lambda_{\mathrm{L}}^* - \lambda_{\mathrm{S}})\langle \mathrm{K_L}|\mathrm{K_S}\rangle = \sum \langle f|\Lambda|\mathrm{K_L}\rangle^* \langle f|\Lambda|\mathrm{K_S}\rangle \tag{1}$$

where we sum over all the decay final states f. The above equation becomes

$$\mathrm{Re}(\epsilon) - i\mathrm{Im}(\delta) = \frac{1}{2(i\Delta m + \frac{1}{2}\gamma)} \times \sum \mathrm{A}_{f_{\mathrm{L}}}\mathrm{A}_{f_{\mathrm{S}}}^* \tag{2}$$

with

$$A_{f_{\mathrm{L}}} = \langle f|\Lambda|\mathrm{K_L}\rangle \,, \qquad A_{f_{\mathrm{S}}} = \langle f|\Lambda|\mathrm{K_S}\rangle \,,$$

$$\sum A_{f_{\mathrm{L}}} A_{f_{\mathrm{S}}}^* = \sum \left(|\mathrm{A_S}|^2 \eta_{\pi\pi}\right) + \sum \left(|\mathrm{A_L}|^2 \eta_{\pi\pi\pi}^*\right) \\ + 2\left[\mathrm{Re}(\epsilon) - \mathrm{Re}(y) - i(\mathrm{Im}(x_+) + \mathrm{Im}(\delta))\right]|f_{\pi\ell\nu}|^2$$

and

$$|\mathrm{A_S}|^2 = \mathrm{BR}^{\mathrm{S}}_{\pi\pi}\Gamma_{\mathrm{S}}$$
$$|\mathrm{A_L}|^2 = \mathrm{BR}^{\mathrm{L}}_{\pi\pi\pi}\Gamma_{\mathrm{L}}$$
$$|f_{\pi\ell\nu}|^2 = \mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}\Gamma_{\mathrm{L}}$$

where BR stands for Branching Ratio with the upper index referring to the decay particle and the lower index to the final state, $\eta_{\pi\pi}$ and $\eta_{\pi\pi\pi}$ are the CP-violation parameters when neutral kaons decay to two and three pions, respectively, and ℓ denotes electrons and muons. The radiative modes, like $\pi^+\pi^-\gamma$, are included in the corresponding parent modes [11]. Channels with $\mathrm{BR}^{\mathrm{S}}_f$ (or $\mathrm{BR}^{\mathrm{L}}_f \times \Gamma_{\mathrm{L}}/\Gamma_{\mathrm{S}}) < 10^{-5}$ do not contribute to Eq. (2) within the accuracy of the present analysis.

From Eq. (2) we obtain an explicit expression for the parameters $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$:

$$\begin{pmatrix} \mathrm{Im}(\delta) \\ \mathrm{Re}(\epsilon) \end{pmatrix} = \frac{\Gamma_{\mathrm{S}}}{(\gamma - 2|f_{\pi\ell\nu}|^2)(\mu^2+1)} \begin{pmatrix} \mu & -1 \\ 1 & \mu \end{pmatrix} \\ \begin{pmatrix} [1 - \gamma_{\mathrm{LoS}}\mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}]\mathrm{Re}(\eta_{\pi\pi}) + [-2\mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}\mathrm{Re}(y) + \mathrm{BR}^{\mathrm{L}}_{\pi\pi\pi}\mathrm{Re}(\eta_{\pi\pi\pi})]\gamma_{\mathrm{LoS}} \\ [1 - \gamma_{\mathrm{LoS}}\mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}]\mathrm{Im}(\eta_{\pi\pi}) - [\mathrm{BR}^{\mathrm{L}}_{\pi\pi\pi}\mathrm{Im}(\eta_{\pi\pi\pi}) + 2\mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}\mathrm{Im}(x_+)]\gamma_{\mathrm{LoS}} \end{pmatrix} \qquad (3)$$

with

$$\mathrm{Re}(\eta_{\pi\pi}) = |\eta_{+-}|\cos(\phi_{+-})\left(1 - [(1-r) + r\sin(\Delta\phi)\tan(\phi_{+-})]\,\mathrm{BR}^{\mathrm{S}}_{\pi^0\pi^0}\right)$$
$$\mathrm{Im}(\eta_{\pi\pi}) = |\eta_{+-}|\sin(\phi_{+-})\left(1 - [(1-r) - r\sin(\Delta\phi)\cot(\phi_{+-})]\,\mathrm{BR}^{\mathrm{S}}_{\pi^0\pi^0}\right)$$
$$\mathrm{BR}^{\mathrm{L}}_{\pi\pi\pi}\mathrm{Re}(\eta_{\pi\pi\pi}) = \mathrm{BR}^{\mathrm{L}}_{\pi^+\pi^-\pi^0}\mathrm{Re}(\eta_{+-0}) + \mathrm{BR}^{\mathrm{L}}_{\pi^0\pi^0\pi^0}\mathrm{Re}(\eta_{000})$$
$$\mathrm{BR}^{\mathrm{L}}_{\pi\pi\pi}\mathrm{Im}(\eta_{\pi\pi\pi}) = \mathrm{BR}^{\mathrm{L}}_{\pi^+\pi^-\pi^0}\mathrm{Im}(\eta_{+-0}) + \mathrm{BR}^{\mathrm{L}}_{\pi^0\pi^0\pi^0}\mathrm{Im}(\eta_{000})$$

and

$$\mu = \frac{2\Delta m}{\gamma - 2|f_{\pi\ell\nu}|^2}\,, \quad \gamma_{\mathrm{LoS}} = \frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{S}}}\,, \quad r = \frac{|\eta_{00}|}{|\eta_{+-}|}\,, \quad \Delta\phi = \phi_{00} - \phi_{+-}\,.$$

Table 1 summarizes the experimental values of the parameters to be entered on the right-hand side of Eq. (3). The value of Δm in Table 1 results from experiments [11] which do not assume CPT invariance in the decay (regeneration experiments). We note that experimental data exist for all the parameters related to two- and three-pion decays, $\eta_{\pi\pi}$ and $\eta_{\pi\pi\pi}$, which contain all the information required for the present analysis, including decay amplitudes. For the semileptonic decays we lack the measurement of the parameter $\mathrm{Re}(y)$, while for the parameter $\mathrm{Im}(x_+)$ the only existing measurement comes from CPLEAR [2].

However, the parameters $\mathrm{Re}(y)$ and $\mathrm{Im}(x_+)$, together with $\mathrm{Re}(\epsilon)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(\delta)$, and $\mathrm{Re}(x_-)$, do enter in the following two semileptonic asymmetries:

$$\frac{\overline{R}_+ - R_-[1 + 4\mathrm{Re}(\epsilon_{\mathrm{L}})]}{\overline{R}_+ + R_-[1 + 4\mathrm{Re}(\epsilon_{\mathrm{L}})]} = 2(\mathrm{Re}(\epsilon) - \mathrm{Re}(y) + \mathrm{Re}(\delta)) \\ + 2\frac{\mathrm{Im}(x_+)\mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\tau}\sin(\Delta m\tau) + \mathrm{Re}(x_-)\mathrm{E}_-(\tau)}{\mathrm{E}_+(\tau) - \mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\tau}\cos(\Delta m\tau)}\,, \qquad (4)$$

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Parameter	Value	
$|\eta_{+-}|$	$(2.283 \pm 0.025) \times 10^{-3}$	Ref. [11, 12]
ϕ_{+-}	$43.6° \pm 0.6°$	Ref. [11, 12]
Δm	$(530.19 \pm 1.54) \times 10^7 \hbar s^{-1}$	see text
$\Delta\phi$	$-0.3° \pm 0.8°$	Ref. [13]
$r = \frac{|\eta_{00}|}{|\eta_{+-}|}$	0.9930 ± 0.0020	Ref. [11]
$\mathrm{Re}(\eta_{+-0})$	$(-2 \pm 8) \times 10^{-3}$	Ref. [7]
$\mathrm{Im}(\eta_{+-0})$	$(-2 \pm 9) \times 10^{-3}$	Ref. [7]
$\mathrm{Re}(\eta_{000})$	0.08 ± 0.11	Ref. [8, 14]
$\mathrm{Im}(\eta_{000})$	0.07 ± 0.16	Ref. [8, 14]
$\mathrm{BR}^{\mathrm{S}}_{\pi^0\pi^0}$	$(31.39 \pm 0.28)\%$	Ref. [11]
$\mathrm{BR}^{\mathrm{L}}_{\pi^+\pi^-\pi^0}$	$(12.56 \pm 0.20)\%$	Ref. [11]
$\mathrm{BR}^{\mathrm{L}}_{\pi^0\pi^0\pi^0}$	$(21.12 \pm 0.27)\%$	Ref. [11]
$\mathrm{BR}^{\mathrm{L}}_{\pi\ell\nu}$	$(65.95 \pm 0.37)\%$	Ref. [11]
τ_{S}	$(0.8934 \pm 0.0008) \times 10^{-10} s$	Ref. [11]
τ_{L}	$(5.17 \pm 0.04) \times 10^{-8} s$	Ref. [11]

Table 1: Experimental status of the neutral-kaon system parameters.

and

$$\frac{\overline{R}_- - R_+[1 + 4\mathrm{Re}(\epsilon_{\mathrm{L}})]}{\overline{R}_- + R_+[1 + 4\mathrm{Re}(\epsilon_{\mathrm{L}})]} = 2(-\mathrm{Re}(\epsilon) + \mathrm{Re}(y) + \mathrm{Re}(\delta))$$

$$+ \frac{-4\mathrm{Re}(\delta)\mathrm{E}_-(\tau) - 2\mathrm{Re}(x_-)\mathrm{E}_-(\tau) + [2\mathrm{Im}(x_+) + 4\mathrm{Im}(\delta)]\mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\tau}\sin(\Delta m\tau)}{\mathrm{E}_+(\tau) + \mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})\tau}\cos(\Delta m\tau)} ,$$

(5)

with

$$\mathrm{E}_\pm(\tau) = \frac{(\mathrm{e}^{-\Gamma_{\mathrm{S}}\tau} \pm \mathrm{e}^{-\Gamma_{\mathrm{L}}\tau})}{2} .$$

Using as constraints the Bell–Steinberger relation, Eq. (3), and the δ_ℓ charge asymmetry in the $\mathrm{K_L}$ semileptonic decays, the determination of the parameters $\mathrm{Re}(\epsilon)$, δ, $\mathrm{Im}(x_+)$, $\mathrm{Re}(x_-)$ and $\mathrm{Re}(y)$ is possible. For the charge asymmetry in the $\mathrm{K_L}$ semileptonic decays we use the value from Ref. [11]:

$$\delta_\ell = 2\mathrm{Re}(\epsilon) - 2\mathrm{Re}(\delta) - 2\mathrm{Re}(y) - 2\mathrm{Re}(x_-) = (3.27 \pm 0.12) \times 10^{-3} .$$

(6)

For the analysis of the asymmetries in Eqs. (4) and (5), we use the normalization procedure described in Ref. [2].

3   Results and systematic errors

We have used the semileptonic decay rates measured by CPLEAR to calculate experimental values for the asymmetries of Eqs. (4) and (5). These experimental data were fitted simultaneously with Eqs. (4) and (5), using Eqs. (3) and (6) as constraints. The values of Table 1 were used in the fit; all known correlations among these quantities were taken into account.

The result of the fit is

$$\mathrm{Im}(x_+) = (-2.0 \pm 2.6) \times 10^{-3}$$
$$\mathrm{Re}(y) = (\ \ 0.3 \pm 3.0) \times 10^{-3}$$
$$\mathrm{Re}(\delta) = (\ \ 2.4 \pm 2.7) \times 10^{-4}$$
$$\mathrm{Re}(x_-) = (-0.5 \pm 3.0) \times 10^{-3} ,$$

and

$$\mathrm{Re}(\epsilon) = (164.9 \pm 2.5) \times 10^{-5} \, ,$$
$$\mathrm{Im}(\delta) = (\quad 2.4 \pm 5.0) \times 10^{-5} \, ,$$

with $\chi^2/\mathrm{d.o.f.} = 1.09$. The correlation coefficients between the various parameters are shown in Table 2.

	$\mathrm{Im}(x_+)$	$\mathrm{Re}(y)$	$\mathrm{Re}(\delta)$	$\mathrm{Re}(x_-)$	$\mathrm{Re}(\epsilon)$	$\mathrm{Im}(\delta)$
$\mathrm{Im}(x_+)$	-	-0.624	-0.555	0.651	-0.142	0.075
$\mathrm{Re}(y)$		-	0.279	-0.997	-0.159	-0.075
$\mathrm{Re}(\delta)$			-	-0.349	0.039	-0.051
$\mathrm{Re}(x_-)$				-	0.060	0.109
$\mathrm{Re}(\epsilon)$					-	-0.256
$\mathrm{Im}(\delta)$						-

Table 2: The correlation coefficients for the parameters $\mathrm{Im}(x_+)$, $\mathrm{Re}(y)$, $\mathrm{Re}(\delta)$, $\mathrm{Re}(x_-)$, $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$.

We studied the contribution to the final errors arising from systematic effects in the CPLEAR semileptonic data. These effects are related to
- background level and background asymmetry,
- normalization correction (parameters $\alpha_{2\pi}$ and η),
- decay-time resolution,
- regeneration correction.

They were determined in exactly the same way as in Ref. [2], and are summarized in Table 3.

Source	$\Delta(\mathrm{Im}(x_+))$ $[10^{-3}]$	$\Delta(\mathrm{Re}(y))$ $[10^{-3}]$	$\Delta(\mathrm{Re}(\delta))$ $[10^{-4}]$	$\Delta(\mathrm{Re}(x_-))$ $[10^{-3}]$	$\Delta(\mathrm{Re}(\epsilon))$ $[10^{-5}]$	$\Delta(\mathrm{Im}(\delta))$ $[10^{-5}]$
Background level	± 0.1	± 0.1	± 0.1	± 0.1	0	0
Background asymmetry	± 0.4	± 0.2	± 0.2	± 0.2	± 0.1	± 0.1
$\alpha_{2\pi}$	0	± 0.1	± 0.5	± 0.1	0	0
η	± 0.02	± 0.5	± 0.02	± 0.02	0	0
Decay-time resolution	± 0.1	± 0.1	0	± 0.1	0	0
Regeneration	± 0.1	± 0.1	± 0.25	± 0.1	0	0
Total syst. error	$\pm \mathbf{0.5}$	$\pm \mathbf{0.6}$	$\pm \mathbf{0.6}$	$\pm \mathbf{0.3}$	$\pm \mathbf{0.1}$	$\pm \mathbf{0.1}$

Table 3: Systematic errors arising from the CPLEAR semileptonic data.

The error on $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$ is dominated by the error on η_{000}. The CPLEAR accuracy on η_{+-0} is such that its contribution becomes negligible. If we assume that there is no $I = 3$ decay amplitude in the three-pion decay, it follows that $\eta_{+-0} = \eta_{000}$ and our analysis yields

$$\mathrm{Re}(\epsilon) = (165.0 \pm 1.9) \times 10^{-5} \, ,$$
$$\mathrm{Im}(\delta) = (-0.5 \pm 2.0) \times 10^{-5} \, ,$$

thus reducing the errors on the parameters $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$ by a factor of two, while the correlation coefficient reduces to -0.003.

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4   Results and conclusions

Our final result, assuming only unitarity, is

$$\mathrm{Re}(\epsilon) = (164.9 \pm 2.5) \times 10^{-5} \,,$$
$$\mathrm{Im}(\delta) = (\;\;\; 2.4 \pm 5.0) \times 10^{-5} \,,$$

and

$$\mathrm{Re}(y) = (\;\;\; 0.3 \pm 3.1) \times 10^{-3} \,,$$
$$\mathrm{Im}(x_+) = (-2.0 \pm 2.7) \times 10^{-3} \,,$$
$$\mathrm{Re}(x_-) = (-0.5 \pm 3.0) \times 10^{-3} \,,$$
$$\mathrm{Re}(\delta) = (\;\;\; 2.4 \pm 2.8) \times 10^{-4} \,.$$

Our results on $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$ are almost one order of magnitude more accurate than those of a previous similar analysis [4] owing to improvements in the accuracy of various measurements where CPLEAR has made significant contributions. The fact that $\mathrm{Re}(\epsilon)$ and $\mathrm{Im}(\delta)$ are essentially determined through the Bell–Steinberger relation allows $\mathrm{Re}(y)$ to be obtained explicitly: a result which could not be achieved from semileptonic data alone. Moreover, the present analysis yields accuracies for the parameters $\mathrm{Im}(x_+)$ and $\mathrm{Re}(x_-)$ which are about one order of magnitude better than those in Ref. [2], while the accuracy on the parameter $\mathrm{Re}(\delta)$ is comparable with that obtained in Ref. [2].

Table 2 shows a strong anticorrelation between the values of $\mathrm{Re}(x_-)$ and $\mathrm{Re}(y)$ given by the fit. If we consider their sum we find

$$\mathrm{Re}(y + x_-) = (-0.2 \pm 0.3) \times 10^{-3} \,. \tag{7}$$

We note that this quantity appears in the asymptotic value of the time-reversal asymmetry measured by CPLEAR [1]. The present result confirms that the possible contribution to this asymmetry, arising from CPT-violating decay amplitudes is negligible.

5   Acknowledgements

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