Quantum-Gravitational Diffusion and Stochastic Fluctuations in the Velocity of Light

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Abstract

We argue that quantum-gravitational fluctuations in the space-time background give the vacuum non-trivial optical properties that include diffusion and consequent uncertainties in the arrival times of photons, causing stochastic fluctuations in the velocity of light *in vacuo*. Our proposal is motivated within a Liouville string formulation of quantum gravity that also suggests a frequency-dependent refractive index of the particle vacuum. We construct an explicit realization by treating photon propagation through quantum excitations of D-brane fluctuations in the space-time foam. These are described by higher-genus string effects, that lead to stochastic fluctuations in couplings, and hence in the velocity of light. We discuss the possibilities of constraining or measuring photon diffusion *in vacuo* via γ-ray observations of distant astrophysical sources.

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1 Introduction

The propagation of light through media with non-trivial optical properties is subject to three important effects: one is a variation in the light velocity with photon energy, namely a frequency-dependent refractive index, a second is a difference between the velocities of light with different polarizations, namely birefringence, and the third is a diffusive spread in the apparent velocity of light. We have argued previously that quantum-gravitational fluctuations in the space-time background may endow the conventional particle vacuum with such non-trivial optical properties, in particular a frequency-dependent refractive index [1]. We have also observed that such an effect may be severely constrained by careful observations of distant astrophysical objects whose emissions exhibit short time structures, such as Gamma-Ray Bursters (GRBs) [2]. The possibility of quantum-gravitational birefringence has been raised within a loop approach to quantum gravity [3]. The purpose of this paper is to propose how quantum-gravitational diffusion may spread the arrival times of photons from distant sources, even if they have the same energies (frequencies), corresponding to stochastic fluctuations in the apparent velocity of light.

An example of this phenomenon in a conventional optical medium, which illustrates clearly the intuition behind our proposal, has been discussed in [4]. Light propagating through ice may encounter air bubbles, which have a different refractive index and hence induce scattering and diffusion. We have argued previously [1] that foamy fluctuations in space-time generate a quantum-gravitational 'aerogel' with an effective refractive index $\sim E/M_{QG}$ *in vacuo*, where M_{QG} is some characteristic scale that may approach the Planck mass M_P. Now we argue that the quantum uncertainties in these foamy quantum-gravitational fluctuations in turn induce fluctuations in the refractive index that act as scattering centres analogous to the air bubbles of [4]. As illustrations of these quantum-gravitational fluctuations, one might consider foaming microscopic black holes that induce local fluctuations in the wave front close to their horizons, leading on larger distance scales to diffusive broadening of any light pulse.

In this paper, we provide a mathematical formulation of this physical intuition, using the framework for quantum gravity adopted in [1], namely that of Liouville strings [5], in which classical conformal string backgrounds are allowed to exhibit quantum space-time fluctuations. These cause departures from criticality and conformal symmetry that may be counterbalanced by introducing on the string world sheet a Liouville field with non-trivial dynamics. This Liouville field may be considered as a dynamical renormalization scale, and conformal invariance may be restored by dressing operators with appropriate Liouville factors [6]. Since the quantum-gravitational space-time fluctuations drive the string super-critical, the Liouville field has negative metric, and we identify it with target time [5, 7].

Interactions of the Liouville field with the conventional string degrees of freedom are controlled by the Zamolodchikov flow [8] of the world-sheet renormalization-group equations. These incorporate the conventional Hamiltonian structure and S-matrix description of the scattering of low-energy particles, but also encode infor-

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mation on the energy-dependent interactions of the probe with non-local quantum-gravitational degrees of freedom. These fail to decouple [5] from the low-energy matter in the presence of foamy singular metric fluctuations. Since a low-energy observer cannot detect such global modes by direct scattering experiments, integrating them out of the low-energy effective theory makes the low-energy system resemble an 'open' quantum system interacting with an unobserved 'environment' of these quantum-gravitational solitonic states.

The outline of this paper is as follows. In Section 2, we review relevant features of this Liouville string framework [5] for space-time foam, discussing in particular the formal basis for the appearance of stochastic fluctuations in theory space. Next, in section 3, we construct a specific realization of this non-critical string approach by considering quantum fluctuations of D-brane excitations in the vacuum. Then, in Section 4, we discuss the propagation of photons through this quantum D-brane foam. Section 5 shows how stochastic fluctuations occur in the context of D branes, and exhibits the expected diffusion effect [1]. Finally, in Section 6, we discuss how this phenomenon may be constrained or detected by observations of distant astrophysical objects.

2   Relevant Aspects of Liouville String

In this Section we review briefly features of Liouville that play roles in our subsequent discussion. In this approach, one treats quantum fluctuations in the space-time background as deviations from a classical string background that is described by supercritical deformations of a conformal field theory on the world sheet. One may restore criticality by Liouville dressing of world-sheet model fields [6]. Specifically, consider a conformal σ model, described by an action S^* on the world sheet Σ, subject to non-conformal deformations $\int_\Sigma g^i V_i d^2\sigma$, with V_i appropriate vertex operators:

$$S_g = S^* + \int_\Sigma g^i V_i d^2\sigma \qquad (1)$$

The non-conformal nature of the couplings g^i implies that their world-sheet renormalization-group β functions β^i do not vanish. The following is the generic structure of such β functions, close to such a fixed point $\{g^{i*} = 0\}$:

$$\beta^i = (h_i - 2)g^i + c^i_{jk}g^j g^k + o(g^3). \qquad (2)$$

One cancels the deviation from criticality by world-sheet 'gravitational' dressing that corresponds to defining renormalized couplings in a curved space. To $\mathcal{O}(g^2)$, one has [6, 9, 10]:

$$\lambda^i(t) = \lambda^i e^{\alpha_i t} + \frac{\pi}{Q \pm 2\alpha_i} c^i_{jk}\lambda^j\lambda^k t e^{\alpha_i t} + O(\lambda^3), \qquad Q^2 = c - 25 \qquad (3)$$

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[1]We do not find any evidence to support the suggestion of birefringence [3].

where t is the zero mode of the Liouville field, Q^2 is the central charge deficit which is ≥ 0 for the supercritical string case of interest here, and the α_i are gravitational anomalous dimensions:

$$\alpha_i(\alpha_i + Q) = h_i - 2 \qquad \text{for} \qquad c \geq 25 \tag{4}$$

The supercriticality implies a Minkowskian signature for the Liouville field [11]. enabling us to identify its zero mode with target time [5, 7] [2]. After the renormalization (3), the critical-string conformal invariance conditions corresponding to the vanishing of flat-space β functions are replaced by [6, 9, 10, 5]:

$$\ddot{\lambda}^i + Q\dot{\lambda}^i = -\beta^i \qquad \text{for} c \geq 25. \tag{5}$$

The minus sign in front of the flat-world-sheet β functions reflects the supercriticality of the string.

The propagation of non-relativistic light-particle modes was examined in [5], where a modification of the quantum Liouville equation for the density matrix was found, as proposed in [12]. The propagation of massless probes was examined in [1], where it was found that the conventional relativistic energy-momentum dispersion relation is modified in non-critical Liouville strings, as a result of the interaction with the quantum-gravitational environment, as we now review briefly for the benefit of non-expert readers.

The first step is to observe that, in the case of interest, the non-criticality of the σ model describing the effective theory is induced by the operator-product-expansion coefficients $c^i_{j\hat{k}}$ that express the interaction of a low-energy probe (latin indices) with quantum-gravitational modes (hatted latin indices). Hence, $Q^2 = 0$ to lowest order, and the Liouville anomalous-dimension coefficients α_i are given simply by the magnitude of the spatial momentum of the massless probe:

$$\alpha_i = |\overline{k}| \tag{6}$$

We use this to rewrite (3) approximately, to order g^2, as [1]:

$$\lambda^i(t) \simeq g^i e^{(\alpha_i + \Delta\alpha_i)t} \tag{7}$$

where the shift $\Delta\alpha_i$ is given by:

$$\Delta\alpha_i \simeq \frac{\pi}{2\alpha_i} c^i_{j\hat{k}} g^{\hat{k}} \tag{8}$$

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[2] We note for later use that [10], in the gravitationally-dressed (Liouville) world-sheet theory, only the leading-order coefficients in the β functions are renormalization-scheme independent. This implies that, when one identifies the Liouville field with target time, one loses general covariance in the foamy ground state. This should be thought of as a spontaneous breaking of the symmetry by quantum-gravitational fluctuations.

We next make the generic hypothesis, which is supported by some explicit examples [13], especially in the context of D branes [14], as discussed in more detail below, that

$$\frac{1}{\alpha_i} c^i_{j\hat{k}} \sim \xi E/M_{QG},$$ (9)

where E is the energy scale of the low-energy probe, $\xi = \pm\mathcal{O}(1)$ and M_{QG} is a characteristic of gravitational interactions, possibly to be identified with the Planck scale $M_P \sim 10^{19}$ GeV. We infer from (6), (7), (8) and (9) a modified dispersion relation

$$E = |\bar{k}| \times (1 + \xi E/M_{QG})$$ (10)

corresponding to an energy- (frequency-)dependent refractive index $\eta = 1 + \mathcal{O}(E/M_{QG})$.

It was observed in [2] that such a quantum-gravitational effect could be probed by observations of distant astrophysical sources, such as GRBs, Active Galactic Nuclei (AGNs) or pulsars, and that these could place important constraints on models in which such a refractive index is suppressed only by a single power of $1/M_{QG}$, potentially with sensitivity to $M_{QG} \sim M_P$. Here we go further, remarking that quantum effects due to world-sheet topology fluctuations [15], which arise in the context of first-quantized strings from the summation over genera, may cause observable stochastic effects that lead to a diffusive spread in apparent velocities even for photons of fixed energy (frequency). This would imply the spreading of an initial pulse, and place limitations on the resolutions of experimental measurements of distant astrophysical sources.

The basic motivation for this suggestion comes from the quantization of the target-space fields $\{g^i\}$ in string theory, which arises from the summation over world-sheet topologies. In our case, this procedure leads to quantum fluctuations in the σ-model couplings g^i felt by the propagating (low-energy) string particle modes. The probability density $\mathcal{P}[g]$ in the space of σ-model theories is given in leading approximation by:

$$\mathcal{P}[g] \sim \exp\left(-g^{\hat{i}} \frac{G_{\hat{i}\hat{j}}}{\Gamma} g^{\hat{j}}\right)$$ (11)

where $G_{ij} \sim <V_i V_j>$ is the Zamolodchikov metric in theory space [8], for vertex operators V_i corresponding to the couplings g^i. Formally, the non-trivial probability density (11) arises [15, 16] as a result of the requiring the cancellation of modular infinities against renormalization-group divergences associated with singular configurations in the sum over genera [15]. The width Γ in (11) depends on some positive power of the string coupling constant g_s, and the precise form is model-dependent [16, 17]. The quantum uncertainties δg^i in the couplings are found by diagonalizing the basis in theory space g^i, for which knowledge of the Zamolodchikov metric is essential.

In the presence of fluctuations in the world-sheet topology of the string, which were not discussed in [1, 2], the shifts (8) associated with the refractive index of the vacuum will fluctuate as:

$$|\delta(\Delta\alpha_i)| \sim |\frac{1}{2\alpha_i}c^i_{j\hat{k}}\delta g^{\hat{k}}|$$

(12)

leading in turn to stochastic quantum fluctuations in the refractive index *in vacuo*. This in turn leads to a characteristic diffusive spread in the arrival times of photons identical energies, to which we return later. Before that, we first compute the Zamolodchikov metric and the related width Γ (11) in a specific model for quantum-gravitational foam based on D particles.

3   Fluctuations in D-particle Foam

We compute the quantum flucuations in (11) in the particular case where the gravitational degrees of freedom $g^{\hat{k}}$ are the collective coordinates and/or momenta of a system of N D particles [14, 17]. As discussed in the literature [18, 19, 16, 17] such collective coordinates can be described by operators characterizing the recoil induced by the scattering of string matter off the D-particle background. The corresponding σ-model deformation is:

$$Y^{ab}_i(x^0) = \ell_s(Y^{ab}_i\ell_s\epsilon + U^{ab}_i x^0)\epsilon\Theta_\epsilon(x^0)$$

(13)

where the spatial coordinates of the D particle are identified with the couplings Y_i and the U_i correspond to their Galilean recoil velocities. These coordinates are to be regarded as σ-model couplings g^i in the sense of the previous section. The parameter $\epsilon \to 0^+$ regulates the ambiguous value of $\Theta(s)$ at $s = 0$, which ensures that the D-particle system starts moving only at the time $x^0 = 0$ [3]. It is related to the world-sheet ultraviolet cutoff scale Λ (measured in units of the world-sheet size Σ) by $\epsilon^{-2} = 2\ell_s^2 \log\Lambda$, where ℓ_s is the fundamental string length. For finite ϵ, the operators Y_i, U_i each have an anomalous dimension $-\frac{1}{2}|\epsilon|^2 < 0$ [18], and thus lead to a relevant deformation of S^*. The corresponding renormalization-group equations are [16]:

$$dY^{ab}_i/dt = U^{ab}_i, \ dU^{ab}_i/dt = 0,$$

(14)

which are the Galilean equations of motion for the D particles, if we identify the time with the world-sheet scale: $t = \ell_s \log\Lambda$.

The natural geometry on the moduli space $\mathcal{M}$ of deformed conformal field theories described by the above recoil operators (13) is given by the following Zamolodchikov

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[3]Throughout this paper, we consider the σ-model 'renormalized' velocity [19, 16, 17], which is related to the 'bare' velocity U_{Bi} by $U_i = U_{Bi}/\epsilon$. It is U_i which is an exactly-marginal deformation in a world-sheet renormalization group sense, and therefore corresponds to uniform D-particle motion for times $t > 0$.

metric $G^{ij}_{ab;cd} = \langle V^i_{ab} V^j_{cd} \rangle$ [17]:

$$
\begin{aligned}
G^{ij}_{ab;cd} &= \frac{4g_s^2}{\ell_s^2} \left[\delta^{ij} I_N \otimes I_N - \frac{g_s^2}{6} \left\{ I_N \otimes \left(U^i U^j + U^j U^i \right) + U^i \otimes U^j \right. \right. \\
&\left. \left. + U^j \otimes U^i + \left(U^i U^j + U^j U^i \right) \otimes I_N \right\} \right]_{db;ca} + \mathcal{O}\left(g_s^6\right),
\end{aligned}
\tag{15}
$$

where I_N is the $N \times N$ identity matrix and we have renormalized g_s to the time-independent coupling $g_s/|\epsilon|\ell_s$. The canonical momentum P^i_{ab} of the D-particle system is given in the Schrödinger picture by the expectation value of $-i\delta/\delta Y^{ab}_i$ evaluated in a σ model deformed by the operator V_{ab}, i.e., $P^i_{ab} \equiv \langle V^i_{ab} \rangle$ [17]:

$$
P^i_{ab} = \frac{8g_s^2}{\ell_s} \left[U^i - \frac{g_s^2}{6} \left(U_k^2 U^i + U_k U^i U^k + U^i U_k^2 \right) \right]_{ba} + \mathcal{O}\left(g_s^6\right)
\tag{16}
$$

which coincides with the contravariant velocity $P^i_{ab} = \ell_s G^{ij}_{ab;cd} \dot{Y}^{cd}_j$ on $\mathcal{M}$.

In agreement with general arguments [5], we note that the moduli space dynamics can be derived [17] from a Lagrangian of the form

$$
\mathcal{L} = -\frac{\ell_s}{2} \dot{Y}^{ab}_i G^{ij}_{ab;cd} \dot{Y}^{cd}_j - \mathcal{C}
\tag{17}
$$

which coincides [17] to leading order with the non-Abelian Born-Infeld effective action [20] for the target-space D-particle dynamics:

$$
\mathcal{L}_{\mathrm{NBI}} = \frac{1}{\ell_s \bar{g}_s} \, Tr \, \mathrm{Sym} \sqrt{\det_{M,N} \left[\eta_{MN} I_N + \ell_s^2 \bar{g}_s^2 \, F_{MN} \right]}
\tag{18}
$$

where the trace Tr is taken over $U(N)$ group indices,

$$
\mathrm{Sym}(M_1, \ldots, M_n) \equiv \frac{1}{n!} \sum_{\pi \in S_n} M_{\pi_1} \cdots M_{\pi_n}
\tag{19}
$$

is the symmetrized matrix product, and the components of the dimensionally-reduced field-strength tensor are given by $F_{0i} = \frac{1}{\ell_s^2} \dot{\bar{Y}}_i$ and $F_{ij} = \frac{\bar{g}_s}{\ell_s^4}[\bar{Y}_i, \bar{Y}_j]$. In the Abelian reduction to the case of a single D particle, the Lagrangian (18) reduces to the usual one describing the free relativistic motion of a massive particle. The leading order F^2 term in the expansion of (18) is just the usual Yang-Mills Lagrangian.

The formalism described in [17] is a non-trivial application of the theory of Liouville string and logarithmic operators. We note, moreover, that the derivation of the Lagrangian (18) was made 'off-shell', in other words we have compared generalized momenta in theory space with those derived from the Born-Infeld lagrangian. The equivalence between the two formalisms extends beyond the equations of motion, which are the conformal-invariance conditions of the σ model. This is a central aspect of the recoil approach: there are deviations from the usual equations of motion for low-energy modes, because conformal invariance is violated by the recoil process [18, 19], and the Zamolodchikov world-sheet renormalization-group flow provides an off-shell treatment of this recoil problem.

The leading contributions to the quantum fluctuations in $\mathcal{M}$ arise from pinched annulus diagrams in the summation over world-sheet genera of the σ model [15]. Symbolically, these lead to contributions in the genus expansion of the form

$$\text{(20)}$$

consisting of thin tubes of width $\delta \to 0$ (world-sheet wormholes) attached to the world-sheet surface Σ. The attachment of each tube corresponds to inserting a bilocal pair $V_{ab}^i(s)V_{cd}^j(s')$ on the boundary $\partial\Sigma$, with interaction strength g_s^2, and computing the string propagator along the thin tubes. There are modular divergences of the form $\log \delta$, which should be identified with world-sheet divergences at lower genera [5], so we set

$$\log \delta = 2g_s^\eta \log \Lambda = \frac{1}{\ell_s^2} g_s^\eta \epsilon^{-2} \tag{21}$$

The exponent $\eta \neq 0$ in this Fischler-Susskind-like [21] relation, which allows one to cancel logarithmic modular divergences by relating the strip widths δ to the world-sheet ultraviolet scale Λ, arises because the relation (21) is induced by the string loop expansion. As we have argued in [17], the case $\eta \neq 0$ allows direct comparison of our results with other results in the string literature, based on alternative approaches [22]. However, we do not determine η here, since we only consider string interactions between D branes. Considering brane exchanges between the system of D branes, may enable the value of η to be fixed (see also below), but this point is not important for the present analysis.

One effect of the dilute gas of world-sheet wormholes is to exponentiate the bilocal operator, leading to a change in the σ-model action [15, 16]. This contribution can be cast into the form of a local action by rewriting it as a Gaussian functional integral over wormhole parameters ρ_i^{ab}, as described in the previous section, and we arrive finally at [17]:

$$\sum_{\text{genera}} Z_N[Y] \simeq \left\langle \int_{\mathcal{M}} D\rho \; e^{-\rho_i^{ab} G_{ab;cd}^{ij} \rho_j^{cd}/2|\epsilon|^2 \ell_s^2 g_s^2 \log \delta} \; W[\partial\Sigma; Y + \rho] \right\rangle_0 \tag{22}$$

We see from (22) that the effect of this resummation over pinched genera is to induce quantum fluctuations of the solitonic background, giving a statistical Gaussian spread to the D-particle couplings. Note that the width of the Gaussian distribution in (22), which we identify as the wave function of the system of D particles [5], is time-independent, and represents not the spread in time of a wave packet on $\mathcal{M}$, but rather the true quantum fluctuations of the D-particle coordinates.

The corresponding spatial uncertainties can be found by diagonalizing the Zamolodchikov metric (15), as was done in [17] and will not be repeated here. For the case

of a single D particle: $a = b$, one arrives at the uncertainties:

$$\left|\Delta X_{aa}^i\right| \equiv \Delta Y^i = \ell_s g_s^{\eta/2}\left(1 + \frac{g_s^2}{8\pi^3}\, u^2\, \delta^{i,1}\right) + \mathcal{O}\left(g_s^4\right) \geq \ell_s\, g_s^{\eta/2} \tag{23}$$

for the individual D-particle coordinates. For $\eta = 0$, the minimal length in (23) coincides with the standard string smearing [23], whereas for $\eta = \frac{2}{3}$ it matches the form of $\ell_{\rm P}$ which arises from the kinematical properties of D particles [22]. A value $\eta \neq 0$ is more natural, because the modular divergences should be small for weakly-interacting strings. We note that the uncertainty (23) is *time-independent* [17], which is important for experimental tests of the phenomenon, as we discuss below. The coordinate uncertainties for $a \neq b$ are responsible for the emergence of a true non-commutative structure of quantum space-time, and represent the genuine non-Abelian characteristics of multi-D-particle dynamics, but we are not concerned with such a case here.

4   Propagation of Photons in a D-Particle Foam Background

After this general discussion, we now discuss the propagation of photons in the background of multiple D particles, taking into account the recoil of the latter due to the scattering of the photons. This is immediate in the context of [17]: one simply adds to the argument of the determinant (18) an Abelian $U(1)$ field strength $f_{MN} = \partial_M a_N - \partial_N a_M$, where a_M denotes the $U(1)$ electromagnetic potential of the photon fields. The corresponding Born-Infeld effective action is considered in four-dimensional space time. We adopt the conventional view point that photons interacting with D particles may be represented by adding to the σ-model action S_σ on the (open) world sheet an electromagnetic potential background as a boundary term [24]:

$$S_\sigma \ni \frac{e}{c}\int_{\partial\Sigma} d\tau\, a_M(X)\partial_\tau X^M \tag{24}$$

in a standard (Neumann) open-string notation [4].

The non-relativistic heavy D-particle [17] corresponds to a time-dependent background (13), whilst the photon propagator depends on the full four-dimensional space time. It is important to note, though, that in our approach the coupled D-particle-photon system is *out of equilibrium*, due to the recoil process and its associated distortion of the surrounding space time. This is reflected in the fact that

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[4]The target-space Born-Infeld action, including photons interacting with D particles, may *alternatively* be considered as the target-space action of a three brane, i.e. a solitonic object in string theory with four coordinates obeying Neumann boundary conditions and the remainder Dirichlet boundary conditions [18]. It is possible that the above Neumann picture can be obtained from the Dirichlet one by a world-sheet T-duality transformation, but there are problems with this transformation at a quantum level [25]. We restrict ourselves here to the Neumann string picture.

the resulting backgrounds do not satisfy the classical equations of motion, as was mentioned earlier in the general context of the departure from conformal symmetry. Moreover [13], the recoil curves the surrounding space time, since it induces - via Liouville dressing and the identification [5, 13] of the Liouville mode with the target time t - graviton excitations for the string, which in a σ-model framework correspond to graviton backgrounds with non-trivial off-diagonal elements:

$$G_{0i} \sim \epsilon^2 U_i t \Theta(t) \tag{25}$$

where U_i is the velocity of the recoiling D particle.

The element (25) is obviously not Lorentz covariant, reflecting the spontaneous breaking of this symmetry by the ground state of the string. The splitting between the quantum-gravitational 'medium' and the propagating particle subsystem is not possible in a Lorentz-invariant way, and there is no formal reason that this should be so. On the contrary, spontaneous violation of Lorentz symmetry is generic in Liouville strings [11, 5]. Thus we may expect the interaction of such graviton modes with the photons to lead to a modification of the photon dispersion relation, analogous to non-Lorentz-covariant, e.g., thermal, effects in conventional media. A key difference between quantum-gravitational effects and those in conventional media [2] is that the former increase with the energy of a probe, whilst the latter attenuate with increasing energy.

Gravitational interactions may be incorporated in the Born - Infeld lagrangian (18) by replacing $\eta_{MN} \to G_{MN}$ and using the non-covariant expression (25) when calculating amplitudes. Thus the complete off-shell Born - Infeld Lagrangian for the interaction of photonic matter with D particles is:

$$\mathcal{L}_{\mathrm{NBI}} = \frac{1}{\ell_s \bar{g}_s} Tr \operatorname{Sym} \sqrt{\det_{M,N} \left[G_{MN} I_N + \ell_s^2 \frac{e^2}{c} f_{MN}(a) I_N + \ell_s^2 \bar{g}_s^2 F_{MN} \right]} \tag{26}$$

which is the basis for our subsequent discussion.

We first re-examine the possible order of magnitude of the frequency-dependent refarctive index (8) induced by such effects, concentrating first on the case of Abelian (single D-particle) defects. The terms in the effective action (18) that are relevant for the modification (8) of the photon dispersion relation arise from the three-point function terms in (3) that involve two photon excitations and one induced-graviton excitation (25). The appropriate term in a derivative expansion of the Born-Infeld action is:

$$\mathcal{L}_{NBI}^{photons,graviton} \ni f_{MN}(a) G_{NA} f_{AM} \sim f_{ij}(a) U_j f_{0i} \epsilon^2 t \tag{27}$$

where latin indices are spatial, and we used (25). Terms that are similar in order of magnitude, but not in tensorial structure, can also be obtained by combining the

f^2_{MN} terms with the determinant $\sqrt{-\det(G_{MN})}$ of the target-space metric [5]. An appropriate order-of-magnitude estimate of the recoil velocity is:

$$U_i \sim g_s|\overline{k}|/M_P \tag{28}$$

reflecting energy-momentum conservation in the scattering process [19, 16, 17], and assuming that the momentum of the recoiling D particle is of the same order as the energy-momentum of the photon. Working at times t after the recoil that are sufficiently large, we may assume [18, 19, 16, 17] that

$$\epsilon^{-2} \sim t \tag{29}$$

on average. Using now for the graviton a linear approximation about flat Minkowski space $G_{MN} \sim \eta_{MN} + h_{MN}$, which allows one to consider a simple Fourier momentum-space decomposition of the pertinent amplitudes, we conclude that the effective photon-graviton interactions in (27) are of order $\mathcal{O}(g_s E|\overline{k}|^2/M_P)$. Such estimates apply to the string amplitudes $c^i_{j\hat{k}} g^{\hat{k}}$ in the shift (8) in the single-defect case, which therefore becomes, to leading order in a low-energy approximation:

$$\Delta\alpha_i = \mathcal{O}(-g_s\frac{E|\overline{k}|}{M_P})) \tag{30}$$

Using (7), we see immediately that this leads to a modified dispersion relation for the photon:

$$E = |\overline{k}| + \mathcal{O}\left(-g_s\frac{E|\overline{k}|}{M_P})\right) \tag{31}$$

The refractive index is then determined from the photon group velocity [2]:

$$v(|\overline{k}|) \equiv \frac{\partial E}{\partial|\overline{k}|} \simeq 1 - \mathcal{O}(\frac{2g_s|\overline{k}|}{M_P}) \simeq 1 - \mathcal{O}(\frac{2g_s E}{M_P}) \tag{32}$$

The sign of the refractive index is determined by the fact that the Born-Infeld action underlying the above analysis prevents superluminal propagation. This sign was not determined in our previous discussion [2]. As discussed there, such a variation in the velocity of light will cause a spread in the arrival times of pulses of photons, according to their energy [2]:

$$\Delta t \sim \frac{\xi L E}{M_P} \tag{33}$$

The possibility of observing experimentally such a shift using distant astrophysical sources appears conceivable, as dicussed in [2] and reviewed in the last section of this paper.

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[5]It is clear that Lorentz-invariant interactions of powers of f^2 terms with generic $\text{Tr}(F^{2n})$ terms in the derivative expansion of (26) do not affect the photon dispersion relation. It is only *non-covariant* terms, such as the the induced-graviton-photon interactions considered above, that do so.

5 Stochastic Fluctuations in the Apparent Velocity of Light

We now extend the above discussion to include an estimate of the stochastic fluctuations in the apparent velocity of light, and hence the refractive index, due to the summation over higher-genus world-sheet topologies. The general theory of this summation in the context of Liouville string, discussed in section 2, leads to stochastic fluctuations in the collective coordinates of the D particles discussed in section 3 [17]:

$$\delta Y^i \geq \ell_s g_s^{\eta/2}, \tag{34}$$

The conventional Heisenberg uncertainty relation between the coordinates Y^i and the corresponding canonical momenta P_j has been shown to take the following form for D particles, after the summation over genera [17]:

$$\delta Y^i \delta P_j \geq 2g_s^{1+\eta/2} \delta_j^i \tag{35}$$

Saturating the bound (34) in (35), we obtain the following estimate of the uncertainty in the associated collective canonical momenta of the D particles:

$$\delta P_i \simeq \frac{2g_s}{\ell_s} \tag{36}$$

We note that this estimate of the uncertainty in the collective momentum is independent of the exponent η, which, as already noted, would be required to take the value $\eta = 2/3$ in order to match results on D particles in conformal string theory.

In our interpretation, the fluctuations arising from the summation over genera lead to a statistical superposition of theories with different values of the couplings $g^{\hat{j}}$. This 'stochastic environment' is characterized by the Gaussian form (11) of the associated probability distribution in the space of σ-model backgrounds. A photon of given energy E propagating through the fluctuating quantum-gravitational medium is subject to stochastic fluctuations in its velocity, which may be obtained from Liouville interactions of the form (12) by endowing the generalized σ-model coordinates $\{g^{\hat{k}}\}$ with the fluctuations (34) found for the collective coordinates Y^i of the D-particle foam.

To see how this effect indeed induces stochastic fluctuations in the refractive index of the photon, one first calculates the amplitudes $c_{j\hat{k}}^i g^{\hat{k}}$ appearing in (12) using the Born-Infeld Lagrangian (26), calculated on world sheets with disc topology. Using (29), one sees that the leading terms in a derivative expansion are the ones given in (27). The summation over world-sheet genera leads to stochastic fluctuations in U_i (28), which are given by (36):

$$\delta U_i = \frac{g_s \delta P_i}{M_P} \sim 2g_s^2 \tag{37}$$

These stochastic fluctuations in U_i generate fluctuations in the overall proportionality coefficient of the Maxwell terms in the effective action for the photon:

$$\delta \mathcal{L}_{NBI}^{photons,graviton} \ni f_{MN}(a) \left(\delta G_{NA}\right) f_{AM} \sim f_{ij}(a) \left(\delta U_j\right) f_{0i} \epsilon^2 t \qquad (38)$$

whose Fourier transform is $\mathcal{O}(2g_s^2 E|\overline{k}|/M_P)$. This does not itself affect the velocity of the photon, but simply renormalizes the energy scale.

However, when one proceeds to higher orders, one picks up contributions that lead to stochastic fluctuations in the refractive index. To see this, we concentrate on terms of quadratic order in U_i, stemming from terms in a derivative expansion of the Born-Infeld lagrangian (26) that are of the generic form $f_{MN}^2 \mathrm{Tr} F_{AB}^2$, where the non-Abelian field strength F_{MN}^{ab} is calculated by considering Y_i^{ab} (13) formally as a 'gauge potential'. For long times of order (29), the only non-vanishing components of F_{AB} are $F_{0i} \sim U_i$ which lead to stochastic fluctuations in the quadratic terms of order

$$f_{MN}^2 \left(\delta U_i^2\right) = 4g_s^2 f_{MN}^2 U_i \qquad (39)$$

Thus, in order to discuss the leading-order effects induced by the stochastic fluctuations of our D-particle foam, as implied by the summation over world-sheet topologies, we should consider corrections to the Liouville-dressed couplings that go beyond quadratic order in the σ-model couplings $\{g^{\hat{i}}\}$. Such corrections have been studied in [10], and are not given here explicitly. It is sufficient for our purposes to point out that some are proportional to four-point amplitudes, c_{jkl}^i divided by 'energy denominators' $\alpha_j + \alpha_k$: $c_{jkl}^i/(\alpha_i + \alpha_j)$, in the limit of vanishingly small central-charge deficits Q that are of interest to us, whereas others are of the form $c_{jm}^i c_{kl}^m$ divided by terms of order α_i^2. The three- and four-point amplitudes are computed from the Born-Infeld lagrangian (26) above [6].

Thus, for long times of order (29), and using (28) for U_i, one finds the following estimate for the corresponding stochastic fluctuations in (8):

$$\delta \left(\Delta \alpha_i\right) = \frac{4g_s^2 E^2}{M_P} \qquad (40)$$

Correspondingly, we find fluctuations in the velocity of light in the quantum-gravitational medium of order $\delta c \sim 8g_s^2 E/M_P$, motivating the following parametrization in the stochastic spread in photon arrival times:

$$(\delta \Delta t) \sim \frac{LE}{\Lambda_{QG}} \qquad (41)$$

where $\Lambda_{QG} \sim M_P/8g_s^2$. We emphasize that, in contrast to the variation (33) in the refractive index, which refers to photons of different energy, the fluctuation (41) characterizes the statistical spread in the velocity of photons *of the same energy*.

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[6]We again reminder the reader that, since such higher-order corrections are world-sheet renormalization-scheme dependent [10], this reflects the (spontaneous) breaking of general covariance by our foamy ground-state when the Liouville field is identified with target time [5].

6 Observational tests

The most important signatures of the refractive index and the stochastic fluctuation in the velocity of light that we find are that they increase *linearly* with the photon energy (frequency). This means that they can, in principle, easily be distinguished from more conventional medium effects, that attenuate with increasing energy. The effects scale inversely with some quantum-gravitational scale characteristic of strings and D branes. We are not in a position to estimate it numerically, but we expect it to be within a few orders of magnitude of $M_P \sim 10^{19}$ GeV. In standard string theories one has $\eta = 2/3$ and $g_s^2/4\pi \sim 1/20$, but the latter may well be modified in a more realistic theory. In principle, one could even envisage using upper limits on (measurements of) the rate of broadening of a radiation spike of definite energy (frequency) to constrain (measure) g_s.

We conclude this paper by mentioning some possible observational tests of these ideas. As has been emphasized previously [2], the figure of merit for constraining the possibility of an energy- (frequency-)dependent refractive index *in vacuo* is the combination $L \times \Delta E/\Delta t$, where L is the distance of a source, ΔE is the range of photon energies studied, and Δt is the observational sensitivity to differences in light-travel times. The latter is limited by the durations of pulses produced by the source as well as by the resolution of the detector. The corresponding figure of merit for testing the new possibility advanced in this paper, namely a stochastic spread in light-travel times for different photons with the same energy E, is simply $L \times E/\Delta t$. In practice, when comparing photons of different energies, ΔE is often dominated by the highest photon energy E that is measured, so that $\Delta E \sim E$, and the two figures of merit are essentially equivalent.

Astrophysical sources offer the largest figures of merit, and the most promising that have been considered include GRBs [2], AGNs [27] and pulsars [28]. These have all been used already to constrain the refractive-index parameter M_{QG}, and offer similar sensitivities to the stochastic-spread parameter Λ_{QG}. As mentioned earlier in this paper, the D-brane Born-Infeld analysis indicates that higher-energy photons should be *retarded* relative to lower-energy photons, rather than advanced, and their arrival times should be more spread out. In the Table below, we list some of the sources that have been considered, and the sensitivities (limits) that have been obtained. For completeness, we have also indicated the sensitivity that might be obtainable from a detailed analysis of the recent GRB 990123. We see from the Table that M_{QG} cannot be much smaller than the Planck scale $M_P \sim 10^{19}$ GeV, and that some of these astrophysical sources may already providing sensitivities to $M_{QG}, \Lambda_{QG} \sim 10^{19}$ GeV. This provides additional fundamental-physics motivation for such γ-ray observatories as AMS [29] and GLAST [30].

Other probes of the signatures of quantum gravity that might be provided by the unorthodox photon propagation proposed here might be possible using labo-

ratory experiments, for example those testing quantum optics and searching for gravitational waves, but we do not explore these possibilities further in this paper [7]. However, we think that the discussion given here demonstrates amply the possibility that at least some quantum-gravity ideas may be accessible to experimental test, and need not remain for ever in the realm of mathematical speculation.

Table: Observational Sensitivities and Limits on M_{QG}, Λ_{QG}

Source	Distance	E	Δt	Sensitivity (Limit)
GRB 920229 [2]	3000 Mpc (?)	200 keV	10^{-2} s	10^{16} GeV (?)
GRB 980425 [2]	40 Mpc	1.8 MeV	10^{-3} s (?)	10^{16} GeV (?)
GRB 920925c [2]	40 Mpc (?)	200 TeV (?)	200 s	10^{19} GeV (?)
Mrk 421 [27]	100 Mpc	2 TeV	280 s	$> 4 \times 10^{16}$ GeV
Crab pulsar [28]	2.2 kpc	2 GeV	0.35 ms	$> 1.8 \times 10^{15}$ GeV
GRB 990123	5000 Mpc	4 MeV	1 s (?)	3×10^{14} GeV (?)

The question marks in the Table indicate uncertain inputs. Hard limits are indicated by inequality signs.

References

[1] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Int. J. Mod. Phys. A12 (1997), 607.

[2] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar, *Nature* 393 (1998), 323; and astro-ph/9810483.

[3] R. Gambini and G. Pullin, gr-qc/9809038.

[4] F. Halzen, astro-ph/9904216.

[5] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37;

International School of Subnuclear Physics: 31st Course: From Supersymmetry to the Origin of Space-Time, Erice, Italy, 4-12 Jul, 1993, Erice Subnuclear Series, Vol. 31, (World Scientific, Singapore, 1995),1.

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[7]However, we do observe that the considerations of [26] concerning interferometric signatures are not applicable in our framework.

[6] F. David, Mod. Phys. Lett. A3 (1988), 1651;
J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509;
see also N.E. Mavromatos and J.L. Miramontes, Mod. Phys. lett. A4 (1989), 1847.

[7] I. Kogan, *Particles and Fields 1991*, eds. D. Axen, D. Bryman and M. Comyn (World Scientific, Singapore, 1992), 837.

[8] A.B. Zamolodchikov, JETP Lett. 43 (1986), 730; Sov. J. Nucl. Phys. 46 (1987), 1090.

[9] I.R. Klebanov, I.I. Kogan and A.M. Polyakov, Phys. Rev. Lett. 71 (1993), 3243;
C. Schmidhuber, Nucl. Phys. B404 (1993), 342.

[10] H. Dorn, Phys. Lett. B343 (1995), 81.

[11] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 383; Nucl. Phys. B328 (1989), 117.

[12] J. Ellis, J. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.

[13] J. Ellis, P. Kanti, N.E. Mavromatos, D.V. Nanopoulos and E. Winstanley, Mod. Phys. Lett. A13 (1998), 303, and references therein.

[14] J. Polchinski, Phys. Rev. Lett. **75** (1995), 184; *TASI lectures on D-branes*, hep-th/9611050;
C.P. Bachas, *Lectures on D-branes*, hep-th/9806199.

[15] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Mod. Phys. Lett. A10 (1995), 1685.

[16] F. Lizzi and N.E. Mavromatos, Phys. Rev. D55 (1997), 7859.

[17] N.E. Mavromatos and R.J. Szabo, Phys. Rev. D59 (1999), 064023; hep-th/9808124, Phys. Rev. D59 (1999) in press.

[18] I.I. Kogan, N.E. Mavromatos and J.F. Wheater, Phys. Lett. B387 (1996), 483.

[19] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Int. J. Mod. Phys. A12 (1997), 2639; *ibid* A13 (1998), 1059.

[20] A.A. Tseytlin, Nucl. Phys. B501 (1997), 41.

[21] W. Fischler and L. Susskind, Phys. Lett. B171 (1986), 383; *ibid.* B173 (1986), 262.

[22] M. Li and T. Yoneya, Phys. Rev. Lett. 78 (1997), 1219;
for a recent review of space-time uncertainty relations in string theory, see M. Li and T. Yoneya, hep-th/9806240.

[23] G. Veneziano, Europhys. Lett. 2 (1986) 199;
D.J. Gross and P.F. Mende, Nucl. Phys. B303 (1988) 407;
D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B216 (1989) 41;
K. Konishi, G. Paffuti and P. Provero, Phys. Lett. B234 (1990) 276.

[24] A. Abouelsaood, C.G. Callan, C.R. Nappi, S.A. Yost, Nucl. Phys. B280 (1987), 599.

[25] H. Dorn, J.High Energy Phys. 4 (1998), 13, and references therein.

[26] G. Amelino-Camelia, Nature 398 (1999), 216 and gr-qc/9903080.

[27] S.D. Biller *et. al.*, gr-qc/9810044.

[28] P. Kaaret, astro-ph/9903464.

[29] S. Ahlen *et al.*, Nucl. Instrum. Meth. A350 (1994), 351.

[30] GLAST Team, E.D. Bloom *et al.*, *Proc. Intern. Heidelberg Workshop on TeV Gamma-Ray Astrophysics*, eds. H.J. Volk and F.A. Aharonian (Kluwer, 1996), 109.