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Four-Dimensional $N = 2$ Superstring Constructions and their (Non-) Perturbative Duality Connections*

Andrea GREGORI [1] and Costas KOUNNAS [2]

[1] *Humboldt-Universität, Institut für Physik,*
D-10115 Berlin, Germany

[2] *LPTH, École Normale Supérieure, 24 rue Lhomond*
F-75231, Paris Cedex 05, France
and
Theory Division, CERN 1211 Geneva 23, Switzerland

Abstract

We investigate the connections between four-dimensional, $N = 2$ M-theory vacua constructed as orbifolds of type II, heterotic, and type I strings. All these models have the same massless spectrum, which contains an equal number of vector multiplets and hypermultiplets, with a gauge group of the maximal rank allowed in a perturbative heterotic string construction. We find evidence for duality between two type I compactifications recently proposed and a new heterotic construction that we present here. This duality allows us to gain insight into the non-perturbative properties of these models. In particular we consider gravitational corrections to the effective action.

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e-mail: gregori@physik.hu-berlin.de
kounnas@nxth04.cern.ch

1. Introduction

In this work we consider five different string constructions, which lead to the same low-energy spectrum. This consists of a four-dimensional supergravity theory with two space-time supersymmetries whose spectrum, besides the gravity multiplet, contains (when the gauge group is broken to its Cartan subgroup) 19 vector multiplets and 20 hypermultiplets. This massless spectrum is obtained via compactification and orbifold projections from all the known perturbative string constructions, namely the types IIA/B, the heterotic, and the type I. The gauge group being a broken phase of $SO(16) \times SO(16)$[1], this spectrum can be obtained from both the heterotic strings, starting either from $E_8 \times E_8$ or from $SO(32)$. In all these constructions the sypersymmetry and/or the gauge group are broken by Z_2 projections.

The type II orbifolds are obtained either with projections which act symmetrically on the right and left movers [1, 2, 3], giving rise to type IIA or IIB models, or with an asymmetric projection, in which all the supersymmetries come only from the left movers. These orbifolds are dual to one another: the type IIA and IIB are trivially related by the inversion of an odd number of radii, the compactification being self-mirror; the type II asymmetric orbifold is instead related to these by a U-duality that exchanges perturbative and non-perturbative moduli (see Ref.[3]).

The type I orbifolds were recently constructed in Ref.[4] as orientifolds of certain type IIB orbifolds, with a spontaneous breaking of the $N = 8$ supersymmetry. In Ref.[4] they are indicated respectively as the "Scherk–Schwarz breaking" and the "M-theory breaking" models.

The heterotic orbifold, which we present here as a new construction, is an interesting example of heterotic compactification in which, although the gauge group has maximal rank, there are always, even away from the Abelian point, an equal number of vector and hypermultiplets, leading to vanishing gauge beta-functions. It therefore behaves as the higher-level, reduced rank models considered in Refs.[5, 6]. We provide evidence of duality between such heterotic orbifold and the above two type I models, which correspond to two different, but continuously related phases of the heterotic theory. This allows us to see the connection between the two models: their relation is non-perturbative from the point of view of type I. Thanks to the duality between these constructions, we are able to determine at least part of the non-perturbative correction to the effective coupling constant of a special, gravitational (R^2) amplitude. As a byproduct, we determine also the U-duality group. In particular, S-duality appears to be broken by the action of a freely acting projection applied to the eleventh coordinate of M-theory.

The type IIA and heterotic constructions, on the other hand, are not dual. This is related to the fact that the type IIA orbifold cannot be regarded as a singular limit of a K3 fibration [2, 3]. However, even though these models cannot be compared for finite values of the moduli, we argue that they correspond to two different phases, or regions in the moduli space, of M-theory. These two regions are connected in the moduli space.

[1] When we speak of gauge group we mean, on the heterotic string, the part that comes from the currents. For the sake of simplicity we always omit to mention the left and right gauge groups which come from the compact space.

The above issues are discussed according to the following order:

In Section 2 we review the type II orbifolds, which were discussed in detail in Ref.[3]. We discuss also the corrections to the R^2 term, which, being a function of the moduli of the vector manifold, will provide us with a quantity on which to test the duality relations.

In Section 3 we recall, from Ref.[4], the type I orientifolds, commenting on the addition of discrete Wilson lines which break the gauge group to the Cartan subgroup and discussing the gravitational corrections.

In Section 4 we then discuss in detail the heterotic construction. As the previous sections, also this ends with a discussion of the R^2 corrections. Through the analysis of this term, we discuss also the duality relation between this model and the type I of Section 3. The duality relations are used to obtain (at least part of) the non-perturbative gravitational corrections, as well as an insight into other non-perturbative properties.

Finally, in Section 5 we comment on the connections between the various phases of these $N = 2$, M-theory compactifications. Our conclusions are given in Section 6.

2. The type II constructions

2.1. The type IIA construction

We start by reviewing the construction of the type IIA, which is obtained by compactification of the ten-dimensional superstring on a Calabi–Yau manifold with Hodge numbers $h^{1,1} = h^{2,1} = 19$ [1, 2]. We recall that this manifold has an orbifold limit in which the $N = 8$ supersymmetry of the type IIA string compactified on T^6 is reduced to $N = 2$ by two Z_2 projections, $Z_2^{(1)}$ and $Z_2^{(2)}$ [2]. The action of these on $T^6 = T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$ is the following: $Z_2^{(1)}$ acts as a rotation in $T^2_{(1)} \times T^2_{(3)}$, while $Z_2^{(2)}$ acts as a rotation on $T^4 = T^2_{(2)} \times T^2_{(3)}$ and as a translation in $T^2_{(1)}$. The partition function of this orbifold reads (see Ref.[3]):

$$\begin{aligned} Z_{\rm II}^{(1,1)} &= \frac{1}{{\rm Im}\,\tau |\eta|^{24}} \frac{1}{4} \sum_{H^1,G^1} \sum_{H^2,G^2} \Gamma_{6,6} \begin{bmatrix} H^1, H^2 \\ G^1, G^2 \end{bmatrix} \\ & \times \frac{1}{2} \sum_{a,b} (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+H^1 \\ b+G^1 \end{bmatrix} \vartheta \begin{bmatrix} a+H^2 \\ b+G^2 \end{bmatrix} \vartheta \begin{bmatrix} a-H^1-H^2 \\ b-G^1-G^2 \end{bmatrix} \\ & \times \frac{1}{2} \sum_{\bar a,\bar b} (-)^{\bar a+\bar b+\bar a\bar b} \bar\vartheta \begin{bmatrix} \bar a \\ \bar b \end{bmatrix} \bar\vartheta \begin{bmatrix} \bar a+H^1 \\ \bar b+G^1 \end{bmatrix} \bar\vartheta \begin{bmatrix} \bar a+H^2 \\ \bar b+G^2 \end{bmatrix} \bar\vartheta \begin{bmatrix} \bar a-H^1-H^2 \\ \bar b-G^1-G^2 \end{bmatrix}, \end{aligned} \qquad (2.1)$$

where the contribution of the compactified bosons, $X^I, \bar X^I, I = 1, \dots, 6$ is contained in the factor $\Gamma_{6,6}\left[{}^{H^1,H^2}_{G^1,G^2}\right] \Big/ |\eta|^{12}$, while the other factors contain the contribution of their fermionic superpartners, Ψ^I, $\bar\Psi^I$, of the left- and right-moving non-compact supercoordinates X^μ, Ψ^μ, $\bar X^\mu$, $\bar\Psi^\mu$ and of the super-reparametrization ghosts b, c, β, γ and $\bar b, \bar c, \bar\beta, \bar\gamma$. (H^1, G^1) refer to the boundary conditions introduced by the projection $Z_2^{(1)}$, and (H^2, G^2) to the projection $Z_2^{(2)}$.

$\Gamma_{6,6}\left[{}^{H^1,H^2}_{G^1,G^2}\right]$ factorizes into the contributions corresponding to the three tori of T^6:

$$\Gamma_{6,6}\begin{bmatrix} H^1, H^2 \\ G^1, G^2 \end{bmatrix} = \Gamma^{(1)}_{2,2}\begin{bmatrix} H^1|H^2 \\ G^1|G^2 \end{bmatrix} \Gamma^{(2)}_{2,2}\begin{bmatrix} H^2|0 \\ G^2|0 \end{bmatrix} \Gamma^{(3)}_{2,2}\begin{bmatrix} H^1+H^2|0 \\ G^1+G^2|0 \end{bmatrix}, \tag{2.2}$$

which are expressed in terms of the twisted and shifted characters of a $c = (2, 2)$ block, $\Gamma_{2,2}\left[{}^{h|h'}_{g|g'}\right]$; the first column refers to the twist, the second to the shift. The non-vanishing components are the following:

$$\begin{aligned} \Gamma_{2,2}\begin{bmatrix} h|h' \\ g|g' \end{bmatrix} &= \frac{4\,|\eta|^6}{\left|\vartheta\left[{}^{1+h}_{1+g}\right]\vartheta\left[{}^{1-h}_{1-g}\right]\right|}\,, \quad \text{for } (h',g') = (0,0) \text{ or } (h',g') = (h,g) \\ &= \Gamma_{2,2}\begin{bmatrix} h' \\ g' \end{bmatrix}, \qquad\qquad \text{for } (h,g) = (0,0)\,, \end{aligned} \tag{2.3}$$

where $\Gamma_{2,2}\left[{}^{h'}_{g'}\right]$ is the Z_2-shifted $(2, 2)$ lattice sum. In this case, the only shift is that due to the $Z^{(2)}_2$ translation on $T^2_{(1)}$, $(-)^{m_2 G^2}$.

For a detailed analysis of the spectrum of this model, we refer to Ref.[3]. Here we simply recall that the $Z^{(1)}_2$ twisted sector has sixteen fixed points, which give rise to eight vector and eight hypermultiplets. The twist of $Z^{(2)}_2$ on the other hand is accompanied by a lattice shift, so there are no massless states from this twisted sector. The other eight vector and eight hypermultiplets come from the sector twisted by $Z^{(1)}_2 \times Z^{(2)}_2$ (the $H^{(1)} + H^{(2)}$-twisted sector).

We now consider the corrections to the R^2 term. They receive a non-zero contribution at one loop, and are related to the infrared-regularized integral of the fourth helicity supertrace, B_4 (for the definition and the details we refer, for instance, to Ref.[3]). The one-loop correction to the coupling constant is:

$$\begin{aligned} \frac{16\,\pi^2}{g^2_{\rm grav}\left(\mu^{\rm (IIA)}\right)} &= -2\log\,{\rm Im}\,T^1|\vartheta_4\left(T^1\right)|^4 - 6\log\,{\rm Im}\,T^2|\eta\left(T^2\right)|^4 - 6\log\,{\rm Im}\,T^3|\eta\left(T^3\right)|^4 \\ &\quad +14\log\frac{M^{\rm (IIA)}}{\mu^{\rm (IIA)}}\,, \end{aligned} \tag{2.4}$$

where T^1, T^2, T^3 are the Kähler class moduli of the three tori of the compact space. The last term in Eq.(2.4), which encodes the infrared running due to the massless contributions, is expressed in terms of the type IIA string scale $M^{\rm (IIA)} \equiv 1/\sqrt{\alpha'_{\rm IIA}}$ and infrared cut-off $\mu^{\rm (IIA)}$. The coefficient, 14, is actually the massless contribution to $(B_4 - B_2)/3$, as is computed in field theory (see Ref.[6]). Observe that there is no limit in the space of moduli T^1, T^2, T^3, at which this correction reproduces the behaviour of an $N = 4$ orbifold, for which it is expected to depend on the Kähler class modulus of only one torus (see for instance Refs.[7, 8]). This implies that, in the space of these three moduli, there is no region in which the $N = 4$ supersymmetry is restored, as happens instead in orbifold constructions with a spontaneous breaking of supersymmetry, such as those considered in Refs.[6, 9]. There is therefore no perturbative connection to an $N = 4$ theory, and therefore this orbifold cannot be seen as a singular limit in the moduli space of a K3 fibration [3].

As explained in Ref.[3], the type IIB dual compactification is trivially obtained by changing the chirality of the right-moving spinors. This is obtained by changing the phase $(-)^{\bar a+\bar b+\bar a\bar b}$ in Eq.(2.1) to $(-)^{\bar a+\bar b}$. The analysis is similar and we obtain analogous results, with the role of the fields T^i, $i=1,2,3$, associated to the Kähler classes of the three tori, interchanged with that of the fields U^i, associated to the complex structures.

2.2. The type II asymmetric dual

The model above described possesses a type II dual constructed as an asymmetric orbifold [3], obtained by combining the projection $Z_2^{(1)}$, acting in the same way as before, with $Z_2^{\rm F}$, which projects out all the right-moving supersymmetries by relating the action of the right fermion number operator, $(-)^{\rm F_R}$, to a translation in the compact space. The partition function reads [3]:

$$
\begin{aligned}
Z_{\rm II}^{(2,0)} \;&=\; \frac{1}{{\rm Im}\,\tau|\eta|^{24}}\frac{1}{4}\sum_{H^{\rm F},G^{\rm F}}\sum_{H^{\rm o},G^{\rm o}}\Gamma_{6,6}\begin{bmatrix}H^{\rm F},H^{\rm o}\\G^{\rm F},G^{\rm o}\end{bmatrix}\\
&\times\frac{1}{2}\sum_{a,b}(-)^{a+b+ab}\vartheta^2\begin{bmatrix}a\\b\end{bmatrix}\vartheta\begin{bmatrix}a+H^{\rm o}\\b+G^{\rm o}\end{bmatrix}\vartheta\begin{bmatrix}a-H^{\rm o}\\b-G^{\rm o}\end{bmatrix}\\
&\times\frac{1}{2}\sum_{\bar a,\bar b}(-)^{\bar a+\bar b+\bar a\bar b}(-)^{\bar aG^{\rm F}+\bar bH^{\rm F}+H^{\rm F}G^{\rm F}}\bar\vartheta^2\begin{bmatrix}\bar a\\\bar b\end{bmatrix}\bar\vartheta\begin{bmatrix}\bar a+H^{\rm o}\\\bar b+G^{\rm o}\end{bmatrix}\bar\vartheta\begin{bmatrix}\bar a-H^{\rm o}\\\bar b-G^{\rm o}\end{bmatrix}\,, \qquad (2.5)
\end{aligned}
$$

with

$$
\Gamma_{6,6}\begin{bmatrix}H^{\rm F},H^{\rm o}\\G^{\rm F},G^{\rm o}\end{bmatrix}=\Gamma_{2,2}^{(1)}\begin{bmatrix}H^{\rm o}\mid H^{\rm F}\\G^{\rm o}\mid G^{\rm F}\end{bmatrix}\Gamma_{2,2}^{(2)}\begin{bmatrix}H^{\rm o}\mid 0\\G^{\rm o}\mid 0\end{bmatrix}\Gamma_{2,2}^{(3)}\begin{bmatrix}0\mid 0\\0\mid 0\end{bmatrix}\,. \qquad (2.6)
$$

As was discussed in Ref.[3], the R^2 gravitational correction, which receives contribution only from one loop, is a function of the moduli $T^{\rm As}$, $U^{\rm As}$, associated respectively to the Kähler class and the complex structure of the third complex plane; it reads

$$
\begin{aligned}
\frac{16\,\pi^2}{g^2_{\rm grav}\left(\mu^{(\rm As)}\right)} \;&=\; -6\log\,{\rm Im}\,T^{\rm As}\left|\eta\left(T^{\rm As}\right)\right|^4-6\log\,{\rm Im}\,U^{\rm As}\left|\eta\left(U^{\rm As}\right)\right|^4+\\
&+14\log\frac{M^{(\rm As)}}{\mu^{(\rm As)}}\,, \qquad (2.7)
\end{aligned}
$$

where we introduced the type II asymmetric mass scale and infrared cut-off, $M^{(\rm As)}$ and $\mu^{(\rm As)}$ respectively. A comparison of Eq.(2.7) and Eq.(2.4) leads directly to the identifications $T^2=T^{\rm As}$, $T^3=U^{\rm As}$ and $T^1=\tau_S^{\rm As}$, where $\tau_S^{\rm As}\equiv 4\pi S^{\rm As}$, $S^{\rm As}$ being the dilaton–axion field of the type II asymmetric models[2]. For later use, we note here that the corrections Eq.(2.4) and Eq.(2.7) remain the same if we correct the R^2 term by adding gauge amplitudes F^2: owing to the absence, in the perturbative type II strings, of gauge charges, these amplitudes are identically vanishing.

[2]We recall that this field belongs to a vector multiplet, as in the heterotic constructions [3].

3. The type I models

There are two type I orbifolds that possess the desired massless spectrum. They were constructed in Ref.[4], as orientifolds of type IIB orbifolds, with the $N = 8$ supersymmetry spontaneously broken to $N = 4$ by a freely acting projection, $Z_2^{(\mathrm{f})}$. The latter acts as a twist on T^4, and, in the first case, as a translation in the momenta, produced by the projection $(-)^{mG^{\mathrm{f}}}$, on a circle of T^2. In Ref.[4] this construction is called the "Scherk–Schwarz breaking" model; in the following we will refer to it as the model **A**. Model **B**, referred to in Ref.[4] as the "M-theory breaking" model, is obtained when the translation is performed on the windings instead. After the orientifold projection, the resulting type I models possess an $N = 4$ supersymmetry spontaneously broken to $N = 2$. In model **A** the $N = 4$ is restored in the limit of decompactification (large radius) of the circle translated by $Z_2^{(\mathrm{f})}$; in model **B**, instead, it is restored when the radius goes to zero. Because of the spontaneous nature of the breaking of the $N = 4$ supersymmetry, the massless spectrum contains an equal number of vector and hypermultiplets[3] in the two models. However, while in model **A** the gauge group consists only of factors arising from the contribution of 99-brane sector states, in model **B** the consistency conditions of the construction, as derived by imposing tadpole cancellation, require the presence of both D9- and D5-branes, and the gauge group is the product of the 99-brane sector and the 55-brane sector contribution.

Model **A**, in which the gauge group is a broken phase of $SO(32)$, is expected to possess a heterotic dual, and indeed we will construct such a dual in the next section. As explained in [4], there is a wide choice of Wilson lines compatible with this Scherk–Schwarz projection, leading to different breakings of the gauge group. In order to compare the type I and the heterotic constructions, we introduce a further set of discrete Wilson lines, which break the gauge group to its Cartan subgroup, $U(1)^{16}$,[4]. In this way we obtain $3 + N_V$ vector multiplets and $4 + N_H$ hypermultiplets, with $N_V = N_H = 16$. It is possible to choose the Wilson lines to act as Z_2 shifts in the directions twisted by $Z_2^{(\mathrm{f})}$. This choice corresponds to a breaking of the gauge group at the $N = 2$, six-dimensional level, before the further torus compactification and supersymmetry breaking via orbifold projection; these Wilson lines therefore do not enter the $\Gamma_{2,18}$ lattice explicitly. The only vector multiplets moduli that appear are those of the $\Gamma_{2,2}$ lattice associated with the two-torus of the compactification from six to four dimensions. We will test the duality with the heterotic construction of the next section, through a comparison of the "holomorphic" gravitational corrections. These are defined as the corrections to the effective coupling constant of a special combination of R^2 and F^2 terms [6, 9], namely

$$\langle R'^2_{\mathrm{grav}} \rangle \equiv \langle R^2_{\mathrm{grav}} \rangle + \frac{1}{12} \langle F_{\mu\nu} F^{\mu\nu} \rangle_{T^2} + \frac{5}{48} \langle F_{\mu\nu} F^{\mu\nu} \rangle_{\mathrm{gauge}} \, . \tag{3.1}$$

This combination was introduced in the above cited works in the context of heterotic orbifolds; it was shown to possess an amplitude smooth in the moduli space, in which the non-harmonic contributions to the R^2 and the F^2 amplitudes cancel each other, leaving only

[3] As usual, we don't count here the three vector multiplets and the four hypermultiplets originating from the compact space, which are common to all the $N = 2$ orbifolds we consider in this paper.

[4] Here also we omit the contribution coming from the compact space.

the $\Gamma_{2,2}$ (Z_2-shifted) lattice sum. This is due to the presence of the full bunch of states, contained in the part of the heterotic spectrum which has, as massless excitations, the states of the $c = (0, 16)$ currents. On the type I side, the pure R^2 amplitude, although smooth in the moduli space, contains, besides the lattice sum provided by the torus $\mathcal{T}$, the contributions of the Klein bottle $\mathcal{K}$, the annulus $\mathcal{A}$ and the Möbius strip $\mathcal{M}$ [10]–[12]. However, only $N = 2$ BPS multiples contribute to such amplitudes, which therefore are all proportional to a "supersymmetric index" [10]; for the combination of gravitational and gauge amplitudes Eq.(3.1), the $\mathcal{K}$, $\mathcal{A}$ and $\mathcal{M}$ contributions cancel (notice that in the case of type I, the gauge amplitude $\langle F_{\mu\nu}F^{\mu\nu}\rangle_{T^2}$ vanishes, because these states come from the RR sector of the type IIB string). The only contribution therefore comes from the torus $\mathcal{T}$.

Since in this model there are no D5-branes, we expect the tree-level effective coupling $\frac{1}{g^2}$ to be given by the imaginary part of only one complex field, $S = S_1 + iS_2$, whose real part S_1 is the scalar dual to $B_{\mu\nu}$, while the imaginary part is

$$S_2 = \mathrm{e}^{-\phi_4} G^{1/4} \omega^2 \ , \tag{3.2}$$

where ϕ_4 is the dilaton of the four-dimensional compactification, $\sqrt{G} \sim R_4 R_5$ is the volume of the two-torus, and ω^4 is the volume of the K3, which in the case at hand is in his T^4/Z_2 limit. This situation has to be contrasted with the most general one [11, 13], in which the coupling is given by a combination

$$v S_2 + v' S_2' \tag{3.3}$$

in which $S_2' = \mathrm{e}^{-\phi_4} G^{1/4} \omega^{-2}$ is part of a complex field S' whose real part S_1' is the dual of B_{45}. The part of the coupling proportional to the inverse of the K3 volume is due to the presence of D5-branes.

The one-loop contribution is given by the complex structure-dependent part of the integral, over the fundamental domain, of the $\Gamma_{2,2}$, Z_2-shifted lattice sum appearing in the $\mathcal{T}$ amplitude. This can be computed in an infrared-regularized background, as in Ref.[14]. This would lead to the introduction of a curvature in the space-time, providing a cut-off μ that, once the flat limit is taken, appears in the running of the effective coupling [14]–[16]. There is no need to go into the details of such a procedure: such a running is in fact fixed by a regularization prescription, which imposes the matching of field theory and string computations in the infrared, and can therefore be determined simply by field-theory arguments. On the other hand, the full dependence of the corrections on the modulus U, the complex structure of the torus T^2 and the only non-trivial modulus that appears in such terms, the Wilson lines being frozen to fixed values, can be easily derived by knowing the action of $Z_2^{(\mathrm{f})}$ on the torus [17]. The total result, including tree-level and one-loop contributions, is therefore given by

$$\frac{16\,\pi^2}{g^2_{\mathrm{grav}}(\mu^{(\mathrm{I})})} = 16\,\pi^2\,\mathrm{Im}\,S - 2\log\,\mathrm{Im}\,U\,|\vartheta_4\,(U)|^4 + 14\log\frac{M^{(\mathrm{I})}}{\mu^{(\mathrm{I})}} \ , \tag{3.4}$$

where the Jacobi function ϑ_4 corresponds to a translation $(-)^{m_2 G^{\mathrm{f}}}$, in the second circle of T^2. The $SL(2,Z)_U$ duality group is broken to a $\Gamma(2)$ subgroup (see Refs.[6, 8, 9, 17]). In the last term we collect the dependence on the infrared cut-off $\mu^{(\mathrm{I})}$ and the type I string mass scale $M^{(\mathrm{I})}$.

We now consider the "M-theory breaking", model **B**. We specialize to the case in which, with reference to the notation of Ref.[4], $n_2 = d_2 = 0$, i.e. the symplectic factors do not appear in the gauge group, which is therefore given by $SO(16)_{99} \times SO(16)_{55}$ (the subscripts indicate the origin of these factors). We now introduce Wilson lines, as we did in the previous case, to break $SO(16)_{99}$ to $U(1)^8$. We also move the D5-branes a bit far from each other, in order to break also the second factor to the Cartan subgroup. We can now repeat the same arguments as before and compute the analogous "gravitational" corrections. In this case, due to the presence of the D5-branes, we expect a dependence of the tree level effective coupling also on the field S'. We therefore obtain:

$$\frac{16\,\pi^2}{g^2_{\rm grav}(\mu^{\rm (I)})} = 16\,\pi^2 v\,{\rm Im}\,S + 16\,\pi^2 v'\,{\rm Im}\,S' - 2\log\,{\rm Im}\,U\,|\vartheta_4\,(U)|^4 + 14\log\frac{M^{\rm (I)}}{\mu^{\rm (I)}}\;, \tag{3.5}$$

where, as in Refs.[11, 12], we allow for the presence of two independent contributions to the effective coupling constant ($vv' \neq 0$). Actually, since the model is symmetric under the exchange of the D9- and the D5-branes sectors, we deduce that $v = v' = 1$. The $\vartheta_4(U)$ is obtained for a translation $(-)^{n_1 G^{\rm f}}$ on the windings of the first circle of T^2.

3. The heterotic construction

We now discuss in detail the heterotic dual. This is constructed as a Z_2 freely acting orbifold of the heterotic string compactified on T^6, with the gauge group broken to $SO(16) \times SO(16)$ by a Z_2, discrete Wilson line. The type I construction corresponds to a special region in the moduli space of the theory, and in order to be able to compare heterotic and type I, we must choose on the heterotic side a special set of Wilson lines. What we need is a set of three other discrete Wilson lines, which act on the compact space as Z_2 translations, and further break $SO(16) \times SO(16)$ to $U(1)^8 \times SO(4)^4$. The $N = 4$ supersymmetry is then reduced to $N = 2$ by a further Z_2 freely acting projection, $Z_2^{\rm (f)}$, which moreover acts as a further Wilson line, which breaks the gauge group, leaving massless only the bosons in the Cartan subgroup. The partition function, $Z_{\rm Het}$, can be easily written in terms of the usual fermionic and bosonic characters for the compact space and the $c = (0, 2)$, $SO(4)$ twisted characters introduced in Ref.[6], F_1 and F_2, for the $c = (0, 16)$ currents. We recall that:

$$F_1\begin{bmatrix}\gamma, h\\ \delta, g\end{bmatrix} \equiv \frac{1}{\eta^2}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_1\\ \delta + g_1\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_2\\ \delta + g_2\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_3\\ \delta + g_3\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma - h_1 - h_2 - h_3\\ \delta - g_1 - g_2 - g_3\end{bmatrix} \tag{3.1}$$

and

$$F_2\begin{bmatrix}\gamma, h\\ \delta, g\end{bmatrix} \equiv \frac{1}{\eta^2}\,\vartheta^{1/2}\begin{bmatrix}\gamma\\ \delta\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_1 - h_2\\ \delta + g_1 - g_2\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_2 - h_3\\ \delta + g_2 - g_3\end{bmatrix}\,\vartheta^{1/2}\begin{bmatrix}\gamma + h_3 - h_1\\ \delta + g_3 - g_1\end{bmatrix}\,, \tag{3.2}$$

where $h \equiv (h_1, h_2, h_3)$ and similarly for g. Under $\tau \to \tau + 1$, F_I transforms as:

$$F_1\begin{bmatrix}\gamma, h\\ \delta, g\end{bmatrix} \quad\to\quad F_1\begin{bmatrix}\gamma, h\\ \gamma + \delta + 1, h + g\end{bmatrix}$$

$$\times \exp -\frac{i\pi}{4}\left(\frac{2}{3}-4\gamma+2\gamma^2+h_1^2+h_2^2+h_3^2+h_1h_2+h_2h_3+h_3h_1\right)\ , \quad (3.3)$$

$$F_2\begin{bmatrix}\gamma,h\\ \delta,g\end{bmatrix} \quad\rightarrow\quad F_2\begin{bmatrix}\gamma,h\\ \gamma+\delta+1,h+g\end{bmatrix}$$
$$\times \exp -\frac{i\pi}{4}\left(\frac{2}{3}-4\gamma+2\gamma^2+h_1^2+h_2^2+h_3^2-h_1h_2-h_2h_3-h_3h_1\right)\ . \quad (3.4)$$

In terms of these, we have

$$\begin{aligned} Z_{\rm Het} &= \frac{1}{{\rm Im}\,\tau|\eta|^4}\frac{1}{2}\sum_{H^{\rm f},G^{\rm f}} Z_{6,22}\begin{bmatrix}H^{\rm f}\\ G^{\rm f}\end{bmatrix} \\ &\times\frac{1}{2}\sum_{a,b}\frac{1}{\eta^4}\,(-)^{a+b+ab}\vartheta\begin{bmatrix}a\\ b\end{bmatrix}^2\vartheta\begin{bmatrix}a+H^{\rm f}\\ b+G^{\rm f}\end{bmatrix}\vartheta\begin{bmatrix}a-H^{\rm f}\\ b-G^{\rm f}\end{bmatrix}, \end{aligned} \quad (3.5)$$

where the second line stands for the contribution of the 10 left-moving world-sheet fermions ψ^μ, Ψ^I and the ghosts β,γ of the super-reparametrization; $Z_{6,22}\left[{H^{\rm f}\atop G^{\rm f}}\right]$ accounts for the $(6,6)$ compactified coordinates and the $c=(0,16)$ conformal system, which is described by the 32 right-moving fermions Ψ_A, $A=1,\ldots,32$:

$$Z_{6,22}\begin{bmatrix}H^{\rm f}\\ G^{\rm f}\end{bmatrix} = \frac{1}{2^5}\sum_{\vec h,\vec g}\frac{1}{\eta^6\bar\eta^6}\Gamma_{2,2}\begin{bmatrix}H^{\rm f},h_1\\ G^{\rm f},g_1\end{bmatrix}\Gamma_{4,4}\begin{bmatrix}H^{\rm f}|\vec h\\ G^{\rm f}|\vec g\end{bmatrix}\overline{\Phi}\begin{bmatrix}H^{\rm f},\vec h\\ G^{\rm f},\vec g\end{bmatrix}, \quad (3.6)$$

and

$$\begin{aligned} \Phi\begin{bmatrix}H^{\rm f},\vec h\\ G^{\rm f},\vec g\end{bmatrix} &= \frac{1}{2}\sum_{\gamma,\delta}F_1^2\begin{bmatrix}\gamma,h_1,h_2,h_3\\ \delta,g_1,g_2,g_3\end{bmatrix}_{(H^{\rm f},G^{\rm f})}F_2^2\begin{bmatrix}\gamma,h_1,h_2,h_3\\ \delta,g_1,g_2,g_3\end{bmatrix}_{(H^{\rm f},G^{\rm f})} \\ &\times F_1^2\begin{bmatrix}\gamma+h_4,H^{\rm f},h_2,h_3\\ \delta+g_4,G^{\rm f},g_2,g_3\end{bmatrix}F_2^2\begin{bmatrix}\gamma+h_4,H^{\rm f},h_2,h_3\\ \delta+g_4,G^{\rm f},g_2,g_3\end{bmatrix}. \end{aligned} \quad (3.7)$$

In Eq.(3.6) we used the twisted-shifted bosonic character for $\Gamma_{4,4}\left[{h|h'\atop g|g'}\right]$, in which the first column indicates the twist (h,g), the second the shift (h',g'), and the doubly-shifted character of the two-torus $\Gamma_{2,2}\left[{h,h'\atop g,g'}\right]$. In Eq.(3.7) the subscripts $(H^{\rm f},G^{\rm f})$ indicate the embedding of the spin connection in the gauge group. This is realized explicitly through a modification of the arguments in the first Ising character, both in F_1 and in F_2:

$$F_1\begin{bmatrix}\gamma,h\\ \delta,g\end{bmatrix}_{(H^{\rm f},G^{\rm f})} \equiv \frac{1}{\eta^2}\,\vartheta^{1/2}\begin{bmatrix}\gamma+h_1+H^{\rm f}\\ \delta+g_1+G^{\rm f}\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma+h_2\\ \delta+g_2\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma+h_3\\ \delta+g_3\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma-h_1-h_2-h_3\\ \delta-g_1-g_2-g_3\end{bmatrix}\ ;$$

$$F_2\begin{bmatrix}\gamma,h\\ \delta,g\end{bmatrix}_{(H^{\rm f},G^{\rm f})} \equiv \frac{1}{\eta^2}\,\vartheta^{1/2}\begin{bmatrix}\gamma+H^{\rm f}\\ \delta+G^{\rm f}\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma+h_1-h_2\\ \delta+g_1-g_2\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma+h_2-h_3\\ \delta+g_2-g_3\end{bmatrix}\vartheta^{1/2}\begin{bmatrix}\gamma+h_3-h_1\\ \delta+g_3-g_1\end{bmatrix}, \quad (3.8)$$

With this embedding, the shift in $\Gamma_{2,2}$ is produced by the projection $(-)^{m_2 G^{\rm f}+n_2 g_1}$ [5]. The conformal blocks in the second line of Eq.(3.7) provide the right-moving part of eight vector multiplets, corresponding to a factor $U(1)^8$ in the gauge group, and sixteen hypermultiplets. From the blocks on the r.h.s. of the first line we get the other eight vectors, to make $U(1)^{16}$, and no further hypermultiplets. In addition to these, the massless spectrum of the model contains the usual three vector multiplets and four hypermultiplets of an ordinary $T^2 \times T^4/Z_2$ compactification of the heterotic string. Thanks to the free action of all the projections, there are no additional massless states coming from the twisted sectors. It is worth remarking that, when some of the Wilson lines are absent, the gauge group is enlarged but still $N_V = N_H$. Notice that, because of the embedding of the spin connection into the gauge group, it is not possible to construct this model at the point $SO(16) \times SO(16)$, but only at a broken phase of it.

It is easy to recognize that the $N = 2$ sector of this orbifold, specified by $(H^{\rm f}, G^{\rm f}) \neq (0,0)$, belongs to the same universality class as the $N = 2$ heterotic constructions with $N_V = N_H$, considered in Refs.[6, 9]. In fact, the modular transformation properties of the untwisted $\Gamma_{2,2}$ lattice, toghether with the condition $N_V = N_H$, as was already pointed out in Ref.[6], are sufficient to fix this orbifold sector uniquely, the difference between the various models residing in the $N = 4$ sector. A consequence of this is that not only the $N = 2$ singularities, but also all the threshold corrections that receive contribution only from this sector, are the same as for the other models of this class; we therefore skip the details about the analysis of this model and go directly to the discussion of the gravitational corrections.

As we anticipated in Section 2, it was pointed out in Refs.[6, 9] that the "pure" gravitational amplitude, $\langle R^2 \rangle$, must be corrected with a term proportional to the gauge amplitude, $\langle F_{\mu\nu} F^{\mu\nu} \rangle$, in order to make it holomorphic and non-singular. We recall here that the precise combination is:

$$\langle R'^2_{\rm grav} \rangle \equiv \langle R^2_{\rm grav} \rangle + \frac{1}{12} \langle F_{\mu\nu} F^{\mu\nu} \rangle_{T^2} + \frac{5}{48} \langle F_{\mu\nu} F^{\mu\nu} \rangle_{\rm gauge} \, . \tag{3.9}$$

The tree-level plus one-loop contribution reads:

$$\begin{aligned} \frac{16\,\pi^2}{g^2_{\rm grav}(\mu^{\rm (Het)})} &= 16\,\pi^2\, {\rm Im}\, S^{\rm (Het)} - 2 \log\, {\rm Im}\, T \, |\vartheta_4\,(T)|^4 - 2 \log\, {\rm Im}\, U \, |\vartheta_4\,(U)|^4 \\ &\qquad + 14 \log \frac{M^{\rm (Het)}}{\mu^{\rm (Het)}} + \ {\rm const.}\,, \end{aligned} \tag{3.10}$$

where $S^{\rm (Het)}$ is the heterotic axion–dilaton field,

$${\rm Im}\, S^{\rm (Het)} = \frac{1}{g^2_{\rm Het}} \tag{3.11}$$

and we used the string scale $M^{\rm (Het)} \equiv 1/\sqrt{\alpha'_{\rm Het}}$ and the infrared cut-off $\mu^{\rm (Het)}$ of the heterotic string.

[5] As in Refs.[6, 9], there exists also an alternative heterotic construction with the same massless spectrum, in which the spin connection is embedded in the right $U(1)^2$ factor from the untwisted two-torus. In this case, the shift in $\Gamma_{2,2}$ is produced by an asymmetric projection, e.g. $(-)^{(m_1+n_1)G^{\rm f}}$. We will not consider this alternative construction, because it is not dual to the type I.

Owing to the free action of the projection $Z_2^{(\mathrm{f})}$, also in this model the $N=4$ supersymmetry is spontaneously broken perturbatively [18]; it is restored when $T\to\infty$, $U\to\infty$. This limit corresponds, as in the type I construction, to a decompactification to five dimensions [6]. In order to compare this model with the type I, "Scherk–Schwarz breaking" model **A** of the previous section, we consider the limit in which T is large while U is kept finite (we stress that in this limit the supersymmetry remains broken to $N=2$). In this limit, the correction Eq.(3.10) depends on T only logarithmically:

$$\frac{16\,\pi^2}{g^2_{\mathrm{grav}}(\mu^{(\mathrm{Het})})} \approx 16\,\pi^2\,\mathrm{Im}\,S^{(\mathrm{Het})} - 2\log\,\mathrm{Im}\,U\,|\vartheta_4\,(U)|^4 + \mathcal{O}(\log\,\mathrm{Im}\,T)\ . \tag{3.12}$$

If we discard this logarithmic dependence for the moment, we see that Eq.(3.12) exactly reproduces the type I correction given in Eq.(3.4), with the identifications $S^{(\mathrm{Het})}\equiv S$ and $U^{(\mathrm{Het})}\equiv U^{(\mathrm{I})}$. We can interpret the logarithmic dependence as due to effects that are non-perturbative from the type I point of view[7]. The limit of restoration of the $N=4$ supersymmetry, in both the theories, corresponds to the decompactification of one radius ($R\to\infty$). In this limit, also $U\to\infty$, and the effective coupling constant, in the $N=4$ phase, depends only on the dilaton $S=S^{(\mathrm{Het})}$, as expected in both the theories[8]. We consider the coincidence of the massless spectrum and the rather non-trivial correspondence of these threshold corrections as compelling evidence of the duality of the heterotic and the type I, model **A** constructions[9].

We now consider the limit $T\to 0$, with U fixed. In this limit, the theory is better described in terms of the inverse modulus $\tilde{T}\equiv -1/T$. By performing an $SL(2,Z)$ inversion, the second term in Eq.(3.10) becomes

$$-2\log\,\mathrm{Im}\,\tilde{T}\left|\vartheta_2\left(\tilde{T}\right)\right|^4\ , \tag{3.13}$$

which diverges linearly when $\mathrm{Im}\,\tilde{T}$ is large:

$$\sim 2\pi\,\mathrm{Im}\,\tilde{T}\ . \tag{3.14}$$

[6]When $\mathrm{Re}\,T=\mathrm{Re}\,U=0$, we have $\mathrm{Im}\,T\sim R_1R_2$ and $\mathrm{Im}\,U\sim R_2/R_1$, where R_1, R_2 are the radii of the two circles of the torus. In this case this limit corresponds to $R_2\to\infty$, with R_1 fixed.

[7]This is the same phenomenon as that appearing in the examples of type IIA/type II asymmetric orbifolds dualities considered in Refs.[3, 8, 9], where the absence of tree level dilaton dependence in the analogous corrections on the asymmetric orbifolds indeed corresponds to a logarithmic dependence due to non-perturbative phenomena.

[8]Still, there is the presence of the logarithmic terms, both in T and U. These terms can be lifted by switching on an appropriate cut-off, as is discussed in Refs.[3, 6, 9, 19].

[9]In the construction with the asymmetric shift in $\Gamma_{2,2}$ referred to in footnote 5, there are lines, $T=f(U)$, in the (T,U) space, along which the "smooth gravitational amplitude" we are considering indeed becomes singular. This is due to the appearance in the massless spectrum of new hypermultiplets, which are uncharged under the gauge group of the torus, and therefore lead to a jump in the β-function of the R^2 term, which is not compensated by an opposite jump in the β-function of F^2 (see Refs.[6, 17]). By duality, these singularities should appear also on the type I side, where new massless states should appear for large or small values of the modulus U. The absence of these rules out the alternative heterotic construction.

Therefore, for large $\mathrm{Im}\, S^{(\mathrm{Het})}$ and small T, Eq.(3.10) becomes

$$\frac{16\,\pi^2}{g^2_{\mathrm{grav}}(\mu^{(\mathrm{Het})})} \approx 16\,\pi^2\, \mathrm{Im}\, S^{(\mathrm{Het})} + 2\pi\, \mathrm{Im}\, \tilde{T} - 2\, \mathrm{Im}\, U |\vartheta_4\,(U)\,|^4\,. \tag{3.15}$$

It is tantalizing to interpret the linear divergence in the field $\tilde{T}$ as corresponding to the appearance of a D5-brane sector in the dual type I theory. Indeed, in this region of the moduli space, the effective coupling constant of the heterotic theory behaves like that of the "M-theory breaking" type I model **B**, Eq.(3.5), provided we identify the field $S^{(\mathrm{Het})}$ with S, as before[10] and $\tilde{T}$ with $8\pi S' = 2\tau'_S$ (we have introduced here as usual the field τ'_S, in analogy with the field $\tau_S = 4\pi S$, the actual dilaton–axion field that enters the $SL(2,Z)$, Montonen–Olive duality transformations). As before, we can identify the complex structure moduli U of the heterotic and type I constructions. It therefore seems that, for large $\tilde{T}$, the heterotic theory is perturbatively dual to the type I construction **B**. This correspondence is better understood in terms of the "T-dual" heterotic theory, obtained by exchanging the two radii of the torus, $R_1 \leftrightarrow R_2$, and then inverting them: $R_1 \to \tilde{R}_1 = 1/R_1$, $R_2 \to \tilde{R}_2 = 1/R_2$. With these inversions, we remain in a spontaneously broken phase of the $SO(16) \times SO(16)$ string, but with the Kähler class modulus of the torus given by $\tilde{T}$ instead of T. The modulus $\tilde{T}$ is coupled to the windings of the torus of this T-dual theory, and in the limit of large $\mathrm{Im}\, \tilde{T}$ all the string states with a non-zero winding number decouple from the spectrum, leaving only the Kaluza–Klein states, of the type I dual string[11].

The above duality implies in this example that the massless states of the 55-branes sector appear on the heterotic side as perturbative states associated to the gauge currents, on the same footing as the states of the 99-branes sector. The situation is therefore rather different from that of the models of Gimon and Polchinski [20], in which the states of the 55-branes sector are non-perturbative on the heterotic side [21, 22]. A solution to this (apparent) puzzle comes from considering the T-dual, type I′ picture [4], in which the two gauge factors are provided by the two "Hořava–Witten" walls of the M-theory, S^1/Z_2 orbifold, where the D9-branes on one wall are wrapped on a T-dualized four-torus, and effectively appear as D5-branes. Nevertheless, because of their origin, the states corresponding to open strings ending on these D-branes are expected to appear as perturbative states on the heterotic theory[12].

We can now try to see what happens from the type I side point of view when the heterotic theory passes from small to large T (or equivalently from large to small $\tilde{T}$). Because of the identifications

$$S^{(\mathrm{Het})} = S = \mathrm{e}^{-\phi_4} G^{1/4} \omega^2\;, \qquad\qquad \tilde{T} = 8\pi S' = 8\pi \mathrm{e}^{-\phi_4} G^{1/4} \omega^{-2}\;, \tag{3.16}$$

we have $8\pi S^{(\mathrm{Het})}/\tilde{T} = S/S' = \omega^4$. We therefore see that such a motion, performed while keeping the field $S^{(\mathrm{Het})}$ fixed, corresponds on the type I side to increasing the volume of

[10]Here, by an abuse of language, we are using the same notation, S, for both the type I constructions.

[11]This limit can also be viewed as the infinite-tension limit of the heterotic string. We have in fact $\mathrm{Im}\, \tilde{T} \sim \tilde{R}_1 \tilde{R}_2/\alpha'$, $\mathrm{Im}\, U \sim \tilde{R}_2/\tilde{R}_1$, and the limit $\mathrm{Im}\, \tilde{T} \to \infty$ with U fixed is equivalent to the limit $\alpha' \to 0$ with fixed radii.

[12]We thank E. Dudas for a clarification of this point.

the $K3$, $\omega^4 \to \infty$. In order to keep the field S fixed, we have also to shrink the volume of the two-torus and/or adjust the value of the dilaton ϕ_4. In order to remain in the phase of broken $N = 4$ supersymmetry, we must shrink the second circle, leaving the first one fixed. Notice that, according to Eq.(3.3), this motion is non-perturbative from the type I point of view, involving a change of the tree-level coupling constant. In this limit, which corresponds to an effective decompactification of the theory to eight dimensions (or nine, if a circle of the two-torus is shrinked), indeed the D5-branes look like D9-branes wrapped around the T^2 torus. The $U(1)$ gauge bosons, which were provided by open strings ending on the same D5-brane, are still there; they contribute for a $U(1)^8$ factor to the gauge group, although they must now be reinterpreted as due to strings ending on D9-branes. What essentially distinguishes the behaviour of this model with respect to the type I constructions, in which supersymmetry is not spontaneously broken, as in Refs.[20, 23], is that there is an unbroken S-duality in those cases; combined with the symmetry under exchange of the fields S and S', i.e. of the 99- and 55-branes sectors, this duality implies that, along such a motion, non-perturbative phenomena enter heavily in the game and, at the limit we are considering, the theory is perturbatively described by the S-, S'-dual, identical theory. In our case, instead, we expect S-duality to be broken, because, as explained in Ref.[4], the type I model **B** indeed corresponds to a Scherk–Schwarz mechanism applied to the 11-th dimension of M-theory[13]. It is then reasonable to find that in this limit, since model **B** is not falling back into itself, it ends up to coincide with model **A**, in which all the gauge bosons of $U(1)^{16}$ are provided by the D9-branes[14].

We now consider the restoration of the $N = 4$ supersymmetry. The higher amount of supersymmetry can be restored essentially in two ways, which correspond on the type I side to the two decompactifications: $R_2 \to \infty$ in model **A** and $R_1 \to 0$ in model **B**. In the first case, the restoration is perturbative in both the type I and the heterotic side (the field $S \to \infty$). In the second case, the restoration is non-perturbative from both the type I and heterotic points of view (S, $S' \to 0$). There are, however, also intermediate possibilities, which involve a change also of ω. In these cases, the restoration, although non-perturbative on the type I side, can look perturbative on the heterotic side. This happens when the product $G^{1/4}\omega^2$ is kept fixed.

Non-perturbative corrections

We have seen that through the duality between heterotic and type I constructions, we gained insight into the non-perturbative behaviour of both of them, at least regarding the restoration of the $N = 4$ supersymmetry. We now try to go further: indeed, through the heterotic dual, we learned that the two type I constructions are actually two realizations of the same theory.

[13] We will come back to this point, which contradicts the results of Ref.[24].

[14] In order to understand how in this limit the D5-branes can look like D9-branes, consider the T-dual situation in which the four circles of T^4/Z_2 are T-dualized. In this case, for finite values of the radii, the D9-branes become D5-branes and vice versa. However, when the radii are shrunk to zero, the theory lives effectively in four extended and two compact dimensions, where there are no 9-branes but only 5-branes wrapped around the compact torus. T-dualizing again the four circles brings us back to the original limit, in which all the D5-branes become D9-branes.

The type I/heterotic duality can be used to get insight into the non-perturbative corrections to the effective coupling constant of the combination of gravitational and gauge amplitudes given in Eq.(3.1). From the heterotic dual we know that the $SL(2,Z)_{S'}$ duality group is broken to a $\Gamma(2)$ subgroup. On the heterotic side the $\Gamma(2)_T \times \Gamma(2)_U$ group is by construction a symmetry that remains valid at any value of the coupling. On the other hand, we know that the type I "M-theory breaking" model **B**, is perturbatively symmetric under the exchange of the fields S and S': this is a consequence of the symmetry under the exchange of the D9- and D5-branes sectors. We claim that this implies that also the $SL(2,Z)_S$ duality group is indeed broken in the same way as the $SL(2,Z)_{S'}$ group. This statement, which promotes a perturbative symmetry to a non-perturbative one, is supported by the observation that, as is discussed in Ref.[4], in the T-dual, type I′ picture, the two contributions come from the two "Hořava–Witten" walls of the M-theory on S^1/Z_2, and the symmetry of the problem under exchange of the two remains true at any value of the 11-th coordinate. Once observed that the symmetry of the theory is $\Gamma(2)_{2S} \times \Gamma(2)_{2S'} \times \Gamma(2)_U$ times the permutations of the three factors, we can write the full, non-perturbative correction, which reduces to Eq.(3.10) in the large-S limit:

$$\begin{aligned}\frac{16\,\pi^2}{g^2_{\rm grav}(\mu)} \quad &= \quad -2\log\,{\rm Im}\,\tau_S\,|\vartheta_2\,(2\tau_S)|^4 - 2\log\,{\rm Im}\,\tau'_S\,|\vartheta_2\,(2\tau'_S)|^4 - 2\log\,{\rm Im}\,U\,|\vartheta_4\,(U)|^4 \\ &\quad +\mathcal{E}(2\tau_S, 2\tau'_S, U) + 14\log\frac{M}{\mu}\ . \end{aligned} \tag{3.17}$$

In this expression we used the type I fields $\tau_S = 4\pi S$, $\tau'_S = 4\pi S'$ and U, which by now we know to be the same for both the type I models and equivalent to the heterotic fields $\tau_S^{\rm Het} = 4\pi S^{\rm Het}$, $\tilde{T}/2$ and U. The infrared cut-off μ and the mass scale M can be indifferently those of the type I or of the heterotic string, their relation being

$$\frac{M^{(\rm Het)}}{\mu^{(\rm Het)}} = \frac{M^{(\rm I)}}{\mu^{(\rm I)}}\ . \tag{3.18}$$

In Eq.(3.17) we allow for the presence of a series, $\mathcal{E}(2\tau_S, 2\tau'_S, U)$, of exponentials symmetric in $2\tau_S, 2\tau'_S, -1/U$. Such a term, always suppressed in the perturbative limit, cannot be excluded by the symmetries of the theory.

In each of the limits in which the $N = 4$ supersymmetry is perturbatively restored (either on the heterotic or on both sides), the contribution of two moduli drops out (it reduces to the already mentioned logarithmic dependence) and the correction Eq.(3.17) diverges linearly as a function of only one modulus (the field S, as expected for $N = 4$ corrections). The term $\mathcal{E}(2\tau_S, 2\tau'_S, U)$ is suppressed in the limits of a restoration of supersymmetry. By using its symmetry properties it is easy to see that it is suppressed also in the non-perturbative limit $S \to 0$. The form of Eq.(3.17) therefore tells us that there exists a limit, which is non-perturbative on both the heterotic and type I sides, in which there is an effective restoration of an $N = 8$ supersymmetry. This takes place when the three moduli contributions drop out, namely when $S \to 0$, $S' \to 0$ and $U \to \infty$. This for instance happens when, in model **B**, we send R_1 to zero while keeping ω, R_2 and the field ϕ_4 fixed. In this case the effective coupling Eq.(3.17) vanishes (modulo the usual logarithmic divergences).

Through the above analysis we have learned that in this theory, which can be interpreted as obtained by a freely acting projection of M-theory, the U-duality group $SL(2,Z)_S \times SL(2,Z)_T \times SL(2,Z)_U$ is indeed broken. In particular, the S-duality is broken to a $\Gamma(2)_{2\tau_S}$ subgroup. This is in contrast with the results of Ref.[24], in which, even in the presence of a Scherk–Schwarz compactification of the 11-th coordinate of M-theory, S-duality remains unbroken. We believe that, once properly treated, a freely acting projection applied on the 11-th coordinate must instead necessarily break the S-duality. The result of Ref.[24] is obtained by using as a starting point the effective Hořava–Witten action, in which the space-time metric appearing in the gauge terms, which live on the two ten-dimensional walls, is a "coordinate-independent" restriction of the eleven-dimensional metric to ten dimensions. In this way, the heterotic dilaton has no dependence on the 11-th radius. This scenario is correct for a genuine Hořava–Witten orbifold of M-theory, in which the original amount of supersymmetry can never be restored by a decompactification of the eleventh dimension. This is, however, not the case of a Scherk–Schwarz compactification of M-theory, in which we expect the supersymmetry to be restored in the large-radius limit (as happens in our present case, where the radius of M-theory is T-dual to the R_1 of model **B**). In this limit, the gauge bosons of the terms introduced to cancel the ten-dimensional anomaly should decouple. This is indeed what happens in our case, and is consistent with general arguments that fix the relation between the four-dimensional couplings and the eleventh radius ρ to be [25, 26]:

$$\mathrm{Im}\, S, \ \mathrm{Im}\, S' \sim \rho^{-2/3} \, , \tag{3.19}$$

where we have omitted the factors that contain all the other parameters.

As a last point, we remark that the model is symmetric under permutations of the fields $X = 8\pi S$, $Y = 8\pi S'$ and $-1/U$. By proceeding as in Ref.[9], it is therefore possible to calculate (at least a part of) the non-perturbative prepotential. This is obtained by a proper symmetrization of the perturbative prepotential, computed, as a function of the moduli T and U, on the heterotic side. The result is the same as in Ref.[9].

5. Discussion

In the previous section we were able to determine part of the non-perturbative behaviour of the gravitational corrections. In order to determine also the term $\mathcal{E}(2\tau_S, 2\tau'_S, U)$ in Eq.(3.17), we would need to identify a type IIA dual, in which these three fields would belong to the perturbative moduli. This would allow us to repeat the analysis of Refs.[6, 7, 9, 27]. In order for the duality to work, such a type IIA dual would have to be constructed as a compactification on (an orbifold limit of) a K3 fibration [28]–[30]. A necessary condition is therefore a symmetric breaking of the supersymmetry; this requirement leads directly to the type IIA orbifold of Section 2. However, the heterotic and the type IIA orbifolds under consideration cannot be compared for finite values of the heterotic moduli T and U. This is related to the fact that the compact space of the type IIA orbifold, a $T^6/(Z_2 \times Z_2)$ limit of $CY^{19,19}$, cannot be seen as a singular limit of a K3 fibration (see Refs.[1]–[3]). Indeed, the spaces of the vector moduli of the two models do not correspond. A signal of the non-coincidence of these subspaces is provided by the absence of a perturbative super-Higgs phenomenon on the type IIA construction: unlike in the heterotic model, on the type IIA side it is not possible to restore the $N = 4$ supersymmetry in a corner of the space of moduli T^i, U^i, $i = 1,2,3$ [15]. However, these models can be viewed as different phases of a single theory, corresponding to different regions of a wider moduli space[16]. These two regions are connected at the limits $\mathrm{Im}\, T^2 \to 0, \infty$, $\mathrm{Im}\, T^3 \to 0, \infty$ of the type IIA moduli space, for any value of the modulus T^1. In these limits, the type II correction, Eq.(2.4), reproduces the heterotic/type I, Eq.(3.17), at the appropriate corners in the space of moduli T and U ($\mathrm{Im}\, T$, $\mathrm{Im}\, U \to 0$ and/or ∞). For any value of T^1, we have the identifications $T^1 = \tau_S^{\mathrm{As}} = -1/2\tau_S$. Indeed, for any value of this modulus, there are two branches of the theory: a branch corresponds to a heterotic/type I phase, with moduli T, U; crossing the borders at $\mathrm{Im}\, T$, $\mathrm{Im}\, U \to 0, \infty$, one passes to another branch, which corresponds to the type II phase, described by the type IIA or type II asymmetric orbifolds, with moduli T^2, T^3. We can gain a deeper understanding of the reason why the heterotic and the type IIA constructions cannot be dual, if we consider the relation between these heterotic/type I constructions and the heterotic/type IIA, S, T, U models with $N_V = H_H = 0$ of Ref.[9], which also possess a spontaneously broken $N = 8$ supersymmetry. In that case, the spectrum is simply the truncation of the $N = 8$ supergravity, and the restoration of the $N = 8$ supersymmetry, which is non-perturbative from the heterotic point of view, being related to a motion in the dilaton field S, necessarily appears as perturbative on the type IIA side, where the field S is the volume form of the K3 fibration. Indeed, on the type IIA side, the spontaneous breaking of the $N = 8$ supersymmetry is due to a perturbative Scherk–Schwarz mechanism realized through freely acting projections, and is therefore directly related to the absence of extra massless states coming from orbifold twisted sectors. On the other hand, the heterotic/type I theory considered in this paper is an interesting example of (non-perturbative) spontaneous breaking of the $N = 8$ supersymmetry, which, possessing an enlarged gauge group, cannot correspond to a truncation of the type IIA, $N = 8$ spectrum, but is directly related to another phase of the M-theory. The two phases are interpolated by switching on and turning off "Wilson lines", which act as freely acting Z_2 projections twisting all the gauge bosons of the two "Hořava–Witten" planes. As is clear from the partition function given in Eqs. (4.2)–(4.8) of Ref.[9], they appear on the heterotic side as "$N = 4$" Wilson lines Y, which act on the twisted coordinates, corresponding to the hypermultiplets. Switching on/off these Wilson lines involves a blowing up of the moduli frozen at the fixed points of T^4/Z_2 and a motion to another T^4/Z_2 singularity[17]. Such a motion is on the other hand non-perturbative from the type IIA point of view, the moduli associated to Y including the dual of the type II dilaton field.

The type II constructions of Section 2, on the other hand, correspond to a limit in the moduli space in which the $N = 8$ supersymmetry is not spontaneously broken; in this limit, according to Ref.[4], on the type I′ picture, the combined action of the Z_2 which acts as a translation on the 11-th coordinate of M-theory, breaking $N = 8$ to $N = 4$, and that

[15] As is discussed in Ref.[3], compactification on a K3 fibration and spontaneous breaking of the $N = 4$ supersymmetry are directly related.

[16] We consider here only the space of moduli belonging to the vector multiplets.

[17] This necessarily involves passing through a $N = 4$ phase of the model, so that the results of Ref.[9] for the R^2 correction cannot be used to rule out the term $\mathcal{E}(2\tau_S, 2\tau_S', U)$ in Eq.(3.17).

of the Z_2 which further breaks to $N = 2$, is no longer free. From a perturbative point of view, the region of the moduli space corresponding to this limit is achieved at the above specified corners in the space of moduli T^2, T^3 (or $T^{(\rm Het)}, U^{(\rm Het)}$), where the two theories can be connected. The above discussion is sketched in Fig. 1.

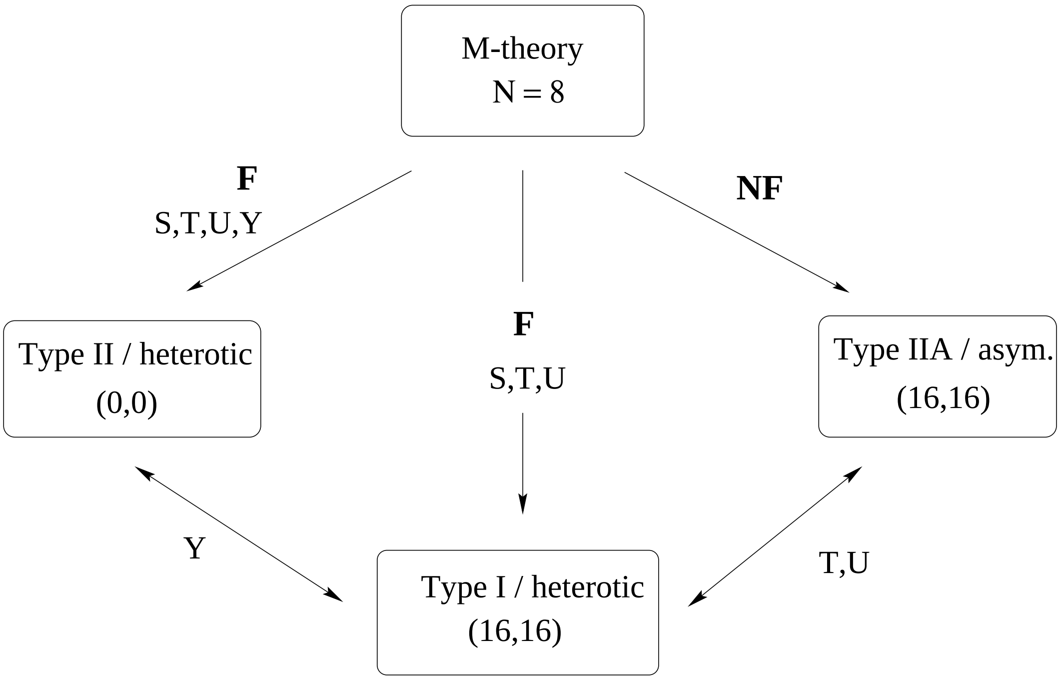

Figure 1: The connections between the $N = 2$ models and M-theory. (**N**)**F** indicates a (non-)freely acting orbifold projection. With (N_V, N_H) we indicate the number of vector and hypermultiplets of the "twisted sector".

6. Conclusions

In this paper we investigated the connections between several four-dimensional string constructions with the same massless spectrum, namely an $N = 2$ supergravity with $3 + N_V$ vector multiplets and $4 + N_H$ hypermultiplets, and $N_V = N_H = 16$. This spectrum is obtained via type IIA/B, heterotic and type I orbifold compactifications. We found evidence that the two type I constructions with spontaneous breaking of supersymmetry, presented in Ref.[4], namely the $N = 2$ "Scherk–Schwarz" and "M-theory" breaking models, indeed constitute two phases of the same theory, and are non-perturbatively related by a motion in the field S', which parametrizes the coupling constant of the gauge fields of the D5-branes sector. This relation appears as perturbative on the heterotic dual construction. Collecting the knowledge coming from the heterotic model and the type I duals, we got some insight into the non-perturbative aspects of this theory. In particular, we discovered the existence of a non-perturbative super-Higgs phenomenon responsible for the spontaneous breaking of the $N = 8$ supersymmetry. This is consistent with the interpretation of the theory as due to a "Scherk–Schwarz" mechanism applied to the 11-th dimension of the M-theory. This mechanism is also responsible for the breaking of the $SL(2, Z)$, Montonen–Olive S-duality, to a $\Gamma(2)$ subgroup, and it reflects on the dilaton dependence of string corrections to effective coupling constants, as the gravitational ones we considered. On the other hand, these constructions do not possess type II duals. Indeed we show that the type IIA, type II asymmetric orbifolds with the same massless spectrum actually correspond to a different phase of the M-theory. The two phases are connected at certain corners in the moduli space.

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