On Large-Angle Bhabha Scattering at LEP

Guido MONTAGNA[a,b], Oreste NICROSINI[b,a] and Fulvio PICCININI[b,a]

[a] Dipartimento di Fisica Nucleare e Teorica, Università di Pavia,
Via A. Bassi 6, 27100, Pavia, Italy
[b] INFN, Sezione di Pavia, Via A. Bassi 6, 27100, Pavia, Italy

Abstract

The theoretical accuracy of the program TOPAZ0 in the large-angle Bhabha channel is estimated. The physical error associated with the full Bhabha cross section and its forward and backward components separately is given for some event selections and several energy points of interest for LEP1 physics, both for the s and non-s contributions to the cross section.

E-mail:
montagna@pv.infn.it
nicrosini@pv.infn.it
piccinini@pv.infn.it

One of the open issues of precision physics at LEP is the determination of the accuracy of the theoretical predictions for the large-angle Bhabha scattering cross section. At present, several computer codes developed for large-angle Bhabha scattering studies can be found in the literature, ranging from semi-analytical to truly Monte Carlo ones. A detailed account of them has been presented in refs. [1, 2]. In particular, in ref. [1] several comparisons have been performed, both for "academic" and realistic event selections (ES's), both for LEP1 and LEP2 energies.

After the publication of ref. [1], a new analysis concerning specifically two codes, namely ALIBABA [3] and TOPAZ0 [4], has been performed [5], where a very detailed comparison between the two programs is developed and the estimate of the theoretical error associated with their predictions is given. If, on the one hand, the comparison is very careful, on the other hand the study is, in the opinion of the authors of the present note, lacking for the following aspects: it considers, as the main source of information on large-angle Bhabha observables, only ALIBABA and TOPAZ0; it is based on a comparison for a "bare" ES, which is far from being realistic, and ignores more realistic ES's such as the ones considered in ref. [1]; it does not fully exploit the detailed comparisons for s-channel annihilation processes, that can be found in the literature [6] and give valuable pieces of information on a significant part of the full Bhabha cross section; it considers only centre of mass energies around the Z^0 resonance, leaving aside LEP2 energies, from which additional information can be extracted concerning the accuracy of the non-s component of the full Bhabha cross section; it addresses the problem of assigning a theoretical error to the full Bhabha cross section both for ALIBABA and TOPAZ0, but the error for the non-s part of the Bhabha cross section is given for ALIBABA only; moreover no information is given concerning the forward and backward components of the cross section itself.

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ^T (pb)	457.08	644.86	912.06	1185.70	1164.82	873.50	476.64	351.80
σ_s^T (pb)	172.94	331.55	590.93	994.27	998.32	820.80	461.49	329.61
σ_{ns}^T (pb)	284.14	313.31	321.13	191.43	166.50	52.70	15.15	22.19
σ_{ns}^A (pb)	284.11	312.69	320.71	192.66	166.93	55.27	16.88	23.82
$\delta\sigma_s^T$ (pb)	0.2	0.3	0.6	1.0	1.0	0.8	0.5	0.3
$\delta\sigma_{ns}^T$ (pb)	2.8	3.1	3.2	1.9	1.7	2.6	1.7	1.6
$\delta\sigma/\sigma$ (%)	**0.7**	**0.5**	**0.4**	**0.2**	**0.2**	**0.4**	**0.5**	**0.5**

Table 1: Estimate of the theoretical error of TOPAZ0 for the full Bhabha cross section at 10° maximum acollinearity. σ^T, σ_s^T and σ_{ns}^T are the full, s and non-s part of the TOPAZ0 cross section. σ_{ns}^A is the non-s part of the ALIBABA cross section. $\delta\sigma_s^T$ and $\delta\sigma_{ns}^T$ are the absolute theoretical error of the s and non-s parts of the TOPAZ0 cross section, as obtained according to the procedure given in the text. $\delta\sigma/\sigma$ is the total relative error.

The aim of the present study is to critically analyze as much as possible of the

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ^T (pb)	485.17	674.89	945.00	1221.13	1200.16	905.25	503.79	377.59
σ^T_s (pb)	176.31	336.84	599.25	1007.03	1011.10	831.43	468.16	334.94
σ^T_{ns} (pb)	308.86	338.05	345.75	214.10	189.06	73.82	35.63	42.65
σ^A_{ns} (pb)	306.62	335.47	343.44	212.36	187.84	73.86	35.16	42.48
$\delta\sigma^T_s$ (pb)	0.2	0.3	0.6	1.0	1.0	0.8	0.5	0.3
$\delta\sigma^T_{ns}$ (pb)	3.1	3.4	3.5	2.1	1.9	0.7	0.5	0.4
$\delta\sigma/\sigma$	**0.7**	**0.5**	**0.4**	**0.3**	**0.2**	**0.2**	**0.2**	**0.2**

Table 2: The same as tab. 1 at 25° maximum acollinearity.

available literature on large-angle Bhabha scattering, in order to give a reliable estimate of the theoretical error associated with TOPAZ0, both for full cross sections and for the forward and backward components, both for the s and non-s parts. The numerical results presented in the following are obtained mostly by elaborating on the ALIBABA and TOPAZ0 predictions shown in ref. [5].

Let us consider first the problem of assigning a theoretical error to the full $s + t$ Bhabha cross section. It can be decomposed into s and non-s contributions. As far as the s part is concerned, TOPAZ0 includes exact $O(\alpha)$ electroweak corrections plus all the relevant and presently under control higher order contributions. From several tuned comparisons discussed in recent literature [6], one can see that the overall difference between TOPAZ0 and ZFITTER [7] is at the scale of 0.01% for s channel QED convoluted cross sections, for both extrapolated and realistic set up. While for completely inclusive s-channel cross sections, or for s-channel cross sections with an s' cut, TOPAZ0 includes $O(\alpha^3 L^3)$ and $O(\alpha^2 L)$ hard photon corrections according to ref. [8], these are not taken into account for s-channel processes with angular acceptance cuts and, in particular, for the s part of the full Bhabha cross section. When taking into account the theoretical error due to neglecting them, and the one due to other minor sources such as the approximate treatment of additional light pairs, one can conclude that the overall theoretical error of the s part of the Bhabha cross section in TOPAZ0 is 0.1%. In setting the theoretical error for the s part, no information coming from ALIBABA is considered, because it is known that the code is not accurate for s-channel processes at the 0.1% level [1, 4] as, for instance, TOPAZ0 and ZFITTER are. As far as the non-s part is concerned, the theoretical error of TOPAZ0 is dominated by missing $O(\alpha)$ non-logarithmic QED corrections, that on the contrary are present in ALIBABA. A way of estimating such an error is to consider the comparisons between TOPAZ0 and BHWIDE [9] performed in ref. [1] at LEP2 energies. Actually, in the LEP2 energy regime Bhabha scattering is essentially a t-channel dominated process. Since BHWIDE contains exact $O(\alpha)$ QED corrections for the s and non-s contributions to the cross section, such a comparison sets the size of the missing non-log contributions in the non-s part of the TOPAZ0 cross section, which is at the 1% level. In order to be as much as conservative as possible, and

not to loose the information contained in ALIBABA for the non-s contributions, a reliable recipe for setting the theoretical error of TOPAZ0 for the non-s part of the Bhabha cross section is to take it as the maximum between 1% of the non-s part of the cross section and the absolute deviation from ALIBABA.

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ_F^T (pb)	67.43	142.90	272.98	497.33	503.20	430.39	253.36	184.99
σ_B^T (pb)	105.51	188.65	317.95	496.94	495.12	390.41	208.13	144.62
$\delta\sigma_F^T$ (pb)	**0.1**	**0.1**	**0.3**	**0.5**	**0.5**	**0.4**	**0.3**	**0.2**
$\delta\sigma_B^T$ (pb)	**0.1**	**0.2**	**0.3**	**0.5**	**0.5**	**0.4**	**0.2**	**0.1**

Table 3: Estimate of the absolute theoretical error of TOPAZ0 for the s part of the Bhabha cross section at $10°$ maximum acollinearity, for the forward (σ_F^T) and backward (σ_B^T) components separately.

By following the above recipe for the estimate of the theoretical error of TOPAZ0 for the full Bhabha cross section, tables 1 and 2 follow. As already stated, the numerical results shown are elaborated from ref. [5], where all the details of the ES and input parameters adopted can be found.

The estimate of the theoretical error for the F+B cross section given in tabs. 1 and 2 refers to a BARE ES. Anyway, it is worth considering also the results of the comparison between BHWIDE and TOPAZ0 shown in ref. [1] for the LEP1 energy range, for both BARE and CALO ES's. Actually, it is known that ALIBABA does not contain the bulk of the $O(\alpha^2 L)$ corrections, while BHWIDE and TOPAZ0 do, by virtue of their factorized formulation [4, 5, 8, 9]. Such missing corrections are, for instance, responsible of part of the theoretical error of ALIBABA, that above the Z peak for s channel processes can be of the order of several 0.1%. When considering the comparison between BHWIDE and TOPAZ0 for LEP1 energies, one realizes that the difference between the two programs for a realistic CALO ES is generally smaller than the errors quoted above[1]. Hence, the estimate of the theoretical error of tabs. 1 and 2 has to be considered as a conservative one, and has its origin in the fact that the recipe adopted aims at using as much information as possible, and in particular the piece of information given by ALIBABA. In the light of the above comments, the error estimate of tabs. 1 and 2 can be considered as a conservative error estimate also for CALO ES's. Less conservative error estimates for the full Bhabha cross section can be found in refs. [2] and [9].

Besides a reliable estimate of the theoretical error for the full $s + t$ Bhabha cross section, it is also interesting to give the forward (F) and backward (B) parts of

---

[1]The authors of BHWIDE consider the program as more reliable for realistic ES's (CALO) rather than for BARE ones [9].

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ_F^T (pb)	68.45	144.77	276.34	503.31	509.24	435.67	256.72	187.62
σ_B^T (pb)	107.86	192.07	322.91	503.72	501.86	395.76	211.44	147.32
$\delta\sigma_F^T$ (pb)	**0.1**	**0.1**	**0.3**	**0.5**	**0.5**	**0.4**	**0.3**	**0.2**
$\delta\sigma_B^T$ (pb)	**0.1**	**0.2**	**0.3**	**0.5**	**0.5**	**0.4**	**0.2**	**0.2**

Table 4: The same as tab. 3 at 25° maximum acollinearity.

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ_F^T (pb)	262.77	289.51	296.69	177.69	154.80	50.39	15.90	22.36
σ_B^T (pb)	21.37	23.80	24.44	13.74	11.70	2.31	-0.75	-0.17
σ_F^{A1} (pb)	263.16	289.26	296.64	179.00	155.58	52.42	17.38	23.90
σ_B^{A1} (pb)	20.95	23.43	24.07	13.66	11.35	2.85	-0.50	-0.08
σ_F^{A2} (pb)	263.54	289.95	296.74	179.00	156.06	52.79	17.91	23.88
σ_B^{A2} (pb)	21.06	23.26	23.95	14.76	11.85	2.24	-0.27	-0.05
$\Delta_1\sigma_F$ (pb)	0.39	0.25	0.05	1.31	0.78	2.03	1.48	1.54
$\Delta_1\sigma_B$ (pb)	0.42	0.37	0.37	0.08	0.35	0.54	0.25	0.09
$\Delta_2\sigma_F$ (pb)	0.77	0.44	0.05	1.31	1.26	2.40	2.01	1.52
$\Delta_2\sigma_B$ (pb)	0.31	0.54	0.49	1.02	0.15	0.07	0.48	0.12
$\delta\sigma_F^T$ (pb)	**2.6**	**2.9**	**3.0**	**1.8**	**1.5**	**2.4**	**2.0**	**1.5**
$\delta\sigma_B^T$ (pb)	**0.4**	**0.5**	**0.5**	**1.0**	**0.4**	**0.5**	**0.5**	**0.1**

Table 5: Estimate of the absolute theoretical error of TOPAZ0 for the non-s part of the Bhabha cross section at 10° maximum acollinearity, for the forward (σ_F^T) and backward (σ_B^T) components separately. The apices $A1$ and $A2$ refer to the results of ALIBABA according to the procedures described in the text. Δ are the absolute differences between ALIBABA and TOPAZ0.

the cross section, together with their theoretical error, for both the s and non-s components. For the program TOPAZ0, the first part of the task can be accomplished by solving the system

$$\sigma = \sigma_F + \sigma_B,$$
$$\sigma A_{FB} = \sigma_F - \sigma_B, \tag{1}$$

where σ and A_{FB} are the cross section and the forward-backward asymmetry, respectively, even if TOPAZ0 has been designed for computing cross sections and asymmetries directly. For the second part, *i.e.* assigning a theoretical error to the F/B components, one should notice that a naive error propagation can lead to artificially overestimated errors. Hence, in the following an alternative procedure is proposed.

First of all, by using the s components of cross section and asymmetry as quoted in ref. [5] one can compute the s components of the forward and backward cross sections. Since in general the s component of the F/B asymmetry is small, one can

$\sqrt{s}$ (GeV)	88.45	89.45	90.20	91.19	91.30	91.95	93.00	93.70
σ_F^T (pb)	285.84	312.6	319.62	198.9	175.87	70.14	35.08	41.50
σ_B^T (pb)	23.02	25.45	26.13	15.20	13.19	3.68	0.55	1.15
σ_F^{A1} (pb)	283.82	310.17	317.71	197.09	174.52	70.09	34.54	41.22
σ_B^{A1} (pb)	22.80	25.30	25.73	15.27	13.32	3.77	0.62	1.26
σ_F^{A2} (pb)	284.18	310.87	317.24	197.64	173.84	69.91	34.80	40.86
σ_B^{A2} (pb)	22.67	24.98	25.59	16.14	13.20	3.43	0.94	1.20
$\Delta_1\sigma_F$ (pb)	2.02	2.43	1.91	1.81	1.35	0.05	0.54	0.28
$\Delta_1\sigma_B$ (pb)	0.22	0.15	0.40	0.07	0.13	0.09	0.07	0.11
$\Delta_2\sigma_F$ (pb)	1.66	1.73	2.38	1.26	2.03	0.23	0.28	0.64
$\Delta_2\sigma_B$ (pb)	0.35	0.47	0.54	0.94	0.01	0.25	0.39	0.05
$\delta\sigma_F^T$ (pb)	**2.9**	**3.1**	**3.2**	**2.0**	**2.0**	**0.7**	**0.5**	**0.6**
$\delta\sigma_B^T$ (pb)	**0.4**	**0.5**	**0.5**	**0.9**	**0.1**	**0.3**	**0.4**	**0.1**

Table 6: The same as tab. 5 at $25°$ maximum acollinearity.

attribute to the F/B components of the s-channel cross section the same theoretical error as the one attributed to the integrated s-channel cross section, namely 0.1%. From this recipe, tabs. 3 and 4 follow.

For the non-s component of the cross section, it should be still desirable to exploit the information provided by ALIBABA. To this aim, it has to be noticed that two procedures can be followed. The first one consists in solving the system above as done for TOPAZ0 (ALIBABA1). The second one consists in computing directly the F/B components of the cross section, both for the full and s parts (ALIBABA2). For the first procedure, the results of ref. [5] have been used. For the second one, ALIBABA has been re-run with high numerical precision in order to neglect the integration error. The theoretical error to be attributed to the non-s part of the F/B cross section, similarly to what has been done for the F+B cross section, has then been defined as the maximum among the 1% of the corresponding cross section and the absolute deviation from ALIBABA1 and ALIBABA2. By following the recipe here described, tabs. 5 and 6 follow.

Two technical comments are in order here. The first one is that the recipe adopted for the estimate of the non-s theoretical error is sensible, since in some energy points the error is fixed to be 1% of the corresponding cross section (typically in the region below the Z peak), whereas in other ones it is fixed by one of the absolute differences (typically in the region at and above the Z peak, where the F/B components are numerically small). The second comment concerns the fact that in reconstructing the full theoretical error of the F+B cross section by adding its F/B components from tabs. 3–6, for the s and non-s parts, one obtains values that are equal or slightly larger than the ones obtained directly in tabs. 1 and 2, as expected. Of course, the same remarks concerning the conservativeness of the error estimate for the F+B cross section apply to the F/B components separately, also.

The present results correspond to an angular acceptance of 40°–140° for the scattered electron. Since the sharing between the s and non-s component of the cross section depends on the angular acceptance, the above estimate of the theoretical error can be considered valid for the presently adopted angular cuts; anyway, an important change in the angular cuts would require a reanalysis of the situation, bearing in mind that for larger/narrower angular acceptances the total error can be expected to increase/decrease, respectively.

To summarize, the estimate of the theoretical error of the Bhabha cross section derived in the present letter is based upon the following pieces of information: tuned comparisons between TOPAZ0 and ZFITTER for s-channel observables [6]; estimate of missing higher-order QED corrections [8]; comparisons between BHWIDE and TOPAZ0 for the full Bhabha cross section in the LEP1 and LEP2 energy range [1]; comparisons between ALIBABA and TOPAZ0 for the non-s part of the cross section. The present paper updates the existing literature for the theoretical error of the full F+B Bhabha cross section [5], and improves it by providing additional information on the theoretical uncertainty to be associated to the F/B components of the s/non-s parts of the Bhabha cross section.

References

[1] S. Jadach, O. Nicrosini et al., "Event Generators for Bhabha Scattering", in *Physics at LEP2*, G. Altarelli, T. Sjöstrand and F. Zwirner, eds., CERN Report **96-01** (Geneva, 1996), vol. 2, p. 229.

[2] G. Montagna, O. Nicrosini and F. Piccinini, "Precision Physics at LEP", Rivista del Nuovo Cimento **21** n. 9 (1998) 1; `hep-ph/9802302`.

[3] W. Beenakker, F. Berends and S.C. van der Mark, Nucl. Phys. **B349** (1991) 323.

[4] G. Montagna, O. Nicrosini, G. Passarino F. Piccinini and R. Pittau, Comput. Phys. Commun. **76** (1993) 328, Nucl. Phys. **B401** (1993) 3;
G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini, TOPAZ0 version 2.0, in Comput. Phys. Commun. **93** (1996) 120;
TOPAZ0 version 4.0, in Comput. Phys. Commun. **117** (1999) 278.

[5] W. Beenakker and G. Passarino, Phys. Lett. **B425** (1998) 199.

[6] D. Bardin, M. Grünewald and G. Passarino, *Precision Calculation Project Report*, `hep-ph/9902452`;
D. Bardin et al., "Electroweak Working Group Report", in *Precision Calculations*

for the Z resonance, D. Bardin, W. Hollik and G. Passarino, eds., CERN Report **95-03** (Geneva, 1995), p. 7, `hep-ph/9709229`.

[7] D. Bardin et al., Z. Phys. **C44** (1989) 493; Comput. Phys. Commun. **59** (1990) 303; Nucl. Phys. **B351** (1991) 1; Phys. Lett. **B255** (1991) 290; CERN-TH 6443/92.

[8] G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. **B406** (1997) 243; Phys. Lett. **B385** (1996) 348;
S. Jadach, M. Skrzypek, B. Pietrzyk, LAPP-EXP-99-01.

[9] S. Jadach, W. Płaczek and B.F.L. Ward, Phys. Lett. **B390** (1997) 298;
W. Płaczek, S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost, CERN-TH/99-07, UTHEP-98-01, `hep-ph/9903381`.