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Generalized Conformal Quantum Mechanics of D0-brane

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Abstract

We study the generalized conformal quantum mechanics of the probe D0-brane in the near horizon background of the bound state of source D0-branes. We elaborate on the relationship of such model to the M theory in the light cone frame.

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I. INTRODUCTION

According to the holographic principle [1–3], the bulk theory (with gravity) and the boundary theory (without gravity) are equivalent. Such correspondence gives insight into one theory from the other. Interest in the holographic principle is revived from the recent observation [4] that the bulk gravity theory in the near horizon geometry of brane configuration is equivalent to the boundary theory described by the corresponding worldvolume field theory in the decoupling limit. (The closely related previous works in Refs. [5–7] study the correspondence between the bulk theory on the AdS space and the boundary supersingleton field theory.) In particular, when the near horizon geometry is the AdS space, i.e. D3-brane, M2-brane and M5-brane cases, the boundary theory is conformal. However, it is found out in Refs. [8,9] that even for other branes, whose near horizon geometry is not the AdS type and therefore the corresponding boundary theory is not a "genuine" conformal theory, one can still define "generalized" conformal theory.

The symmetry group of such generalized conformal theory can be manifestly seen in the bulk theory in the so-called "dual" frame [10,6,7,11–13]. The dual frame is regarded as a preferred frame for supergravity probes in the near horizon background of the source branes. In this frame, the near horizon geometry of a p-brane supergravity solution takes the $\mathrm{AdS}_{p+2} \times S^n$ form. The $SO(p+1,2)$ isometry of the AdS_{p+2} part is realized in the boundary theory as the "generalized" conformal symmetry of Refs. [8,9]. The dual frame is also called a "holographic" frame, since an UV/IR connection between the bulk and the boundary theory is manifest in this frame.

In this paper, we study the D0-brane case of such generalized conformal theory. D0-brane is particularly interesting for its relevance to M theory. An important discovery of the M(atrix) model [1] is that difficult problems of quantum M theory is reduced to non-relativistic quantum mechanics. Originally, it is conjectured [16] that M-theory in the infinite momentum frame (IMF) [17] is exactly described by the $U(\infty)$ $D=1$ super-Yang-Mills (SYM) theory, which is the worldvolume theory of the bound state of the infinite number ($N \to \infty$) of D0-branes. The $D=1$ $U(N)$ SYM theory, which is the supersymmetric matrix model description of the supermembrane theory [18], is nothing but the maximally supersymmetric $U(N)$ Yang-Mills theory dimensionally reduced from 9+1 to 0+1 dimensions (i.e. $N \times N$ hermitian matrix quantum mechanics). It is further conjectured [19] that the equivalence of (M)atrix theory to M-theory is also valid for a finite N. The conjecture states that M-theory compactified on a light-like circle with finite momentum along the circle is exactly described by a $U(N)$ matrix theory. The quantization of such theory is called the discrete light-cone quantization (DLCQ). One can think of this light-like circle as a small space-like circle that is boosted by a large amount [20]. Just like M-theory in the IMF, M-theory in the light-cone frame (LCF) is described purely by D0-branes with positive momentum and has the Galilean invariance in the transverse space [19]. Therefore, M-theory on a light-like circle can be viewed as a theory of the finite number of D0-branes

[1] This model was proposed [14] long ago as the $N=16$ supersymmetric gauge quantum mechanics. See also Ref. [15].

in the low velocity (non-relativistic) limit.

This idea can also be understood from the bulk/boundary duality as follows. When compactified on a light-like circle, the supergravity M-wave solution (viewed as the supergraviton with the momentum number N) becomes the near horizon limit of supergravity solution for N coinciding D0-branes [21]. Since M-theory in the LCF in the supergravity solution level is related to the near horizon limit of the supergravity D0-brane solution, by the generalized AdS/CFT duality [22,8,9] M-theory in the LCF has to be related to the boundary theory of the bound state of N D0-branes. This boundary theory is the maximally supersymmetric $SU(N)$ Yang-Mills theory dimensionally reduced from $9+1$ to $0+1$ dimensions, as stated in the (M)atrix theory conjecture.

In this paper, we view the non-relativistic (matrix) quantum mechanics of bound states of D0-brane as a system of the probe D0-brane moving in the background of large number of source D0-brane bound state. This view of (M)atrix model is also taken in Ref. [21], which reproduces the (M)atrix model graviton-graviton scattering calculation from the effective action for a probe moving in the background of the M-wave supergravity solution. It is further shown in Refs. [8,9] that the Dirac-Born-Infeld (DBI) action for a radially moving probe D0-brane in the background of the source D0-brane bound state can also be determined by imposing the generalized conformal symmetry of the boundary theory, which is just the $D=1$ $U(N)$ SYM theory describing the M theory in the LCF.

The quantum mechanics of the probe D0-brane in the near horizon background of the source D0-brane is a reminiscence of the conformal quantum mechanics of the charged test particle moving in the near horizon background of the $D=4$ extreme Reissner-Nordström black hole studied in Ref. [23]. The radial motion of such test particle is described by the relativistic conformal quantum mechanics with the $SL(2,\mathbf{R}) \cong SU(1,1)$ symmetry. This is a generalized version of (non-relativistic) conformal mechanics studied in Refs. [24–26]. One can view such generalized conformal mechanics of the radial motion of the test particle as the boundary conformal field theory counterpart of the bulk gravity theory in the near horizon Ads_2 geometry of the extreme Reissner-Nordström black hole, since the $SO(1,2) \cong SU(1,1)$ isometry of the AdS_2 space is realized as the symmetry of the dynamics of the test particle in the spacetime with one lower dimension. Since the near horizon geometry of the source D0-brane supergravity solution in the dual frame is $\mathrm{AdS}_2 \times S^8$, one would expect that the dynamics of the probe D0-brane in this background has the $SL(2,\mathbf{R}) \cong SO(2,1)$ symmetry. In the D0-brane case, since this symmetry does not extend to the genuine conformal symmetry of the boundary theory but only to the so-called generalized conformal symmetry [8,9] as pointed out in the above, the quantum mechanics of the probe D0-brane will have a generalized conformal symmetry.

The paper is organized as follows. In section 2, we summarize the properties of the near horizon geometry of the supergravity D0-brane solution. In section 3, we study the "generalized" conformal quantum mechanics of D0-brane, elaborating its relation to M theory as pointed out in the above.

II. $D0$-BRANE SOLUTION IN THE NEAR HORIZON REGION

In this section, we survey aspects of the $D0$-brane supergravity solution and its near horizon geometry, illuminating their relations.

The string-frame type-IIA effective supergravity action for $D0$-brane solution is given by

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G^{str}} [e^{-2\phi}(\mathcal{R}^{str} + 2\partial_M \phi \partial^M \phi) - \frac{1}{2 \cdot 2!} F_{MN} F^{MN}], \tag{1}$$

where G_{10} is the 10-dimensional gravitational constant, $\mathcal{R}^{str}$ and G^{str} are the Ricci scalar and the determinant for the the string-frame metric tensor G^{str}_{MN}, ϕ is the dilaton in the NS-NS sector and F_{MN} is the field strength for the RR 1-form potential A_M that $D0$-branes couple to.

The supergravity solution for the $D0$-brane has the following form:

$$\begin{aligned} ds^2_{str} &= G^{str}_{MN} dx^M dx^N = -H^{-\frac{1}{2}} dt^2 + H^{\frac{1}{2}} (dx_1^2 + \ldots + dx_9^2), \\ e^{\phi} &= g_s H^{\frac{3}{4}}, \quad A_t = -g_s^{-1} H^{-1}; \quad H = 1 + \frac{Q}{r^7} \equiv 1 + \left(\frac{\mu}{r}\right)^7, \end{aligned} \tag{2}$$

where g_s is the string coupling constant, which is just the vacuum expectation value or the asymptotic value of the dilaton e^{ϕ}. Here, the $D0$-brane charge Q is related to the string theory quantities as

$$Q = (\alpha')^{\frac{7}{2}} g_s N, \tag{3}$$

where α' is related to the string length scale l_s as $l_s = \sqrt{\alpha'}$ and N is the number of the $D0$-branes. By expanding the Dirac-Born-Infeld (DBI) action for the D0-brane to the lowest order in α', one finds the following relation of the string theory quantities to the Yang-Mills gauge coupling g_{YM}:

$$g^2_{YM} = \frac{g_s}{(\alpha')^{\frac{3}{2}}}. \tag{4}$$

Therefore, the constant Q in the harmonic function is rewritten in terms of the SYM quantities as $Q = \mu^7 = (\alpha')^5 g^2_{YM} N$.

In order to decouple the massive string modes (whose masses are proportional to $1/\alpha'$) and the gravity modes (whose strength goes as $g_s^2(\alpha')^4$) from the massless open string modes, which describe the Yang-Mills theory, one has to take $\alpha' \to 0$, while keeping g_{YM} as a finite constant, which means for the D0-brane case $g_s \to 0$ as well (Cf. Eq. (4)). Furthermore, the above metric (2) can be regarded as the gravitational field felt by a probe D0-brane in the background of the collection of N numbers of source D0-branes with the radial coordinate r being interpreted as the distance between and probe and source $D0$-branes [27]. So, in order to keep the mass $m_{string} = r/\alpha'$ of the state of open string, which stretches between the source and the probe, finite, one has also take the limit $r \to 0$ while keeping the following combination as a finite constant:

$$U \equiv \frac{r}{\alpha'}, \tag{5}$$

thereby going to the near horizon region of the supergravity solution (2). This combination also corresponds to the conventional super-Yang-Mills scalar $\Phi^I = X^I/l_s^2$, whose vacuum expectation value sets the energy scale. In this decoupling limit, the supergravity solution (2) takes the following form:

$$\begin{aligned}
ds^2_{str} &= -\left(\frac{r}{\mu}\right)^{\frac{7}{2}} dt^2 + \left(\frac{\mu}{r}\right)^{\frac{7}{2}} (dr^2 + r^2 d\Omega_8^2) \\
&= \alpha' \left[-\frac{U^{\frac{7}{2}}}{g_{YM}\sqrt{N}} dt^2 + \frac{g_{YM}\sqrt{N}}{U^{\frac{7}{2}}} (dU^2 + U^2 d\Omega_8^2) \right], \\
e^\phi &= g_s \left(\frac{\mu}{r}\right)^{\frac{21}{4}} = g_{YM}^2 \left(\frac{g_{YM}^2 N}{U^7}\right)^{\frac{3}{4}}, \\
A_t &= g_s^{-1} \left(\frac{r}{\mu}\right)^7 = \frac{\sqrt{\alpha'}}{g_{YM}^2} \frac{U^7}{g_{YM}^2 N}.
\end{aligned} \tag{6}$$

The above supergravity solution (6) can be trusted when the spacetime curvature $\mathcal{R} \sim U^3/(g_{YM}^2 N)$ is much smaller than the string scale $(\alpha')^{-1}$ and string coupling e^ϕ is very small, leading to the following constraints on U and the original radial coordinate r:

$$g_{YM}^{\frac{2}{3}} N^{\frac{1}{7}} \ll U \ll g_{YM}^{\frac{2}{3}} N^{\frac{1}{3}}, \tag{7}$$

$$\sqrt{\alpha'} g_s^{\frac{1}{3}} N^{\frac{1}{7}} \ll r \ll \sqrt{\alpha'} g_s^{\frac{1}{3}} N^{\frac{1}{3}}. \tag{8}$$

On the other hand, the near horizon condition $r \ll \mu = Q^{\frac{1}{7}}$ is expressed in terms of string theory quantities as $r \ll \sqrt{\alpha'}(g_s N)^{\frac{1}{7}}$. So, for sufficiently large N and small g_s, the supergravity solution (6) can be trusted in the overlapping region of U.

Note, the near horizon geometry of the D0-brane supergravity solution in the string frame is not $\mathrm{AdS}_2 \times S^8$, since when the metric is expressed in the following suggestive form

$$ds^2_{str} = \alpha' \left[-\frac{U^2}{\sqrt{\rho_0}} dt^2 + \sqrt{\rho_0} \frac{dU^2}{U^2} + \sqrt{\rho_0} d\Omega_8^2 \right], \tag{9}$$

the radius $\rho_0 \equiv Q/((\alpha')^5 U^3)$ of the would-be AdS space depends on the coordinate U.

However, with the suitable choice of frame, called the "dual" frame [10,6,7,13], the spacetime metric in the near-horizon region takes the $\mathrm{AdS}_2 \times S^8$ form. Namely, if one applies the Weyl transformation [2] on the metric as $G^{str}_{MN} \to G^{dual}_{MN} = e^{-\frac{2}{7}\phi} G^{str}_{MN}$, then the metric in (6) transforms to

$$ds^2_{dual} = G^{dual}_{MN} dx^M dx^N = g_s^{-\frac{2}{7}} \left[-\left(\frac{r}{\mu}\right)^5 dt^2 + \left(\frac{\mu}{r}\right)^2 dr^2 + \mu^2 d\Omega_8^2 \right]$$

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[2] In this paper, we do not include a factor involving N in the Weyl transformation of the metric, unlike Ref. [13], so that the metric in the standard AdS form in the horospherical coordinates has explicit dependence on N.

$$= N^{\frac{2}{7}}\alpha' \left[-\frac{U^5}{g_{YM}^2 N} dt^2 + \frac{dU^2}{U^2} + d\Omega_8^2 \right]. \quad (10)$$

By further redefining the radial coordinate as $\bar{r} = (\frac{25}{4} g_s^{\frac{4}{7}} \mu^3)^{-\frac{1}{2}} r^{\frac{5}{2}}$ or $u = (\frac{25}{4} g_{YM}^2 N^{\frac{3}{7}})^{-\frac{1}{2}} U^{\frac{5}{2}}$, one can bring the metric to the following standard $\mathrm{AdS}_2 \times S^8$ form in the horospherical coordinates:

$$\begin{aligned} ds^2_{dual} &= -\left(\frac{5 g_s^{\frac{1}{7}}}{2\mu}\right)^2 \bar{r}^2 dt^2 + \left(\frac{2\mu}{5 g_s^{\frac{1}{7}}}\right)^2 \frac{d\bar{r}^2}{\bar{r}^2} + \left(\frac{\mu}{g_s^{\frac{1}{7}}}\right)^2 d\Omega_8^2 \\ &= \alpha' \left[-\left(\frac{5}{2N^{\frac{1}{7}}}\right)^2 u^2 dt^2 + \left(\frac{2N^{\frac{1}{7}}}{5}\right)^2 \frac{du^2}{u^2} + N^{\frac{2}{7}} d\Omega_8^2 \right], \end{aligned} \quad (11)$$

where $u = \bar{r}/\alpha'$. In the dual frame, in which the metric in the near horizon takes the above AdS form, the effective action (1) takes the following form:

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G^{dual}} \left[e^{-\frac{6}{7}\phi} (\mathcal{R}^{dual} + \frac{16}{49} \partial_M \phi \partial^M \phi) - \frac{1}{4} e^{\frac{6}{7}\phi} F_{MN} F^{MN} \right]. \quad (12)$$

On the other hand, one can view the $D0$-brane solution (2) as being magnetically charged under the Hodge-dual field strength to the field strength of the 1-form potential in the RR sector. Namely, the supergravity solution (2) solves the equations of motion of the following effective action and is magnetically charged under the 7-form potential $A_{M_1 \cdots M_7}$:

$$S' = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G^{str}} [e^{-2\phi} (\mathcal{R}^{str} + 2\partial_M \phi \partial^M \phi) - \frac{1}{2 \cdot 8!} F_{M_1 \cdots M_8} F^{M_1 \cdots M_8}], \quad (13)$$

where $F_{M_1 \cdots M_8}$ is the field strength of the 7-form potential $A_{M_1 \cdots M_7}$. In the dual frame with the spacetime metric $G^{dual}_{MN} = e^{-\frac{2}{7}\phi} G^{str}_{MN}$, the action (13) takes the following form [13]:

$$S' = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G^{dual}} e^{-\frac{6}{7}\phi} \left[\mathcal{R}^{dual} + \frac{16}{49} \partial_M \phi \partial^M \phi - \frac{1}{2 \cdot 8!} F_{M_1 \cdots M_8} F^{M_1 \cdots M_8} \right]. \quad (14)$$

As in Ref. [13], if one properly includes the factor involving N in the Weyl transformation of the metric, i.e. $G^{str}_{MN} \to G^{dual}_{MN} = (Ne^{\phi})^{-\frac{2}{7}} G^{str}_{MN}$, then the new radial coordinate u (with which the near horizon metric takes the AdS form in the horospherical coordinates) is related to U as $u = \frac{2}{5} \frac{U^{5/2}}{g_{YM} N^{1/2}}$. This is reminiscence of the holographic UV/IR connection between the bulk and the boundary theory [28,29], if one identifies u with the energy scale E of the boundary theory [13].

In this case, the effective action in the dual frame takes the following form [13]:

$$S' = \frac{N^2}{16\pi G_{10}} \int d^{10}x \sqrt{-G^{dual}} (Ne^{\phi})^{-\frac{6}{7}} \left[\mathcal{R}^{dual} + \frac{16}{49} \partial_M \phi \partial^M \phi - \frac{1}{2 \cdot 8!} \frac{1}{N^2} F_{M_1 \cdots M_8} F^{M_1 \cdots M_8} \right]. \quad (15)$$

The near horizon form of D0-brane solution to the associated equations of motion is:

$$ds^2_{dual} = -\frac{u^2}{u_0^2}dt^2 + u_0^2\frac{du^2}{u^2} + d\Omega_8^2,$$
$$e^\phi = \frac{1}{N}(g^2_{YM}N)^{\frac{7}{10}}\left(\frac{u}{u_0}\right)^{-\frac{21}{10}},$$
$$F_8 = 7N\, vol(S^8), \tag{16}$$

where $u_0 = 2/5$. From this solution, one can see that there is the Freund-Rubin compactification [30] on S^8 of the $D = 10$ action (15) to the following 2-dimensional effective gauged supergravity action [13]:

$$S' = N^2 \int d^2x\sqrt{-g}(Ne^\phi)^{-\frac{6}{7}}\left[\mathcal{R} + \frac{16}{49}\partial_\mu\phi\partial^\mu\phi + \frac{63}{2}\right]. \tag{17}$$

The near-horizon D0-brane supergravity solution (16) is reduced under this S^8 compactification to a domain solution [31–33], which is supported by the cosmological term in the action (17).

III. GENERALIZED CONFORMAL MECHANICS OF D0-BRANES

We consider a probe D0-brane with mass m and charge q moving in the near horizon geometry of the source N D0-brane bound state. The gravitational field felt by the probe D0-brane in the string [dual] frame is given by Eqs. (6) and (9) [Eqs. (10) and (11)]. Here, once again, r is the radial distance between the source and the probe D0-branes. The probe D0-brane moves with the 10-momentum $p = (p_M) = (p_t, p_1, ..., p_9)$ and its time-component is the (static-gauge) Hamiltonian $H = -p_t$. The expression for the Hamiltonian of the probe D0-brane in the near horizon background of the source D0-brane can be obtained by solving the mass-shell constraint of the probe D0-brane.

Unlike the case of Ref. [23], which studies a charged particle in the Einstein-Maxwell theory, the probe D0-brane satisfies the mass-shell constraint which is different from the ordinary constraint $0 = (p - qA)^2 + m^2 = G^{MN}(p_M - qA_M)(p_N - qA_N) + m^2$. This is due to the non-trivial dilaton field that is present for the D0-brane solution. So, here we rederive the mass-shell condition for the case of D0-branes. The action for the probe D0-brane with the mass m and the charge q moving in the background of the source D0-brane has the following form:

$$S = \int d\tau\, L = \int d\tau \left(me^{-\phi}\sqrt{-G^{str}_{MN}\dot{x}^M\dot{x}^N} - q\dot{x}^M A_M\right), \tag{18}$$

where $v^M = \dot{x}^M \equiv \frac{dx^M}{d\tau}$ is the 10-velocity of the probe D0-brane. Note, since we choose the static gauge for the action, the worldline time τ and the target-space time t are set to equal. Once again, G^{str}_{MN} and A_M are the fields produced by the source D0-brane. Note, in this action the metric G^{str}_{MN} is in the string frame. The (generalized) momentum conjugate to $x^M(\tau)$ is

$$P_M = -\frac{\delta L}{\delta\dot{x}^M} = \frac{me^{-\phi}\dot{x}_M}{\sqrt{-G^{str}_{MN}\dot{x}^M\dot{x}^N}} + qA_M. \tag{19}$$

As usual, $p_M = m\dot{x}_M/\sqrt{-G^{str}_{MN}\dot{x}^M\dot{x}^N}$ is the ordinary 10-momentum of the D0-brane. From this, one obtains the following mass-shell constraint for the probe D0-brane in the string-frame background of the probe D0-brane:

$$G^{str\,MN}(P_M - qA_M)(P_N - qA_N) + m^2 e^{-2\phi} = 0. \tag{20}$$

To obtain the expression for the Hamiltonian $H = -P_t$ for the probe D0-brane mechanics, we solve this mass-shell constraint (20). We consider the following general spherically symmetric $D = 10$ metric Ansatz:

$$\begin{aligned} G_{MN}dx^M dx^N &= -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega_8^2 \\ &= -A(r)dt^2 + B(r)dr^2 + C(r)[d\theta^2 + \cos^2\theta d\psi_1^2 + \cos^2\theta\cos^2\psi_1 d\psi_2^2 \\ &\quad + \cos^2\theta\cos^2\psi_1\cos^2\psi_2 d\psi_3^2 + \mu_i^2 d\phi_i^2], \end{aligned} \tag{21}$$

where

$$\mu_1 = \sin\theta, \quad \mu_2 = \cos\theta\sin\psi_1, \quad \mu_3 = \cos\theta\cos\psi_1\sin\psi_2, \quad \mu_4 = \cos\theta\cos\psi_1\cos\psi_2\sin\psi_3. \tag{22}$$

Then, the expression for the Hamiltonian H takes the following form:

$$H = \frac{P_r^2}{2f} + \frac{g}{2f}, \tag{23}$$

where

$$\begin{aligned} f &\equiv \frac{1}{2}A^{-\frac{1}{2}}Be^{-\phi}\left[\sqrt{m^2 + e^{2\phi}(P_r^2 + BC^{-1}\vec{L}^2)/B} + qA^{-\frac{1}{2}}A_t e^{\phi}\right], \\ g &\equiv Be^{-2\phi}\left[(m^2 - q^2A^{-1}A_t^2 e^{2\phi}) + C^{-1}e^{2\phi}\vec{L}^2\right]. \end{aligned} \tag{24}$$

Here, A_t is the time component of the 1-form field A_M of the near-horizon source D0-brane supergravity solution (6) and $\vec{L}^2$ is the angular momentum operator of the probe D0-brane given by

$$\vec{L}^2 = P_\theta^2 + \frac{P_{\psi_1}^2}{\cos^2\theta} + \frac{P_{\psi_2}^2}{\cos^2\theta\cos^2\psi_1} + \frac{P_{\psi_3}^2}{\cos^2\theta\cos^2\psi_1\cos^2\psi_2} + \sum_{i=1}^{4}\frac{P_{\phi_i}^2}{\mu_i^2}. \tag{25}$$

First, we consider the probe D0-brane moving in the string-frame near horizon background of the source D0-brane. In the original work [23] of the superconformal mechanics of a test charged particle in the near horizon geometry of the Reissner-Nordström black hole, it was necessary to redefine the radial coordinate so that the metric components A and B in Eq. (21) satisfy the relation $A = B^2$ for the purpose of setting the factor $A^{-\frac{1}{2}}B$ in Eq. (23) equal to 1. However, in the D0-brane case, as we will see, it is more convenient to work with the original form (6) of near horizon metric, since the expression for the Hamiltonian becomes simpler in the original radial coordinate.

By substituting the string-frame near horizon metric into the general formulae (23) and (24), one obtains the following Hamiltonian for the probe D0-brane in the string-frame near horizon background of the source D0-brane bound state:

$$H = \frac{P_r^2}{2f} + \frac{g}{2f}, \tag{26}$$

where f and g are given by

$$f = \frac{1}{2} g_s^{-1} \left[\sqrt{m^2 + g_s^2 \left(\frac{\mu}{r}\right)^7 \left(P_r^2 + \frac{\vec{L}^2}{r^2}\right)} + q \right],$$
$$g = g_s^{-2} \left(\frac{\mu}{r}\right)^{-7} (m^2 - q^2) + \frac{\vec{L}^2}{r^2}, \tag{27}$$

or in terms of the SYM theory variables

$$H = \frac{P_U^2}{2f} + \frac{g}{2f}, \tag{28}$$

where f and g are given by

$$f = \frac{1}{2} \alpha'^{\frac{1}{2}} g_{YM}^{-2} \left[\sqrt{m^2 + \frac{\alpha'^{-1} g_{YM}^6 N}{U^7} \left(P_U^2 + \frac{\vec{L}^2}{U^2}\right)} + q \right],$$
$$g = \left(\frac{\alpha'^{-1} g_{YM}^6 N}{U^7}\right)^{-1} (m^2 - q^2) + \frac{\vec{L}^2}{U^2}. \tag{29}$$

It is interesting that the term $A^{-1} A_t^2 e^{2\phi}$ in Eq. (24) becomes 1 for the near horizon D0-brane solution (6). So, the expressions for f and g becomes greatly simplified. And, in particular, in the extreme limit $(m - q \to 0)$ of the probe D0-brane, the first term in g drops out. This also generally holds for any dilatonic 0-brane supergravity solutions.

Just as in the case of the test charged particle in the Reissner-Nordsröm black hole background, the mechanics of the probe D0-brane has the $SL(2, \mathbf{R})$ symmetry with the following generators:

$$H = \frac{P_r^2}{2f} + \frac{g}{2f}, \qquad K = -\frac{1}{2} f r^2, \qquad D = \frac{1}{2} r P_r, \tag{30}$$

where the Hamiltonian H generates the time translation, K generates the special conformal transformation and D generates the scale transformation or the dilatation. These generators satisfy the following $SL(2, \mathbf{R})$ algebra:

$$[D, H] = H, \qquad [D, H] = -K, \qquad [H, K] = 2D. \tag{31}$$

This is the D0-brane generalization of conformal quantum mechanics studied in Refs. [24,26,23].

In fact, the near horizon solution (6), when uplifted as a solution of the 11-dimensional gravity (i.e. the 11-dimensional plane wave solution), is invariant under the $SU(1,1) \cong$

$SL(2,\mathbf{R})$ isometry [3] generated by the scale transformation δ_D, the special coordinate transformation δ_K and the time translation δ_H [8]. Under the time translation, $\delta_H t = 1$, $\delta_H U = 0$ and $\delta_H g_s = 0$. Under the special coordinate transformation, $\delta_K t = -(t^2 + k\frac{g_{YM}^2}{U^5})$, $\delta_K U = 2tU$ and $\delta_K g_s = 6tg_s$. Finally, under the dilatation, $\delta_D t = -t$, $\delta_D U = U$ and $\delta_D g_s = 3g_s$. And A_t transforms as a conformal field of dimension 1. These infinitesimal transformations satisfy the following $SL(2,\mathbf{R})$ algebra just like the symmetry generators (30) of the probe D0-brane mechanics:

$$[\delta_D, \delta_H] = \delta_H, \qquad [\delta_D, \delta_K] = -\delta_K, \qquad [\delta_H, \delta_K] = 2\delta_D. \tag{32}$$

Note, the string coupling g_s changes under the dilatation and the special coordinate transformation, and especially g_s becomes time dependent after the special coordinate transformation is applied. Thereby, this $SL(2,\mathbf{R})$ isometry of the near horizon geometry does not extend to a conformal symmetry of the complete supergravity solution. The corresponding boundary theory, i.e. $0+1$ dimensional SYM matrix quantum mechanics, has the same $SL(2,\mathbf{R})$ symmetry, but the string coupling g_s also transforms under this symmetry, unlike the case of D3-brane. However, as noted in Ref. [8], since the dilaton coupling g_s is related to the matrix model coupling constant, one can still think of 'generalized' $SL(2,\mathbf{R})$ conformal symmetry in which the string coupling is now regarded as a part of background fields that transform under the symmetry. The 'generalized' conformal symmetry therefore transforms a matrix model at one value of the coupling constant to another. Note, as pointed out in the previous section, D0-brane in the dual frame has description in terms of domain-wall solution after the compactification on S^8. So, this is also related to the fact that in the domain-wall/QFT correspondence the choice of horosphere (the hypersurface of constant u) for the Minkowski vacuum corresponds to a choice of coupling constant of a non-conformal QFT [13]. In the non-conformal case, the interpolation between the AdS Killing horizon (in the dual frame) and its boundary therefore corresponds to an interpolation between strong and weak coupling, or vice versa.

The extreme limit of the probe D0-brane ($(m-q) \to 0$) can be interpreted as the M theory in the LCF, since in this case both the source and the probe D0-branes are in the BPS limit. In this case, the functions f and g are simplified to

$$f = \frac{1}{2} g_s^{-1} \left[\sqrt{m^2 + g_s^2 \left(\frac{\mu}{r}\right)^7 \left(P_r^2 + \frac{\vec{L}^2}{r^2}\right)} + m \right], \qquad g = \frac{\vec{L}^2}{r^2}, \tag{33}$$

or in terms of the SYM theory variables

$$f = \frac{1}{2} \alpha'^{\frac{1}{2}} g_{YM}^{-2} \left[\sqrt{m^2 + \frac{\alpha'^{-1} g_{YM}^6 N}{U^7} \left(P_U^2 + \frac{\vec{L}^2}{U^2}\right)} + m \right], \qquad g = \frac{\vec{L}^2}{U^2}. \tag{34}$$

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[3] At the 10-dimensional level, such $SL(2,\mathbf{R})$ transformations act on the near-horizon supergravity D0-brane solution in such a way that the constant Q in the harmonic function H transforms as if it is a "field" on the worldvolume and then is set to a constant after the transformations [9].

Unlike the case of a charged test particle in the near horizon Reissner-Nordsröm black hole background studied in Ref. [23], we do not let the mass of the source D0-brane bound state go to infinity, since the number N of the D0-branes in the M theory on the LCF is kept finite. The fact that we are considering the near horizon geometry of the D0-brane supergravity solution means that we are in the LCF of M theory, since the near horizon geometry of the D0-brane supergravity solution is also the null reduction of the M wave supergravity solution [21], which is interpreted as M theory in the LCF. Taking N to infinity, i.e. infinitely massive source D0-brane, corresponds to M theory in the IMF. Note, the IMF is defined as the limit in which N and R_{11} go to infinity such that the momentum $P_{11} = N/R_{11}$ also goes to infinity. Since $P_{11} \sim g_{YM}^{-7/2} N^{1/4} U^{21/4}$ and $R_{11} \sim g_{YM}^{7/2} N^{3/4} U^{-21/4}$, one can take both P_{11} and R_{11} to infinity while taking $N \to \infty$, if U is in the range of $g_{YM}^{2/3} N^{-1/21} \ll U \ll g_{YM}^{2/3} N^{1/7}$. However, this range of U is beyond the range of validity (8) of the near horizon solution (6). In other words, one cannot go to the IMF of M theory while keeping the parameters within the validity of the near horizon solution (6). Anyway, the IMF means decompactification ($R_{11} \to \infty$) to the eleven dimensions. So, we should rather consider the M wave solution in the case $N \to \infty$.

By expanding this Hamiltonian for an extreme $D0$-brane probe, one obtains a Hamiltonian of the form which is the sum of the non-relativistic kinetic term for the probe and the velocity dependent potential given by the sum of terms of the form $\sim \frac{v^{2n+2}}{r^{7n}}$ ($n \in \mathbf{Z}^+$). This should reproduce the (M)atrix theory calculation of graviton-graviton scattering and its supergravity calculation reproduction of Ref. [21]. This is because the above action (18) is just the reduction of the action for the probe in the background of $D = 11$ plane wave (describing the motion of the probe graviton in the background of the moving heavy source graviton) on a light-like circle to 10 dimensions, i.e. M theory in the LCF. Note also that since M theory in the LCF has "Galilean" invariance (or is described by non-relativistic quantum mechanics) in the transverse space, the above Hamiltonian for an extreme probe D0-brane has the non-relativistic structure. These above arguments, together with the fact that the generalized conformal mechanics of probe D0-brane and the boundary $SU(N)$ SYM theory (i.e. the (M)atrix model of M theory in the LCF) satisfy the same $SL(2,\mathbf{R})$ symmetry, implies the equivalence between the (M)atrix model with finite N and the generalized conformal mechanics of D0-brane.

Next, we consider the probe D0-brane moving in the background of the dual-frame metric of the source D0-brane bound state. Since the dual frame can be considered as a "holographic frame" describing supergravity probes [13], it is worthwhile to consider the case of the dual frame. In this frame, the near horizon metric takes the $\mathrm{AdS}_2 \times S^8$ form. So, one should expect that the 'generalized' conformal mechanics with the $SL(2,\mathbf{R})$ symmetry can also be realized in the dual frame. The action for the probe D0-brane in the dual frame background of the source D0-brane is given by (18) with the string-frame metric G^{str}_{MN} replaced by the dual-frame metric G^{dual}_{MN} through the relation $G^{str}_{MN} = e^{\frac{2}{7}\phi} G^{dual}_{MN}$. So, for the probe D0-brane in the dual frame, the dilaton factor $e^{-\phi}$ in Eqs. (18) and (19) [the dilaton factor $e^{-2\phi}$ in Eqs. (20) and (24)] is replaced by $e^{-\frac{6}{7}\phi}$ [$e^{-\frac{12}{7}\phi}$].

Substituting the dual-frame near horizon solution (10) into this general expression for the Hamiltonian corresponding to the dual frame, one finds that the Hamiltonian in the dual frame has the same form as the string-frame Hamiltonian (26) with the same f and g (27).

In fact, in general the Hamiltonian describing probe D0-brane is independent of the near horizon spacetime frame of the source D0-brane. So, the dynamics of the probe D0-brane in the dual frame also has the $SL(2,\mathbf{R})$ symmetry with the same symmetry generators (30) and algebra (31) as the string-frame case.

As pointed out in the previous section, the source D0-brane solution also has the (Hodge) dual description in terms of the 8-form field strength, whose magnetic charge now is carried by the source D0-brane. In this case, by compactifying the D0-brane solution on S^8 one obtains a domain wall solution in $1+1$ dimensions [33,13]. This domain wall solution solves the equations of motion of a $D=2$ $SO(9)$ gauged maximal supergravity theory, which is an S^8 compactification of the type IIA supergravity. This $SO(9)$, which is the largest subgroup of the $SO(16)$ and the isometry group of S^8 upon which the D0-brane is compactified, is also the R-symmetry group of the corresponding boundary $D=1$ QFT. In general, the R-symmetry of the supersymmetric QFT on the domain wall worldvolume matches the gauge group, which is the isometry group of the compactification manifold, of the equivalent gauged supergravity [13].

Putting together all the above facts, namely (*i*) generalized conformal mechanics of the probe D0-brane in the string-frame near horizon background of the source D0-brane is related to the M theory in the LCF, (*ii*) the generalized conformal mechanics of the probe D0-branes in the string and the dual frames of the near horizon source D0-branes are described by the same Hamiltonian, (*iii*) upon the dimensional reduction on S^8 the bulk theory of the source D0-brane in the dual frame in the Hodge-dual description (in terms of the 8-form field strength) is the $D=2$ $SO(9)$ gauged maximal supergravity theory, one arrives at the speculation that the M theory in the LCF is related to the $D=2$ $SO(9)$ gauged maximal supergravity theory (i.e. a $D=2$ Kaluza-Klein supergravity theory with domain wall vacuum).

REFERENCES

[1] G. 't Hooft, "Dimensional reduction in quantum gravity", gr-qc/9310026.

[2] L. Susskind, "Strings, black holes and Lorentz contraction", Phys. Rev. **D49** (1994) 6606, hep-th/9308139.

[3] L. Susskind, "The world as a hologram", J. Math. Phys. **36** (1995) 6377, hep-th/9409089.

[4] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity", Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.

[5] M.J. Duff, "Supermembranes: The first fifteen weeks", Class. Quant. Grav. **5** (1988) 189.

[6] G.W. Gibbons and P.K. Townsend, "Vacuum interpolation in supergravity via super p-branes", Phys. Rev. Lett. **71** (1993) 3754, hep-th/9307049.

[7] M.J. Duff, G.W. Gibbons and P.K. Townsend, "Macroscopic superstrings as interpolating solitons", Phys. Lett. **B332** (1994) 321, hep-th/9405124.

[8] A. Jevicki and T. Yoneya, "Space-time uncertainty principle and conformal symmetry in D-particle dynamics", Nucl. Phys. **B535** (1998) 335, hep-th/9805069.

[9] A. Jevicki, Y. Kazama and T. Yoneya, "Generalized conformal symmetry in D-brane matrix models", Phys. Rev. D59 (1999) 066001, hep-th/9810146.

[10] M.J. Duff and J.X. Lu, "Black and super p-branes in diverse dimensions", Nucl. Phys. **B416** (1994) 301, hep-th/9306052.

[11] H.J. Boonstra, B. Peeters and K. Skenderis, "Duality and asymptotic geometries", Phys. Lett. **B411** (1997) 59, hep-th/9706192.

[12] H. J. Boonstra, B. Peeters and K. Skenderis, "Brane intersections, anti-de Sitter spacetimes and dual superconformal theories", Nucl. Phys. **B533** (1998) 127, hep-th/9803231.

[13] H.J. Boonstra, K. Skenderis and P.K. Townsend, "The domain-wall/QFT correspondence", JHEP **9901** (1999) 003, hep-th/9807137.

[14] M. Claudson and M.B. Halpern, "Supersymmetric ground state wave functions", Nucl. Phys. **B250** (1985) 689.

[15] M.B. Halpern and C. Schwartz, "Asymptotic search for ground states of $SU(2)$ matrix theory", Int. J. Mod. Phys. **A13** (1998) 4367, hep-th/9712133.

[16] T. Banks, W. Fischler, S.H. Shenker, and L. Susskind, "M theory as a matrix model: a conjecture", Phys. Rev. **D55** (1997) 5112, hep-th/9610043.

[17] S. Weinberg, "Dynamics at infinite momentum", Phys. Rev. **150** (1966) 1313.

[18] B. de Wit, J. Hoppe and H. Nicolai, "On the quantum mechanics of supermembranes", Nucl. Phys. **B305** (1988) 545.

[19] L. Susskind, "Another conjecture about M(atrix) theory", hep-th/9704080.

[20] N. Seiberg, "Why is the matrix model correct?", Phys. Rev. Lett. **79** (1997) 3577, hep-th/9710009.

[21] K. Becker, M. Becker, J. Polchinski, and A. Tseytlin, "Higher order graviton scattering in M(atrix) theory", Phys.Rev. **D56** (1997) 3174, hep-th/9706072.

[22] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, "Supergravity and the large N limit of theories with sixteen supercharges", Phys. Rev. **D58** (1998) 046004, hep-th/9802042.

[23] P. Claus, M. Derix, R. Kallosh, J. Kumar, P.K. Townsend, and A. Van Proeyen, "Black holes and superconformal mechanics", Phys. Rev. Lett. **81** (1998) 4553, hep-th/9804177.
[24] V. de Alfaro, S. Fubini and G. Furlan, "Conformal invariance in quantum mechanics", Nuovo Cim. **34A** (1976) 569.
[25] V.P. Akulov and A.I. Pashnev, "Quantum superconformal model in (1,2) space", Theor. Math. Phys. **56** (1983) 862.
[26] S. Fubini and E. Rabinovici, "Superconformal quantum mechanics", Nucl. Phys. **B245** (1984) 17.
[27] J. M. Maldacena, "Branes probing black holes", Nucl. Phys. Proc. Suppl. **68** (1998) 17, hep-th/9709099.
[28] L. Susskind and E. Witten, "The holographic bound in Anti-de Sitter space", hep-th/9805114.
[29] A. W. Peet and J. Polchinski, "UV/IR relations in AdS dynamics", Phys. Rev. **D59** (1999) 065006, hep-th/9809022.
[30] P. G. O. Freund and M. A. Rubin, "Dynamics of dimensional reduction", Phys. Lett. **97B** (1980) 233.
[31] H. Lu, C.N. Pope, E. Sezgin, and K.S. Stelle, "Stainless super p-branes", Nucl.Phys. **B456** (1995) 669, hep-th/9508042.
[32] H. Lu, C.N. Pope, E. Sezgin, and K.S. Stelle, "Dilatonic p-brane solitons", Phys.Lett. **B371** (1996) 46, hep-th/9511203.
[33] H. Lu, C.N. Pope and P.K. Townsend, "Domain walls from Anti-de Sitter spacetime", Phys. Lett. **B391** (1997) 39, hep-th/9607164.