Hybrid Inflation in Supergravity with $(SU(1,1)/U(1))^m$ Kähler Manifolds

C. Panagiotakopoulos

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

and

Physics Division, School of Technology, Aristotle University of

Thessaloniki, 54006 Thessaloniki, Greece

Abstract

In the presence of fields without superpotential but with large vevs through D-terms the mass-squared of the inflaton in the context of supergravity hybrid inflation receives positive contributions which could cancel the possibly negative Kähler potential ones. The mechanism is demonstrated using Kähler potentials associated with products of $SU(1,1)/U(1)$ Kähler manifolds. In a particularly simple model of this type all supergravity corrections to the F-term potential turn out to be proportional to the inflaton mass allowing even for an essentially completely flat inflationary potential. The model also allows for a detectable gravitational wave contribution to the microwave background anisotropy. Its initial conditions are quite natural largely due to a built in mechanism for a first stage of "chaotic" D-term inflation.

The hybrid inflationary scenario [1] has many advantages compared to most other inflationary models [2]. It does not involve tiny inflaton self-couplings and succeeds in reconnecting inflation with phase transitions in grand unified theories (GUTs). In its simplest realization it involves a gauge singlet inflaton and a possibly gauge non-singlet non-inflaton field. During inflation the non-inflaton field finds itself trapped in a false vacuum state and the universe expands quasi-exponentially dominated by the almost constant false vacuum energy density. Inflation ends with (or just before) a very rapid phase transition when the non-inflaton field rolls to its true vacuum state ("waterfall").

The simplest supersymmetric (SUSY) particle physics model implementing the above scenario in the context of a "unifying" gauge group G (of rank ≥ 5) which breaks spontaneously directly to the Standard Model (SM) gauge group $G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ at a scale $M_X \sim 10^{16}\,GeV$ is described by a superpotential which includes the terms [3]

$$W = S(-\mu^2 + \lambda \Phi \bar{\Phi}). \tag{1}$$

Here Φ, $\bar{\Phi}$ is a conjugate pair of left-handed SM singlet superfields which belong to N_d-dimensional representations of G and reduce its rank by their vacuum expectation values (vevs), S is a gauge singlet left-handed superfield, μ is a superheavy mass scale related to M_X and λ a real and positive coupling constant. The superpotential terms in Eq. (1) are the dominant couplings involving the superfields S, Φ, $\bar{\Phi}$ which are consistent with a continuous R-symmetry under which $W \to e^{i\gamma}W$, $S \to e^{i\gamma}S$, $\Phi \to \Phi$ and $\bar{\Phi} \to \bar{\Phi}$. The potential obtained from W has, in the supersymmetric limit, a SUSY vacuum at

$$< S >= 0, < \Phi >< \bar{\Phi} >= \frac{\mu^2}{\lambda} = \left(\frac{M_X}{g}\right)^2, \ |< \Phi >|=|< \bar{\Phi} >|, \tag{2}$$

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields, M_X is the mass acquired by the gauge bosons and g the G gauge coupling constant. By an appropriate R-trasformation we can bring the complex field S on the real axis, i.e. $S \equiv \frac{1}{\sqrt{2}}\sigma$, where σ is a real scalar field. For any fixed value of $\sigma^2 > \sigma_c^2$, where $\sigma_c = \sqrt{2}\mu/\sqrt{\lambda}$, the potential of the globally supersymmetric model has a minimum at Φ

$= \bar{\Phi} = 0$ with a σ-independent value $V_{gl} = \mu^4$ and the universe expands quasi-exponentially. When σ^2 falls below σ_c^2 the false vacuum state at $\Phi = \bar{\Phi} = 0$ becomes unstable and Φ, $\bar{\Phi}$ roll rapidly to their true vacuum.

If global supersymmetry is promoted to local one expects that the potential will become very steep and an effective mass for the inflaton, large compared to the inflationary Hubble parameter H, will be generated forbidding inflation even at small inflaton field values. In our case we can investigate the consequences that $N = 1$ supergravity has on our simple hybrid inflationary model by restricting ourselves to the inflationary trajectory ($\Phi = \bar{\Phi} = 0$). Then, we are allowed to use the simple superpotential

$$W = -\mu^2 S \tag{3}$$

involving just the gauge singlet superfield S. [Throughout our subsequent discussion we make use of units in which the reduced Planck scale $m_{Pl} \equiv M_{Pl}/\sqrt{8\pi} \simeq 2.4355 \times 10^{18} \, GeV$ is equal to 1 ($M_{Pl} \simeq 1.221 \times 10^{19} \, GeV$ is the Planck mass).] If the minimal Kähler potential $K = |S|^2$ leading to canonical kinetic terms for σ is employed the "canonical" potential V_{can} acquires a slope and becomes [4] [5] [6]

$$V_{can} = \mu^4 \left(1 - |S|^2 + |S|^4\right) e^{|S|^2} = \mu^4 \sum_{k=0}^{\infty} \frac{(k-1)^2}{k!} |S|^{2k}. \tag{4}$$

V_{can} of Eq. (4) does not allow inflation unless $|S|^2 \ll 1$. From its expansion as a power series in $|S|^2$ we see that, due to an "accidental" cancellation, the linear term in $|S|^2$ is missing and no mass-squared term is generated for σ. Small deviations from the minimal form of the Kähler potential respecting the R-symmetry lead to a Kähler potential [7]

$$K = |S|^2 - \frac{\alpha}{4} |S|^4 + \ldots. \tag{5}$$

which, in turn, gives rise to a potential admitting an expansion

$$V = \mu^4 \left(1 + \alpha |S|^2 + \ldots\right) \qquad \left(|S|^2 \ll 1\right) \tag{6}$$

in which a linear term in $|S|^2$ proportional to the small parameter α is now generated. All higher powers of $|S|^2$ are still present in the series with coefficients only slightly different from

the corresponding ones of Eq. (4). Almost all studies of hybrid inflation in the context of the simplest model of Eq. (1) rely, explicitly or implicitly, on this "miraculous" cancellation of the mass of σ in $N = 1$ canonical supergravity.

Our discussion so far seems to indicate that the only potential source of mass for σ is the next to leading term in the expansion of the Kähler potential in powers of $|S|^2$ which must have a small and negative coefficient. This conclusion is certainly correct if all other fields are assumed to play absolutely no role during inflation. In the present paper we show that fields which do not contribute to the superpotential and are singlets under the "unifying" group G, and therefore could be regarded as "playing no role", do contribute to the mass-squared of σ if they acquire large vevs. Then, in the presence of such fields the "miraculous" properties of the minimal Kähler potential are lost [1] but other "miraculous" cancellations might occur involving other, possibly better motivated, types of Kähler potentials.

Let us consider a G-singlet chiral superfield Z which does not contribute to the superpotential at all because, for instance, it has non-zero charge, let us say -1, under an "anomalous" $U(1)$ gauge symmetry and, as we assume, all other superfields which have a $U(1)$ charge can be safely ignored. Also let us assume that $\frac{\partial^2 K}{\partial S \partial Z^*} = \frac{\partial^2 K}{\partial Z \partial S^*} = 0$ such that $K = K_1(|S|^2) + K_2(|Z|^2)$. Then, with the parameters μ and λ in W renamed as μ' and λ', the scalar potential becomes

$$V = \mu'^4 \left\{ \left| 1 + S \frac{\partial K}{\partial S} \right|^2 \left(\frac{\partial^2 K}{\partial S \partial S^*} \right)^{-1} + \left(\left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} - 3 \right) |S|^2 \right\} e^K + \frac{1}{2} g_1^2 \left(\frac{\partial K}{\partial Z} Z - \xi \right)^2,$$
(7)

where the first(second) term is the F(D)-term, $\xi > 0$ is a Fayet-Iliopoulos term and g_1 the gauge coupling of the "anomalous" $U(1)$ gauge symmetry. Minimization of such a potential for fixed $|S|^2$ not much larger than unity, assuming $|S|^2$ takes values away from any points

---

[1]The danger that the "miraculous" cancellation of the inflaton mass might be destroyed by fields acquiring large vevs has also been pointed out in [8].

where the potential is singular and $\mu'^2 \ll \xi$, typically gives rise to a $< \mid Z \mid^2 > \equiv v^2 \sim \xi$ with $\left(\left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \sim v^2 \sim \xi$ and therefore a contribution to the mass-squared of σ

$$\delta m_\sigma^2 = \left(\left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \mu'^4 e^{K_2(v^2)} \tag{8}$$

of the order of ξ in units of the false vacuum energy density. For the sake of convenience we absorb the factor $e^{K_2(v^2)}$ appearing in the F-term potential in the reintroduced parameters $\mu = \mu' e^{K_2(v^2)/4}$ and $\lambda = \lambda' e^{K_2(v^2)/2}$ obeying the relation $\frac{\mu}{\sqrt{\lambda}} = \frac{\mu'}{\sqrt{\lambda'}}$.

It would be very interesting if the contribution of Z to the mass-squared of σ in units of the false vacuum energy density were independent of the value of Z. This is exactly the case if Z enters the Kähler potential through a function K_2 of the "no-scale" type [9] [10]

$$K_2 = -n \ln |f(Z) + f^*(Z^*)|, \tag{9}$$

where n is an integer, $f(Z)$ an analytic function of Z and $f^*(Z^*)$ its complex conjugate which is a function of Z^*. The corresponding Kähler manifold is an Einstein-Kähler manifold of constant scalar curvature $2/n$ with a non-compact $SU(1,1)$ global symmetry and a local $U(1)$ symmetry, namely the coset space $SU(1,1)/U(1)$. Different choices of $f(Z)$ correspond to Z field redefinitions. Since, however, we want Z to transform non-trivially under a continuous $U(1)$ symmetry the function $f(Z)$ must be a linear function of $\ln Z$. Thus, without loss of generality, we are led to the Kähler potentials

$$K_2(|Z|^2) = -n \ln \left(-\ln |Z|^2 \right) \qquad \left(0 < |Z|^2 < 1 \right). \tag{10}$$

Such a choice makes the contribution of Z to the mass-squared of σ

$$\delta m_\sigma^2 = n\mu^4, \tag{11}$$

an integer multiple of the false vacuum energy density.

In order for the above discussion to be of any use we should, of course, find Kähler potentials $K_1(|S|^2)$ whose expansion in powers of $|S|^2$ has a positive next to leading term. A class of such Kähler potentials is given by

$$K_1(|S|^2) = -N \ln \left(1 - \frac{|S|^2}{N} \right) \qquad \left(|S|^2 < N \right), \tag{12}$$

where N is an integer. The corresponding Kähler manifold is again the coset space $SU(1,1)/U(1)$ with constant scalar curvature $2/N$. [Actually the kinetic terms of Z and S with Kähler potentials given by Eqs. (9) and (12), respectively (with $n = N$) transform into each other under the redefinition $f(Z) = \frac{\sqrt{N}+S}{\sqrt{N}-S}$.] Expanding K_1 of Eq. (12) in powers of $|S|^2$

$$K_1(|S|^2) = |S|^2 + \frac{1}{2N} |S|^4 + \ldots \tag{13}$$

and comparing with Eqs. (5) and (6) we see that the mass-squared of σ is

$$m_\sigma^2 = \left(-\frac{2}{N} + \left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \mu^4 \equiv \beta \mu^4. \tag{14}$$

For all N we can make m_σ^2 positive (or, by fine tuning, zero) through appropriately chosen vevs (ξ parameters) of Z-type fields. The most interesting cases, however, occur for $N = 1$ or $N = 2$ because $2/N$ is an integer and the option of naturally making m_σ^2 vanish by employing Z-type fields with Kähler potentials given by Eq. (10) (with $n = 2$ or $n = 1$, respectively) becomes now available. A small positive m_σ^2 could be subsequently generated through additional Z-type fields which acquire vevs of the order of appropriately chosen ξ parameters.

It is important to realize that $\beta \equiv m_\sigma^2/\mu^4$ is approximately constant only for $\left(1 - \frac{|S|^2}{N} \right)^N \gg \mu^4/g_1^2\xi$. As $\left(1 - \frac{|S|^2}{N} \right)^N$ approaches zero the Z fields get displaced from their vevs at small $|S|^2$ and β will eventually vanish. Further decrease of $\left(1 - \frac{|S|^2}{N} \right)^N$ will result in a negative β. This can be avoided in the cases $N = 1, 2$ provided the cancellation mechanism involving the Kähler potentials of Eq. (10) is used.

The choices $N = 1$ or $N = 2$ deserve further study because, as we now show, in these cases all supergravity corrections to the F-term potential are proportional to the mass-squared m_σ^2 of the field σ or, equivalently, to the parameter β. This offers the possibility of suppressing or even eliminating all supergravity corrections to the inflationary trajectory

by suppressing or making vanish the parameter β. Indeed, a straightforward substitution of the Kähler potential $K_1(|S|^2)$ of Eq. (12) in Eq. (7) gives

$$V \simeq \mu^4 \left\{ \left(1 - |S|^2 + \frac{(N-1)^2}{N^2} |S|^4 \right) + \beta |S|^2 \right\} \left(1 - \frac{|S|^2}{N} \right)^{-N} \tag{15}$$

(up to terms $\sim \mu^8$) which, for $N = 1, 2$ only, becomes

$$V \simeq \mu^4 \left\{ 1 + \beta |S|^2 \left(1 - \frac{|S|^2}{N} \right)^{-N} \right\} \qquad (N = 1, 2) \tag{16}$$

independently of the mechanism chosen to make $\beta \geq 0$. In particular, the combinations of $K_1(|S|^2)$ of Eq. (12) with $N = 1$ and $K_2(|Z|^2)$ of Eq. (10) with $n = 2$ (or two Z-type fields each entering a K_2 with $n = 1$) or $K_1(|S|^2)$ with $N = 2$ and $K_2(|Z|^2)$ with $n = 1$ give $\beta = 0$ and consequently a completely flat potential.

Because of the possibility of suppressing the supergravity corrections through a single parameter β we expect that the models with $N = 1, 2$ will lead to some novel features compared to the quasi-canonical case [7], mostly due to the anticipated extension of inflation to σ values close or even slightly larger than unity. In particular, one might hope for the possibility of predicting a detectable gravitational wave signal in the cosmic microwave background anisotropy which requires an inflaton field variation of order unity [11]. In order to decide which of the two models $(N = 1, 2)$ has a flatter potential and consequently a better chance for novel features we express their potentials in terms of the canonically normalized inflaton field σ_{infl}. The relation between σ and σ_{infl} is

$$\frac{\sigma}{\sqrt{2N}} = \tanh \frac{\sigma_{infl}}{\sqrt{2N}} . \tag{17}$$

Then, the potential of the $N = 1$ model becomes

$$V \simeq \mu^4 \left[1 + \frac{\beta}{2} \left(\cosh \left(\sqrt{2} \sigma_{infl} \right) - 1 \right) \right] = \mu^4 \left[1 + \frac{\beta}{2} \sum_{k=1}^{\infty} \frac{2^k}{(2k)!} \sigma_{infl}^{2k} \right] \qquad (N = 1), \tag{18}$$

whereas the potential of the $N = 2$ model becomes

$$V \simeq \mu^4 \left[1 + \frac{\beta}{4} \left(\cosh \left(2\sigma_{infl} \right) - 1 \right) \right] = \mu^4 \left[1 + \frac{\beta}{2} \sum_{k=1}^{\infty} \frac{2^{2k-1}}{(2k)!} \sigma_{infl}^{2k} \right] \qquad (N = 2). \tag{19}$$

Comparing the coefficients of the corresponding terms of the series in Eqs. (18) and (19) we clearly conclude that the model with $N = 1$ has flatter potential and is more promising.

In the following we concentrate on the model with $N = 1$. The derivative of the potential with respect to σ_{infl} expressed as a function of σ is

$$V' = \frac{\mu^4}{2\sigma} \left[N_d \left(\frac{\lambda}{2\pi} \right)^2 \left(1 - \frac{\sigma^2}{2} \right) + 2\beta\sigma^2 \left(1 - \frac{\sigma^2}{2} \right)^{-1} \right], \tag{20}$$

where the effect of the radiative corrections [3] due to the conjugate pair of non-inflaton fields Φ, $\bar{\Phi}$ belonging to N_d-dimensional representations of the gauge group G are included. Their vev is fixed at the Minimal Supersymmetric Standard Model (MSSM) value $\mu/\sqrt{\lambda} = M_X/g \simeq 0.011731$ ($M_X \simeq 2 \times 10^{16}$ GeV, $g \simeq 0.7$) and correspondingly the critical value of σ is $\sigma_c \simeq 0.01659$.

For the quadrupole anisotropy $\frac{\Delta T}{T}$ we employ the standard formula [12]

$$\left(\frac{\Delta T}{T} \right)^2 \simeq \frac{1}{720\pi^2} \left[\frac{V^3}{V'^2} + 6.9V \right]_{\sigma_H}, \tag{21}$$

where σ_H is the value of σ when the scale ℓ_H, corresponding to the present horizon, crossed outside the inflationary horizon. The first term in Eq. (21) is the scalar component $\left(\frac{\Delta T}{T} \right)^2_S$ of $\left(\frac{\Delta T}{T} \right)^2$ whereas the second is the tensor one $\left(\frac{\Delta T}{T} \right)^2_T$ which represents the gravitational wave contribution. Their ratio r is

$$r \equiv \left(\frac{\Delta T}{T} \right)^2_T \bigg/ \left(\frac{\Delta T}{T} \right)^2_S \simeq 6.9 \left(\frac{V'}{V} \right)^2_{\sigma_H} \simeq 27.6 \left(\frac{\beta\sigma_H}{2 - (1-\beta)\sigma_H^2} \right)^2, \tag{22}$$

assuming, as it turns out to be the case, that the effect of radiative corrections is negligible at the point where the spectrum of temperature fluctuations is normalized.

The number of e-foldings $\Delta N(\sigma_{in}, \sigma_f)$ for the time period that σ varies between the values σ_{in} and σ_f ($\sigma_{in} > \sigma_f$) is given, in the slow roll approximation, by

$$\Delta N(\sigma_{in}, \sigma_f) = -\int_{\sigma_{in}}^{\sigma_f} \frac{V}{V'} \left(1 - \frac{\sigma^2}{2} \right)^{-1} d\sigma. \tag{23}$$

The evaluation of the above integral is straightforward but the result is rather lengthy and will not be given.

8

Let ℓ_H be the scale corresponding to our present horizon and ℓ_o another length scale. Also let σ_o be the value that σ had when ℓ_o crossed outside the inflationary horizon. We define the average spectral index $n(\ell_o)$ for scales from ℓ_o to ℓ_H as

$$n(\ell_o) \equiv 1 + 2ln\left[\left(\frac{\delta\rho}{\rho}\right)_{\ell_o} / \left(\frac{\delta\rho}{\rho}\right)_{\ell_H}\right] / ln\left(\frac{\ell_H}{\ell_o}\right) = 1 + 2ln\left[\left(\frac{V^{3/2}}{V'}\right)_{\sigma_o} / \left(\frac{V^{3/2}}{V'}\right)_{\sigma_H}\right] / \Delta N(\sigma_H, \sigma_o).$$
$$(24)$$

Here $(\delta\rho/\rho)_\ell$ is the amplitude of the energy density fluctuations on the length scale ℓ as this scale crosses inside the postinflationary horizon and $\Delta N(\sigma_H, \sigma_o) = ln(\ell_H/\ell_o)$.

It should be clear that all quantities characterizing inflation depend on just two parameters, namely μ and β. Therefore for each value of μ we can determine σ_H (> 0) and β by normalizing the spectrum according to Eq. (21) and requiring that $N_H \equiv \Delta N(\sigma_H, \sigma_c)$ takes the appropriate value. We choose $\frac{\Delta T}{T} = 6.6 \times 10^{-6}$ and $N_H \simeq 50$.

Table 1 gives the values of β, σ_H, σ_{infl_H}, $n \equiv n(\ell_1)$, $n_{COBE} \equiv n(\ell_2)$ and r, where ℓ_1 (ℓ_2) is the scale that corresponds to 1 Mpc (2000 Mpc) today, for different values of μ assuming that the present horizon size is 12000 Mpc and $N_d = 1$. Comparing with the quasi-canonical case [7] we see clearly an increase in the values of β. The spectrum of density perturbations is again blue but the spectral index, which here also shows a strong scale dependence, is severely lowered. As a result of this drastic decrease of the spectral index the parameter μ now could be, for the first time, very close to the MSSM gauge coupling unification scale M_X. This has as a consequence a dramatic increase of r which reaches values that could be detectable in the foreseeable future. As already mentioned all these effects are due to the suppression of the supergravity corrections which allows inflation to take place at larger values of the inflaton field and makes our model approach the original hybrid model [1].

Table 2 gives, for fixed $\mu = 7.3 \times 10^{-3}$, the values of β, σ_H, σ_{infl_H}, n, n_{COBE} and r as functions of the dimensionality N_d of the representations to which Φ, $\bar{\Phi}$ belong. We see that as N_d increases β and the spectral index decrease whereas σ_H and σ_{infl_H} increase. Remarkably enough r remains essentially unaltered.

As σ_{infl} (> 0) increases the constant term in V of Eq. (18) becomes gradually irrelevant

9

and the potential along the inflationary trajectory can be approximated, for $\sigma^c_{infl} \gtrsim \sigma_{infl} \gg 1$ (with σ^c_{infl} a model-dependent critical value), by the well-known one [2]

$$V = V_0 e^{\sqrt{2}\sigma_{infl}} \qquad (\sigma^c_{infl} \gtrsim \sigma_{infl} \gg 1) \qquad (25)$$

(with $V_0 = \mu^4 \beta/4$). The equation of motion of $\sigma_{infl}(t)$ with such a potential in a σ_{infl}-dominated universe admits a solution with

$$\frac{d}{dt}\sigma_{infl} = -\sqrt{V} = -\mu^2 \frac{\sqrt{\beta}}{2} e^{\frac{\sigma_{infl}}{\sqrt{2}}} \qquad (\sigma^c_{infl} \gtrsim \sigma_{infl} \gg 1), \qquad (26)$$

meaning that eventually σ_{infl} acquires a "terminal" velocity which is a function of σ_{infl} independently of the initial conditions. Then, the scale factor $R(t)$ of the universe expands like $R \sim t$ and the energy density ρ falls like $\rho \sim R^{-2}$ or, equivalently, the Hubble length $H^{-1} \sim R$. Thus, although the expansion is certainly slower than inflation it leaves invariant the volume of any space region in units of H^{-3} thereby leading to the onset of inflation at $\rho \sim \mu^4 \ll 1$. This will happen provided the inflationary trajectory extends almost up to $\rho \sim 1$ and a natural set of initial conditions develops sufficiently early into a set of field values and velocities describing motion on the inflationary trajectory.

To address these issues we need to further specify the model. We choose to introduce, in addition to the superfields S, Φ, $\bar{\Phi}$, two G-singlet superfields Z_1, Z_2 with charges $(-1, 0)$ and $(0, -q)$, respectively (with $q > 0$) under two "anomalous" $U(1)$ gauge symmetries. The Kähler potential is chosen to be

$$K = -\ln\left(1 - |S|^2\right) - 2\ln\left(-\ln|Z_1|^2\right) - \ln\left(1 - |Z_2|^2\right) + |\Phi|^2 + \left|\bar{\Phi}\right|^2 \qquad (27)$$

$(|S|^2, |Z_2|^2 < 1, 0 < |Z_1|^2 < 1)$ with the superpotential being $W = S(-\mu'^2 + \lambda' \Phi\bar{\Phi})$.

To discuss the (extension of the) inflationary trajectory at large $|\sigma_{infl}|$ values we ignore the superfields Φ, $\bar{\Phi}$ and we obtain the potential

---

[2]The possibility that the evolution of the universe before the onset of "slow-roll" inflation could be described by potentials of this type has been considered in [8].

$$V = \mu'^4 \frac{\xi_1^2}{4} e^{2\zeta} \left[1 + \frac{1}{4} \left(\cosh\left(\sqrt{2}\chi\right) - 1 \right) \left(\cosh\left(\sqrt{2}\sigma_{infl}\right) + 1 \right) \right]$$

$$+ \frac{1}{2} g_1^2 \xi_1^2 \left(e^\zeta - 1 \right)^2 + \frac{1}{2} g_2^2 \left[\frac{q}{2} \left(\cosh\left(\sqrt{2}\chi\right) - 1 \right) - \xi_2 \right]^2, \tag{28}$$

where g_1, $g_2 \sim 1$ are the gauge couplings of the "anomalous" $U(1)$' s and $\xi_1 \sim 1$, $\xi_2 \sim 10^{-1} - 10^{-2}$ the corresponding (positive) Fayet-Iliopoulos terms. The canonically normalized real scalar fields ζ and χ are defined through the relations

$$e^{-\zeta} \equiv -\frac{\xi_1}{2} \ln|Z_1|^2, \quad \tanh\frac{\chi}{\sqrt{2}} \equiv \operatorname{Re} Z_2, \tag{29}$$

with the complex scalar fields Z_1, Z_2 brought to the real axis by gauge transformations. For fixed $|\sigma_{infl}| < \sigma_{infl}^c$, where

$$\sigma_{infl}^c = \frac{1}{\sqrt{2}} \operatorname{arccos} h \left[\frac{8 g_2^2 \xi_2 q}{\mu'^4 \xi_1^2} \left(1 + \frac{\mu'^4}{2 g_1^2} \right)^2 - 1 \right], \tag{30}$$

V is minimized at $\zeta = \zeta_{\min} < 0$ with $|\zeta_{\min}| \ll 1$ and $\chi = \chi_{\min}$ with

$$\chi_{\min} \simeq \frac{1}{\sqrt{2}} \operatorname{arccos} h \left[1 + \frac{2\xi_2}{q} \left(1 - \frac{\cosh\left(\sqrt{2}\sigma_{infl}\right) + 1}{\cosh\left(\sqrt{2}\sigma_{infl}^c\right) + 1} \right) \right]. \tag{31}$$

If $|\sigma_{infl}| \ll \sigma_{infl}^c$ its value at the minimum is given by Eq. (18) with $\beta \simeq \frac{\xi_2}{q} \left(1 + \frac{\xi_2}{q} \right)^{-1}$ and $\mu^4 \simeq \mu'^4 \frac{\xi_1^2}{4} \left(1 + \frac{\xi_2}{q} \right)$. [Also the coupling λ is related to the coupling λ' appearing in W through the relation $\lambda \simeq \lambda' \frac{\xi_1}{2} \left(1 + \frac{\xi_2}{q} \right)^{\frac{1}{2}}$ such that $\frac{\mu}{\sqrt{\lambda}} = \frac{\mu'}{\sqrt{\lambda'}} = \frac{M_X}{g}$.] For $|\sigma_{infl}| \geqslant \sigma_{infl}^c$ instead, $\zeta_{\min} = -\ln\left(1 + \frac{\mu'^4}{2 g_1^2} \right)$, $\chi_{\min} = 0$ and the potential along the inflationary trajectory is completely flat

$$V = \mu'^4 \frac{\xi_1^2}{4} \left(1 + \frac{\mu'^4}{2 g_1^2} \right)^{-1} + \frac{1}{2} g_2^2 \xi_2^2 \simeq \frac{1}{2} g_2^2 \xi_2^2 \quad \left(|\sigma_{infl}| \geqslant \sigma_{infl}^c \right) \tag{32}$$

(neglecting the tiny radiative corrections). Thus, the inflationary trajectory could provide a description of the evolution of the universe only after $\rho \lesssim \frac{1}{2} g_2^2 \xi_2^2 \sim 10^{-2} - 10^{-4}$ provided, of course, the initial conditions are appropriate. If this is the case the universe undergoes a huge period of D-term inflation [13], essentially governed by quantum fluctuations, along

11

the flat region of the potential followed by the expansion described approximately by the potential of Eq. (25) before the "observable" inflation takes place.

An investigation of the initial conditions necessarily involves the fields Φ, $\bar{\Phi}$. To simplify the discussion we keep only one out of the four real scalars in Φ, $\bar{\Phi}$ by setting $\Phi = \bar{\Phi} = \frac{\varphi}{2}$, where φ is a canonically normalized real scalar field, and we consider a trancated version of the complete scalar potential

$$V = \frac{\lambda^2}{4}\varphi^2 \left(\cosh\left(\sqrt{2}\sigma_{infl}\right) - 1\right) e^{2\zeta} + \frac{1}{2}g_1^2\xi_1^2 \left(e^\zeta - 1\right)^2 \tag{33}$$

(technically justified for $\frac{\mu^2}{\lambda} \ll \varphi^2 \lesssim 10^{-1}$ and $\chi^2 \simeq 2\beta$) possessing all its salient features. We assume that initially $|\sigma_{infl_0}| \gg 1$, $e^{\zeta_0} \ll 1$, $\varphi_0^2 \sim 10^{-1}$ and the initial time derivatives of all fields vanish. Notice that $e^{\zeta_0} \ll 1$ is required in order for $\rho_0 \lesssim 1$ if $|\sigma_{infl_0}|$ is sufficiently large. Then, e^ζ starts decreasing further unless the F-term in Eq. (33) is smaller than $\rho_0 e^{\zeta_0}$ to begin with. To ensure a sufficiently fast decrease of φ^2 we assume that $\frac{\partial^2 V}{\partial \varphi^2} \gtrsim \rho$ holds from the beginning which, for the initial conditions adopted, translates into

$$\left(\cosh\left(\sqrt{2}\sigma_{infl_0}\right) - 1\right) e^{2\zeta_0} \gtrsim \frac{g_1^2\xi_1^2}{\lambda^2}. \tag{34}$$

With φ^2 decreasing fast the relation $\frac{1}{V}\frac{\partial V}{\partial \zeta} \simeq \frac{1}{V}\frac{\partial^2 V}{\partial \zeta^2} \simeq -2e^\zeta$ $(e^\zeta \ll 1)$ is soon established and the universe experiences a stage of "chaotic" D-term inflation with $H = H_1 \simeq \frac{1}{\sqrt{6}}g_1\xi_1(1 - e^\zeta)$ which begins when $\zeta = \zeta_{beg} \lesssim \zeta_0 < 0$. The total number of e-foldings N_{tot} as ζ varies from ζ_{beg} towards its minimum at $\zeta_{min} \simeq 0$ is

$$N_{tot} \simeq \frac{1}{2}\left(e^{-\zeta_{beg}} - e\right) \gtrsim \frac{1}{2}\left(e^{-\zeta_0} - e\right) \tag{35}$$

(assuming inflation ends at $\zeta_{end} = -1$). Moreover, $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \simeq \varphi^2 \frac{\partial^2 V}{\partial \varphi^2}$. Consequently, even if initially $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \gtrsim \rho$ (i.e. $|\sigma_{infl_0}| \gg 1$), very soon $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \ll \rho$ and $|\sigma_{infl}|$ stays large with φ^2 becoming very small. Thus, when the "chaotic" D-term inflation is over the field configuration is close to the inflationary trajectory but σ_{infl} does not reach its terminal velocity as long as ρ is dominated by the field ζ which oscillates coherently about its minimum. It is understood that χ (which plays a secondary role in the beginning) will also oscillate about

its minimum. Whether this minimum is at $\chi = 0$ or not depends on whether at the end of the "chaotic" D-term inflation $|\sigma_{infl}|$ is larger or smaller than σ_{infl}^c. In the former case a second stage of D-term inflation with $H = H_2 \simeq \frac{1}{\sqrt{6}} g_2 \xi_2$ will take place as discussed earlier. [Of course, $|\sigma_{infl}| \gtrsim \sigma_{infl}^c$ requires relatively large initial values of $|\sigma_{infl}|$ ($|\sigma_{infl_0}| > 10$) and large negative values of ζ_0.] In the latter case, in which we concentrate, there will be no second stage of D-term inflation. The Hubble length H^{-1} will grow like $R^{\frac{3}{2}}$ as long as the universe is matter dominated and gradually will switch to an expansion law $H^{-1} \sim R$, as discussed earlier. Overall, between the "chaotic" D-term inflation and the "observable" inflation, H^{-1}/R grows only during the era of matter domination like $R^{\frac{1}{2}} \sim \rho^{-\frac{1}{6}}$ or by at most a factor $\sim 20 - 90$ for $\mu \sim (7.3 - 1) \times 10^{-3}$. The effect of this growth on the onset of "observable" inflation [14] can be "compensated for" if the "homogeneous" region of volume $\sim H_1^{-3}$ required for the onset of the "chaotic" D-term inflation experiences at least $3 - 4.5$ e-foldings of expansion during this first Planck-scale inflationary stage [15] [16]. Notice that $\rho_0 \sim 1$ is obtained without invoking enormous initial field values and therefore suppression of the gradient energy density in the above "homogeneous" region is not unnatural.

So far we neglected the effects of the quantum fluctuations during the stage of "chaotic" D-term inflation. These fluctuations generate at the end of this first inflationary stage classical perturbations $\Delta \sigma_{infl} \sim \frac{H_1}{2\pi}$ of the (nearly) massless field σ_{infl} and perturbations $\Delta \varphi \sim c \frac{H_1^2}{m_\varphi}$ of the massive field φ (with H_1 the Hubble parameter towards the end of the "chaotic" D-term inflation, $c \sim 10^{-1}$ and m_φ the mass of φ). $\Delta \sigma_{infl}$ gives rise to a spatial gradient energy density $\sim \frac{H_1^4}{4\pi^2}$ which falls like R^{-2}. Under the extreme assumption of matter domination during the whole intermediate stage before the onset of "observable" inflation this gradient energy density remains subdominant until $\rho \sim \mu^4$ provided $\left(\frac{3H_1^2}{\mu} \right)^{\frac{4}{3}} \ll 36\pi^2$. $\Delta \varphi$ gives rise to $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \simeq (\Delta \varphi)^2 \frac{\partial^2 V}{\partial \varphi^2} \sim c^2 H_1^4$ at the end of the first stage of inflation which, assuming constant σ_{infl}, falls like R^{-3} during the subsequent expansion. Thus, the contribution of the quantum fluctuations to $\frac{\partial^2 V}{\partial \sigma_{infl}^2}$ is at most $\sim c^2 H_1^2 H^2 \ll H^2$ and therefore negligible.

A numerical investigation reveals that even more natural initial conditions than the

ones adopted in the above somewhat simplified discussion do lead to evolution along the inflationary trajectory. Let us choose $q = 1$ and $g_1 = g_2 = \xi_1 = \frac{1}{\sqrt{2}} \simeq 0.7$. Then, with Φ, $\bar{\Phi}$ along the D-flat direction $\Phi = \bar{\Phi} = \frac{1}{2}(\varphi + i\psi)$, where φ, ψ are canonically normalized real scalar fields, it is possible to set $\varphi_0 = \psi_0 = \chi_0 = 1$ and $\zeta_0 = -2.5$ (in order to obtain sufficient expansion during the "chaotic" D-term inflation), assuming zero initial time derivatives for these fields. Moreover, we start with $\frac{d}{dt}\sigma_{infl} = -1$ and determine σ_{infl_0} (> 0) by requiring $\rho_0 = 1$. For $\mu = 7.3 \times 10^{-3}$ and $\beta = 0.033$ the initial value of σ_{infl} turns out to be $\sigma_{infl_0} \simeq 3.45$, for $\mu = 3 \times 10^{-3}$ and $\beta = 0.044$ the value $\sigma_{infl_0} \simeq 6$ is obtained whereas for $\mu = 10^{-3}$ and $\beta = 0.027$, $\sigma_{infl_0} \simeq 9.05$ [3]. Thus, our scenario allows for a quite natural starting point involving field values which are neither very small nor very large and an initial energy density of order unity equally partitioned into kinetic and potential.

We conclude by summarizing our results. In the presence of fields which without appearing in the superpotential acquire large vevs the inflaton mass in the context of supergravity hybrid inflation receives additional contributions. These contributions allow the construction of new and simple hybrid inflationary scenarios in supergravity with non-compact Kähler manifolds like products of coset spaces $SU(1,1)/U(1)$ which often occur in superstring compactifications. A particularly simple model with quite natural initial conditions allows for a detectable gravitational wave contribution to the microwave background anisotropy.

ACKNOWLEDGMENTS

---

[3]In order to lower the relatively high value of σ_{infl_0} for "small" μ we could increase χ_0 to compensate the decrease of the initial F-term potential energy density with an increase of the D-term one and choose a somewhat smaller ξ_1 to suppress the quantum fluctuations during the "chaotic" D-term inflation. Taking, e.g, $\mu = 10^{-3}$, $\beta = 0.027$ and $\xi_1 = \frac{1}{\sqrt{6}}$ we obtain $\sigma_{infl_0} \simeq 3$ if $\chi_0 = 1.42$.

REFERENCES

[1] A. D. Linde, Phys. Rev. D **49** 748 (1994).

[2] A. D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic, 1990).

[3] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. **73** 1886 (1994).

[4] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D **49** 6410 (1994).

[5] C. Panagiotakopoulos, Phys. Rev. D **55** R7335 (1997).

[6] A. D. Linde and A. Riotto, Phys. Rev. D **56** R1841 (1997).

[7] C. Panagiotakopoulos, Phys. Lett. B **402** 257 (1997).

[8] D. H. Lyth and A. Riotto, Phys. Rep. **314** 1 (1999).

[9] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133** 61 (1983).

[10] J. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **241** 406 (1984).

[11] D. H. Lyth, Phys. Rev. Lett. **78** 1861 (1997).

[12] A. R. Liddle and D. H. Lyth, Phys. Rep. **231** 1 (1993).

[13] J. A. Casas and C. Munoz, Phys. Lett. B **216** 37 (1989); J. A. Casas, J. M. Moreno, C. Munoz and M. Quiros, Nucl. Phys. B **328** 272 (1989); E. D. Stewart, Phys. Rev. D **51** 6847 (1995); P. Binetruy and G. Dvali, Phys. Lett. B **388** 241 (1996); E. Halyo, Phys. Lett. B **387** 43 (1996).

[14] N. Tetradis, Phys. Rev. D **57** 5997 (1998).

[15] C. Panagiotakopoulos and N. Tetradis, Phys. Rev. D **59** 083502 (1999).

[16] G. Lazarides and N. Tetradis, Phys. Rev. D **58** 123502 (1998).

μ	β	σ_H	σ_{infl_H}	n	n_{COBE}	r
7.3×10^{-3}	0.033	1.1411	1.5810	1.177	1.237	7.2×10^{-2}
7.0×10^{-3}	0.034	1.1082	1.4914	1.168	1.222	6.0×10^{-2}
6.8×10^{-3}	0.035	1.0817	1.4253	1.163	1.212	5.3×10^{-2}
6.5×10^{-3}	0.036	1.0421	1.3345	1.155	1.197	4.3×10^{-2}
6.0×10^{-3}	0.038	0.9643	1.1776	1.142	1.175	3.1×10^{-2}
5.0×10^{-3}	0.041	0.7810	0.8791	1.120	1.138	1.4×10^{-2}
4.0×10^{-3}	0.043	0.5657	0.5991	1.104	1.112	5.8×10^{-3}
3.0×10^{-3}	0.044	0.3464	0.3536	1.093	1.096	1.8×10^{-3}
2.0×10^{-3}	0.040	0.1755	0.1764	1.081	1.081	3.6×10^{-4}
1.0×10^{-3}	0.027	0.06633	0.06638	1.054	1.054	2.3×10^{-5}

Table 1. The values of β, σ_H, σ_{infl_H}, n, n_{COBE} and r as functions of μ for $N_d = 1$.

N_d	β	σ_H	σ_{infl_H}	n	n_{COBE}	r
0	0.079	0.8528	0.9869	1.221	1.271	7.1×10^{-2}
1	0.033	1.1411	1.5810	1.177	1.237	7.2×10^{-2}
2	0.026	1.1937	1.7469	1.171	1.233	7.2×10^{-2}
9	0.0125	1.3031	2.2603	1.160	1.225	7.3×10^{-2}
16	0.0086	1.3366	2.5232	1.157	1.222	7.3×10^{-2}

Table 2. The values of β, σ_H, σ_{infl_H}, n, n_{COBE} and r as functions of N_d for $\mu = 7.3 \times 10^{-3}$.