Precision Tests of Electroweak Physics

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We review the current status of precision electroweak measurements and the constraints they impose on new physics. We perform a model independent analysis using the *STU*–formalism of Ref. 1, and then discuss how the *Z*–pole data from LEP and SLD can be used to constrain models that are not covered within that framework.

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1   Introduction

There are two reasons for studying the data from precision electroweak measurements. First, we would like to know how well the Standard Model (SM) agrees with the experimental data: any significant deviation between the two would be a signal of new physics beyond the SM. Also, since the SM predictions depend on the mass of the still unobserved Higgs boson, the value of m_H which best reproduces the experimental data serves as its prediction. Second, we would like to know if any of the proposed theories of new physics beyond the SM is viable, i.e. does not make theoretical predictions inconsistent with the experimental data (or perhaps makes the agreement even better than with the SM).

The way to kill these two birds with one stone is to constrain the sizes of radiative corrections coming from new physics using the precision electroweak data. If the data prefers a non–zero value for the size of these extra corrections for some choice of m_H, it would signal that: (1) the agreement between the SM and data is not perfect for that particular choice of Higgs mass, and (2) any new physics which predicts such corrections would make the agreement between theory and experiment better.

2   Constraints on Oblique Electroweak Corrections

In order to constrain the size of radiative corrections coming from new physics, we must first make some assumptions about the new physics giving rise to them (since it is impossible to consider all possible corrections from all possible new physics all at once) and express them in terms of a few model independent parameters.

The simplest assumptions that would sufficiently constrain the number of corrections one must consider while still encompassing a large class of models are the following:

1. The electroweak gauge group is the standard $SU(2)_L \times U(1)_Y$. The only electroweak gauge bosons are the photon, the $W^{\pm}$, and the Z.
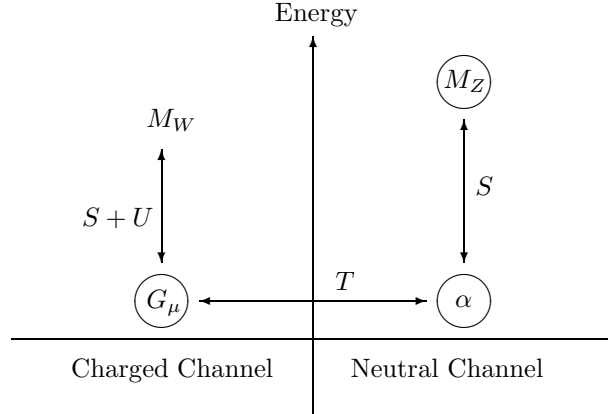

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Figure 1. The S, T, and U variables quantify the extra vacuum polarization corrections due to new physics when relating physics in different channels or physics at different energy scales. Low energy and neutral current observables depend only on S and T because α, G_μ, and M_Z are used as inputs to make the SM predictions.

2. The couplings of new physics to light fermions are highly suppressed. Since all precision electroweak measurements only involve four fermion processes with light external fermions, this means that vertex and box corrections from new physics can be neglected. Only vacuum polarization (i.e. oblique) corrections need to be considered.

3. The mass scale of new physics is large compared to the W and Z masses.

The first two assumptions let us focus our attention on just four vacuum polarization corrections: $\Pi_{\mathrm{WW}}(q^2)$, $\Pi_{\mathrm{ZZ}}(q^2)$, $\Pi_{\mathrm{Z}\gamma}(q^2)$, and $\Pi_{\gamma\gamma}(q^2)$. Here, $\Pi_{\mathrm{XY}}(q^2)$ is the transverse part of the vacuum polarization function between gauge bosons X and Y. The third assumption lets us expand these vacuum polarization functions around $q^2 = 0$ and neglect the higher order terms since they are suppressed by powers of q^2/M_{new}^2 where $q^2 \le M_Z^2$ for the processes under consideration:

$$\Pi_{\mathrm{WW}}(q^2) = \Pi_{\mathrm{WW}}(0) + q^2\Pi'_{\mathrm{WW}}(0) + \cdots$$
$$\Pi_{\mathrm{ZZ}}(q^2) = \Pi_{\mathrm{ZZ}}(0) + q^2\Pi'_{\mathrm{ZZ}}(0) + \cdots$$
$$\Pi_{\mathrm{Z}\gamma}(q^2) = q^2\Pi'_{\mathrm{Z}\gamma}(0) + \cdots$$
$$\Pi_{\gamma\gamma}(q^2) = q^2\Pi'_{\gamma\gamma}(0) + \cdots$$

Therefore, we can express the radiative corrections from new physics in terms of just six parameters: $\Pi_{\mathrm{WW}}(0)$, $\Pi'_{\mathrm{WW}}(0)$, $\Pi_{\mathrm{ZZ}}(0)$, $\Pi'_{\mathrm{ZZ}}(0)$, $\Pi'_{\mathrm{Z}\gamma}(0)$, and $\Pi'_{\gamma\gamma}(0)$. Of these six, three will be absorbed into the renormalization of the three input parameters α, G_μ and M_Z, leaving us with three observables parameters which can be taken to be: [1]

$$\alpha S = 4s^2c^2 \left[\Pi'_{\mathrm{ZZ}}(0) - \frac{c^2 - s^2}{sc}\Pi'_{\mathrm{Z}\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right] ,$$
$$\alpha T = \frac{\Pi_{\mathrm{WW}}(0)}{M_W^2} - \frac{\Pi_{\mathrm{ZZ}}(0)}{M_Z^2} ,$$

Observable	SM prediction	Measured Value	Reference
$\nu_\mu e$ and $\bar\nu_\mu e$ scattering			
$g_V^{\nu e}$	-0.0365	-0.041 ± 0.015	3
$g_A^{\nu e}$	-0.5065	-0.507 ± 0.014	3
Atomic Parity Violation			
$Q_W(^{133}_{55}\mathrm{Cs})$	-73.19	-72.41 ± 0.84	3
$Q_W(^{205}_{81}\mathrm{Tl})$	-116.8	-114.8 ± 3.6	3
$\nu_\mu N$ and $\bar\nu_\mu N$ DIS			
g_L^2	0.3031	0.3009 ± 0.0028	3
g_R^2	0.0304	0.0328 ± 0.0030	3
NuTeV	0.2289	0.2277 ± 0.0022	4
LEP/SLD			
$\Gamma_{\ell^+\ell^-}$	0.08392 GeV	0.08390 ± 0.00010 GeV	5
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (LEP)	0.23200	0.23153 ± 0.00034	5
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (SLD)	0.23200	0.23109 ± 0.00029	5
W mass			
M_W ($p\bar p$ + LEP2)	80.315 GeV	80.39 ± 0.06 GeV	5

Table 1. The data used for the oblique correction analysis. The value of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ for LEP is from leptonic asymmetries only. The SM predictions for the W mass and the LEP/SLD observables were obtained using the program ZFITTER 4.9 [6]. The predictions for the low energy observables were calculated from the formulae given in Ref. 7. The parameter choice for the reference SM was $M_Z = 91.1867$ GeV[5], $m_t = 173.9$ GeV[8], $m_H = 300$ GeV, $\alpha^{-1}(M_Z) = 128.9$[9], and $\alpha_s(M_Z) = 0.120$.

$$\alpha U = 4s^2 \left[\Pi'_{\mathrm{WW}}(0) - c^2\Pi'_{\mathrm{ZZ}}(0) - 2sc\Pi'_{\mathrm{Z}\gamma}(0) - s^2\Pi'_{\gamma\gamma}(0) \right] .$$

Here, α is the fine structure constant and s and c are the sine and cosine of the weak mixing angle. Only the contribution of new physics to these functions are to be included. The parameters T and U are defined so that they vanish if new physics does not break custodial $SU(2)$ symmetry. See Ref. 2 for a discussion on the symmetry properties of S.

The fact that three parameters are necessary to describe the effects of new physics can also be understood as follows: Since vacuum polarizations modify the gauge boson propagators, their presence can be seen when comparing the exchange of different electroweak gauge bosons, or when comparing the exchange of the same boson at different energy scales. In order to make predictions based on the three input parameters α, G_μ, and M_Z, which are neutral current–low energy, charged current–low energy, and neutral current–high energy observables, respectively, one must compare the theory in the charged (W exchange) and the neutral (Z and photon exchange) channels, as well as in the same channel at different energy scales as shown by arrows in Fig. 1. New physics effects will manifest themselves in each of the three arrows shown in Fig. 1. The parameter S quantifies the extra correction from new physics one must include when comparing neutral current processes at different energy scales while the parameter T quantifies the extra corrections that

must be included when comparing charged and neutral current processes at low energy. A third parameter, in this case $S+U$, is necessary to quantify the correction due to new physics when comparing charged channel processes at different energy scales.

This discussion also shows that all low energy and neutral current observables will receive extra corrections from only S and T. Of all the precision electroweak measurements, the only quantity which receives a correction from U is the W mass. Therefore, the majority of the precision data can be used to constrain just two parameters S and T, which in turn can be calculated for each model of new physics beyond the SM.

The details of how to calculate the corrections to various observables from S, T, and U can be found elsewhere[1]. One obtains expressions such as

$$M_W / [M_W]_{\text{SM}} = 1 + \frac{\alpha}{2(c^2 - s^2)} \left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U \right],$$

where $[M_W]_{\text{SM}}$ is the SM prediction of M_W. These expressions can be compared directly with the experimental data to place constraints on S, T, and U.

In Table 1 we list the data we used in our analysis. The definitions of the parameters $g_{V/A}^{e\nu}$ for $e\nu_\mu$ and $e\bar{\nu}_\mu$ scattering, the weak charge Q_w for atomic parity violation, and $g_{L/R}^2$ for ν_μ and $\bar{\nu}_\mu$ deep inelastic scattering (DIS) can be found in the Review of Particle Physics[3] from which the data were taken. The quantity measured by the NuTeV collaboration[4] is a linear combination of g_L^2 and g_R^2 for which the uncertainty due to the charm threshold cancels. The rest of the data is from Ref. 5.

Comparing the experimental data to SM predictions with $m_t = 173.9$ GeV[8], $m_H = 300$ GeV, and $\alpha^{-1}(M_Z) = 128.9$[9], we obtain the constraints shown in Fig. 2. Note that Figs. 2d and 2f are drawn at a different scale from the other four. Fig. 2e shows the 90% confidence limits on S and T due to the four classes of experiments separately, and Fig. 2f shows the 68% and 90% confidence limits from all experiments combined. As is evident from Fig. 2, the LEP/SLD measurements provide the tightest constraints on S and T. All of the other observables combined have little effect on the final result, which is

$$S = -0.30 \pm 0.13,$$
$$T = -0.14 \pm 0.15,$$
$$U = 0.15 \pm 0.21.$$

These limits of course depend on the values of m_t, m_H, and $\alpha^{-1}(M_Z)$ used as input to calculate the SM predictions. The dependence of the limits on these input parameters is shown by arrows in Fig. 2f. We can see that the current data favor either a small value of the Higgs mass or a larger value of $\alpha^{-1}(M_Z)$.

3 Limits on Topcolor Assisted Technicolor

The limits on S and T are useful in constraining new physics models which satisfy the three initial assumptions but are less useful for other theories. As an example of
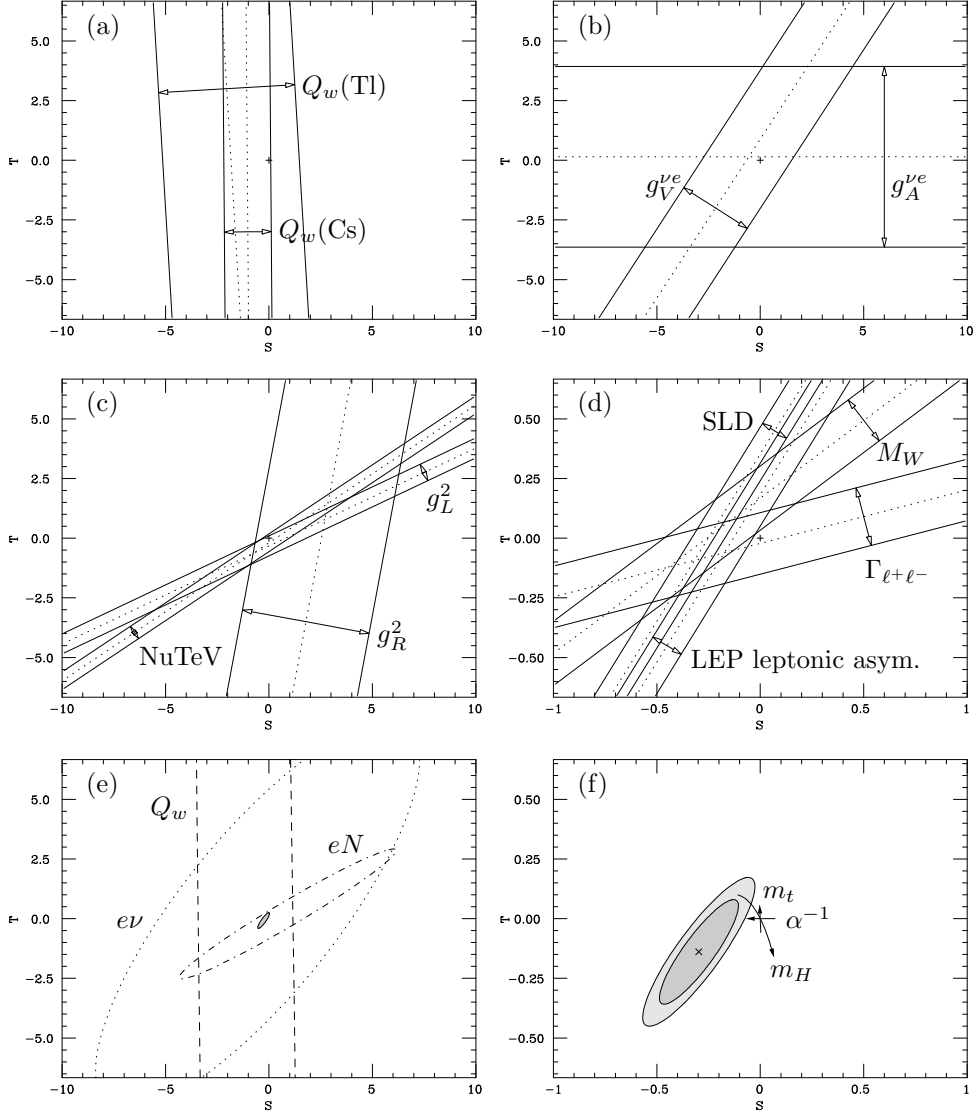
Figure 2. The 1–σ limits on S and T from (a) atomic parity violation, (b) $e\nu_\mu$ and $e\bar{\nu}_\mu$ scattering, (c) ν_μ and $\bar{\nu}_\mu$ DIS, and (d) LEP/SLD + M_W. (e) The 90% confidence contours for the four classes of experiments. The LEP/SLD + M_W contour is the small shaded area in the middle. (f) The 68% and 90% confidence limits on S and T, all experiments combined. The arrows indicate how the SM point will move relative to the contours for $m_t = 173.9 \pm 5$ GeV, $m_H = 300^{+700}_{-210}$ GeV, $\alpha^{-1}(M_Z) = 128.9 \pm 0.1$.

such a theory, let us consider topcolor assisted technicolor[10] with the gauge group

$$SU(3)_s \times SU(3)_w \times U(1)_s \times U(1)_w \times SU(2)_L$$

	$SU(3)_s$	$SU(3)_w$	$U(1)_s$	$U(1)_w$	$SU(2)_L$
$(t,b)_L$	3	1	$\frac{1}{3}$	0	2
$(t,b)_R$	3	1	$\left(\frac{4}{3}, -\frac{2}{3}\right)$	0	1
$(\nu_\tau, \tau^-)_L$	1	1	-1	0	2
τ_R^-	1	1	-2	0	1
$(c,s)_L,\ (u,d)_L$	1	3	0	$\frac{1}{3}$	2
$(c,s)_R,\ (u,d)_R$	1	3	0	$\left(\frac{4}{3}, -\frac{2}{3}\right)$	1
$(\nu_\mu, \mu^-)_L,\ (\nu_e, e^-)_L$	1	1	0	-1	2
$\mu_R^-,\ e_R^-$	1	1	0	-2	1

Table 2. Charge assignments of the ordinary fermions in topcolor assisted technicolor.

The coupling constants for the two $SU(3)$'s and the two $U(1)$'s are assumed to satisfy $g_{3s} \gg g_{3w}$ and $g_{1s} \gg g_{1w}$. The charges of the ordinary fermions under these groups are shown in Table 2.

At a scale of about one TeV, it is assumed that technicolor breaks the two $SU(3)$'s and the two $U(1)$'s to their diagonal subgroups:

$$SU(3)_s \times SU(3)_w \to SU(3)_c, \qquad U(1)_s \times U(1)_w \to U(1)_Y,$$

which are identified with the SM color and hypercharge groups. Because of the assumption $g_{3s} \gg g_{3w}$ and $g_{1s} \gg g_{1w}$, the broken $SU(3)$ gauge bosons (the *colorons*) and the broken $U(1)$ gauge boson (the Z') couple strongly to the third generation fermions but only weakly to the first and second generation fermions. Coloron exchange is attractive in both the $t\bar{t}$ and $b\bar{b}$ channels, while Z' exchange is attractive for $t\bar{t}$ but repulsive for $b\bar{b}$. The combined strength of the coloron and Z' interactions is assumed to be strong enough to condense the top but not so strong as to condense the bottom. As a result, only the top quark becomes heavy.

It is easy to see that this model does not fit into the STU framework since (1) it has an extra electroweak gauge boson, the Z', and (2) coloron and Z' exchange can lead to large vertex corrections for the third generation fermions (b, τ, and ν_τ). How would we place constraints on such a model?

A naive extension of the STU formalism to include the Z' vacuum polarization functions turns out to be too complicated to be illuminating. A much better way is to concentrate our attention on the vertex corrections at the Z mass scale, where we have a wealth of data from LEP and SLD. Recall from our previous discussion that S and T are relevant only when comparing processes at different energy scales or in different channels. If we only look at the LEP/SLD data, which come from neutral current processes at the Z mass scale, we can make our analysis completely blind to the vacuum polarization corrections and obtain limits on the vertex corrections only. Another way to see this is to notice that most of the observables at LEP/SLD are asymmetries and branching fractions which are just *ratios of coupling constants* at the Z mass scale. The SM predictions for these observables can be fixed by using only one of them as input to predict all the others, and any deviations must come

Figure 3. Limits on κ_1 and κ_3 from LEP/SLD observables. Δ parametrizes the size of the top–pion correction to the $Zb\bar{b}$ vertex. The shaded region is where the combined coloron and Z' interaction can condense the top without condensing the bottom or the τ.

from the vertex corrections due to new physics.[a]

Since the details of our analysis has been presented elsewhere[12], we give only an outline here. In the topcolor assisted technicolor model considered above, vertex corrections come in two classes: (1) gauge boson mixing terms, and (2) proper vertex corrections. Gauge boson mixing modifies the current to which the Z couples to from $J_Z^0 = J_{I_3} - s^2 J_Q$ to

$$ J_Z = J_{I_3} - (s^2 + \delta s^2)J_Q + \epsilon J_{1s}. $$

The parameters δs^2 and ϵ quantify the amount of Z–photon and Z–Z' mixing, respectively. The relevant proper vertex corrections are the coloron and Z' corrections to the third generation fermion vertices[13], the sizes of which we parametrize by

$$ \kappa_i = \frac{g_{is}^2}{4\pi}\left(\frac{g_{is}^2}{g_{is}^2 + g_{iw}^2}\right), \qquad (i = 1,3), $$

and a correction to the left–handed coupling of the b to the Z from the top–pion loop[14] which we denote Δ.

The corrections to various LEP/SLD obserables from δs^2, ϵ, κ_1, κ_3, and Δ were calculated and compared to the experimental data. In performing the fit, we kept the value of Δ fixed and let the four other parameters and the QCD coupling constant $\alpha_s(M_Z)$ float. In Fig. 3, we show only the results in the κ_1–κ_3 plane for two choices of the value of Δ: 0.003 and 0.006. These correspond to top–pion masses of $m_+ = 1000$ GeV and $m_+ = 600$ GeV, respectively. κ_1 and κ_3 must fall into the shaded region in order for the coloron and Z' interactions to condense the top while not condensing the bottom. Clearly the $\Delta = 0.003$ case is viable while the $\Delta = 0.006$ case is ruled out.

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[a]We have used a similar technique in Ref. 11.

4 Conclusions

Precision electroweak measurements provide stringent constraints on new physics beyond the SM. For models which satisfy the three conditions listed in section 2, the limits can be described in a model independent way using the STU–formalism of Ref. 1. Currently, the tightest limits come from the LEP/SLD observables, with all other observables having little effect. Current data also favors a smaller Higgs mass or a larger $\alpha^{-1}(M_Z)$. For models which are not encompassed within the STU framework, in particular, those with extra electroweak gauge bosons and/or large vertex corrections, one can still obtain stringent limits, albeit in a model dependent way, by focussing only on the vertex corrections at the Z mass scale and using the LEP/SLD observables to constrain them.

Acknowledgments

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