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Constraints on the parameters of the CKM matrix by End 1998

F. Parodi[(a)], P. Roudeau[(b)] and A. Stocchi[(b)]

[(a)] **Dipartimento di Fisica, Universitá di Genova and INFN**
Via Dodecaneso 33, 16146 Genova, Italy

[(b)] **Laboratoire de l'Accélérateur Linéaire**
IN2P3-CNRS et Université de Paris-Sud, BP 34, F-91898 Orsay Cedex

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Fabrizio Parodi[(a)], **Patrick Roudeau**[(b)] **and Achille Stocchi**[(b)]

[(a)] *Dipartimento di Fisica, Universitá di Genova and INFN, Via Dodecaneso 33, IT-16146 Genova, Italy*
[(b)] *Laboratoire de l'Accélerateur Linéaire (LAL), IN2P3-CNRS and Université de Paris-Sud, B.P. 34-91898 Orsay Cedex, France*

Abstract

A review of the current status of the Cabibbo-Kobayashi-Maskawa matrix (CKM) is presented. This paper is an update of the results published in [1]. The experimental constraints imposed by the measurements of $|\epsilon_K|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d and from the limit on Δm_s are used. Values of the constraints and of the parameters entering into the constraints, which restrict the range of the $\overline{\rho}$ and $\overline{\eta}$ parameters, include recent measurements presented at 1998 Summer Conferences and progress obtained by lattice QCD collaborations. The results are:

$$\overline{\rho} = 0.202^{+0.053}_{-0.059},\ \overline{\eta} = 0.340 \pm 0.035$$

from which the angles α, β and γ of the unitarity triangle are inferred :

$$\sin 2\alpha = -0.26^{+0.29}_{-0.28},\ \ \sin 2\beta = 0.725^{+0.050}_{-0.060},\ \ \gamma = (59.5^{+8.5}_{-7.5})^\circ$$

Without using the constraint from $|\epsilon_K|$, $\sin 2\beta$ has been obtained: $\sin 2\beta = 0.72^{+0.07}_{-0.11}$ Several external measurements or theoretical inputs have been removed, in turn, from the constraints and their respective probability density functions have been obtained. Central values and uncertainties on these quantities have been compared with actual measurements or theoretical evaluations. In this way it is possible to quantify the importance of the different measurements and the coherence of the Standard Model scenario for CP violation. An important result is that Δm_s is expected to be between $[12.0 - 17.6] ps^{-1}$ with 68% C.L. and $< 20\ ps^{-1}$ at 95% C.L. Finally relations between the CKM parameters and the quark masses are examined within a given model.

1 Introduction.

In a previous publication [1], uncertainties on the determination of the C.K.M. parameters A, $\overline{\rho}$ and $\overline{\eta}$, [1] in the framework of the Wolfenstein parametrization, have been reviewed. This study was based on measurements and on theoretical estimates available at the beginning of 1997. Present results have been obtained using new measurements, coming

[1] $\overline{\rho}$ and $\overline{\eta}$ are related to the original ρ and η parameters: $\overline{\rho}(\overline{\eta}) = \rho(\eta)(1 - \frac{\lambda^2}{2})$ [2]

mainly from LEP experiments and theoretical analyses available by end 1998.
Details on the determination of the values of the different quantities entering into the present analysis are explained in Section 2. Section 2.1 explains the evaluation of A through the measurement of the $|V_{cb}|$ element of the C.K.M. matrix and, in Section 2.2, the new results on $\left|\frac{V_{ub}}{V_{cb}}\right|$ from LEP and CLEO collaborations are presented. In Section 2.3 the new limit on Δm_s obtained by LEP experiments is recalled. Section 2.4 is dedicated to a review of the present determination of the non-perturbative QCD parameters, contributing to this analysis, from lattice QCD calculations. The present value of f_{B_d}, through the measurement of f_{D_s}, and the use of lattice QCD is explained [1]. New results from lattice QCD relating the B^0_d and B^0_s decay constants are reported and finally the value used for B_K is commented.
Using the constraints on $\overline{\rho}$ and $\overline{\eta}$ provided by the measurements of $|\epsilon_K|$, Δm_d, Δm_s and $\left|\frac{V_{ub}}{V_{cb}}\right|$, in the framework of the Standard Model, the region selected in the ($\overline{\rho}$, $\overline{\eta}$) plane and the determination of the angles α, β and γ of the unitarity triangle, are analyzed in Section 3. Similar studies prior to the present analysis can be found in [3].
Results have been also obtained by removing the constraint from $|\epsilon_K|$ [4]. The information coming from one constraint or from an external parameters (Δm_s, m_t, $|V_{cb}|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, B_K, and $B_{B_d}\sqrt{f_{B_d}}$) has been removed, in turn, and the probability density distribution, determined by the other parameters and constraints, has been determined in Section 4.
Finally a test on a possible relation between the values of the CKM matrix parameters and quark masses is shown in Section 5, using the framework of the parametrization of the C.K.M. matrix proposed in [5].

2 Evaluation of the values of the parameters entering into this analysis.

In the following, two sets of values have been used for some of the parameters. The first set corresponds to our best estimate (Scenario I). The purpose of using a second set (Scenario II) is to illustrate the variation of the uncertainties on the parameters with a more conservative evaluation of theoretical errors.

2.1 The A parameter.

The value of the parameter A is obtained from the determination of $|V_{cb}|$ in exclusive and inclusive semileptonic decays of B hadrons. By definition:

$$|V_{cb}| \ = \ A\ \lambda^2, \text{ with } \lambda \ = \ \sin\theta_c. \tag{1}$$

Using exclusive decays $\overline{B} \rightarrow D^*\ell^-\overline{\nu_\ell}$, the value of $|V_{cb}|$ is obtained by measuring the differential decay rate $\frac{d\Gamma_{D^*}}{dq^2}$ at maximum value of q^2 [6]. q^2 is the mass of the charged lepton-neutrino system. At $q^2 \ = \ q^2_{max}$, the D^* is produced at rest in the B rest frame and HQET can be invoked to obtain the value of the corresponding form factor: $F_{D^*}(w=1)$. The variable w is usually introduced; it is the product of the four-velocities of the B and D^* mesons:

$$w \ = \ v_B \cdot v_{D^*} \ = \ \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}, \ w=1 \text{ for } q^2 = q^2_{max}. \tag{2}$$

In terms of w, the differential decay rate can be written:

$$\frac{dBR_{D^*}}{dw} = \tau_{B_d^0} \frac{G_F^2}{48\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 K(w) \sqrt{w^2-1} F_{D^*}^2(w) \, |\mathrm{V}_{cb}|^2 \quad (3)$$

in which $K(w)$ is a kinematic factor.

As the decay rate is zero for $w = 1$, the w dependence has to be adjusted over the measured range, using the previous expression and a parametrization of $F_{D^*}(w)$[7].

The following LEP average has been obtained:

$$(F_{D^*}(1) \, |\mathrm{V}_{cb}|) \times 10^3 = 35.3 \pm 1.7 \pm 1.7 \quad (4)$$

which includes a new measurement from DELPHI [8] based on a large sample of events and an improved treatment of the contribution from D** decays. The accuracy of this measurement is still expected to improve by adding new analyses from ALEPH and OPAL collaborations. In Table 1 the values measured by the different experiments included in this average have been reported. Central values have been corrected so that they correspond to the values of the parameters given in Table 2. The statistical uncertainty on the average value of each $F_{D^*}(1) \times |\mathrm{V}_{cb}|$ measurement has been multiplied by 1.7 so that the corresponding $\chi^2/n.d.f.$ be equal to unity.

Collaboration	Measured value (10^3)	Corrected value (10^3)	Ref.
ALEPH	31.9 ± 1.8 ± 1.9	31.5 ± 1.8 ± 1.9	[9]
DELPHI1	35.4 ± 1.9 ± 2.4	35.0 ± 1.9 ± 2.4	[10]
DELPHI2	37.95 ± 1.34 ± 1.67	37.95 ± 1.34 ± 1.67	[8]
OPAL	32.8 ± 1.9 ± 2.2	32.4 ± 1.9 ± 2.2	[11]

Table 1: *Measured and corrected values of the quantity $F_{D^*}(1) \times |\mathrm{V}_{cb}|$ included in the LEP average (4). The corrected values have been obtained using the parameters given in Table 2.*

Results have been combined, including the CLEOII measurement [12] and taking into account common systematics between LEP and CLEO results, coming from $BR(\mathrm{D}^0 \rightarrow \mathrm{K}^-\pi^+)$, $\tau_{B_d^0}$, $BR(\mathrm{D}^{*+} \rightarrow \mathrm{D}^0\pi^+)$ and the D** rate in semileptonic decays (see Tables 1, 2). Using $F_{D^*}(1) = 0.91 \pm 0.03$ [6], $|\mathrm{V}_{cb}|$ is obtained from these exclusive measurements :

$$(|\mathrm{V}_{cb}|)(exclusive) \times 10^3 = 38.8 \pm 2.1 \pm 1.3(theory) \quad (5)$$

Inclusive semileptonic decays of B hadrons can be used also to measure $|\mathrm{V}_{cb}|$ [13] in the framework of the H.Q.E. (Heavy Quark Expansion) :

$$(|\mathrm{V}_{cb}|)(inclusive) \times 10^3 = 41.0 \pm 1.1 \pm 0.9(theory) \quad (6)$$

The last error has been obtained by summing in quadrature the different theoretical contributions. If these contributions are summed linearly the theoretical error has to be increased by a factor 1.5. These two values of $|\mathrm{V}_{cb}|$ are in agreement and the corresponding theoretical uncertainties are uncorrelated. The global average which has been used in the present analysis is:

$$(|V_{cb}|) \times 10^3 = 40.4 \pm 1.2 \tag{7}$$

which gives :

$$A = 0.831 \pm 0.029 \tag{8}$$

It has to be noticed that this result does not yet constitute really an accurate measurement because, experimentally, the present uncertainty corresponds to a 6% error on measured quantities. It has to be also stressed that there is not yet a general consensus, among theorists, concerning the importance of theoretical errors affecting $|V_{cb}|$. As experimental result become more and more accurate this point needs to be clarified soon.

The more conservative evaluation of the uncertainty on $|V_{cb}|$, has been obtained by multiplying theoretical errors by 1.5 :

$$(|V_{cb}|) \times 10^3 = 40.4 \pm 1.5 \tag{9}$$

which gives :

$$A = 0.831 \pm 0.035 \tag{10}$$

This last evaluation will be used in the second scenario. In [14] the following value is given :

$$(|V_{cb}|) \times 10^3 = 39.5 \pm 1.7 \tag{11}$$

It agrees with the present evaluation and the quoted error is slightly larger than the present values (7,9) because this average does not include more recent measurements.

Parameter	Value	Ref.
$BR(D^0 \rightarrow K^- \pi^+)$	$(3.85 \pm 0.09)\%$	[14]
$\tau_{B_d^0}$	$(1.57 \pm 0.04) ps$	[15]
$BR(B_d^0 \rightarrow \ell X)$	$(10.2 \pm 0.5)\%$	(see legenda)
$P(b \rightarrow B_d^0)$	$(39.5^{+1.6}_{-2.0})\%$	[16]
R_b	21.58%	Std. Model

Table 2: *Values of the parameters used in the present determination of* $|V_{cb}|$. *The semileptonic branching fraction for the* B_d^0 *meson has been obtained using the inclusive semileptonic branching fraction measurement done at the* $\Upsilon(4S)$ *[17] and correcting for the contribution of charged B mesons by taking into account the difference between* B_d^0 *and* B^- *lifetimes.*

2.2 Present value of $|V_{ub}|/|V_{cb}|$.

LEP Collaborations [18] have provided new results on $|V_{ub}|$ which have different systematics and a similar accuracy as the inclusive CLEO measurement (using the endpoint of the lepton spectrum (inclusive) [19]). Using the value of $|\mathrm{V}_{cb}|$ given in (7) it follows :

$$\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|} = 0.104 \pm 0.011(exp.) \pm 0.015(theo.) \qquad \text{LEP average} \tag{12}$$

A recent result has been obtained by the CLEO Collaboration on $|\mathrm{V}_{ub}|$ by measuring the $B \rightarrow \pi\ell\nu$ and $B \rightarrow \rho(\omega)\ell\nu$ branching fractions [20]. Using the value of $|\mathrm{V}_{cb}|$ given in (7) it follows :

$$\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|} = (0.081 \pm 0.011 \pm 0.029(theo.)) \quad \text{CLEO} - \text{exclusive} \quad \mathrm{B} \rightarrow \pi(\rho,\omega)\ell\nu \tag{13}$$

The quoted theoretical error is estimated to be the maximal excursion of the values of $\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|}$ evaluated in different models [21]. From the measured ratio of $\frac{\rho}{\pi}$ production ratio the (KS) model is disfavoured [22]. This is quite important since the value of $|\mathrm{V}_{ub}|$ obtained in this model deviated the most from the other estimates both in the exclusive and inclusive measurements [19]. After having removed this model, the maximal excursion on the values of $\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|}$, evaluated in different models from the endpoint measurement (CLEO-inclusive), is 0.008. A conservative approach has been used in this paper which consists in doubling this error. The result is then :

$$\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|} = 0.080 \pm 0.006 \pm 0.016(theo.) \qquad \text{CLEO} - \text{inclusive} \tag{14}$$

In the following, only the results from LEP and from the inclusive CLEO measurement are used. Theoretical errors between these two measurements are largely uncorrelated and the two results will be used as independent constraints on the ratio $\frac{|\mathrm{V}_{ub}|}{|\mathrm{V}_{cb}|}$.

2.3 Present limit on Δm_s.

A new limit on Δm_s, $\Delta m_s > 12.4 ps^{-1}$ at $95\% C.L.$, has been derived by the "B Oscillation Working Group" [16]. The sensitivity of present measurements is at $13.8 ps^{-1}$.
This limit has been obtained in the framework of the amplitude method [23] which consists in measuring, for each value of the frequency Δm_s, an amplitude a and its error $\sigma(a)$. The parameter a is introduced in the time evolution of pure B^0_s or $\overline{\mathrm{B}^0_s}$ states so that the value $a = 1$ corresponds to a genuine signal for oscillation:

$$\mathcal{P}(\mathrm{B}^0_s \rightarrow (\mathrm{B}^0_s,\ \overline{\mathrm{B}^0_s})) = \frac{1}{2\tau_s} e^{-\frac{t}{\tau_s}} \times (1 \pm a\ cos(\Delta m_s t))$$

The values of $\Delta \mathrm{m_s}$ excluded at 95% C.L. are those satisfying the condition $\mathrm{a}(\Delta \mathrm{m_s}) + 1.645\ \sigma_a(\Delta \mathrm{m_s}) < 1$. With this method it is easy to combine different experiments and to treat systematic uncertainties in an usual way since, at each value of $\Delta \mathrm{m_s}$, a value for a with a Gaussian error σ_a, is measured. Furthermore the sensitivity of the experiment can be defined as the value of $\Delta \mathrm{m_s}$ corresponding to $1.645\ \sigma_a(\Delta \mathrm{m_s}) = 1$ (for $\mathrm{a}(\Delta \mathrm{m_s}) = 0$, namely supposing that the "true" value of $\Delta \mathrm{m_s}$ is well above the limit. The sensitivity is the limit which would be reached in 50% of the experiments.
However the set of measurements $a(\Delta m_s)$ contains more information than the 95% C.L. limit. It is possible to build a χ^2, which quantifies the compatibility of $a(\Delta m_s)$ with the value $a = 1$, defined as:

$$\chi^2(\Delta m_s) = \frac{(a(\Delta m_s) - 1)^2}{\sigma^2(a(\Delta m_s))^2}$$

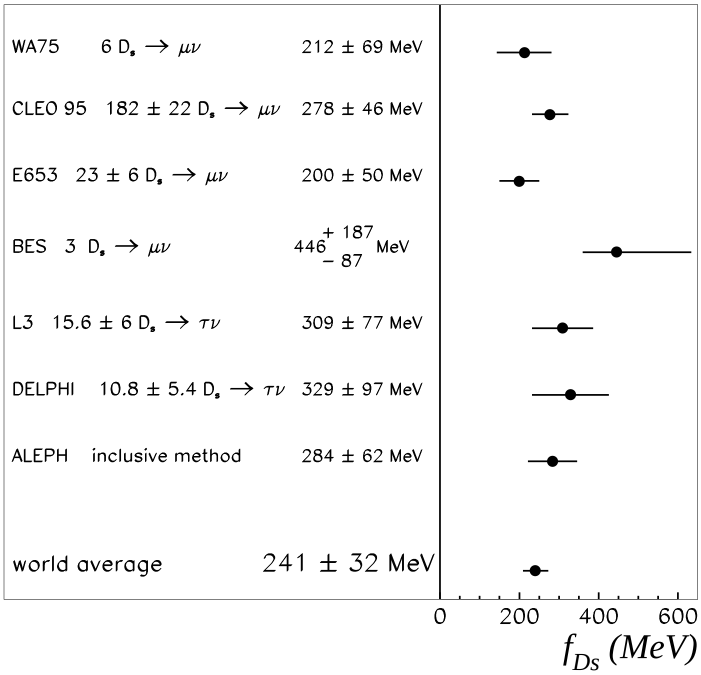

Figure 1: *Summary of the evaluation of f_{D_s} from the measurements of $Br(D_s \to \mu\nu)$ and $Br(D_s \to \tau\nu)$.*

and which can be used as a constraint.

2.4 Present values for the non-perturbative QCD parameters.

Important improvements have been achieved in the last few years in the evaluation of the non-perturbative QCD parameters, entering into this analysis, in the framework of lattice QCD. As a consequence, in this paper, only most recent results are used. Motivations for this attitude have been given in [24]. Recent reviews on lattice QCD can be found in [24],[25],[26].

2.4.1 Present value of f_{B_d}.

Following the proposal made in [1], f_{B_d} is evaluated from the measurements of f_{D_s} and using the extrapolation from the D to the B sector as predicted by lattice QCD [27]. The value of f_{D_s} is deduced from the measurements of the branching fractions : $\mathrm{D_s^+} \to \tau^+\nu_\tau$ and $\mathrm{D_s^+} \to \mu^+\nu_\mu$. The different determinations of f_{D_s} [28] are shown in Figure 1. The recent measurements from CLEO and ALEPH collaborations have been included [28].

The average is:

$$f_{D_s} = (241 \pm 32)\ MeV \tag{15}$$

This value is in agreement with those most recently obtained from lattice QCD calculations, quoted in Table 3.

In recent publications, from lattice QCD, the ratio between the D_s^+ and the $\mathrm{B_d^0}$ decay constants has been evaluated, results are also given in Table 3. All evaluations are in

f_{D_s} [MeV]	f_{B_d} [MeV]	f_{B_d}/f_{D_s}	f_{B_s}/f_{B_d}	ref.
$224 \pm 2 \pm 16 \pm 12$	$173 \pm 9 \pm 8 \pm 11$			[29]
$213 \pm^{+14}_{-11} \pm 11^{+21}_{-0}$	$164^{+14}_{-11} \pm 8\ ^{+16}_{-0}$	0.78 ± 0.04	$1.13\ ^{+5}_{-4}$	[30]
$213 \pm 9^{+23+17}_{-9\ -0}$	$159 \pm 11^{+22+21}_{-9\ -0}$	$0.75 \pm 0.03^{+0.04\ +0.07}_{-0.02\ -0.00}$	$1.10 \pm 0.02^{+0.05+0.03}_{-0.03-0.02}$	[31]
$231 \pm 12^{+6}_{-1}$	$179 \pm 17^{+26}_{-9}$	$0.71 \pm 0.04^{+0.07}_{-0.00}$	$1.14 \pm 0.03^{+0.00}_{-0.01}$	[32]
	$147 \pm 11 \pm 11^{+8}_{-12}$			[33]

Table 3: *f_{D_s}, f_{B_d}, f_{B_d}/f_{D_s} and f_{B_s}/f_{B_d} parameters from recent lattice QCD calculations. For the f_{D_s} and f_{B_d} parameters the first two errors are of statistical and systematical origins respectively, the third comes from the use of the quenched approximation.*

agreement. The errors coming from the quenched approximation are estimated to be of the order of 10% [31],[32] and tend to increase the value of the ratio f_{B_d}/f_{D_s}.

Using the experimental value $f_{D_s} = (241 \pm 32)\ MeV$, and the theoretical evaluation given in Table 3 from [31], it follows :

$$f_{B_d} = 181 \pm 24(exp.) \pm 7(theo.stat.)^{+20}_{-5}(theo.\ \ non.\ \ stat.)MeV \qquad (16)$$

This result can be compared with the most recent evaluations of f_{B_d}, from lattice QCD, which are also given in Table 3. The values obtained for f_{B_d} are in relative agreement and are also in agreement with the extrapolation from the f_{D_s} measurement (16). In the following, the value used for f_{B_d} is the one coming from the extrapolation of the f_{D_s} measurement (16).

2.4.2 Present value of $f_{B_d}\sqrt{B_{B_d}}$.

Present evaluations of the B_{B_d} parameter are given in Table 4. With respect to the situation concerning the previously discussed parameters, f_{D_s} and f_{B_d}, most recent results on B_{B_d} are not nicely in agreement. On the other hand the error coming from the quenched approximation evaluated in [27] is small, of the order of 4%.
A conservative approach consists in using :

$$B_{B_d} = 1.35 \pm 0.15 \qquad (17)$$

Combining this result with the value of f_{B_d} given in (16) the value used in this analsyis for $f_{B_d}\sqrt{B_{B_d}}$ is :

$$f_{B_d}\sqrt{B_{B_d}} \ = \ 210 \pm 29(stat.)^{+23}_{-6}(theo.\ \ non\ \ stat) \pm 12(from\ B_{B_d})\ MeV \qquad (18)$$

In table 5 the contributions of different parameters to the error on $f_{B_d}\sqrt{B_{B_d}}$ are given.

2.4.3 Present value for ξ.

Significant improvements have been achieved in the determination of the ξ parameter, defined as $\frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$. Several authors agree on a relative precision better than 10% on the

B_{B_d}	Ref.
1.53 ± 0.19	[34]
1.42 ± 0.07	[35]
$1.44 \pm 0.09 \pm 0.06$	[27]
$1.29 \pm 0.08 \pm 0.06$	[36]
$1.40 \pm 0.06^{+0.04}_{-0.26}$	[37]
$1.17 \pm 0.09 \pm 0.05$	[38]

Table 4: *Values of the B_{B_d} parameter from recent lattice QCD calculations. The second error in [27] is an estimate of the error which comes from the use of the quenched approximation.*

parameter (error)	error on $f_{B_d}\sqrt{B_{B_d}}$
f_{D_s} (stat.) (16%)	8%
$Br(D_s \to \phi\pi)$ (25%)	10%
other systematics	5%
f_{B_d}/f_{D_s} (stat.) (4%)	4%
f_{B_d}/f_{D_s}(non stat.) ($^{+11\%}_{-3\%}$)	$^{+11\%}_{-3\%}$
B_{B_d} (11%)	6%

Table 5: *Contributions to the precision on $f_{B_d}\sqrt{B_{B_d}}$ from different quantities. The main contribution in "other systematics" comes from the assumption that the mechanism of production of strange B and D mesons in jets is the same.*

ratio f_{B_s}/f_{B_d} (see Table 3). The error coming from the quenched approximation seems to be controlled at the level of 3%. The ratio of the bag factors $\frac{B_{B_s}}{B_{B_d}}$ is known very precisely ($\frac{B_{B_s}}{B_{B_d}} = 1.01 \pm 0.01$ [39]).

The value from reference [31] has been used in the following :

$$\xi = 1.11 \pm 0.02^{+0.06}_{-0.04} \tag{19}$$

2.4.4 Present value of B_K.

The two most recent values are in agreement :

$$\begin{aligned} B_K(\text{ at 2 GeV}) &= 0.62 \pm 0.02 \pm 0.02[40] \\ &= 0.628 \pm 0.042[41] \end{aligned} \tag{20}$$

The scale-invariant value for B_K is then evaluated to be [40]: $B_K = 0.86 \pm 0.03 \pm 0.03$. The error due to the quenched approximation is evaluated to be less than 10%. For this analysis the following value is used :

$$B_K = 0.86 \pm 0.06 \pm 0.08(theo.\ \ from\ \ quenching) \tag{21}$$

A more conservative evaluation of theoretical uncertainties has been proposed in [26]. Differences on the central value and on the last error with respect to (21) are coming from the evaluation of the quenched approximation. The value is:

$$B_K = 0.94 \pm 0.06 \pm 0.14(theo.\ \ from\ \ quenching) \tag{22}$$

This second evaluation has been used in the conservative scenario (Scenario II).

parameter	Values	Gaussian − Flat error	Ref.
λ	0.2205 ± 0.0018	$\pm 0.0018 - \pm 0.000$	[1]
$\|V_{cb}\|$(Scenario I)	$(40.4 \pm 1.2) \times 10^{-3}$	$(\pm 1.2 - \pm 0.0) \times 10^{-3}$	this paper
$\|V_{cb}\|$(Scenario II)	$(40.5 \pm 1.5) \times 10^{-3}$	$(\pm 1.5 - \pm 0.0) \times 10^{-3}$	this paper
$\frac{\|V_{ub}\|}{\|V_{cb}\|}$ (CLEO)	0.080 ± 0.017	$\pm 0.006 - \pm 0.016$	this paper
$\frac{\|V_{ub}\|}{\|V_{cb}\|}$ (LEP)	0.104 ± 0.019	$\pm 0.011 - \pm 0.015$	this paper
Δm_d	$(0.472 \pm 0.016)\ ps^{-1}$	$\pm 0.016 - \pm 0.000$	[16]
Δm_s	> 12.4 ps^{-1} at 95% $C.L.$	see text (2.3)	[16]
$\overline{m_t}(m_t)$	$(167 \pm 5)\ GeV/c^2$	$\pm 5 - \pm 0$	[42]
B_K (Scenario I)	0.86 ± 0.09	$\pm 0.06 - \pm 0.08$	this paper
B_K (Scenario II)	0.94 ± 0.15	$\pm 0.06 - \pm 0.14$	this paper
$f_{B_d}\sqrt{B_{B_d}}$	$(210^{+39}_{-32})\ MeV$	$\pm 29\ -^{+23}_{-6}$ and ± 12	this paper
$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$	$1.11^{+0.06}_{-0.04}$	$\pm 0.02 -^{+0.06}_{-0.04}$	this paper
$\overline{m_c}(m_c)$	$(1.3 \pm 0.1)\ GeV/c^2$	$\pm 0.1 - \pm 0.000$	[14]
η_1	1.38 ± 0.53	$\pm 0.53 - \pm 0.000$	[43]
η_3	0.47 ± 0.04	$\pm 0.04 - \pm 0.000$	[43]
η_2	0.574 ± 0.004	fixed	[43]
f_K	$0.161\ GeV/c^2$	fixed	[14]
Δm_K	$(0.5301 \pm 0.0014) \times 10^{-2}\ ps^{-1}$	fixed	[14]
$\|\epsilon_K\|$	$(2.280 \pm 0.019) \times 10^{-3}$	fixed	[14]
η_B	0.55 ± 0.01	fixed	[43]
G_F	$(1.16639 \pm 0.00001) \times 10^{-5} GeV^{-2}$	fixed	[14]
m_W	$80.41 \pm 0.10\ GeV/c^2$	fixed	[14]
$m_{B^0{}_d}$	$5.2792 \pm 0.0018\ GeV/c^2$	fixed	[14]
$m_{B^0{}_s}$	$5.3693 \pm 0.0020\ GeV/c^2$	fixed	[14]
m_K	$0.493677 \pm 0.000016\ GeV/c^2$	fixed	[14]

Table 6: *Values of the quantities entering into the expressions of* $|\epsilon_K|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d *and* Δm_s. *In the third column the Gaussian and the flat part of the error are given explicitely. Two scenarios are considered depending on the expected theoretical uncertainties on the values of the parameters* B_K *and* $|\mathrm{V}_{cb}|$.

3 Results with present measurements.

The central values and uncertainties for the relevant parameters used in this analysis are given in Table 6.
For the $|V_{cb}|$ and B_K parameters two sets of values are quoted corresponding to the present best estimate (scenario I) and to a more conservative evaluation of theoretical uncertainties (scenario II). In this paper all figures correspond to scenario I, but numerical results are given for both scenarios.
It has to be reminded that the approach consists in building up the two dimensional probability distribution for $\overline{\rho}$ and $\overline{\eta}$ [1]. This is done as follows :

- a point uniformly distributed in the $(\overline{\rho}, \overline{\eta})$ plane, is chosen,
- values for the different parameters entering into the equations of constraints are obtained using random generations extracted from Gaussian/uniform distributions depending on the source of the error. As an example, for the non-perturbative QCD parameters (as $f_{B_d}\sqrt{B_{B_d}}$, ξ, B_K) the error coming from the quenching approximation is extracted from a flat distribution.
- the predicted values for the four quantities, $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|^2$, Δm_d and $\frac{\Delta m_d}{\Delta m_s}$ are then obtained and compared with present measurements which can have Gaussian/non Gaussian errors. As an example, for the constraint provided by the measurement of $\frac{|V_{ub}|}{|V_{cb}|}$, theoretical errors are treated as a flat distribution. A weight is then computed which takes into account the expected shape of the probability distribution for the constraint (Gaussian, flat or a convolution of the two distributions). The final weight is equal to the product of all independent weights.
- the sum of all weights, over the $(\overline{\rho}, \overline{\eta})$ plane, is normalized to unity and contours corresponding to 68% and 95% confidence levels have been defined.

The region of the $(\overline{\rho},\ \overline{\eta})$ plane selected by the measurements of $|\epsilon_K|$, $\left|\frac{V_{ub}}{V_{cb}}\right|$, Δm_d and from the limit on Δm_s has been obtained and is given in Figure 2. The measured values for the two parameters are:

$$\overline{\rho} = 0.202^{+0.053}_{-0.059},\ \overline{\eta} = 0.340 \pm 0.035 \qquad \text{Scenario I} \tag{23}$$

$$\overline{\rho} = 0.214^{+0.055}_{-0.062},\ \overline{\eta} = 0.328 \pm 0.040 \qquad \text{Scenario II} \tag{24}$$

The errors on $\overline{\rho}$ and $\overline{\eta}$, between the two scenarios, differ by about 10%.
It is clear that the allowed region for $\overline{\rho}$ is not symmetric around zero, negative values for $\overline{\rho}$ being clearly disfavoured : $\mathcal{P}_{\rho<0} = 0.8\%$.

3.1 Measured values of $\overline{\rho}$ and $\overline{\eta}$ without the $|\epsilon_K|$ constraint.

Following the idea proposed in [4], a region of the $(\overline{\rho},\ \overline{\eta})$ plane can be selected without using the $|\epsilon_K|$ constraint. The result is shown in Figure 3, where the contours corresponding to 68 % and 95 % confidence levels are also indicated. This test shows that the $(\overline{\rho},\ \overline{\eta})$

[2] the formula given in [1] has been corrected to $|\epsilon_K| = C_\epsilon\ \mathrm{B}_K\ A^2\lambda^6\overline{\eta}[-\eta_1(1-\frac{\lambda^2}{2})S(x_c) + A^2\lambda^4(1-\overline{\rho}-(\overline{\rho}^2+\overline{\eta}^2-\overline{\rho})\lambda^2)\eta_2 S(x_t) + \eta_3 S(x_c,x_t)]$. The term $\overline{\rho}$ was missing in the $(\overline{\rho}^2+\overline{\eta}^2-\overline{\rho})$ term.

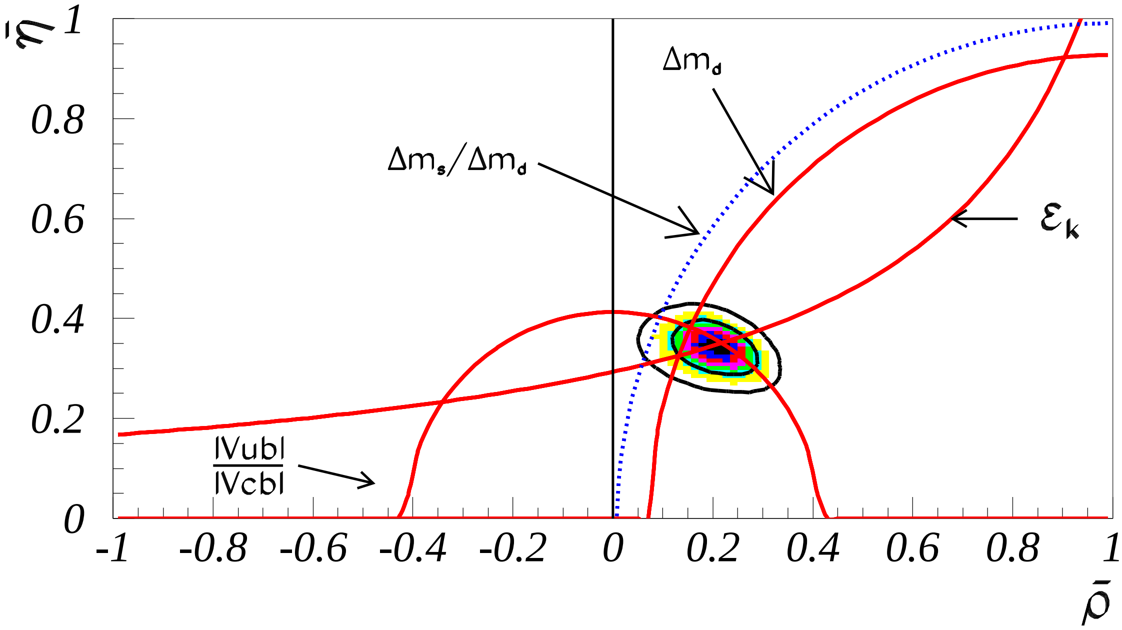

Figure 2: *The allowed region for $\overline{\rho}$ and $\overline{\eta}$ using the parameters listed in Table 6. The contours at 68 % and 95 % are shown. The full lines correspond to the central values of the constraints given by the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|$ and Δm_d. The dotted curve corresponds to the 95 % C.L. upper limit obtained from the experimental limit on Δm_s.*

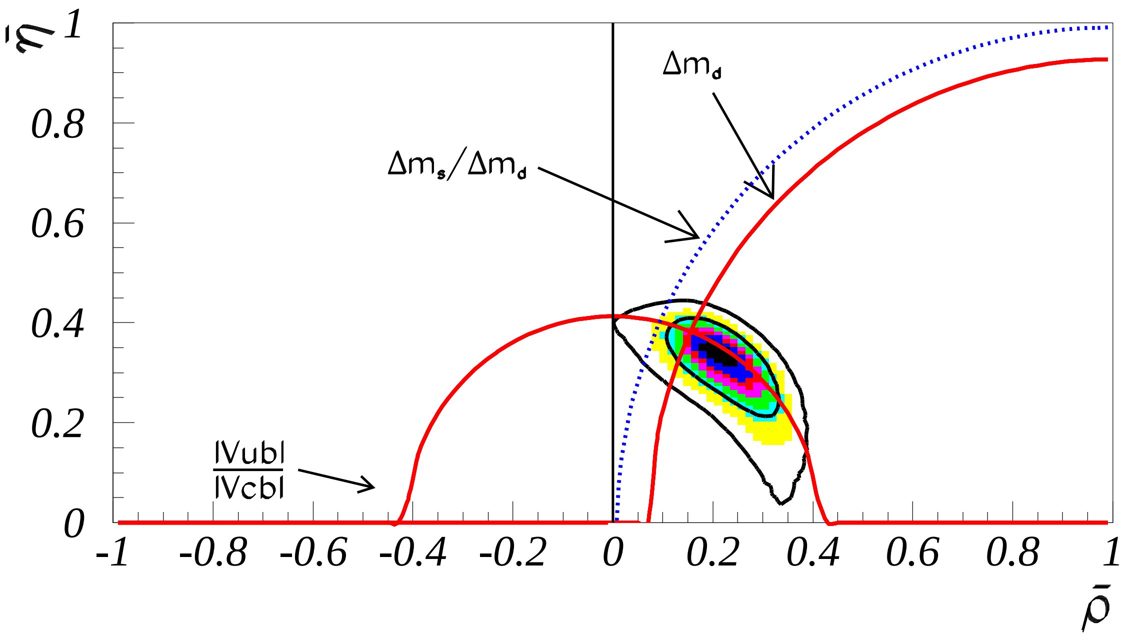

Figure 3: *The allowed region for $\overline{\rho}$ and $\overline{\eta}$ using the parameters listed in Table 6 without using the $|\epsilon_K|$ constraint. The contours at 68 % and 95 % are shown. The dotted curve corresponds to the 95 % C.L. upper limit obtained from the experimental limit on Δm_s.*

region selected by the measurements in the B sector is well compatible with the region selected from the measurement of the direct CP violation in the kaon sector.
The value $\overline{\eta} = 0$ is situated in the region excluded at 96% C.L. in scenario I and at 95% C.L. in scenario II. Another approach which consists in testing the hypothesis of a real C.K.M matrix by setting $\overline{\eta}$=0 can be found in [44].

3.2 Measured values of $\sin 2\alpha$, $\sin 2\beta$ and γ.

It is of interest to determine the central values and the uncertainties on the quantities $\sin 2\alpha$, $\sin 2\beta$ and γ which can be measured directly at future facilities like, CDF, D0, HERA-B, B factories and LHC experiments. The results are :

$$\sin 2\alpha = -0.26^{+0.29}_{-0.28},\ \sin 2\beta = 0.725^{+0.050}_{-0.060},\ \gamma = (59.5^{+8.5}_{-7.5})^{\circ} \qquad \text{Scenario I} \tag{25}$$

$$\sin 2\alpha = -0.36^{+0.33}_{-0.30},\ \sin 2\beta = 0.715^{+0.055}_{-0.065},\ \gamma = (56.5^{+9.5}_{-8.5})^{\circ} \qquad \text{Scenario II} \tag{26}$$

Figure 4 gives the correlation between the measurements of $\sin 2\alpha$ and $\sin 2\beta$ and the contours at 68% and 95 % C.L.. Figure 5 gives the distribution of the angle γ.
The preliminary result from the CDF Collaboration is $\sin 2\beta = 0.79^{+0.41}_{-0.44}$ [45].

3.2.1 Measurement of $sin 2\beta$.

The value of $sin2\beta$ is rather precisely determined, with an accuracy already at a level expected after 3-4 years of running at B factories. The situation will improve in the current year (1999) with better measurements of $|V_{cb}|$, with a possible improvement of the sensitivity of LEP analyses on Δm_s and with an expected progress from lattice QCD calculations.
Without using the constraint on $|\epsilon_K|$: $\sin 2\beta = 0.72^{+0.07}_{-0.11}$.

3.2.2 Measurement of $sin 2\alpha$.

In our previous analysis [1], it was concluded that there was no restriction on the domain of variation of $sin2\alpha$ between -1 and +1. The present study, see Figure 4, allows to identify now a favoured domain for this parameter.

3.2.3 Measurement of the angle γ.

It has been proposed in [46] to restrict the range of variation of the angle γ using the measurement of the ratio, R_1, of the branching fractions of charged and neutral B mesons into Kπ final states. In the hypothesis that this ratio is below unity, the following constraint has to be satisfied:

$$\sin^2 \gamma < R_1 \tag{27}$$

The present result from CLEO [47]:

$$R_1 = \frac{BR(\mathrm{B}^0\ (\overline{\mathrm{B}^0}) \rightarrow \pi^{\pm}\mathrm{K}^{\mp})}{BR(\mathrm{B}^{\pm} \rightarrow \pi^{\pm}\mathrm{K}^0)} = 0.65 \pm 0.40, \tag{28}$$

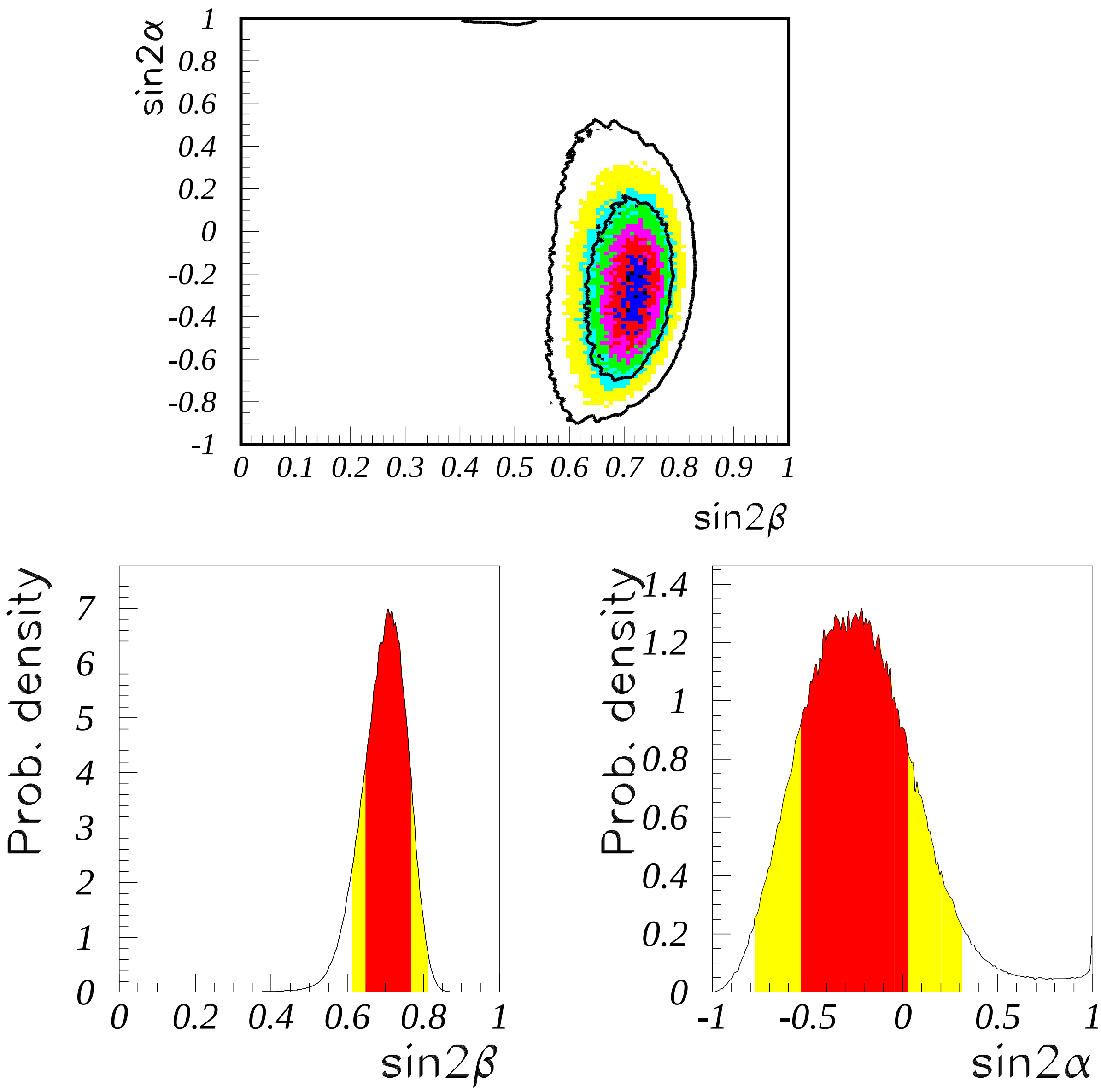

Figure 4: *The $\sin 2\alpha$ and $\sin 2\beta$ probability distributions have been obtained using the contraints corresponding to the values of the parameters listed in Table 6. The dark-shaded and the clear-shaded intervals correspond, respectively, to 68% and 95 % confidence level regions.*

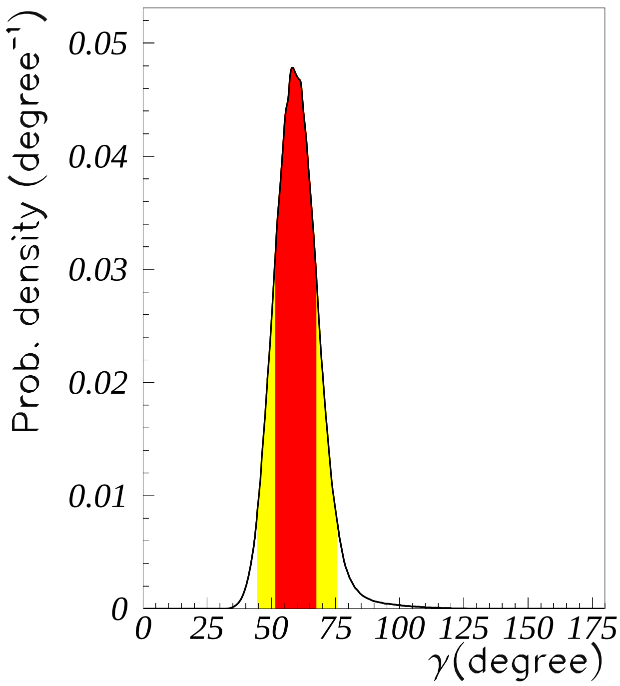

Figure 5: *The γ angle probability distribution obtained using the same constraints as in Figure 2. The dark-shaded and the clear-shaded intervals correspond to 68% and 95 % confidence level regions respectively.*

has a too large uncertainty to be really constraining on γ. This bound excludes a region which is symmetric around $\gamma = 90°$. In fact, as explained already in Section 3, negative values of $\overline{\rho}$ are already excluded and the region around $\gamma = 90°$ has a low probability. These restrictions are clearly apparent in Figure 5 which gives the expected density probability distribution for the angle γ, which is determined to be : $\gamma = (59.5^{+8.5}_{-7.5})°$

At present, theorists do not agree on the effects of hadronic interactions on this analysis [48] of the ratio R_1. But, considering that these effects are under control, the needed experimental accuracy on R_1 has been evaluated such that this measurement provides an information on $\overline{\rho}$ of similar precision as the one obtained at present. The model of [49] has been used in which the authors have studied the variation of R_1 with the $\overline{\rho}$ parameter (Figure 6). The present determination of $\overline{\rho}$ corresponds to:

$$R_1 \;=\; \frac{BR(\mathrm{B}^0\ (\overline{\mathrm{B}^0}) \rightarrow \pi^{\pm}\mathrm{K}^{\mp})}{BR(\mathrm{B}^{\pm} \rightarrow \pi^{\pm}\mathrm{K}^0)} = 0.87 \pm 0.07 \tag{29}$$

4 Tests of the internal consistency of the Standard Model for CP violation.

Four constraints, three measurements and one limit, have been used until now to measure the values of the two parameters $\overline{\rho}$ and $\overline{\eta}$. It is also possible to remove, from the construction of the two dimensional probability distribution for $\overline{\rho}$ and $\overline{\eta}$, the external information on the value of one of the constraint or of another parameter entering into the

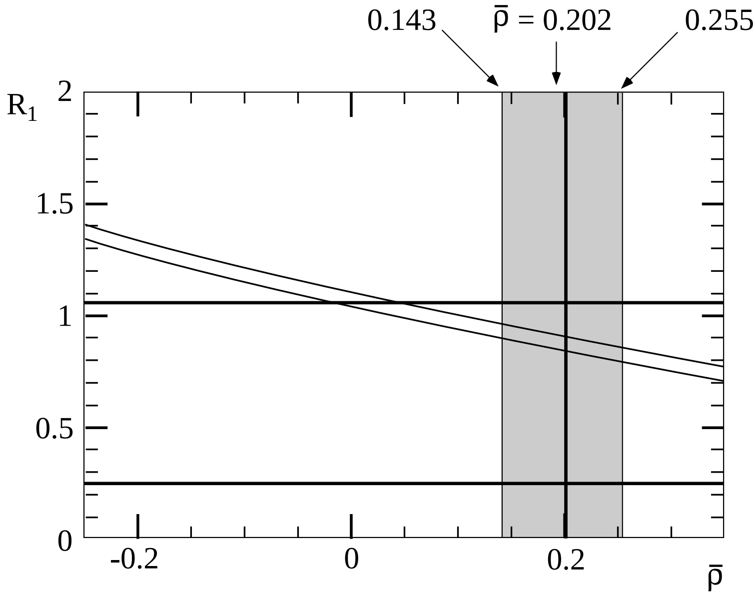

Figure 6: *The ratio* $R_1 = \frac{BR(\mathrm{B}^0\ (\overline{\mathrm{B}^0}) \to \pi^\pm \mathrm{K}^\mp)}{BR(\mathrm{B}^\pm \to \pi^\pm \mathrm{K}^0)}$ *as a function of the parameter* $\overline{\rho}$ *taken from [49], for* $\overline{\eta} = 0.25$ *(lower curve) and* $\overline{\eta} = 0.52$ *(upper curve). The horizontal thick lines show the CLEO measurement (with* $\pm 1\sigma$ *errors). The shaded vertical band corresponds to the* $\pm 1\sigma$ *interval for* $\overline{\rho}$ *obtained in the present analysis.*

Standard Model expressions for the constraints. The results will have some dependence in the central values taken for all the other parameters but, the main point in this study, is to compare the uncertainty on a given quantity determined in this way to its present experimental or theoretical error. This comparison allows to quantify the importance of present measurements of the different quantities entering into the definition of the allowed region in the $(\overline{\rho},\ \overline{\eta})$ plane. Results have been summarized in Table 7.

4.1 Expected value for the $\mathrm{B_s^0} - \overline{\mathrm{B_s^0}}$ oscillation parameter, Δm_s.

Removing the constraint from the measured limit on the mass difference between the strange B meson mass eigenstates, Δm_s, the density probability distribution for Δm_s is given in Figure 7. Δm_s is expected to be between [12.0 - 17.6]ps^{-1} within one sigma and $< 20\ ps^{-1}$ at 95% C.L. The present limit excludes already a large fraction of this distribution. Present analyses at LEP are situated in a high probability region for a positive signal and this is still a challenge for LEP collaborations.

4.2 Top mass measurement.

If the information on the top mass measurement by CDF and D0 collaborations is removed, the value for $\overline{m_t}(m_t)$ is : $\overline{m_t}(m_t) = (179^{+52}_{-34})$ GeV. The present determination of m_t with a ± 5 GeV error has thus a large impact for the present analysis.

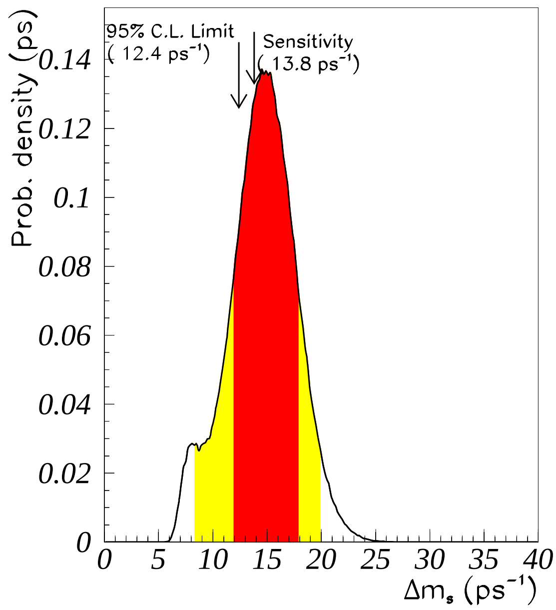

Figure 7: *The Δm_s probability distribution obtained with the same constraints as in Figure 2 once the constraint from the experimental limit on Δm_s has been removed. The dark-shaded and the clear-shaded intervals correspond to 68% and 95 % confidence level regions respectively.*

4.3 Measurement of $|V_{cb}|$.

The central value determined for $|V_{cb}|$ is close to the direct measurement: $|V_{cb}|=(42.0^{+8.0}_{-4.0}) \times 10^{-3}$. Direct measurements of $|V_{cb}|$ are thus important to constrain the allowed region in the $(\overline{\rho},\ \overline{\eta})$ plane once their relative error is below 10%. The density probability distribution for the parameter $|V_{cb}|$ is given in Figure 8.

4.4 Measurement of $\left|\frac{V_{ub}}{V_{cb}}\right|$.

The central value determined for $\left|\frac{V_{ub}}{V_{cb}}\right|$ is close to the direct measurement: $\left|\frac{V_{ub}}{V_{cb}}\right|=0.097^{+0.033}_{-0.022}$. This indirect measurement shows the importance of having a precision on $\left|\frac{V_{ub}}{V_{cb}}\right|$ better than 30%. The density probability distribution for $\left|\frac{V_{ub}}{V_{cb}}\right|$ is given in Figure 8.

4.5 Measurement of B_K.

The density distribution for the parameter B_K is given in Figure 8. It indicates that:

- values of B_K smaller than 0.6 are excluded at 98.4% C.L.,
- large values of B_K are compatible with the other constraints over a large domain.

The present estimate of B_K, from lattice QCD, with a 10% relative error has thus a large impact for the present analysis.

parameter	Results in Scenario I	Results in Scenario II	Present value
Δm_s	$14.8^{+2.8}_{-2.8}\ ps^{-1}$	$15.0^{+3.3}_{-3.3}\ ps^{-1}$	> 12.4 ps^{-1} at 95% $C.L.$
$\left\|\frac{V_{ub}}{V_{cb}}\right\|$	$0.097^{+0.033}_{-0.022}$	$0.091^{+0.033}_{-0.024}$	$0.080 \pm 0.017/0.104 \pm 0.019$
B_K	$0.87^{+0.34}_{-0.20}$	$0.90^{+0.34}_{-0.22}$	0.86 ± 0.09
$f_{B_d}\sqrt{B_{B_d}}$	(223 ± 13) MeV	(228 ± 14) MeV	$(210^{+39}_{-32})\ MeV$
$\overline{m_t}(m_t)$	(179^{+52}_{-34}) GeV	(179^{+56}_{-39}) GeV	(167 ± 5) GeV
$\|V_{cb}\|$	$(42.0^{+8.0}_{-4.0}) \times 10^{-3}$	$(41.0^{+9.5}_{-4.0}) \times 10^{-3}$	$(40.4 \pm 1.2) \times 10^{-3}$

Table 7: *Values of the different parameters obtained after having removed, in turn, their contribution into the different constraints. The results in the two scenarios are reported.*

4.6 Measurement of $f_{B_d}\sqrt{B_{B_d}}$

A rather accurate value is obtained:

$$f_{B_d}\sqrt{B_{B_d}} = (223 \pm 13)\,MeV \tag{30}$$

This result is, in practice, in agreement and more precise than the present evaluation of this parameter (18) which has been determined measuring f_{D_s} and using results from lattice QCD on $\frac{f_{B_d}}{f_{D_s}}$. Table 5 gives the contributions of the uncertainties, on the different parameters, to the error on $f_{B_d}\sqrt{B_{B_d}}$ extracted from the ratio f_{B_d}/f_{D_s}. An effort has to be done to improve both the experimental and the theoretical precisions. Finally, from the experimental point of view, a τ/Charm factory could provide the ultimate precision on f_{D_s} and f_{D^+}. The density probability distribution for the parameter $f_{B_d}\sqrt{B_{B_d}}$ is given in Figure 8.

5 Measurement of the angles θ, θ_u, θ_d and ϕ.

This section is an update of the results presented in [1], a similar study can be found also in [50]. There are nine possibilities to introduce the CP violation phase into the elements of the CKM matrix [5]. The authors of [5] have argued for a parametrization, based on the observed hierarchy in the values of quark masses. Several theoretical works [51] show that the observed pattern of fermion masses and mixing angles could originate from unified theories with an U(2) flavour symmetry. The authors of [5] have introduced four angles which have simple physical interpretations. θ corresponds to the mixing between the families 2 and 3. $\theta_{u(d)}$ is the mixing angle between families 1 and 2, in the up(down) sectors. Finally ϕ is responsible for CP violation and appears only in the elements of the C.K.M. matrix relating the first two families.

This parametrization is given below:

$$V_{CKM} = \begin{pmatrix} s_u s_d c + c_u c_d e^{-i\phi} & s_u c_d c - c_u s_d e^{-i\phi} & s_u s \\ c_u s_d c - s_u c_d e^{-i\phi} & c_u c_d c + s_u s_d e^{-i\phi} & c_u s \\ -s_d s & -\, c_d s & c \end{pmatrix} \tag{31}$$

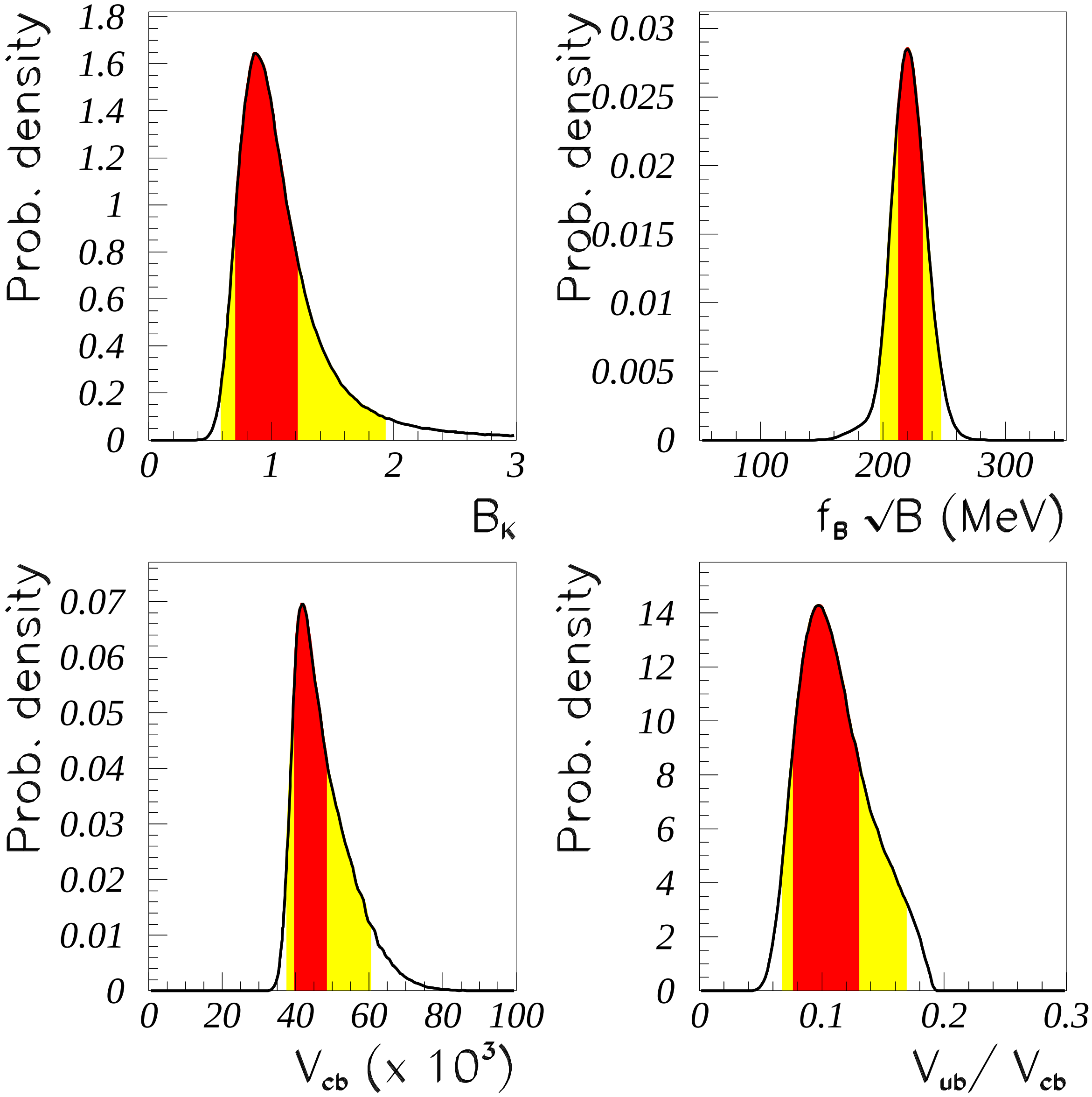

Figure 8: *The B_K, $f_{B_d}\sqrt{B_{B_d}}$, A and $\left|\frac{V_{ub}}{V_{cb}}\right|$ probability distributions obtained with the same constraints as in Figure 2 once the concerned parameter has been removed. The dark-shaded and the clear-shaded intervals correspond to 68% and 95 % confidence level regions respectively.*

where c_x and s_x stand for $cos\theta_x$ and $sin\theta_x$ respectively.

The four angles are related to the modulus of the following C.K.M. elements:

$$\sin\theta = |\mathrm{V}_{cb}|\sqrt{1+\left|\frac{V_{ub}}{V_{cb}}\right|^2} \tag{32}$$

$$\tan\theta_u = \left|\frac{V_{ub}}{V_{cb}}\right| \tag{33}$$

$$\tan\theta_d = \left|\frac{V_{td}}{V_{ts}}\right| \tag{34}$$

and

$$\phi = \arccos\left(\frac{\sin^2\theta_u\cos^2\theta_d\cos^2\theta+\cos^2\theta_u\sin^2\theta_d-|\mathrm{V}_{us}|^2}{2\sin\theta_u\cos\theta_u\sin\theta_d\cos\theta_d\cos\theta}\right) \tag{35}$$

The first three equations illustrate the direct relation between the angles θ, θ_u and θ_d and the measurements of B decay and oscillation parameters.

The angle ϕ has also a nice interpretation because, in the limit where $\theta = 0$ (in practice $\theta \simeq 2^\circ$), the elements V_{us} and V_{cd} have the same modulus, equal to $\sin\theta_c$ (θ_c is the Cabibbo angle) and can be represented in a complex plane by the sum of two vectors, of respective lengths $\sin\theta_u\cos\theta_d$ and $\sin\theta_d\cos\theta_u$, making a relative angle ϕ. It can be shown that this triangle is congruent to the usual unitarity triangle [5] and that $\phi \simeq \alpha$.

Angle	measured value	value expected from quark masses [52]
θ	$(2.35 \pm 0.07)^\circ$	
θ_u	$(5.00 \pm 0.45)^\circ$	$(3.4 \pm 0.4)^\circ$
θ_d	$(11.2 \pm 0.7)^\circ$	$(12.6 \pm 1.2)^\circ$
ϕ	$(96.7 \pm 8.1)^\circ$	$(85 \pm 21)^\circ$

Table 8: *Values for the angles of the parametrization [5], compared with those obtained using the values of the quark masses as given in [52] evaluated at* Q^2=M_W^2 *(the values used for the quark masses are:* $m_u = 2.35^{+0.42}_{-0.45} MeV/c^2$, $m_d = 4.73^{+0.61}_{-0.67} MeV/c^2$, $m_s = 94.2^{+11.9}_{-13.1} MeV/c^2$ *and* $m_c = 684^{+56}_{-61} MeV/c^2$ *)*

Using the constraints defined previously, the respective probability distributions for the four angles have been given in Figure 9; the numerical values are summarized in Table 8.

In this parametrization of the C.K.M. matrix the relation between the mixing angles and the quark masses can be written as [53]:

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}},\ \ \tan\theta_d = \sqrt{\frac{m_d}{m_s}}. \tag{36}$$

Using the values for the quark masses given in [52] evaluated at Q^2=M_W^2, the values for the angles θ_u and θ_d are given in Table 8. In this interpretation, the angle ϕ can be obtained using (35) and (36). Present measurements support a value of ϕ close to 90° which corresponds to the maximal CP violation scenario of [5].

The present analysis indicates that a higher value for the ratio m_u/m_c is favoured or that the expression relating θ_u to the u and c quark masses has to be corrected.

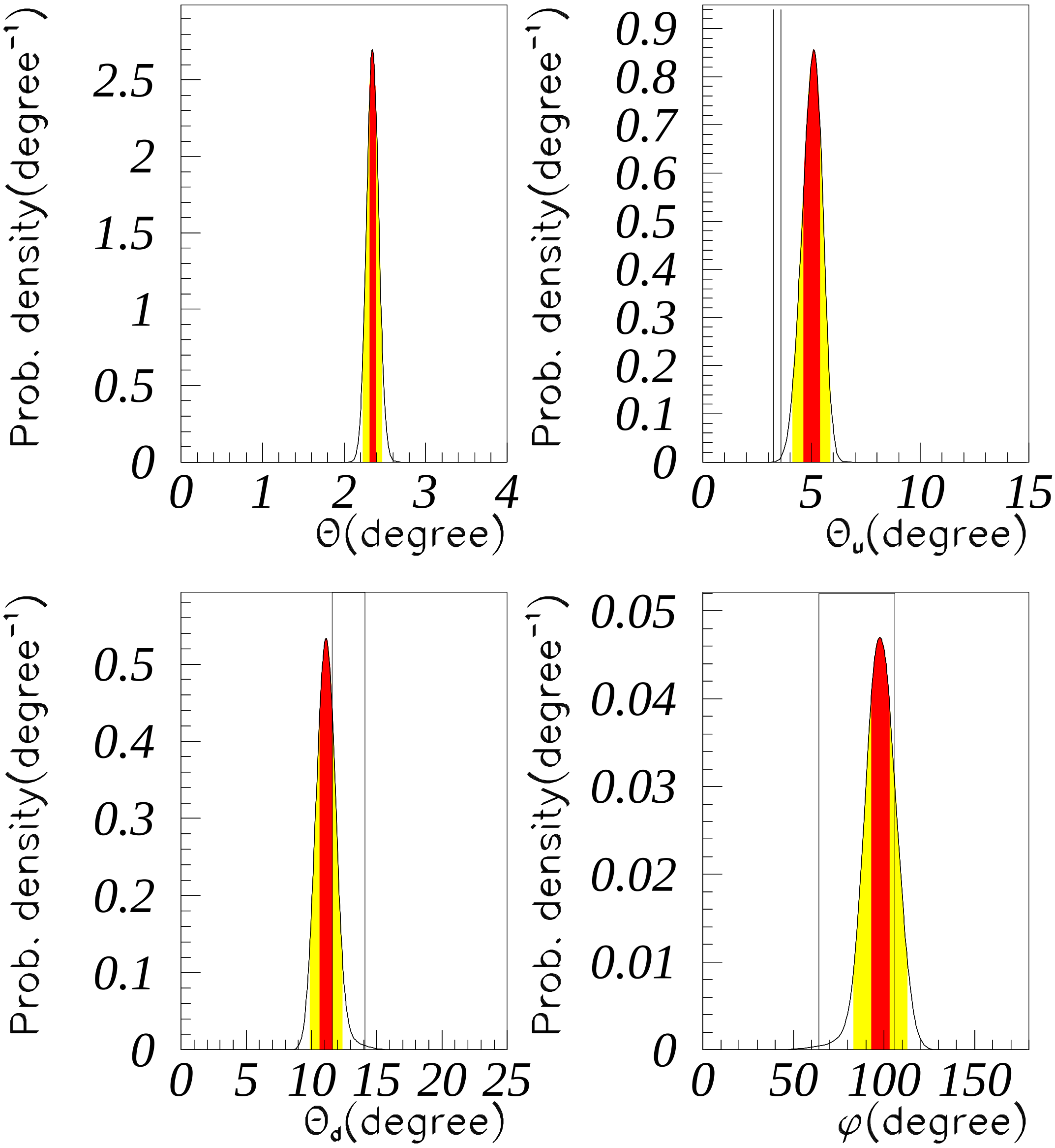

Figure 9: *The distributions of the angles θ, θ_u, θ_d and ϕ proposed in the parametrisation [5]. The dark-shaded and the clear-shaded intervals correspond to 68% and 95 % confidence level regions respectively. The lines represent the ± 1 σ region corresponding to the values of the angles obtained using the values for quark masses given in [52].*

6 Conclusions.

The $\overline{\rho}$ and $\overline{\eta}$ parameters have been determined using the constraints from the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, $|\epsilon_K|$, Δm_d and from the limit on Δm_s:

$$\overline{\rho} = 0.202^{+0.053}_{-0.059},\ \overline{\eta} = 0.340 \pm 0.035$$

Contrary to similar studies in this field, which claim a rather symmetric interval of variation for $\overline{\rho}$, around zero [54], the negative $\overline{\rho}$ region is excluded at 99.2% C.L.. The present analysis assumes the unitarity of the CKM matrix. In this framework the values of $sin2\beta$, $sin2\alpha$ and γ have been also deduced. They are :

$$\sin 2\alpha = -0.26^{+0.29}_{-0.28},\ \sin 2\beta = 0.725^{+0.050}_{-0.060},\ \gamma = (59.5^{+8.5}_{-7.5})^\circ$$

The value of $sin2\beta$ has an accuracy similar to the one expected after 3-4 years of running at B factories. An interesting outcome from this study is that a more conservative approach in the choice of the parameters (Scenario II) gives a less than 10% variation on the uncertainties attached to the various measured quantities. The internal consistency of the Standard Model expectation for CP violation, expressed by a single phase parameter in the C.K.M. matrix, has been verified by removing, in turn, the different constraints imposed by the external parameters. No anomaly has been noticed with respect to the central values used in the present analysis. This study has mainly quantified the needed accuracy on the determinations of these parameters so that they bring useful constraints in the determination of $\overline{\rho}$ and $\overline{\eta}$. In this respect, present uncertainties on m_t, $|\mathrm{V}_{cb}|$,$\left|\frac{V_{ub}}{V_{cb}}\right|$ and B_K have important contributions. Low values of B_K, below 0.6, are not compatible with the present analysis at 98.4% C.L.. Δm_s is expected to lie, with 68% C.L., between 12.0 and 17.6 ps^{-1}. More accurate measurements, still expected at CLEO and LEP, and more precise evaluations of non perturbative QCD parameters from lattice QCD, will improve these results in the coming years. A Tau/Charm factory providing accurate values for f_{D^+} and f_{D_s} is expected to have important contributions in this analysis.
Another parametrization of the CKM matrix, proposed in [5] has been studied. The four corresponding parameters, which are angles, have been determined. The present analysis indicates that a lower value for the ratio m_u/m_c is favoured or that the expression relating θ_u to the u and c quark masses has to be corrected.

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