Genetic Algorithms for the Optimal Design of Superconducting Accelerator Magnets

S. Ramberger, S. Russenschuck
CERN, Geneva, Switzerland

Abstract

The paper describes the use of genetic algorithms with the concept of niching for the optimal design of superconducting magnets for the Large Hadron Collider, LHC at CERN. The method provides the designer with a number of local optima which can be further examined with respect to objectives such as ease of coil winding, sensitivity to manufacturing tolerances and local electromagnetic force distribution. A 6 block dipole coil was found to have advantages compared to the standard 5 block version which was previously designed using deterministic optimization methods. Results were proven by a short model magnet recently built and tested at CERN.

1 INTRODUCTION

The 5-block coil cross-section for the LHC main dipoles as described in the “Yellow Book” [1] was designed at the time, using deterministic optimization algorithms. The advantage of this coil design is a higher average margin (inner and outer layer) for a given amount of superconductor material. However, as the conductor and coil design has been subject to a number of changes, e.g. change in cable dimensions and insulation and a part compensation of the persistent current multipoles at injection, the 5 block design had become too inflexible to accommodate further adjustments if they became necessary at a later stage. This was due to geometrical constraints, i.e. copper wedges becoming too small at the inner edge. Moreover, the performance of the model magnets seemed to indicate that the force distribution in the inner block of the inner layer with its 4 turns and an adjacent copper wedge of large dimensions is not favourable. Beam simulations have also shown the dodecapole field component to be a limiting factor for the machine performance.

The classical method of designing superconducting magnets includes the application of mathematical optimization techniques based on vector-optimization and deterministic algorithms. The limitations of deterministic optimization methods are that the number of conductors in each of the coil blocks has to be constant, as the optimization of coupled problems of discrete and continuous variables is not possible with search routines. For the given requirements and cable dimensions no alternative design with a six block structure could be found by the standard design procedure. The aim was therefore to utilize an optimization method that overcomes the deficiencies of the common deterministic algorithms and to apply it to the main dipole coil for the LHC.

2 THE OPTIMIZATION PROBLEM

The main objectives of the dipole design are a small content of unwanted field errors in the main field, low sensitivity of the field quality to manufacturing errors, easy manufacturing, and possibility to tune the geometry after the pre-series manufacture of the magnets. Not all of these objectives can be included into the mathematically formulated objective function.

The amount of unwanted multipoles is expressed by the coefficients of the Fourier series expansion of the radial field component in the aperture where in the assumed symmetric case only the odd b_n (coefficients of the sine terms) are to be minimized. However, using goal programming methods the weight for the components showing different sensitivity has to be found in an iterative procedure, as the effects of the components have to be examined using beam tracking. As the electro-magnetic forces are enormous (about 4000 kN/m radially) the local force distribution in the coil collar structure has to be optimized. However, this requires computations of coupled electro-magnetic mechanical problems. Manufacturing considerations include ease of the coil winding and collaring, i.e. geometrical constraints on the pole angle. Setting too many geometrical constraints, however, results in ill-conditioned optimization problems. During manufacturing, systematic errors occur due to the applied tooling. After the pre-series construction of the magnets the coils have to be repositioned to compensate for these systematic errors. It will be impossible to change the topology of the coil which therefore has to have sufficient flexibility for adjustments.

Therefore an optimization method is required that not only converges towards a “global” minimizer of the objective function, which is in our case just a weighted sum of the Fourier coefficients of the flux density in the aperture of the magnet (at r=10 mm), but provides the user with a number of design proposals (local minima) which can then be examined in more detail.

The design variables for the optimization problem are the number of turns per coil block, the positioning and inclination angles of the blocks, and the current in each turn. The current has to be included as a design variable in the optimization in order to guarantee a feasible solution not exceeding the load-line limit of the superconducting wires, which depends on the local magnetic field. The magnetization of the iron yoke with an inner radius of 98.5 mm is calculated by means of a reduced vector potential finite-element formulation [2] which does not require the meshing of the coil, as the source terms can easily be calculated using Biot-Savart's law. The fact that the coils don't have to be meshed is important because the topology of the coil is changed during the optimization process by omitting and adding conductors to coil blocks.

3 GENETIC OPTIMIZATION

For the minimization of the resulting objective function genetic algorithms are used [3]. As our problem is mixed continuous and integer, the different parameters are combined by linear sampling of the floating point parameters and Gray-encoding of the resulting integers into a binary string.

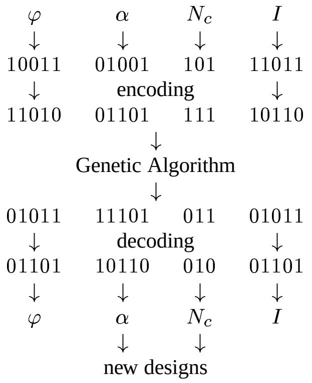

Figure 1: Parameter encoding and decoding

The current in the conductors and the angles of the coil-blocks were encoded by 5 bits each. The number of turns of the outer and inner blocks used 4 bit strings each, thus resulting in "chromosomes" of typically 50 to 60 bits.

Genetic algorithms then proceed by 3 main operators, selection, crossover and mutation. The selection operator guarantees convergence to an optimum by keeping the better chromosomes and discarding the less fit ones. Using the standard operation of fairy-wheel selection or simply retaining the better half of the chromosomes reduces diversity generation by generation thus leading to a single solution. The applied concept of niching, however, provides the designer with a set of solutions rather than only one, which can then be examined in detail. After a new offspring is generated by one of the three operators, the chromosome with the smallest hamming-distance $HD = \sum_i u_i \oplus v_i$ (least number of different bits) is located and replaced if its fitness is worse than that of the offspring.

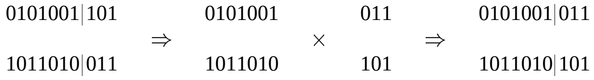

Figure 2: Crossover

Crossover is a recombination of bit strings of two chromosomes by swapping the strings at a random point. This is the major influence directing the search process to good solutions. Although the crossover point is chosen at random, the offspring created by crossover do not cover the entire search space, as can be seen in Fig. 3.

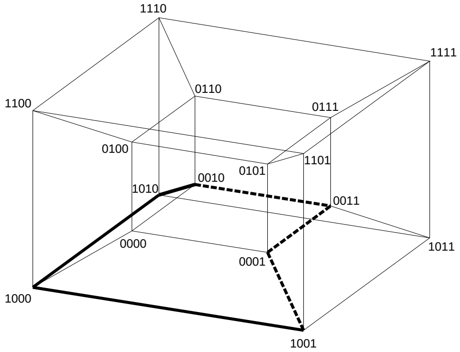

Figure 3: Hypercube with transition patterns for single point crossover of 0010 and 1001

Crossover is therefore not a pure Monte-Carlo process as is mutation, which avoids preliminary convergence of the entire population towards a local minimum and improves solutions close to a minimum at the same time.

A new offspring is generated by crossover with a rate of 0.8 or mutation with a rate of 0.15. After each application of an operator the offspring is evaluated and selected strings are introduced into the chromosome pool for immediate participation. A population size of 60 chromosomes is found to be sufficient, which determines also the number of final solutions.

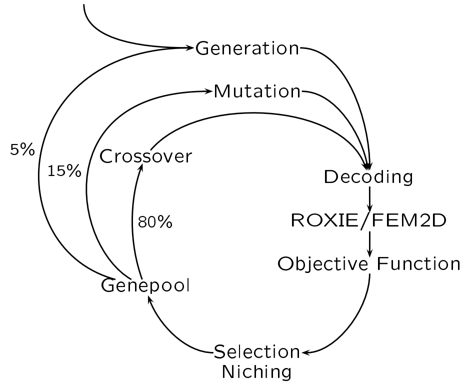

Figure 4: Genetic algorithm with niching

4 RESULTS

Two 6-block coil designs and an alternative 5 block solution were found using genetic algorithms. The two 6 block solutions were studied in detail and compared to the standard 5 block coil cross-section as described in the yellow-book. Table 1 gives the characteristic data for the three de-

Table 1: Characteristic data for the three designs.

	V6-3	V6-1	VY
Turns (coil)	38	40	41
% on LL outer	81.05	84.92	82.5
% on LL inner	86.15	85.64	86.5
PF / MF outer	0.83	0.89	0.87
PF / MF inner	1.03	1.03	1.052
I_{nom} (A) (8.36T)	11879.	11532.	11224.
B_{ss} (T)	9.70	9.76	9.65
L (mH/m)	6.64	7.17	7.47
b_3 (pers)	-4.11	-3.67	-4.17
b_5 (pers)	0.20	0.15	0.21
b_7 (pers)	-0.021	-0.022	-0.036
b_9 (pers)	0.003	0.0035	0.0073
b_3 (geo)	1.0	1.41	0.1
b_5 (geo)	-0.198	-0.1055	-0.19
b_7 (geo)	0.0122	0.0255	0.0342
b_9 (geo)	-0.0087	0.0014	-0.01
b_{11} (geo)	0.0037	0.0029	0.0088
Pole angle (deg)	70.5	70.99	57.4
Pole size (mm)	7.1	7.43	8.7
F_p (N/m)	16400.	17239.	33877.
σa_2	0.637	0.590	0.741
σb_2	0.591	0.572	0.796
σa_3	0.265	0.242	0.304
σb_3	0.239	0.235	0.318

F_p = electro-magnetic force parallel to broad face of cable no. 41 (VY), no. 40 (V6-1), no.38 (V6-3). PF/MF = Peak-field (in the coil) to main-field (in the aperture) ratio. b_n in units of 10^{-4} at a radius of $r = 10$mm. pers = errors due to persistent currents at injection, geo= geometrical field errors.

signs which were studied in detail. The multipole content is given in units of 10^{-4} at a radius of 10mm.

The V6-1 design has a B_{ss} which is about 0.1 T higher than in the VY version. This is remarkable as it can be achieved with 1 turn less. The explanation is the reduced peak-field to main-field ratio in the inner layer. At the same time the margin in the outer layer blocks is reduced with respect to the VY version but is still higher than for the inner layer. The b_{11} is considerably reduced. The radial forces on the two inner turns (turn 39 and 40 for the V6-1 version) are reduced. The random multipole errors were calculated using 500 identically distributed random errors on the block positioning and inclination angles, and their radial positions, between ± 0.05 mm. Analysis of the multipole content of these 500 random magnets yields a normal distribution function where the mean value and the standard deviation σ can be calculated. As can be seen in Table 1 the random multipoles are slightly lower for the V6-1 design.

5 CONCLUSION

Genetic algorithms with niching can be applied efficiently to the design of magnetic devices supplying the designer with a number of local optima, whereas the more cumbersome standard procedure would result in a single solution only. These local optima can then be evaluated in detail. The method therefore supports the designer's creativity. A new dipole coil cross-section was found this way, which proved advantageous compared to previous designs. The results have been validated by a model magnet which was successfully tested in December 1997. It reached 9.2 T at the first and about 9.5 T at the second quench, well above the nominal field of 8.4 T [4].

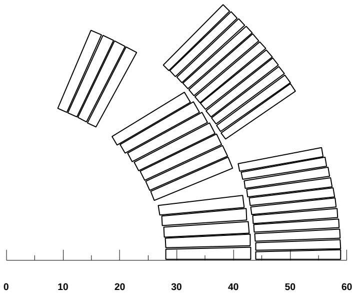

Figure 5: 5-block alternative to the classical 5 block design

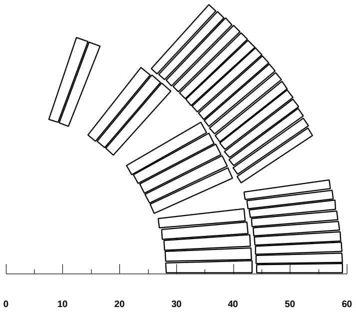

Figure 6: 6-block (40 turns) design (V6-1)

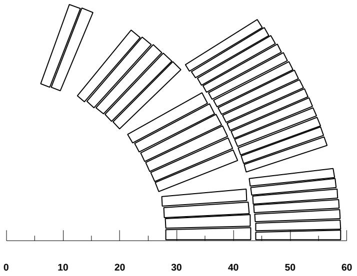

Figure 7: 6-block (38 turns) design (V6-3)

6 REFERENCES

[1] *The Large Hadron Collider, Conceptual Design,* CERN/AC/95-05. CERN, European Organisation for Nuclear Research, 1995.

[2] C. Paul, K. Preis, and S. Russenschuck. Saturation induced field errors in the LHC main dipoles. LHC-project note 84, CERN, 1997.

[3] J. H. Holland. *Adaption in natural and artificial systems.* MIT Press, 1992.

[4] N. Siegel et al. State of the short dipole model program for the LHC. In *EPAC'98, Stockholm,* June 1998.