

## DIRECT $\mathcal{T}$ -VIOLATION MEASUREMENTS AND $\mathcal{T}$ -ODD EFFECTS IN DECAY EXPERIMENTS

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Motivated by the recent experimental announcements for direct measurements of time-reversal non-invariance in the neutral kaon system, we make a comparative discussion of the CPLEAR and KTeV measurements. The most suitable way to consistently incorporate the mixing, the time evolution and the decays of kaons, is to describe the neutral kaon system as a system with a non-Hermitian Hamiltonian. In this framework, the physical (decaying) incoming and outgoing states are distinct and belong to dual spaces. Moreover, since they are eigenstates of the full Hamiltonian, they never oscillate. This is directly manifest in the orthogonality conditions of the physical states, which entirely determine the evolution of the kaon system. Along these lines we conclude: CPLEAR studies  $K^0$ - $\bar{K}^0$  oscillations, a process where initial and final states can be reversed, the CPLEAR asymmetry being an effect directly related to the definition of time-reversal. Conclusively, CPLEAR provides a direct measurement of  $\mathcal{T}$ -violation without any assumption either on unitarity or on  $CPT$ -invariance. The KTeV experiment studies in particular the process  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ , where they measure a  $\mathcal{T}$ -odd effect. However, using unitarity together with estimates of the final state interactions, it should be possible to determine whether this effect can be identified with a genuine  $\mathcal{T}$ -reversal violation .

## 1 Introduction

Recently, the CPLEAR experiment at CERN, reported the first direct observation of time-reversal violation in the neutral kaon system<sup>1</sup>. This observation is made by comparing the probabilities of a  $\bar{K}^0$  state transforming into a  $K^0$  and vice-versa. Moreover, the KTeV experiment at Fermilab, similarly reported evidence for  $\mathcal{T}$ -violation in the decay  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ .<sup>2</sup> In the present note, we will discuss the experimental asymmetries used by both collaborations and interpret their measurements on  $\mathcal{CP}$ ,  $\mathcal{T}$  and/or  $\mathcal{CPT}$ -violation.

The discrete symmetry properties of the neutral kaon system have been extensively studied in the literature<sup>4</sup>. To analyse this issue, in a consistent way one needs to study a system with a non-hermitean Hamiltonian. This is clear, because of the following: Although the physical kaons at rest coincide with the strong interaction (strangeness) eigenstates  $|K^0\rangle = |d\bar{s}\rangle$  and  $|\bar{K}^0\rangle = |\bar{d}s\rangle$ , the latter are not the eigenstates of the the full Hamiltonian. Since however weak interactions do not conserve strangeness (but also allow  $K^0$ - $\bar{K}^0$  oscillations) the full Hamiltonian eigenstates, denoted by  $|K_S\rangle$  and  $|K_L\rangle$ , are different from the strangeness eigenstates, and obey the relations

$$\begin{aligned} H |K_S\rangle &= \lambda_S |K_S\rangle, & |K_S(t)\rangle &= e^{-i\lambda_S t} |K_S\rangle, \\ H |K_L\rangle &= \lambda_L |K_L\rangle, & |K_L(t)\rangle &= e^{-i\lambda_L t} |K_L\rangle, \end{aligned} \quad (1)$$

with  $\lambda_L = m_L - i\Gamma_L/2$  and  $\lambda_S = m_S - i\Gamma_S/2$ , where  $m_{S,L}$  denotes the masses of the physical kaons and  $\Gamma_{S,L}$  their decay widths. The complexity of the eigenvalues, implies the non-hermiticity of the full Hamiltonian of the neutral kaon system.

Non-hermiticity of  $H$  implies that the physical incoming and outgoing states ( $|K_{S,L}^{in}\rangle$  and  $|K_{S,L}^{out}\rangle \equiv \langle K_{S,L}^{out}|^\dagger$  respectively), are not identical, but instead belong to two distinct (dual) spaces<sup>3</sup>. In the Heisenberg representation (where the states are time-independent), the physical incoming and outgoing states coincide with the left- and right-eigenstates of the full Hamiltonian:

$$\begin{aligned} H |K_{S,L}^{in}\rangle &= \lambda_{S,L} |K_{S,L}^{in}\rangle, & \langle K_{S,L}^{in}| H^\dagger &= \langle K_{S,L}^{in}| \lambda_{S,L}^*, \\ H^\dagger |K_{S,L}^{out}\rangle &= \lambda_{S,L}^* |K_{S,L}^{out}\rangle, & \langle K_{S,L}^{out}| H &= \langle K_{S,L}^{out}| \lambda_{S,L}, \end{aligned} \quad (2)$$

where

$$|K_{S,L}^{out}\rangle \equiv \langle K_{S,L}^{out}|^\dagger \neq |K_{S,L}^{in}\rangle, \quad \langle K_{S,L}^{in}| \equiv |K_{S,L}^{in}\rangle^\dagger \neq \langle K_{S,L}^{out}|. \quad (3)$$

Notice that *only if*  $H = H^\dagger$ ,  $\lambda_{S,L} = \lambda_{S,L}^*$  and  $|K_{S,L}^{out}\rangle = |K_{S,L}^{in}\rangle$ , thus the incoming and outgoing states are identical. In the generic case ( $H \neq H^\dagger$ ), the time evolution of the incoming and outgoing states  $|\Psi_I^{in}(t_i)\rangle$  and  $|\Psi_I^{out}(t_f)\rangle$  are obtained from  $|\Psi_I^{in}\rangle$  and  $|\Psi_I^{out}\rangle$ , using the evolution operators  $e^{-iHt_i}$  and  $e^{-iH^\dagger t_f}$  respectively:

$$|K_{S,L}^{in}(t_i)\rangle = e^{-iHt_i} |K_{S,L}^{in}\rangle, \quad |K_{S,L}^{out}(t_f)\rangle = e^{-iH^\dagger t_f} |K_{S,L}^{out}\rangle. \quad (4)$$

From the above equations, follows the evolution of the conjugate states:

$$\langle K_{S,L}^{in}(t_i)| = \langle K_{S,L}^{in}| e^{iH^\dagger t_i}, \quad \langle K_{S,L}^{out}(t_f)| = \langle K_{S,L}^{out}| e^{iH t_f}. \quad (5)$$

An important point to stress here, is that the physical incoming and outgoing eigenstates have to obey *at all times* the orthogonality conditions<sup>3</sup>

$$\langle K_I^{out}(t_f)|K_J^{in}(t_i)\rangle = \langle K_I^{out}|e^{-iH\Delta t}|K_J^{in}\rangle = e^{-i\lambda_I \Delta t} \delta_{IJ}, \quad (6)$$

and in particular for  $\Delta t = 0$

$$\begin{aligned} \langle K_L^{out}|K_S^{in}\rangle &= 0, & \langle K_S^{out}|K_L^{in}\rangle &= 0, \\ \langle K_S^{out}|K_S^{in}\rangle &= 1, & \langle K_L^{out}|K_L^{in}\rangle &= 1. \end{aligned} \quad (7)$$

unlike what has been stated in a wide part of the literature. These conditions express the fact that the Hamiltonian eigenstates cannot oscillate to each-other at any time, and therefore an initial  $|K_S^{in}\rangle$  may not be transformed to a final  $|K_S^{out}\rangle$ . Moreover, it follows that the inner products among incoming (outgoing) states *do not obey* the usual orthogonality conditions

$$\langle K_J^{in} | K_J^{in} \rangle \neq \delta_{IJ} \quad \text{and} \quad \langle K_J^{out} | K_J^{out} \rangle \neq \delta_{IJ} . \quad (8)$$

Finally, in the basis of the states  $K_L$  and  $K_S$ ,  $H$  can be expressed in terms of a diagonal  $2 \times 2$  matrix

$$H = |K_S^{in}\rangle \lambda_S \langle K_S^{out}| + |K_L^{in}\rangle \lambda_L \langle K_L^{out}| , \quad (9)$$

where the unity operator  $\mathbf{1}$  takes the form:

$$\mathbf{1} = \sum_{I=S,L} |K_I^{in}\rangle \langle K_I^{out}| . \quad (10)$$

## 2 Study of discrete symmetries in the neutral kaon system

Having clarified our formalism, we may now proceed to study particle-antiparticle mixing in the neutral kaon system. A convenient representation to study the action of  $\mathcal{CP}$ ,  $\mathcal{T}$  and  $\mathcal{CPT}$ , is the  $K^0, \bar{K}^0$  (particle-antiparticle) base. In this representation

$$\begin{aligned} \mathcal{CP} |K_0^{in}\rangle &= |\bar{K}_0^{in}\rangle , \\ \mathcal{T} |K_0^{in}\rangle &= \langle K_0^{out}| , \\ \mathcal{CPT} |K_0^{in}\rangle &= \langle \bar{K}_0^{out}| . \end{aligned} \quad (11)$$

Without loss of generality, we can express the physical incoming states in terms of  $|K_0^{in}\rangle$  and  $|\bar{K}_0^{in}\rangle$  as:

$$\begin{aligned} |K_S^{in}\rangle &= \frac{1}{N} \left( (1 + \alpha) |K_0^{in}\rangle + (1 - \alpha) |\bar{K}_0^{in}\rangle \right) , \\ |K_L^{in}\rangle &= \frac{1}{N} \left( (1 + \beta) |K_0^{in}\rangle - (1 - \beta) |\bar{K}_0^{in}\rangle \right) , \end{aligned} \quad (12)$$

where  $\alpha$  and  $\beta$  are complex variables associated with  $\mathcal{CP}$ ,  $\mathcal{T}$  and  $\mathcal{CPT}$ -violation (usually denoted by  $\epsilon_S$  and  $\epsilon_L$  respectively), and  $N$  a normalization factor. Then, the respective equations for the outgoing states are not independent, but are determined by the orthogonality conditions for the physical states<sup>3</sup>

$$\begin{aligned} \langle K_S^{out}| &= \frac{1}{N} \left( (1 - \beta) \langle K_0^{out}| + (1 + \beta) \langle \bar{K}_0^{out}| \right) , \\ \langle K_L^{out}| &= \frac{1}{N} \left( (1 - \alpha) \langle K_0^{out}| - (1 + \alpha) \langle \bar{K}_0^{out}| \right) . \end{aligned} \quad (13)$$

where the normalisation factor  $N$  can always be chosen equal to  $N = \sqrt{2(1 - \alpha\beta)}$ <sup>3</sup> Using the equations(9, 12, 13) the Hamiltonian can be expressed in the basis of  $K^0, \bar{K}^0$  as

$$H = \frac{1}{2} \begin{pmatrix} (\lambda_L + \lambda_S) - \Delta\lambda \frac{\alpha - \beta}{1 - \alpha\beta} & \Delta\lambda \frac{1 + \alpha\beta}{1 - \alpha\beta} + \Delta\lambda \frac{\alpha + \beta}{1 - \alpha\beta} \\ \Delta\lambda \frac{1 + \alpha\beta}{1 - \alpha\beta} - \Delta\lambda \frac{\alpha + \beta}{1 - \alpha\beta} & (\lambda_L + \lambda_S) + \Delta\lambda \frac{\alpha - \beta}{1 - \alpha\beta} \end{pmatrix} , \quad (14)$$

where  $\Delta\lambda = \lambda_L - \lambda_S$ .

From eq.(14), we can identify the  $\mathcal{T}$ -,  $\mathcal{CP}$ - and  $\mathcal{CPT}$ - violating parameters. Indeed:

- Under  $\mathcal{T}$ -transformations,

$$\langle K_0^{out} | H | \bar{K}_0^{in} \rangle \leftrightarrow \langle \bar{K}_0^{out} | H | K_0^{in} \rangle ,$$

thus, the off-diagonal elements of  $H$  are interchanged. This indicates that the parameter  $\epsilon \equiv (\alpha + \beta)/2$ , which is related to the difference of the off-diagonal elements of  $H$ , measures the magnitude of the  $T$ -violation<sup>5</sup>.

$$\frac{2}{N^2} \epsilon = \frac{\langle K_0^{out} | H | \bar{K}_0^{in} \rangle - \langle \bar{K}_0^{out} | H | K_0^{in} \rangle}{2 \Delta \lambda} . \quad (15)$$

• Under  $CPT$ -transformations,

$$\langle K_0^{out} | H | K_0^{in} \rangle \leftrightarrow \langle \bar{K}_0^{out} | H | \bar{K}_0^{in} \rangle ,$$

and therefore, the parameter  $\delta \equiv (\alpha - \beta)/2$ , related to the difference of the diagonal elements of  $H$ , measures the magnitude of  $CPT$ -violation.

$$\frac{2}{N^2} \delta = \frac{\langle \bar{K}_0^{out} | H | \bar{K}_0^{in} \rangle - \langle K_0^{out} | H | K_0^{in} \rangle}{2 \Delta \lambda} . \quad (16)$$

• Under  $CP$ -transformation,

$$\langle K_0^{out} | H | K_0^{in} \rangle \leftrightarrow \langle \bar{K}_0^{out} | H | \bar{K}_0^{in} \rangle ,$$

and simultaneously

$$\langle K_0^{out} | H | \bar{K}_0^{in} \rangle \leftrightarrow \langle \bar{K}_0^{out} | H | K_0^{in} \rangle ,$$

thus, *both* the diagonal and the off-diagonal elements of  $H$  are interchanged. Then, the parameters  $\alpha = \epsilon + \delta$  and  $\beta = \epsilon - \delta$ , are the ones which measure the magnitude of  $CP$ -violation in the decays of  $K_S$  and  $K_L$ , respectively.

### 3 $CPLEAR$ direct measurement of time-reversibility

Having identified the  $CP$ ,  $T$  and  $CPT$ -violating operations, one may construct asymmetries that measure discrete symmetry-violations. For instance, a time-reversal operation interchanges initial and final states, with identical positions and opposite velocities:

$$T [ \langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle ] = \langle K_0^{out}(-t_i) | \bar{K}_0^{in}(-t_f) \rangle . \quad (17)$$

Assuming time-translation invariance

$$T [ \langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle ] = \langle K_0^{out}(t_f) | \bar{K}_0^{in}(t_i) \rangle . \quad (18)$$

The time evolution from  $t_i$  to  $t_f$  implies that

$$\langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle = \frac{1}{N^2} (1 - \alpha)(1 - \beta) (e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}) , \quad (19)$$

$$\langle K_0^{out}(t_f) | \bar{K}_0^{in}(t_i) \rangle = \frac{1}{N^2} (1 + \alpha)(1 + \beta) (e^{-i\lambda_S \Delta t} - e^{-i\lambda_L \Delta t}) . \quad (20)$$

Then, by definition, the magnitude of  $T$ -violation is directly related to the Kabir asymmetry<sup>5</sup>

$$\begin{aligned} A_T &= \frac{|\langle K_0^{out}(t_f) | \bar{K}_0^{in}(t_i) \rangle|^2 - |\langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle|^2}{|\langle K_0^{out}(t_f) | \bar{K}_0^{in}(t_i) \rangle|^2 + |\langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle|^2} , \\ &= \frac{|(1 + \alpha)(1 + \beta)|^2 - |(1 - \alpha)(1 - \beta)|^2}{|(1 + \alpha)(1 + \beta)|^2 + |(1 - \alpha)(1 - \beta)|^2} \approx 4 \operatorname{Re}[\epsilon] , \end{aligned} \quad (21)$$

<sup>5</sup>  $2/N^2 \approx 1$ , in the linear approximation.

which is time-independent. Any non-zero value for  $A_T$  signals a direct measurement of  $T$ -violation without any assumption about  $CPT$  invariance. Here, we should note that in linear order in  $\epsilon$  and  $\delta$ , the approximate equality

$$\langle K_S^{in} | K_L^{in} \rangle + \langle K_L^{in} | K_S^{in} \rangle \approx 4Re[\epsilon], \quad (22)$$

holds. This follows directly from the non-orthogonality of the adjoint states  $\langle K_S^{in} |$  and  $\langle K_L^{in} |$  that is manifest in the equations

$$\begin{aligned} \langle K_S^{in} | K_S^{in} \rangle &= \frac{1 + |\alpha|^2}{|1 - \alpha\beta|}, & \langle K_L^{in} | K_L^{in} \rangle &= \frac{1 + |\beta|^2}{|1 - \alpha\beta|}, \\ \langle K_S^{in} | K_L^{in} \rangle &= \frac{\alpha^* + \beta}{|1 - \alpha\beta|}, & \langle K_L^{in} | K_S^{in} \rangle &= \frac{\alpha + \beta^*}{|1 - \alpha\beta|}. \end{aligned} \quad (23)$$

However, although the time-reversal asymmetry can *in the linear approximation* be expressed in terms of only incoming states, the conceptual issue of reversing the time-arrow for any  $T$ -violation measurement is unambiguous. For this reason, the CPLEAR collaboration searched for  $T$ -violation through  $K^0$ - $\bar{K}^0$  oscillations, a process where initial and final states can be interchanged.

CPLEAR produces initial neutral kaons with defined strangeness from proton-antiproton annihilations at rest, via the reactions

$$p\bar{p} \longrightarrow \begin{cases} K^-\pi^+K^0 \\ K^+\pi^- \bar{K}^0, \end{cases} \quad (24)$$

and tags the neutral kaon strangeness at the production time by the charge of the accompanying charged kaon. Since weak interactions do not conserve strangeness, the  $K^0$  and  $\bar{K}^0$  may subsequently transform into each-other via oscillations with  $\Delta S = 2$ . The final strangeness of the neutral kaon is then tagged through the semi-leptonic decays

$$\begin{aligned} K^0 &\rightarrow e^+\pi^-\nu, & \bar{K}^0 &\rightarrow e^-\pi^+\bar{\nu}, \\ K^0 &\rightarrow e^-\pi^+\bar{\nu}, & \bar{K}^0 &\rightarrow e^+\pi^-\nu. \end{aligned} \quad (25)$$

Among them, the first two are characterized by  $\Delta S = \Delta Q$  while the other two are characterized by  $\Delta S = -\Delta Q$  and would therefore indicate either (i) explicit violations of the  $\Delta S = \Delta Q$  rule, or (ii) oscillations between  $K^0$  and  $\bar{K}^0$  that even if  $\Delta S = \Delta Q$  holds, would lead at a final state similar to (i) (with the “wrong-sign” leptons). The CPLEAR experimental asymmetry is given by

$$A_T^{e\pi p} = \frac{\bar{R}_+(\Delta t) - R_-(\Delta t)}{\bar{R}_+(\Delta t) + R_-(\Delta t)},$$

with

$$\begin{aligned} \bar{R}_+(\Delta t) &= |\langle e^+\pi^-\nu(t_f) | \bar{K}_0^{in}(t_i) \rangle \\ &+ \langle e^+\pi^-\nu(t_f) | K_0^{in}(t_f) \rangle \langle K_0^{out}(t_f) | \bar{K}_0^{in}(t_i) \rangle|^2, \end{aligned} \quad (26)$$

$$\begin{aligned} R_-(\Delta t) &= |\langle e^-\pi^+\bar{\nu}(t_f) | K_0^{in}(t_i) \rangle \\ &+ \langle e^-\pi^+\bar{\nu}(t_f) | \bar{K}_0^{in}(t_f) \rangle \langle \bar{K}_0^{out}(t_f) | K_0^{in}(t_i) \rangle|^2. \end{aligned} \quad (27)$$

where the first term in each sum stands for (i) and the second for (ii) (thus containing the kaon oscillations multiplied by the matrix element for semileptonic decays through  $\Delta S = \Delta Q$ ). The experimental asymmetry  $A_T^{e\pi p}$  therefore, besides  $\epsilon$ , also contains the parameters  $x_-$  and  $y$ , where  $x_-$  measures  $\Delta Q = -\Delta S$ , while  $y$  stands for  $CPT$  violation in the decays.

$$A_T^{e\pi p} = 4Re[\epsilon] - 2Re[x_-] - 2Re[y]. \quad (28)$$

In the CPLEAR experiment with the proper experimental normalisations the measured asymptotic asymmetry is <sup>6</sup>:

$$\bar{A}_T^{exp} = 4Re [\epsilon] - 4Re [x_-] - 4Re [y] . \quad (29)$$

The average value of  $\bar{A}_T^{exp}$  was found to be  $= (6.6 \pm 1.6) \times 10^{-3}$ , which is to be compared to the recent CPLEAR measurement of  $(Re [x_-] + Re [y]) = (-2 \pm 3) \times 10^{-4}$ , indicating that the measured asymmetry is related to the violation of time-reversal invariance. Conclusively, CPLEAR made a direct measurement of time-reversal violation, as we had already stated <sup>3</sup>. Similar arguments have been presented <sup>7</sup>, using the density matrix formalism for the description of the kaon system.

An interesting question to ask at this stage, is what information one could obtain from previous measurements plus unitarity <sup>8</sup>. Unitarity implies the relations

$$\begin{aligned} \langle K_L^{in} | K_S^{in} \rangle &= \Sigma_f \langle K_L^{in} | f^{in} \rangle \langle f^{out} | K_S^{in} \rangle , \\ \langle K_S^{in} | K_L^{in} \rangle &= \Sigma_f \langle K_S^{in} | f^{in} \rangle \langle f^{out} | K_L \rangle , \end{aligned} \quad (30)$$

where  $f$  stands for *all* possible decay channels. Making the additional assumption that the final decay modes satisfy the relation  $|f^{in}\rangle = |f^{out}\rangle \gg \langle f^{out} | \dagger$  (which is equivalent to making use of *CPT*-invariance of the final state interactions), it is possible to calculate the sum  $\langle K_L^{in} | K_S^{in} \rangle + \langle K_S^{in} | K_L^{in} \rangle$ , by *measuring only the branching ratios of kaon decays*. This is what can be done in  $K_L, K_S$  experiments, where only the *incoming kaon states* are used. (Note here, however, that in the next section we discuss a  $T$ -odd asymmetry that can be measured in a single decay channel). In the linear approximation, this sum is equal to  $4 Re [\epsilon]$  (see eq. (22)). However, this is an *indirect* determination of  $T$ -violation, and would not have been possible if invisible decays were present. This is to be contrasted with the results of CPLEAR, which use only one out of the possible decaying channels, and does not rely at all on unitarity and or the knowledge of other decay channels than the one used in the analysis <sup>3</sup>.

#### 4 $T$ -odd effects versus $T$ -reversal violation

The KTeV experiment looks at the rare decay  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$  of which they have collected more than 2000 events. In particular, they measure the asymmetry in the differential cross section, with respect to the angle  $\phi$  between the pion and electron planes <sup>9</sup>. To give to the angle an unambiguous sign, they define  $\phi$  according to

$$\sin \phi \cos \phi = (\vec{n}_e + \vec{n}_\pi) \cdot \hat{z} , \quad (31)$$

where  $\vec{n}_e(\vec{n}_\pi)$  is the unit vector in the direction  $\vec{p}_{e^-} \times \vec{p}_{e^+} (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})$ , and  $\hat{z}$  is the unit vector in the direction of the sum of the two pion momenta <sup>9</sup>. A  $T$ -odd observable is one that changes sign under the reversal of all incoming and outgoing three-momenta and polarisations. By construction,  $\phi$  satisfies this property. The operation of  $T$ -reversal, involves in addition to the operations mentioned, a flip of the arrow of time (i.e. exchanging initial and final states). The KTeV collaboration observes an asymmetry of nearly 14% about  $\phi = 0$ , thus identifying a  $T$ -odd effect.

The important issue is to assess when such an effect can be interpreted as a direct measurement of  $T$ -reversal violation, since nowhere have the initial and final states been interchanged <sup>10</sup>. The key ingredient that effectively allows one to invert the arrow of time in such a process, is the hypothesis of the unitarity of the  $S$ -matrix:  $SS^\dagger = 1$ . The  $S$ -matrix can be written in terms of the  $T$ -matrix for a process  $i \rightarrow f$ , as

$$S_{if} = \delta_{if} + iT_{if} , \quad (32)$$

where a *delta*-function for energy-momentum conservation is included in  $T_{if}$ . Unitarity now implies:

$$T_{fi}^* = T_{if} - iA_{if} , \quad (33)$$

where  $T_{fi}$  is the amplitude for a process  $f \rightarrow i$  ( $i, e$  exchanging initial and final states), and  $A_{if}$  is the absorptive part of the  $i \rightarrow f$  process:

$$A_{if} = \sum_k T_{ik} T_{fk}^* , \quad (34)$$

and the sum extends over all possible on-shell intermediate states. Taking the absolute square of (33):

$$|T_{fi}|^2 = |T_{if}|^2 + 2Im(A_{if} T_{if}^*) + |A_{if}|^2 . \quad (35)$$

If  $\bar{i}, \bar{f}$  denote the initial and final states with three-momenta and polarisations reversed,  $\mathcal{T}$ -reversal invariance would imply

$$|T_{fi}|^2 = |T_{\bar{i}\bar{f}}|^2 , \quad (36)$$

and from (35) we can construct

$$\begin{aligned} |T_{if}|^2 - |T_{\bar{i}\bar{f}}|^2 &= -2Im(A_{if} T_{if}^*) - |A_{if}|^2 \\ &+ (|T_{fi}|^2 - |T_{\bar{i}\bar{f}}|^2) . \end{aligned} \quad (37)$$

The left-hand side of (37) is precisely a  $\mathcal{T}$ -odd probability, for instance the one measured by KTeV. However on the right-hand side we have two contributions. The first contribution arises from the terms in the first line corresponding to final-state interactions (for instance the exchange of a photon between the  $\pi$ 's and  $e$ 's) which can affect the dependence on the angle  $\phi$  and generate a  $\mathcal{T}$ -odd effect through  $\mathcal{T}$ -reversal conserving interactions. The other contribution, the last line of (37), is a genuine  $\mathcal{T}$ -reversal violating contribution. To identify a  $\mathcal{T}$ -odd effect with a violation of  $\mathcal{T}$ -reversal, it is thus necessary to estimate the effect of the final state interactions for the process concerned and to determine how big these contributions are with respect to the measured  $\mathcal{T}$ -odd effect. If these effects are small, then we can say that using unitarity (and  $\mathcal{CPT}$  invariance of the final state interactions, which results in  $\langle \pi^+ \pi^- e^+ e^- |^{out} = (|\pi^+ \pi^- e^+ e^- \rangle^{in})^\dagger$ ), we are effectively interchanging the roles of past and future and it is legitimate to identify the  $\mathcal{T}$ -odd effect with a measurement of  $\mathcal{T}$ -reversal violation.

## 5 Conclusions

In the light of the recent data by the CPLEAR and KTeV collaborations, we discuss violations of discrete symmetries in the neutral kaon system, with particular emphasis to  $\mathcal{T}$ -reversal violation versus  $\mathcal{T}$ -odd effects. Since decaying kaons correspond mathematically to a system with a non-hermitean Hamiltonian, we use the dual space formalism, where the physical (decaying) incoming and outgoing states are distinct and dual of each-other. This reflects the fact that the eigenstates of the full Hamiltonian may never oscillate to each-other and have to be orthogonal at all times. The orthogonality conditions of the physical states, entirely determine the evolution of the kaon system. In this framework, we study both the asymmetries reported by CPLEAR and KTeV and conclude the following: CPLEAR, through  $K^0-\bar{K}^0$  oscillations, effectively reverses the arrow of time and thus its measured asymmetry is directly related to the definition of  $\mathcal{T}$ -reversal. Having measured in the same experiment that additional effects which enter in the experimental asymmetry (arising by tagging the final kaon strangeness by semileptonic decays, i.e. violations of the  $\Delta S = \Delta Q$  rule and  $\mathcal{CPT}$ -invariance in the decays) are small, it is concluded that CPLEAR indeed made the first direct measurement of  $\mathcal{T}$ -violation. Since the experiment uses only one out of the possible decaying channels, its results are also independent of any unitarity assumption, and the possible existence of invisible decay modes.

On the other hand, KTeV studies the decay  $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ , which being an irreversible process measures  $\mathcal{T}$ -odd effects. These are not necessarily the same as  $\mathcal{T}$ -violating effects, since they reverse momenta and polarisations but not the time-arrow. It is straightforward to demonstrate that  $\mathcal{T}$ -odd and  $\mathcal{T}$ -violating effects are two different concepts. Non-vanishing  $\mathcal{T}$ -odd effects due to final state interactions, may arise even if unitarity and  $\mathcal{T}$ -invariance hold. However, since unitarity implies the

inversion of the arrow of time, a  $T$ -odd effect could be interpreted as time-reversal violation, provided  $CPT$ -invariance of the final states holds and final state interactions are negligible.

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