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# Rapidity veto effects in the NLO BFKL equation

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## Abstract

We examine the effect of suppressing the emission of gluons which are close by in rapidity in the BFKL framework. We show that, after removing the unphysical collinear logarithms which typically arise in formally higher orders of the perturbative expansion, the effect of the rapidity veto is greatly reduced. This is an important result, since it supports the use of multi-Regge and quasi-multi-Regge kinematics which are implemented in the leading and next-to-leading order BFKL formalism.

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# 1 Eliminating unphysical logarithms

In the case of fixed coupling the leading eigenvalue of the BFKL kernel, to next-to-leading logarithmic accuracy, is determined by solving the equation

$$\omega = \chi(\gamma, \bar{\alpha}_s) = \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) \quad (1)$$

where  $\bar{\alpha}_s = N_c \alpha_s / \pi$ . The leading order kernel is given by

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) \quad (2)$$

and  $\chi_1$  has recently become available [1, 2]:

$$\begin{aligned} \chi_1(\gamma) = & -\frac{1}{8} \left( \frac{11}{3} - \frac{2n_f}{3N_c} \right) \chi_0(\gamma)^2 + \left( \frac{67}{36} - \frac{\pi^2}{12} - \frac{5n_f}{18N_c} \right) \chi_0(\gamma) + \frac{3}{2} \zeta(3) \\ & - \frac{\pi^2 \cos \pi \gamma}{4(1-2\gamma) \sin^2 \pi \gamma} \left( 3 + \left( 1 + \frac{n_f}{N_c^3} \right) \frac{2 + 3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)} \right) \\ & + \frac{\psi''(\gamma) + \psi''(1-\gamma)}{4} + \frac{\pi^3}{4 \sin \pi \gamma} - \phi(\gamma) \end{aligned} \quad (3)$$

where

$$\phi(\gamma) = \sum_{n=0}^{\infty} (-1)^n \left[ \frac{\psi(n+1+\gamma) - \psi(1)}{(n+\gamma)^2} + \frac{\psi(n+2-\gamma) - \psi(1)}{(n+1-\gamma)^2} \right]. \quad (4)$$

We take  $n_f = 3$  although our results depend only very weakly upon  $n_f$ .

For our purposes, it is sufficient to consider the gluon Green's function:

$$f(s, k_1, k_2) = \int \frac{d\omega}{2\pi i} \frac{d\gamma}{2\pi i} \left( \frac{s}{s_0} \right)^\omega \left( \frac{k_1^2}{k_2^2} \right)^\gamma \frac{1}{\omega - \chi(\gamma, \bar{\alpha}_s)} \quad (5)$$

With  $s_0 = k_1 k_2$  the kernel is as given explicitly above.

It has been pointed out that the NLO kernel induces unphysical logarithms in the ratio  $k_1/k_2$  at NNLO and beyond [2, 3]. If  $k_1 \gg k_2$  then the unphysical logs are induced by the singular behaviour of the kernel at  $\gamma = 0$ , i.e.  $\chi_0 \approx 1/\gamma$  and  $\chi_1 \approx -1/(2\gamma^3)$  (the symmetry under interchange of  $k_1$  and  $k_2$  is reflected in the fact that the kernel is even about  $\gamma = 1/2$ ). Although these logarithms lie formally beyond the boundary of the NLO approach, they can induce spurious large effects, especially if the external momenta are strongly ordered ( $k_1 \gg k_2$  or  $k_1 \ll k_2$ ). For consistency with the DGLAP approach we know that they must really be absent. At NLO all spurious logs are indeed cancelled. Salam demonstrated how to extend the NLO kernel so as to guarantee the cancellation of unphysical logs to all orders. The prescription for modifying the kernel is ambiguous and Salam investigated a variety of different schemes [3]. Let us summarise the four schemes used in [3].

In scheme 1, one replaces  $\chi(\gamma, \bar{\alpha}_s)$  of (1),(5) with

$$\begin{aligned}
\bar{\chi}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s [ 2\psi(1) - \psi(\gamma + \omega/2) - \psi(1 - \gamma + \omega/2) ] \\
&+ \bar{\alpha}_s^2 \left[ \chi_1(\gamma) + \frac{1}{2} \chi_0(\gamma) (\psi'(\gamma) + \psi'(1 - \gamma)) \right. \\
&+ \left. \left( \delta_1 + \frac{\pi^2}{6} \right) [\psi(\gamma) + \psi(1 - \gamma) - \psi(\gamma + \omega/2) - \psi(1 - \gamma + \omega/2)] \right. \\
&- \left. \delta_2 [\psi'(\gamma) + \psi'(1 - \gamma) - \psi'(\gamma + \omega/2) - \psi'(1 - \gamma + \omega/2)] \right]
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
\delta_1 &= -\frac{5n_f}{18N_c} - \frac{13n_f}{36N_c^3} \\
\delta_2 &= -\frac{11}{8} + \frac{n_f}{12N_c} - \frac{n_f}{6N_c^3}
\end{aligned} \tag{7}$$

are the coefficients of the singular parts of the NLO kernel, i.e.

$$\text{Lim}_{\gamma \rightarrow 0} \chi_1(\gamma) = -\frac{1}{2\gamma^3} + \frac{\delta_2}{\gamma^2} + \frac{\delta_1}{\gamma}. \tag{8}$$

This new kernel is identical to the NLO kernel to order  $\bar{\alpha}_s^2$  but is completely free from any singularities as  $\gamma \rightarrow 0, 1$ .

The leading  $\omega$ -plane singularity is found by solving

$$\omega = \bar{\chi}(\gamma, \omega, \bar{\alpha}_s). \tag{9}$$

In scheme 2, the new kernel is

$$\begin{aligned}
\bar{\chi}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s \left[ \chi_0(\gamma) - \frac{1}{\gamma} - \frac{1}{1 - \gamma} + \frac{1}{\gamma + \omega/2} + \frac{1}{1 - \gamma + \omega/2} \right] \\
&+ \bar{\alpha}_s^2 \left[ \chi_1(\gamma) + \frac{1}{2} \chi_0(\gamma) \left( \frac{1}{\gamma^2} + \frac{1}{(1 - \gamma)^2} \right) \right. \\
&- \left. \left( \delta_1 + \frac{1}{2} \right) \left( \frac{1}{\gamma} + \frac{1}{1 - \gamma} - \frac{1}{\gamma + \omega/2} - \frac{1}{1 - \gamma + \omega/2} \right) \right. \\
&- \left. \delta_2 \left( \frac{1}{\gamma^2} + \frac{1}{(1 - \gamma)^2} - \frac{1}{(\gamma + \omega/2)^2} - \frac{1}{(1 - \gamma + \omega/2)^2} \right) \right].
\end{aligned} \tag{10}$$

In scheme 3, it is

$$\begin{aligned}
\bar{\chi}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s (1 - \bar{\alpha}_s A) [2\psi(1) - \psi(\gamma + \omega/2 + \bar{\alpha}_s B) - \psi(1 - \gamma + \omega/2 + \bar{\alpha}_s B)] \\
&+ \bar{\alpha}_s^2 \left[ \chi_1(\gamma) + \left( \frac{1}{2} \chi_0(\gamma) + B \right) (\psi'(\gamma) + \psi'(1 - \gamma)) + A \chi_0(\gamma) \right]
\end{aligned} \tag{11}$$

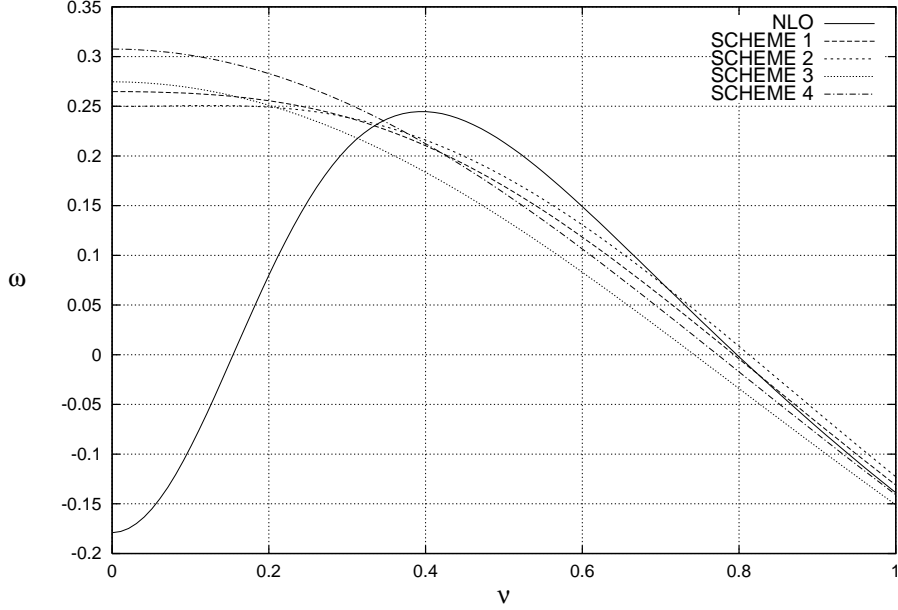


Figure 1: Behaviour of the collinear improved kernels described in the text

where

$$\begin{aligned}
 A &= -\delta_1 - \frac{\pi^2}{6} \\
 B &= -\delta_2
 \end{aligned}
 \tag{12}$$

Finally, in scheme 4, the new kernel is

$$\begin{aligned}
 \bar{\chi}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s \left[ \chi_0(\gamma) - \frac{1}{\gamma} - \frac{1}{1-\gamma} \right. \\
 &+ \left. (1 - \bar{\alpha}_s A') \left( \frac{1}{\gamma + \omega/2 + \bar{\alpha}_s B} + \frac{1}{1 - \gamma + \omega/2 + \bar{\alpha}_s B} \right) \right] \\
 &+ \bar{\alpha}_s^2 \left[ \chi_1(\gamma) + \left( B + \frac{1}{2} \chi_0(\gamma) \right) \left( \frac{1}{\gamma^2} + \frac{1}{(1-\gamma)^2} \right) + A' \left( \frac{1}{\gamma} + \frac{1}{1-\gamma} \right) \right]
 \end{aligned}
 \tag{13}$$

where  $B$  is as above and where

$$A' = -\delta_1 - \frac{1}{2}.
 \tag{14}$$

We note that other schemes have been advocated [4]. It was demonstrated in [3] that the all orders kernel is much better behaved than the NLO kernel. In particular, in schemes 3 and 4 it was shown that the subsequent integral over  $\gamma$  is again dominated by the saddle point at  $\gamma = 1/2$  and that the leading eigenvalue of the kernel does not pick up huge higher order corrections (although they do still lead to a significant reduction in its value). To illustrate

these points, in Fig. 1 we show the behaviour of the all order kernels as a function of  $\nu$  where  $\gamma = 1/2 + i\nu$ , with  $\bar{\alpha}_s = 0.2$ , and compare with the behaviour of the NLO kernel (which no longer has the saddle point at  $\gamma = 1/2$  [5]). It can be seen that for these modest values of  $\bar{\alpha}_s$  the result is not very sensitive to the scheme used. In this paper we follow Salam in maintaining that schemes 3 and 4 are the most realistic schemes and focus upon them.

## 2 The rapidity veto

Imposing the constraint that subsequent gluon emissions must be separated by some minimum interval in rapidity,  $b$ , upon the LO BFKL equation leads to the new kernel:

$$\chi_{\text{veto}}^{\text{LO}}(\gamma, \omega, \bar{\alpha}_s) = \bar{\alpha}_s e^{-b\omega} \chi_0(\gamma). \quad (15)$$

This kernel encodes the rapidity veto to all orders in  $\bar{\alpha}_s$ .

It has been suggested that a veto of  $b \sim 1$  may well be a good approximation to real physics and as such may account for a large part of the NLO corrections to the leading eigenvalue of the kernel [6, 7, 8]. The effect of imposing the veto on the LO BFKL equation leads to a leading eigenvalue which is determined by the solution to

$$\omega = \bar{\alpha}_s e^{-b\omega} \chi_0(\gamma). \quad (16)$$

Expanding in  $\bar{\alpha}_s$  gives

$$\begin{aligned} \omega &\approx (1 - b \omega) \omega_0 \\ \omega &\approx \frac{\omega_0}{1 + b \omega_0} \\ &\approx \omega_0(1 - b \omega_0) \end{aligned} \quad (17)$$

and  $\omega_0 = \bar{\alpha}_s \chi_0(1/2)$ , which is relevant if we assume that the saddle point at  $\gamma = 1/2$  is reliable. This is to be compared with the NLO kernel, which gives

$$\omega = \omega_0(1 - 2.4 \omega_0). \quad (18)$$

If a veto of 2 units were physics then we see that it saturates most of the NLO correction, leaving behind a genuinely small correction. The study of a rapidity veto has been pursued in [9].

At NLO, the imposition of the veto leads to

$$\chi_{\text{veto}}^{\text{NLO}}(\gamma, \omega, \bar{\alpha}_s) = \bar{\alpha}_s e^{-b\omega} [\chi_0(\gamma) + \bar{\alpha}_s \chi_1(\gamma) + \bar{\alpha}_s b \chi_0(\gamma)^2]. \quad (19)$$

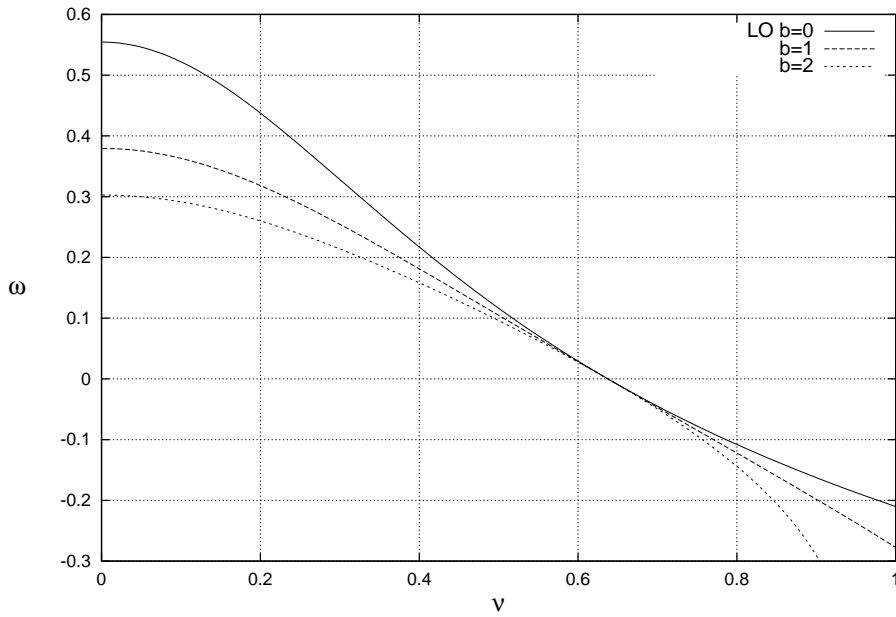


Figure 2: Dependence of the LO kernel plus veto upon  $\nu$ , for  $\bar{\alpha}_s = 0.2$

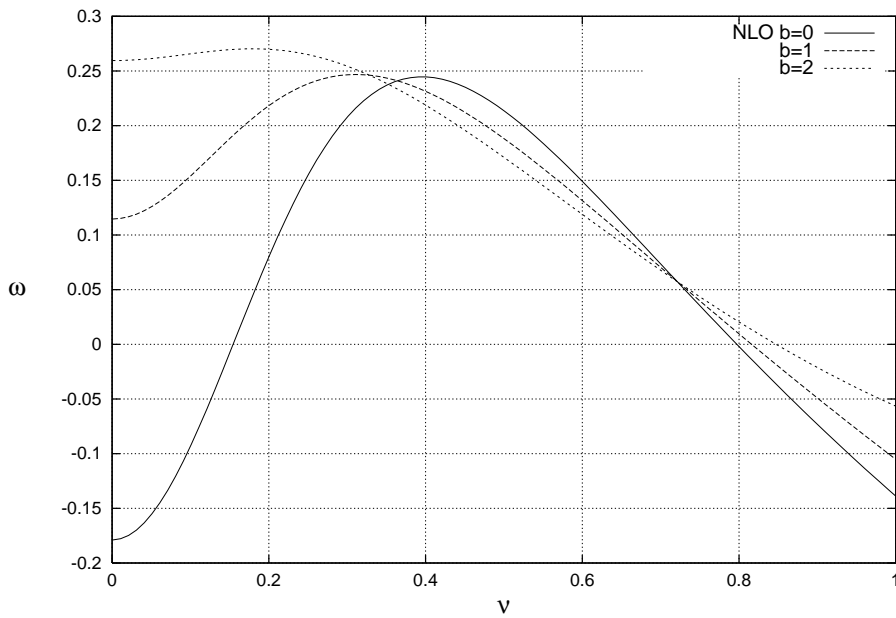


Figure 3: Dependence of the NLO kernel plus veto upon  $\nu$ , for  $\bar{\alpha}_s = 0.2$

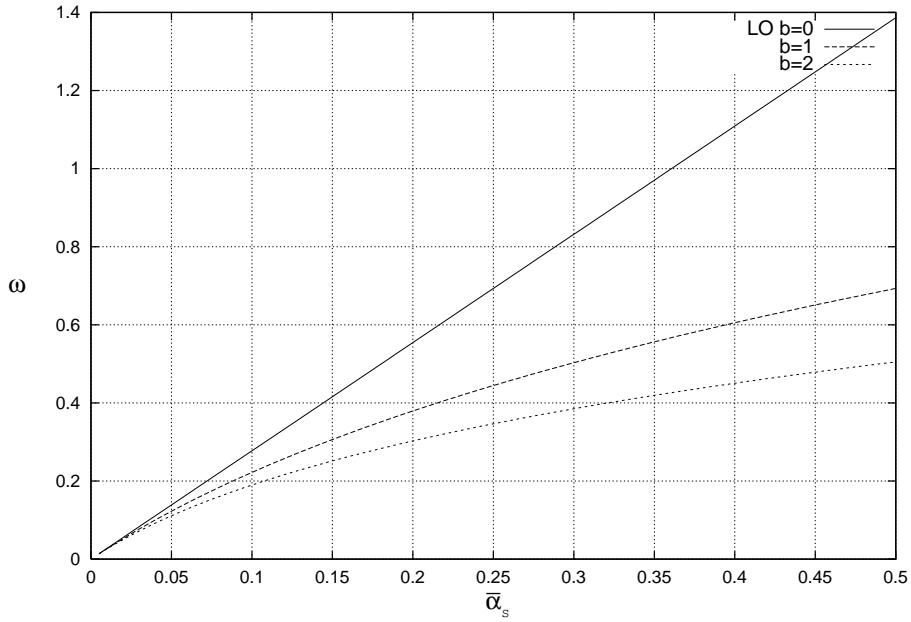


Figure 4: Dependence of the LO kernel plus veto upon  $\bar{\alpha}_s$ , for  $\nu = 0$

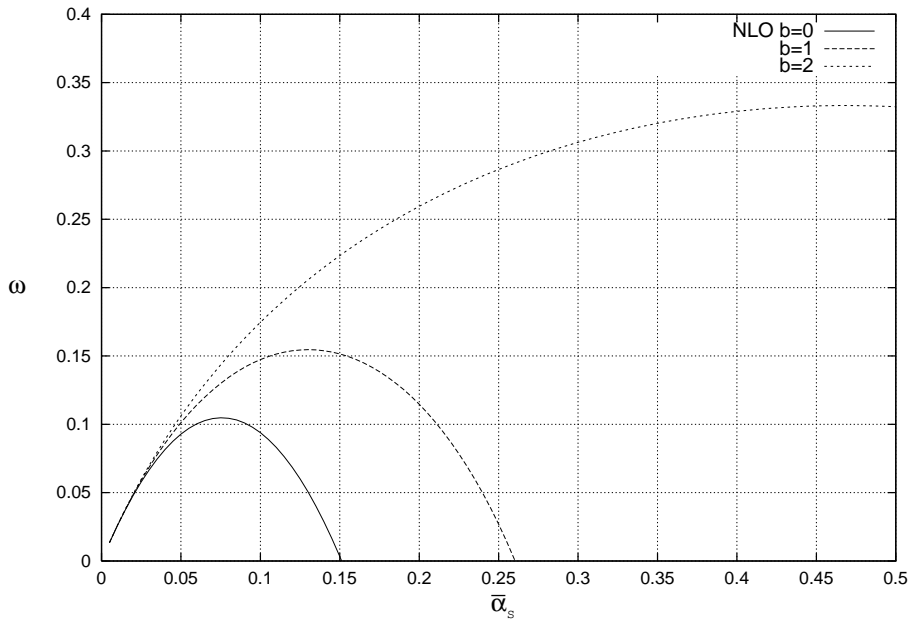


Figure 5: Dependence of the NLO kernel plus veto upon  $\bar{\alpha}_s$ , for  $\nu = 0$

In Figs. 2–5 we show the effect of imposing the veto on the LO and NLO BFKL equations. We show separately the dependence of the kernels upon  $\nu$  and the variation of the kernels evaluated at  $\nu = 0$  with  $\bar{\alpha}_s$ . In each case we show results for three different values of  $b$ ,  $b = 0$  (no veto),  $b = 1$  and  $b = 2$ . Clearly the imposition of the veto has a dramatic effect upon the behaviour of the kernel.

In this paper, we wish to investigate the effect of imposing a rapidity veto in conjunction with the removal of the unphysical logarithms. Our philosophy is as follows. If, after removing the unphysical logarithms, the imposition of a rapidity veto has little effect then we have internal consistency. This is because the BFKL approach relies on the assumption of multi-Regge kinematics, i.e. that successive emissions are infinitely far apart in rapidity, at LO and it assumes that subsequent corrections should be small. It follows that cutting out emissions which are nearby in rapidity should have a relatively small effect. We show that this is indeed the case.

The rapidity veto can be implemented in a way which ensures that the cancellation of the unphysical logarithms is not disturbed. The procedure is scheme dependent, just as in the case without any veto.

For scheme 3 we have

$$\begin{aligned} \chi_{\text{veto}}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s e^{-b\omega} \{ (1 - \bar{\alpha}_s A) [2\psi(1) \\ &\quad - \psi(\gamma + \omega/2 + \bar{\alpha}_s B') - \psi(1 - \gamma + \omega/2 + \bar{\alpha}_s B')] \\ &\quad + \bar{\alpha}_s \left[ \chi_1(\gamma) + b\chi_0(\gamma)^2 + \left( \frac{1}{2}\chi_0(\gamma) + B' \right) (\psi'(\gamma) + \psi'(1 - \gamma)) + A\chi_0(\gamma) \right] \} \end{aligned} \quad (20)$$

where  $A$  is as before but  $B'$  is now

$$B' = -\delta_2 - b, \quad (21)$$

which is required to ensure that the introduction of the rapidity gap does not destroy the elimination of the unphysical logarithms.

For scheme 4 we have:

$$\begin{aligned} \chi_{\text{veto}}(\gamma, \omega, \bar{\alpha}_s) &= \bar{\alpha}_s e^{-b\omega} \left\{ \left[ \chi_0(\gamma) - \frac{1}{\gamma} - \frac{1}{1-\gamma} \right. \right. \\ &\quad \left. \left. + (1 - \bar{\alpha}_s A') \left( \frac{1}{\gamma + \omega/2 + \bar{\alpha}_s B'} + \frac{1}{1 - \gamma + \omega/2 + \bar{\alpha}_s B'} \right) \right] \right. \\ &\quad \left. + \bar{\alpha}_s \left[ \chi_1(\gamma) + b\chi_0(\gamma)^2 + \left( B' + \frac{1}{2}\chi_0(\gamma) \right) \left( \frac{1}{\gamma^2} + \frac{1}{(1-\gamma)^2} \right) + A' \left( \frac{1}{\gamma} + \frac{1}{1-\gamma} \right) \right] \right\}. \end{aligned} \quad (22)$$

These new kernels implement the veto, are matched to NLO and free from any singularities as  $\gamma \rightarrow 0, 1$ .



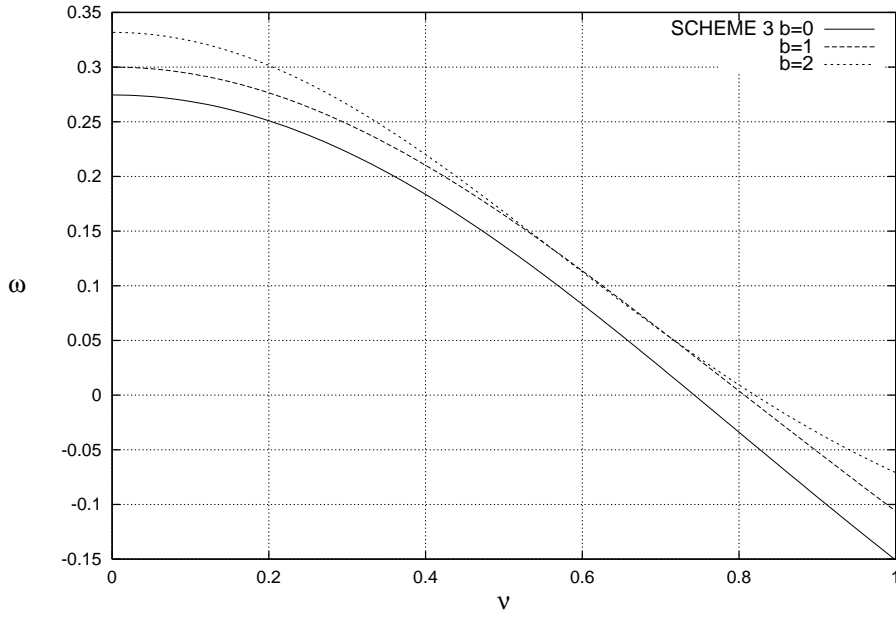


Figure 6: Dependence of the ‘Scheme 3 plus veto’ NLO kernel upon  $\nu$ , for  $\bar{\alpha}_s = 0.2$

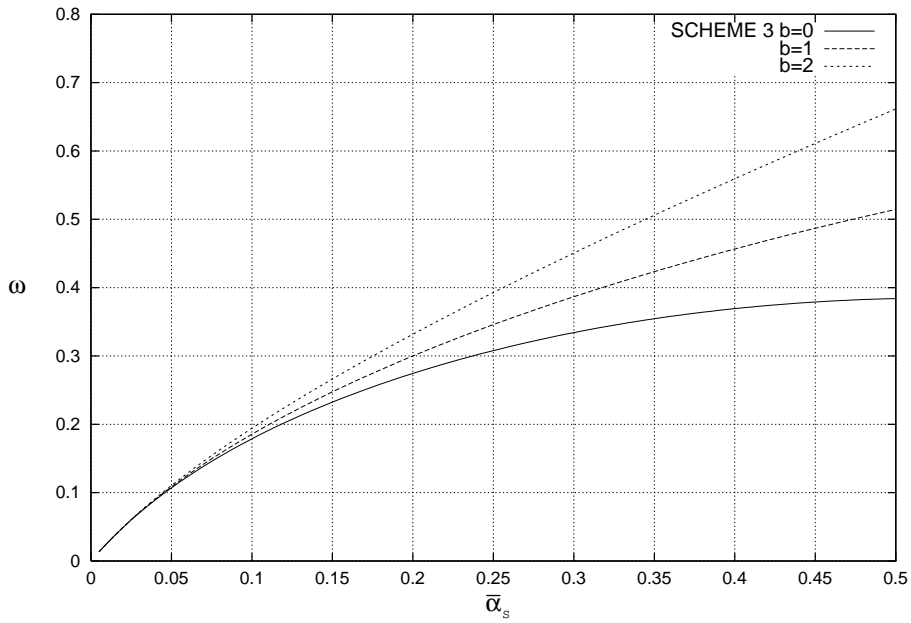


Figure 7: Dependence of the ‘Scheme 3 plus veto’ NLO kernel upon  $\bar{\alpha}_s$ , for  $\nu = 0$

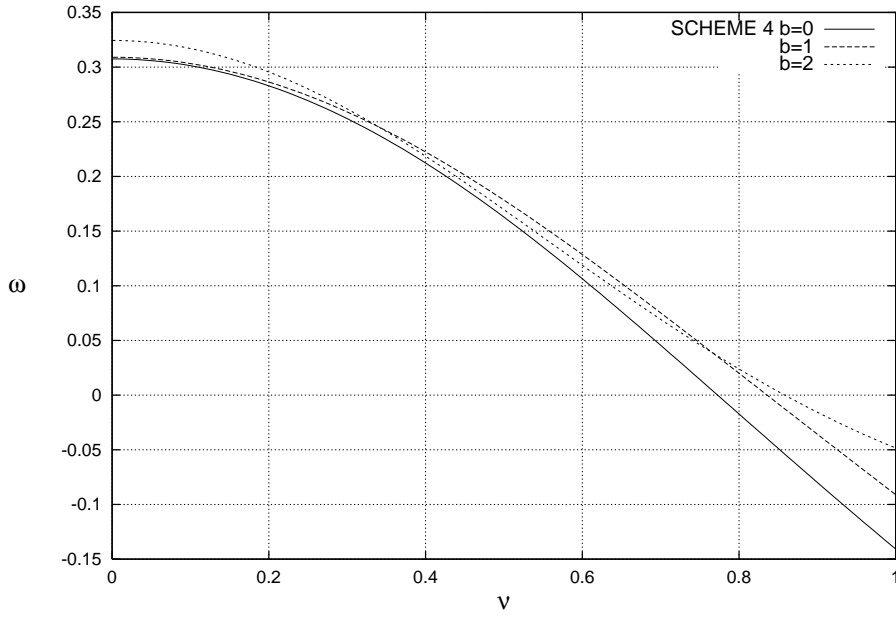


Figure 8: Dependence of the ‘Scheme 4 plus veto’ NLO kernel upon  $\nu$ , for  $\bar{\alpha}_s = 0.2$

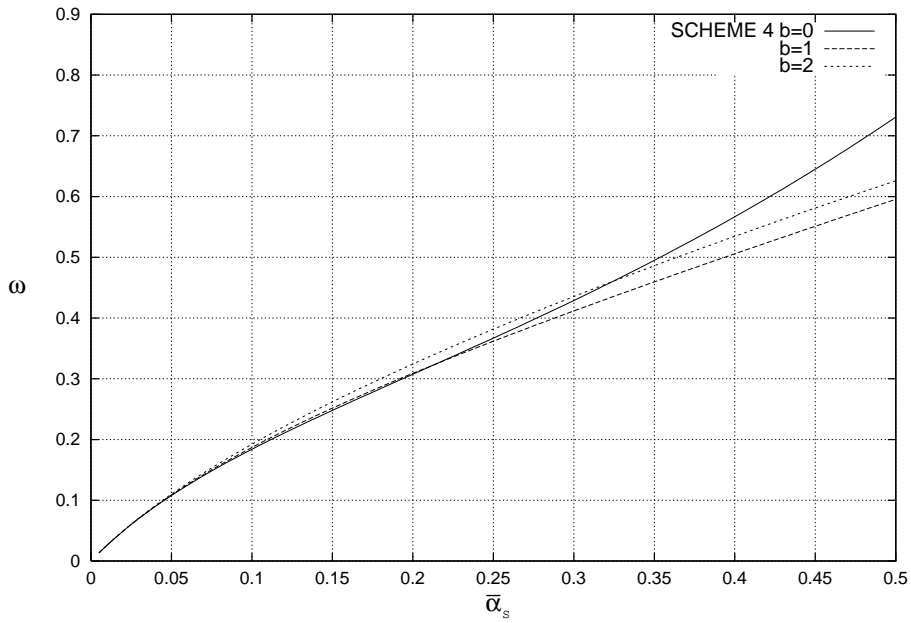


Figure 9: Dependence of the ‘Scheme 4 plus veto’ NLO kernel upon  $\bar{\alpha}_s$ , for  $\nu = 0$

In Figs. 6,7, we do as in Figs. 2–5 but using the new Scheme 3 kernel and in Fig. 8,9 we show the results for the Scheme 4 kernel. Having subtracted the unphysical logarithms we can see clearly that the dependence upon the rapidity veto is greatly diminished. We interpret this as supporting the use of (quasi-)multi-regge kinematics.

In the case of the pure LO and NLO kernels and their collinear corrected variants, the behaviour of the kernel near  $\nu = 0$  drives the asymptotic ( $s/s_0 \rightarrow \infty$ ) behaviour of the Green function of (5). However, the situation is far from clear after implementing the veto.

For definiteness, let us take the kernel of (15). We note that the solution to (16) develops cuts along the line  $\gamma = 1/2$  with branch points at  $\pm\nu_0$ :

$$\chi_0\left(\frac{1}{2} + i\nu_0\right) = -\frac{e^{-1}}{\bar{\alpha}_s b}. \quad (23)$$

However, this cut has no physical origin. It arises because we summed over arbitrarily large numbers of gluon emissions with each emission some minimum distance in rapidity from its neighbours. For any finite energy this is not possible. The result of truncating the number of emissions is to remove the cut. To see this, we first note that after imposing the veto and performing the integral over  $\omega$  the Green function of (5) becomes

$$f(s, k_1, k_2) = \int \frac{d\gamma}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega_0(\gamma)} \left(\frac{k_1^2}{k_2^2}\right)^\gamma \frac{1}{1 + b\omega_0(\gamma)} \quad (24)$$

where  $\omega_0(\gamma)$  is the solution to (16). We can now expand the RHS as a power series in  $\bar{\alpha}_s$  [9]:

$$\frac{e^{\omega_0(\gamma)y}}{1 + b\omega_0(\gamma)} = \sum_{n=0}^{\infty} \frac{[\bar{\alpha}_s \chi_0(\gamma) (y - nb)]^n}{n!} \quad (25)$$

where  $y \equiv \ln(s/s_0)$ . Limiting the number of emitted gluons forces us to truncate the summation of the RHS of (25) and since the only singularities of  $\chi_0(\gamma)$  are poles at  $\gamma = 0, -1, -2, \dots$  it follows that the cut is absent.

Our claim that imposing the veto has a small effect is still not complete. We must also show that the asymptotic behaviour is driven by the behaviour of the kernel near  $\nu = 0$ . This is not as obvious as in the cases without a veto since the solution to (16),  $\omega_0(\gamma)$ , does not fall monotonically as  $\gamma \rightarrow \infty$ . In fact it grows slowly as

$$\omega_0(\gamma) \sim \ln[\ln \gamma(1 - \gamma)].$$

However, the presence of the  $(k_1^2/k_2^2)^\gamma$  term in the integral ensures that the integrand falls monotonically from its maximum in the vicinity of  $\nu = 0$ . For example, for  $k_1 > k_2$  any  $\gamma$ -plane contour which heads to  $\text{Re } \gamma \rightarrow -\infty$ , e.g. the contour  $C$  shown in Fig. 10, is suitable. For large enough  $y$  we can therefore approximate (24) by

$$f(s, k_1, k_2) \approx \frac{e^{\tilde{\omega}_0 y}}{1 + b\tilde{\omega}_0} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \left(\frac{k_1^2}{k_2^2}\right)^{1/2+i\nu} e^{-Ay\nu^2} \quad (26)$$

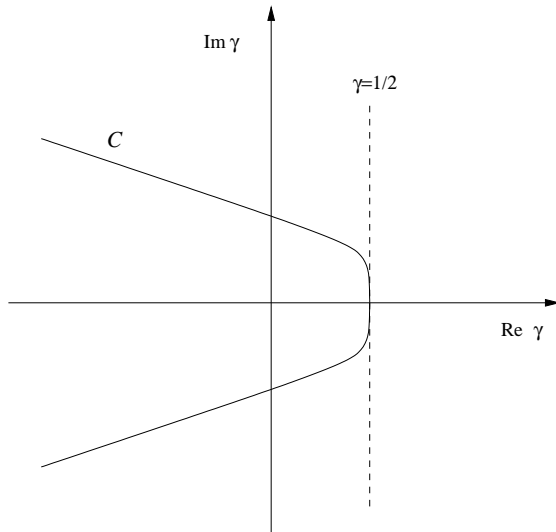


Figure 10: A suitable contour of integration for  $k_1 > k_2$ .

where  $\omega_0(\gamma) \approx \tilde{\omega}_0 - A\nu^2$  in the vicinity of  $\nu = 0$ . The argument runs through in precisely the same fashion for the other kernels which include a veto. We have demonstrated numerically that the results obtained from the approximation (26) are compatible with the results obtained from truncating the series (25) and inserting into (24).

Thus, for large enough  $s/s_0$  the truncated sum can be approximated by the infinite sum<sup>†</sup> and the integral over  $\gamma$  can be approximated using the saddle point method, i.e. the asymptotic behaviour is driven by the value of the kernel at  $\nu = 0$  (i.e.  $\gamma = 1/2$ ). It is this precisely this region of the kernel which we have shown to be unaffected by the imposition of the veto.

### 3 Conclusions

We have demonstrated that, after taking care to eliminate unphysical collinear logarithms in the BFKL formalism, the resulting kernel is insensitive to the imposition of the restriction that successive gluon emissions not be too close together in rapidity. Insensitivity to such a veto supports the application of the (quasi-)multi-Regge kinematics provided that the logarithms in transverse momenta are treated in a consistent manner.

### 4 Acknowledgements

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<sup>†</sup>In practice, the infinite sum is a good approximation down to quite low values of  $y$ , e.g.  $y \gtrsim 1$  for  $\bar{\alpha}_s = 0.2$ .

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