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PRECISION CALCULATION OF BHABHA SCATTERING AT LEP[†]

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For the small-angle Bhabha-scattering process, we consider the error budget for the calculation of the LEP/SLC luminosity in the Monte Carlo event generator BHLUMI 4.04, from the standpoint of new calculations of exact results for the respective $\mathcal{O}(\alpha^2)$ photonic corrections in the context of the Yennie-Frautchi-Suura exponentiation. We find that an over-all precision tag for the currently available program BHLUMI 4.04 can be reduced from 0.11% to 0.061% at LEP1 and from 0.25% to 0.122% at LEP2. For the large-angle Bhabha process, we present the Monte Carlo program BHWIDE and compare its predictions with predictions of other Monte Carlo programs as well as semi-analytical calculations.

1 Introduction

At LEP, for practical purposes, the process of Bhabha scattering, $e^+e^- \to e^+e^-$, is divided into two classes depending on the kinematical regions[a]: the small-angle Bhabha (SABH) scattering, for the scattering angle $1^\circ \lesssim \theta_e \lesssim 6^\circ$, and the large-angle Bhabha (LABH) scattering, for $\theta_e \gtrsim 10^\circ$. In general, this process is mediated by both the γ and Z bosons, exchanged in both s- and t-channels. Thus, at the lowest order, there are 4 pure s- and t-channel contributions and 6 interference terms (between γ and Z and between the s- and

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[a] In the following we shall always consider the case with both the e^+ and e^- detected.

t-channels). At low angles, however, the Bhabha scattering is almost completely dominated ($\gtrsim 99\%$ of a cross section) by the pure QED process of the t-channel γ-exchange for which a very high accuracy in theoretical predictions can be achieved. For this reason SABH was chosen at LEP1 and LEP2 for the luminosity monitoring. At large angles, physical features of the Bhabha process at LEP1 and LEP2 are very different, as different Feynman-diagram contributions dominate at these two energy regimes. At LEP1 energies, a dominating role is played by the s-channel Z-exchange, so this process is used, in parallel to other fermion-pair production, to determine properties of the Z-boson as well as to measure other important electroweak (EW) parameters [1]. At LEP2, LABH is dominated by the t-channel γ exchange, so at these energies it resembles more SABH than the other two-fermion processes. Therefore, LABH is not very useful at LEP2 for testing the EW sector of the Standard Model (SM). It is considered mainly as a background for those processes that are of the main experimental interest at LEP2. The e^+e^--channel is investigated particularly in searches for possible “new physics”, such as SUSY, contact interactions, etc.

Our discussion is organized as follows. In Section 2, we concentrate on the SABH scattering and discuss the new error budget for the Monte Carlo (MC) program BHLUMI 4.04 [2] based on the exact $\mathcal{O}(\alpha^2)$ calculations. The LABH process is discussed in Section 3, where we give a brief description of the MC event generator BHWIDE [3] and discuss its cross-checks and comparisons with those of other programs. Finally, Section 4 contains our conclusions and outlook.

2 Small-Angle Bhabha Scattering

Currently, new luminometers at LEP [4] have given results on the luminosity process $e^+e^- \rightarrow e^+e^- + n(\gamma)$ at experimental precision tags below 0.1%. This should be compared with the prediction by the Kraków-Knoxville Collaboration in the Monte Carlo program BHLUMI 4.04 [2], wherein the theoretical precision tag of 0.11% is realized for this process in the ALEPH SICAL-type [5] acceptance. If one combines the experimental results, one arrives at an experimental precision of $\lesssim 0.05\%$. Evidently, for the final EW precision tests data analysis for LEP1, it would be desirable to reduce the theoretical precision tag on the luminosity cross section prediction, at least to the comparable 0.05%-regime, in order not to obscure unnecessarily the comparison between experiment and the respective Standard Model of the electroweak interaction. With this as our primary motivation, we examined the error budget arrived at in Refs. [1,6] in view of recent exact results impacting both the technical and physical precision of the errors quoted in that budget. More precisely, if one looks into the error budget shown in Table 1 of Ref. [6], one sees that the largest contribution is associated with the $\mathcal{O}(\alpha^2)$ photonic corrections, which contribute 0.1% in quadrature to the total 0.11% quoted for the total precision of the BHLUMI 4.04 prediction in these references for the ALEPH SICAL-type acceptance. Accordingly, we have used the exact results of Refs. [7,8,9] and the exact result of Ref. [10] to make a more realistic estimate of the true size of this dominant error quoted in Refs. [1,6].

In re-examining the photonic corrections used in BHLUMI 4.04 at the $\mathcal{O}(\alpha^2)$, which is the relevant order of the corrections, one needs to look at the approximations made in the matrix element used in the calculation encoded in the program in comparison to available exact results. This will allow us to re-assess the physical precision of the corresponding part of the BHLUMI 4.04 matrix element, which is the exact $\mathcal{O}(\alpha^2)$ LL (leading-log) Yennie-Frautschi-Suura (YFS) exponentiated matrix element. The phase-space integration of two hard photon emission in BHLUMI is exact (the LL approximations are only in the matrix element). Nevertheless, this four-body phase-space integration should be cross-checked with another, independent, exact integration method. This check, which we have recently completed, will allow us to give a more realistic estimate of the technical precision of the realization of the corresponding aspect of the matrix element in BHLUMI 4.04. The net result is a new estimate of the total precision of the prediction of the luminosity cross section by BHLUMI 4.04 at LEP1 and LEP2 energies.

Considering now the exact $\mathcal{O}(\alpha)$ correction to the single hard bremsstrahlung in the luminosity process, we have implemented the results of Ref. [7] into BHLUMI 4.xx and made a systematic study of the net change in the prediction for the luminosity relative to the prediction of BHLUMI 4.04 in which this correction is treated to the LL level. What we find is illustrated in Fig. 1a for the ALEPH SICAL-type acceptance at the Z peak. In the language of the YFS theory, this correction enters the hard-photon residuals as $\bar{\beta}_1^{(2)}$, the $\mathcal{O}(\alpha^2)$ contribution to the one-hard-photon residual $\bar{\beta}_1$. In Fig. 1a, we show the difference between the corresponding LL result in BHLUMI 4.04 and: (1) our exact result as given in Ref. [7], (2) the approximate ansatz in Eq. (3.25) of Ref. [7], (3) the result (NLLB) of Ref. [11], which is supposed to include the dominant non-LL effects, in ratio to the respective Born cross section. What we see is that the BHLUMI 4.04 results are within 0.02% of the exact result throughout the experimentally interesting regime $0.2 \leq 1 - z_{min} \leq 1.0$. This is the main reason why we will be able to reduce the estimated precision of the BHLUMI 4.04 prediction in comparison to Refs. [1,6].

Turning next to the technical precision of the 2γ bremsstrahlung calculation in BHLUMI 4.04, we have constructed a completely independent real-

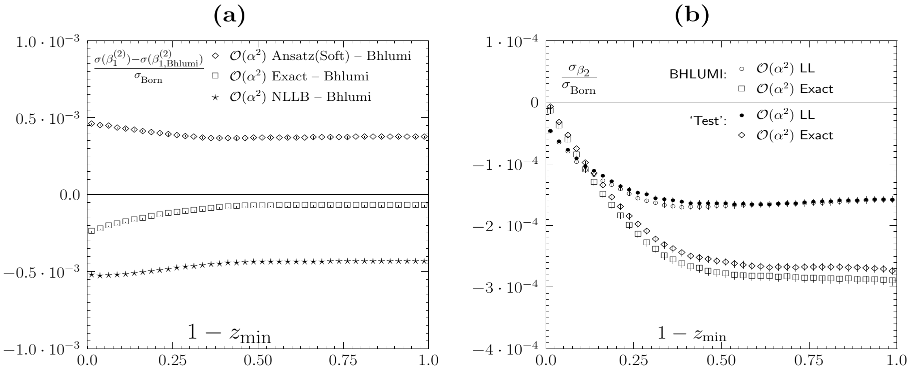

Figure 1: **(a)** The pure second order Monte Carlo result for $\bar{\beta}_1^{(2)} - \bar{\beta}_{1,Bhlumi}^{(2)}$ differences for the SICAL Wide-Narrow trigger, divided by the Narrow-Narrow Born cross section; z_{min} is as it is defined in Fig. 2 of Phys. Lett. **B353** (1995) 362. **(b)** Comparison of Monte Carlo results for $\bar{\beta}_2^{(2)}$ for the LL and exact matrix elements. The results are shown for the BHLUMI generator and for an alternative 'Test' generator for a technical precision test.

ization of the two-photon phase-space integration compared to what is used in BHLUMI 4.04 by way of an independent MC algorithm. We have implemented this new MC realization of the exact two-photon phase-space and compared its result with that of BHLUMI 4.04's for the hard-photon residual $\bar{\beta}_2$ contribution to the luminosity cross section, both for the LL matrix element in BHLUMI 4.04 and for the exact matrix element for the two-photon bremsstrahlung of Refs. [8,9]. What we find is shown in Fig. 1b for the ALEPH SICAL-type acceptance at the Z-peak. We find that the difference between the two realizations of the exact 2γ bremsstrahlung is below 0.003% of the Born cross section. Moreover, we get an estimate of the physical precision of the LL approximation for this part of the cross section from comparing the LL and exact results as 0.012%, in agreement with our estimate in Refs. [1,6].

Finally, we turn to the exact result for the two-loop contribution of the hard-photon residual $\bar{\beta}_0$ to the cross section in comparison to the LL result used for it in BHLUMI 4.04. We have analytically continued the result of Ref. [10] from the s-channel to the t-channel for the required two-loop contribution to the respective charge form factor in QED. In this way, using the YFS theory, we have found that the difference between the LL result in BHLUMI 4.04 and the exact result corresponds to the shift of the function v in Eq. (2) of Ref. [12] by

$$\Delta v^{(2)} = \left(\tfrac{\alpha}{\pi}\right)^2 \left[\left(6 + 6\zeta(3) - \tfrac{45}{8} - \tfrac{\pi^2}{2}\right) L + 6 - 9\zeta(3) + \left(\tfrac{17}{8} - 2\ln 2\right)\pi^2 - \tfrac{8}{45}\pi^4\right],$$

where the big logarithm is defined as $L = \ln|t|/m_e^2$ and $\zeta(3)$ is the Riemann ζ-function of the argument 3. For the ALEPH SICAL-type acceptance at the Z-peak, this corresponds to 0.014% in the cross section.

Collecting the above results in quadrature, we obtain the result that the current calculation of the $\mathcal{O}(\alpha^2)$ photonic corrections in BHLUMI 4.04 are accurate to **0.027**%. Using this result in Table 1 of Ref. [6] we arrive at the precision tag **0.061**% for the currently available calculation in BHLUMI 4.04 at the Z peak. At the LEP2 energy of 176 GeV, if we repeat the analysis just described, we find that the corresponding precision of BHLUMI 4.04, for both the SICAL- and LCAL-type acceptances, is now reduced to **0.122**%, to be compared to the estimate of 0.25% in Refs. [1,6]. The current situation is now summarized in Table 1. A more detailed exposition of the results in this paper will appear elsewhere [13].

	LEP1		LEP2	
Type of correction/error	Past [1,6]	Present	Past [1,6]	Present
(a) Missing photonic $\mathcal{O}(\alpha^2)$ [13]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$ [14]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [15,16]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [17]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [18]	0.015%	0.015%	0.0%	0.0%
Total	0.11%	0.061%	0.25%	0.122%

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within 1°–3°, and for LEP2 it covers energies up to 176 GeV, and the angular ranges within 1°–3° and 3°–6°.

Our result on the size of the error associated with the missing sub-leading bremsstrahlung correction at $\mathcal{O}(\alpha^2)$ in BHLUMI 4.04, which is 0.027%, agrees with the estimate of 0.03% made by Montagna *et al.* [19], using a structure function convolution of a hard collinear external photon with an acollinear internal photon. As these authors have argued, while such a pairing of convolutions does not represent a complete set of photonic $\mathcal{O}(\alpha^2 L)$ corrections, one expects it to contain the bulk of such corrections. Indeed, our exact result of 0.027% shows that the approximation made in Ref. [19] does give the bulk of the respective $\mathcal{O}(\alpha^2 L)$ correction. Evidently, the fact that we now have two independent results, one exact, that presented by us in this paper, and one approximate, that in Ref. [19], which agree on the size of the error associated with the missing photonic $\mathcal{O}(\alpha^2 L)$ correction in BHLUMI 4.04, enhances the results in this paper.

3 Large-Angle Bhabha Scattering

At LEP1, the main physical quantities measured in LABH are: the total cross section σ_e and the forward–backward charge asymmetry $\mathcal{A}_{FB}$. A value of σ_e is used to extract the partial Z decay width Γ_e, while $\mathcal{A}_{FB}$ is sensitive to the important EW parameters, such as the top and Higgs masses. The experimental precision for LABH, after the final LEP1 data analysis is completed, is expected to be $\lesssim 0.5\%$ at the Z peak and $\sim 1\%$ at ± 2 GeV off the peak[20]. At LEP2, the e^+e^--channel is considered mainly for the "new physics" searches, and there, the experimental precision is expected to reach 0.5%.[21]. On the theory side, several programs (both the MC and semi-analytical ones) for LABH have been developed, see e.g. Ref.[1], but a comprehensive analysis of the theoretical error is still missing[b]. An important step in this direction was made during the '95 Workshop "Physics at LEP2"[1], where comparisons of several codes for LABH at LEP1 and LEP2 energies were performed. They showed, however, that the predictions of various programs can differ by as much as 2% at LEP1 and 4% at LEP2. In this section, we briefly describe our MC event generator for LABH called BHWIDE and discuss some important cross-checks of the program as well as comparisons of its predictions with the results of other programs.

BHWIDE is based on the YFS exclusive exponentiation procedure[22], where all the IR singularities are summed-up to infinite order and cancelled out properly in the so-called YFS form factor. The remaining non-IR residuals, $\bar{\beta}_n^{(l)}$, corresponding to the emission of n-real photons, are calculated perturbatively up to a given order l, where $l \geq n$, and $(l-n)$ is a number of loops in the $\bar{\beta}_n^{(l)}$ calculation. In BHWIDE an arbitrary number n of real photons with non-zero p_T are generated according to the YFS MC method of Ref.[23]. The non-IR residuals $\bar{\beta}_n^{(l)}$ are calculated up to $\mathcal{O}(\alpha)$, i.e. $\bar{\beta}_0^{(1)}$ and $\bar{\beta}_1^{(1)}$ corresponding to zero-real (one-loop) and one-real (zero-loop) photons, respectively, are included. In $\bar{\beta}_0^{(1)}$ we implemented two libraries of the $\mathcal{O}(\alpha)$ virtual EW corrections: (1) the older one of Ref.[24], which is not up to date but can be useful for some tests, and (2) the more recent one of Ref.[25]. When the genuine weak corrections are switched off (or numerically negligible) they are equivalent. In $\bar{\beta}_0^{(1)}$ we implemented two independent matrix elements for single-hard-photon radiation: (1) our calculation[3] in terms of helicity amplitudes, and (2) the formula of CALKUL[26] for the squared matrix element. We have checked that the above two representations agree numerically up to at least 6 digits on an event-by-event basis. This constitutes a very important technical cross-check

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[b] Recently, the analysis of the theoretical errors of two semi-analytical programs, ALIBABA and TOPAZ0, has been presented[20], but only for the LEP1 energies and for the so-called BARE acceptance.

of the implementation of the hard-photon matrix element in BHWIDE.

The MC algorithm of BHWIDE is based on the algorithm of the program BHLUMI for SABH[23] with a few important modifications: (1) QED interferences between the electron and positron lines ("up-down" interferences) had to be reintroduced as they are important in LABH; (2) the full YFS form factor for the $2 \rightarrow 2$ process, including all s-, t- and u-channels, was implemented[3]; (3) the exact $\mathcal{O}(\alpha)$ matrix element for the full BHABHA process was included. The multiphoton radiation is generated at the low-level MC stage as for the t-channel process, while the s-channel as well as all interferences are reintroduced through appropriate MC weights. This means that the program is more efficient when the t-channel contribution is dominant, as e.g. at LEP2 energies; however, it proved to work well also at the Z peak.

Having all necessary ingredients in the program and an appropriate algorithm for the MC event generation, we had to subject BHWIDE to several tests in order to check that it gives correct predictions. First, we wanted to know whether it reproduces the small-angle limit correctly. To this end we compared BHWIDE with BHLUMI 4.04[2] whose precision in the SABH regime, as shown in the previous section, is under the per-mille level. We found that for the angular acceptance of $1^\circ < \theta_e < 10^\circ$ the two programs agree within 0.1% (statistical error) for both the pure $\mathcal{O}(\alpha)$ QED corrections and the full YFS exponentiated cross sections at the energies of 5, 10, and 91.19 GeV. Then, we turned to large angles ($40^\circ < \theta_e < 140^\circ$) and compared the pure QED (Z-exchange switched off) $\mathcal{O}(\alpha)$ predictions of BHWIDE with the ones of the MC program OLDBIS[23] (a modernized version of the program OLDBAB[27]) whose technical precision was shown to be at the level of 0.02%[23]. We found an agreement between these two programs up to 0.05% (stat. error) for both the BARE and CALO acceptances as defined in Ref.[1] at the Z-peak energy. This should also remain valid at LEP2 energies, since without the Z contribution there is no qualitative difference in LABH at these two energy regimes. The above result is a very important technical cross-check of BHWIDE, both for the implementation of the $\mathcal{O}(\alpha)$ QED corrections and for the correctness of the MC algorithms at large angles (the $\mathcal{O}(\alpha)$ result was extracted in BHWIDE from the full multiphoton YFS calculation).

Finally, with all contributions/corrections at $\mathcal{O}(\alpha)^{YFS}_{exp}$ included, we compared BHWIDE with several MC and semi-analytical programs for LABH. These comparisons were first presented in Ref.[1], and then updated in Ref.[3]. At LEP1 energies, the program TOPAZ0[28] is considered a semi-analytical benchmark for LABH. Its theoretical errors for the BARE-type acceptance, as recently estimated[20], are 0.4% (0.2%) at the Z peak and 0.3% (0.7%) on the wings (±2 GeV off the peak) for the maximum acollinearity of the final e^+ and

e^-: $\theta_{acol}^{max} = 10^\circ$ (25°). From the tables and figures of Refs. [1,3], we observe that for the BARE trigger BHWIDE agrees with TOPAZ0 within 0.4% (0.55%) at the Z peak and within 0.4% (0.95%) on the wings, for $\theta_{acol}^{max} = 10^\circ$ (25°). For the CALO trigger the agreement between these two programs is within 0.15% (0.25%) at the Z peak and 0.2% (0.55%) on the wings. We can see from the above results that the agreement between the two calculations improves considerably, as it should, for the CALO-type acceptance, which is closer to the real experiment.

For LEP2 energies, the comparisons in Refs. [1,3] were done for the CALO trigger only. Here, the discrepancies between various calculations are larger than at LEP1. This can be explained by the fact that most of the programs were constructed for the Z peak region, i.e. assuming that the s-channel contribution is dominant, while at the LEP2 energies this is not the case. Here, a program designed for the t-channel process, as e.g. SABSPV [19], should be more reliable. The results in Refs. [1,3] show that BHWIDE is within 1.5% of SABSPV for the whole LEP2 energy range. We have also checked that for the BARE trigger BHWIDE agrees with the semi-analytical code ALIBABA [25] within 0.3% at the pure $\mathcal{O}(\alpha)$ level and within 1% when all corrections are included, for the same energy range.

From the above comparisons we can see that BHWIDE is in a better agreement with the semi-analytical benchmarks for LEP1 and LEP2 energies than any other MC event generator for LABH. A more detailed analysis of the BHWIDE theoretical precision is in progress now.

4 Conclusions and Outlook

We have discussed some aspects of the Bhabha scattering at LEP1 and LEP2. For the small-angle Bhabha process, we have presented the new error budget of the program BHLUMI 4.04 based on the exact $\mathcal{O}(\alpha^2)$ calculations. We have shown that the theoretical error for the luminosity measurement can be reduced now from 0.11% to 0.061% at LEP1 and from 0.25% to 0.122% at the LEP2 energy of 176 GeV. The predictions of BHLUMI 4.04 remain unchanged. The exact calculations can be included in the future version of the program if necessary. For the large-angle Bhabha process, we have presented the MC event generator BHWIDE and discussed some of its cross-checks and comparisons with other programs. From this we conclude that BHWIDE is the most precise MC event generator for LABH at LEP1 and LEP2. A comprehensive analysis of its theoretical errors is in progress.

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