On Asymptotic Freedom and Confinement from Type-IIB Supergravity

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Abstract

We present a new type-IIB supergravity vacuum that describes the strong coupling regime of a non-supersymmetric gauge theory. The latter has a running coupling such that the theory becomes asymptotically free in the ultraviolet. It also has a running theta angle due to a non-vanishing axion field in the supergravity solution. We also present a worm-hole solution, which has finite action per unit four-dimensional volume and two asymptotic regions, a flat space and an $AdS^5 \times S^5$. The corresponding $\mathcal{N} = 2$ gauge theory, instead of being finite, has a running coupling. We compute the quark-antiquark potential in this case and find that it exhibits, under certain assumptions, area-law behaviour for large separations.

1 Introduction

One of the well-known vacua of type-IIB supergravity is the $AdS_5 \times S^5$ one, first described in [1]. The non-vanishing fields here are the metric and a Freund-Rubin type anti-self-dual five-form. This background has received a lot of attention recently due to its conjectural connection to $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills (SYM) theory at large N [2, 3]. According to this conjecture [2], the large N limit of certain superconformal field theories (SCFT) can be described in terms of Anti de-Sitter (AdS) supergravity and correlation functions of the SCFT theory which lives in the boundary of AdS can be expressed in terms of the bulk theory [3]. In particular, the four-dimensional $\mathcal{N} = 4$ $SU(N)$ SYM theory is described by the type-IIB string theory on $AdS_5 \times S^5$, where the radius of both the AdS_5 and S^5 are proportional to N.

However, one is interested for obvious reasons in non-supersymmetric YM theories. In particular, the existence of supergravity duals of such theories will help in understanding their strong-coupling behaviour. There are a number of proposals towards this direction. One of them is within the type-II0 theories [4, 5, 6] which, although non-supersymmetric, are consistent string theories [7]. These theories have a tachyon in their spectrum due to lack of supersymmetry. The tachyon is coupled to the other fields of the theory, namely, the graviton, the dilaton and the antisymmetric-form fields. In particular, the coupling of the tachyon to the dilaton is such that it drives the latter to smaller values in the ultraviolet (UV), a hint for asymptotic freedom.

However, a different approach has been proposed in [8]. According to this, *the supergravity duals of non-supersymmetric gauge theories are non-supersymmetric background solutions in type-IIB theory.* Based on this, a solution with a non-constant dilaton which corresponds to a gauge theory with an UV-stable fixed point has been found [8]. The coupling approaches the fixed point with a power-law behaviour. The solution is valid for strong 't Hooft coupling g_H^2 which is consistent with the fact that there are no known perturbative field theories with UV-stable fixed point. This scenario has also been followed in [9]. The same power law behaviour as in [8] have also been found in the case of an $\mathcal{N} = 2$ boundary gauge theory from a supergravity vacuum with both D3- and D7-branes [10].

2 A non-supersymmetric solution

Here, we will try to describe a celebrated feature of gauge theories, namely the asymptotic freedom, within the type-IIB theory. As we will see, this is possible if we make a "minimal" extension to the solution we found in [8] by turning on the other scalar field of type-IIB, the axion. The anti-self-dual five-form F_5 is given by the Freund-Rubin type ansatz which is explicitly written as

$$F_{\mu\nu\rho\kappa\lambda} = -\frac{\sqrt{\Lambda}}{2}\epsilon_{\mu\nu\rho\kappa\lambda} , \qquad \mu,\nu,\ldots = 0, 1, \ldots, 4 ,$$

$$F_{ijkpq} = \frac{\sqrt{\Lambda}}{2}\epsilon_{ijkpq} , \qquad i,j,\ldots = 5,\ldots,9 , \tag{1}$$

and it is clearly anti-self dual. We will also assume for the metric, four-dimensional Poincaré invariance $ISO(1,3)$ since we would like a gauge theory defined on Minkowski space-time. In addition, we will preserve the $SO(6)$ symmetry of the supersymmetric $AdS_5 \times S^5$ vacuum. As a result, the $ISO(1,3) \times SO(6)$ invariant ten-dimensional metric, in the Einstein frame is of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{ij}dx^i dx^j , \tag{2}$$

where

$$g_{\mu\nu}dx^\mu dx^\nu = dr^2 + K(r)^2 dx_\alpha dx^\alpha , \qquad \alpha = 0,1,2,3 , \tag{3}$$

and g_{ij} is the metric on S^5. The dilaton and the axion, by $ISO(1,3) \times SO(6)$ invariance, can only be a function of r. In the Euclidean regime, the supergravity equations for the the metric, the dilaton and the axion in the Einstein frame follow from the action [11]

$$S = \frac{1}{\kappa^2}\int d^{10}x\sqrt{g}\left(R - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2\right) . \tag{4}$$

Note the minus sign in front of the axion kinetic term which is the result of the Hodge-duality rotation of the type-IIB nine-form [11]. Then, the field equations, taking into account the anti-self-dual form as well, are

$$R_{MN} = \frac{1}{2}\partial_M\Phi\partial_N\Phi - \frac{1}{2}e^{2\Phi}\partial_M\chi\partial_N\chi + \frac{1}{6}F_{MKLPQ}F_N{}^{KLPQ} , \tag{5}$$

$$\nabla^2\Phi = -e^{2\Phi}(\partial\chi)^2 , \tag{6}$$

$$\nabla^2\chi = 0 . \tag{7}$$

The equation of motion of the five-form is the anti- self-duality condition which is satisfied for the ansatz (1). For the particular case we are considering here, the above equations turn out to be

$$R_{\mu\nu} = -\Lambda g_{\mu\nu} + \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}e^{2\Phi}\partial_\mu\chi\partial_\nu\chi , \tag{8}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = -e^{2\Phi}\partial_\nu\chi\partial_\mu\chi g^{\mu\nu} , \tag{9}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}e^{2\Phi}\partial_\nu\chi\right) = 0 , \tag{10}$$

and

$$R_{ij} = \Lambda g_{ij} . \tag{11}$$

The equation (11) is automatically solved for a five sphere of radius $2/\sqrt{\Lambda}$ and a first integral of the axion equation (10) is

$$\chi' = \chi_0 K^{-4}e^{-2\phi} , \tag{12}$$

where the prime denotes derivation with respect to r, and χ_0 is a dimensionless integration constant. With no loss of generality it can be taken to be positive by appropriately changing the sign of r. Using this expression for χ in (9) we obtain the differential equation

$$K^4(K^4\Phi')' = -e^{-2\Phi}\chi_0^2 , \qquad (13)$$

a first integral of which is given by

$$K^8\Phi'^2 = \chi_0^2 e^{-2\Phi} + \mu . \qquad (14)$$

Equations (12) and (14) are enough to proceed and solve for the function $K(r)$ that appears in the metric (3). The non-zero components of the Ricci tensor for the metric (3) are

$$
\begin{aligned}
R_{\rho\rho} &= -4\frac{K''}{K^2} , \\
R_{\alpha\beta} &= -\eta_{\alpha\beta}\left(KK'' + 3K'^2\right) ,
\end{aligned}
\qquad (15)
$$

and (8) is equivalent to the following differential equation

$$K'^2 = \frac{\mu}{24} K^{-6} + \frac{\Lambda}{4} K^2 . \qquad (16)$$

The solution of the above equation for $\mu = \alpha^2 > 0$, as a function of r, is

$$K^4 = \frac{\alpha}{\sqrt{6\Lambda}} \sinh\left(2\sqrt{\Lambda}(r_0 - r)\right) , \qquad r \le r_0 , \qquad (17)$$

where r_0 is an integration constant. It should be noted that the case $\mu < 0$ gives rise to a dilaton with bad asymptotics and it will not be considered here. Substituting the expression in (17) in (14) and (12) we find that the string coupling and the axion are given by[1]

$$e^\Phi = \frac{\chi_0}{2\alpha}\left(\left(\coth\sqrt{\Lambda}(r_0 - r)\right)^{\sqrt{3}/2} - \left(\tanh(\sqrt{\Lambda}(r_0 - r))\right)^{\sqrt{3}/2}\right) , \qquad (18)$$

$$\chi = -\frac{\alpha}{\chi_0} \frac{\left(\coth\sqrt{\Lambda}(r_0 - r)\right)^{\sqrt{3}} + 1}{\left(\coth\sqrt{\Lambda}(r_0 - r)\right)^{\sqrt{3}} - 1} . \qquad (19)$$

It is convenient to identify the parameters α and Λ in such a way that in the limit $r \to -\infty$ the Einstein metric becomes that of $AdS_5 \times S^5$. This requires that $\alpha = 4\sqrt{6}R^3 e^{-4r_0/R}$ and $\Lambda = 4/R^2$. On the other hand, r cannot take arbitrarily large values since at $r = r_0$

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[1]The computation is greatly facilitated if we first change variables as

$$z = \int K^{-4}dr = \sqrt{\frac{3}{2\alpha^2}} \ln\coth\left(\sqrt{\Lambda}(r_0 - r)\right) .$$

the function $K(r)$ has a zero and the space is singular both in Einstein and in string frame. The presence of this singularity makes the extrapolation from the UV region, where the space is $AdS_5 \times S^5$, to the IR region problematic and it is ultimate related to the lack of supersymmetry. As we will see next, there are supersymmetric backgrounds which are non-singular in the sense that there are geodesically complete.

3   A supersymmetric solution

The aspect we would like to address here is supersymmetry. For this we need the fermionic variations [11] which in our background are written as

$$
\delta\lambda = -\frac{1}{2}\left(e^{\Phi}\chi' - \Phi'\right)\gamma_r\epsilon^* \,,
$$

$$
\delta\lambda^* = -\frac{1}{2}\left(e^{\Phi}\chi' + \Phi'\right)\gamma_r\epsilon \,,
\tag{20}
$$

$$
\delta\psi_M = \left(\nabla_M + \frac{1}{4}e^{\Phi}\partial_M\chi - \frac{\sqrt{\Lambda}}{4}\gamma_M\right)\epsilon \,,
$$

$$
\delta\psi_M^* = \left(\nabla_M - \frac{1}{4}e^{\Phi}\partial_M\chi - \frac{\sqrt{\Lambda}}{4}\gamma_M\right)\epsilon^* \,,
\tag{21}
$$

where λ, ψ_M are the dilatino and the gravitino, respectively and γ_M are the γ-matrices. We may easily check, using (14), that the background is supersymmetric for

$$
\mu = 0 \,, \qquad \chi' = \pm e^{-\Phi}\Phi' \,,
\tag{22}
$$

In that case, there exist Killing spinors $\epsilon = e^{\pm\Phi/4}\zeta$ where ζ are Killing spinors on the (Euclidean) $AdS^5 \times S^5$. By choosing $\chi' = e^{-\Phi}\Phi'$, the background breaks half of the supersymmetries and we find that

$$
K(r) = e^{-r/R} \,,
\tag{23}
$$

$$
e^{\Phi} = g_s + \frac{R}{4}\chi_0 e^{4r/R} \,.
\tag{24}
$$

$$
\chi = a - e^{-\Phi} \,,
\tag{25}
$$

After changing coordinates as $\rho = Re^{r/R}$, the metric for the supersymmetric solution, in the string frame becomes

$$
ds^2 = \left(g_s + \frac{\chi_0}{4R^3}\rho^4\right)^{1/2}\left(\frac{R^2}{\rho^2}\left(d\rho^2 + dx^\alpha dx_\alpha\right) + R^2 d\Omega_5^2\right)
\tag{26}
$$

$$
= R^2\left(\frac{\chi_0}{4R^3}\right)^{1/2}\left(1 + \frac{4g_s R^3}{\chi_0}\frac{1}{\rho^4}\right)^{1/2}\left(d\rho^2 + \rho^2 d\Omega_5^2 + dx^\alpha dx_\alpha\right) \,.
\tag{27}
$$

4

Thus, the background is conformally flat and it has two asymptotic regions

$$\rho \to \infty \ , \qquad \text{Flat space} \ , \tag{28}$$

$$\rho \to 0 \ , \qquad AdS_5 \times S^5 \ . \tag{29}$$

The action of this vacuum is easily found, following [11] to be

$$S = -\frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} \nabla^2 \Phi = \frac{c}{g_s} \ , \qquad c = \frac{1}{2\kappa^2} \chi_0 R^5 V(S^5) \int d^4 x \ . \tag{30}$$

Thus, it has a finite value per unit four-dimensional volume. In addition it has the standard $1/g_s$ scaling of D-branes. As a result, this solution can be interpreted as an interpolating soliton between flat space and Euclidean $AdS_5 \times S^5$. We also note that this solution corresponds to the large size limit of the D-instanton solutions of [12, 13].

Concerning the above solution, we would like to stress that it breaks half of the supersymmetries even at the two asymptotic regions, where the space is either flat or $AdS_5 \times S^5$. Thus, the corresponding boundary gauge theory will be an $\mathcal{N} = 2$ SYM theory which however, will have running coupling, as we will see. Hence, it should be different than the one obtained by orbifolding the $\mathcal{N} = 4$ theory [14].

4 Running couplings and the quark-antiquark potential

In order to discuss running of couplings we must identify the coordinate in the bulk that corresponds to the energy in the boundary gauge theory. We identify the energy in the bulk as $U = R^2 e^{-r/R}$ and also define an energy scale $U_1 = R^2 e^{-r_0/R}$ which, since $U > U_1$, may be considered as the IR cutoff. The identification of the energy follows from the fact [15] that if one considers the massless scalar equation $\frac{1}{\sqrt{G}} \partial_M e^{-2\Phi} \sqrt{G} G^{MN} \partial_N \Psi = 0$, where G_{MN} is the σ-model metric, this takes the form of the usual scalar equation in $AdS_5 \times S^5$ to leading order in the expansion for large U. According to the AdS/CFT correspondence, the dependence of the bulk fields on U may be interpreted as energy dependence of the boundary theory such that long (short) distances in the AdS space corresponds to low (high) energies in the CFT side. In particular, the U-dependence of the dilaton defines the energy dependence of the YM coupling $g_{\text{YM}}^2 = 4\pi e^\Phi$ as well as of the 't Hooft coupling $g_{\text{H}}^2 = g_{\text{YM}}^2 N$. According to this we find the running of the 't Hooft coupling to be

$$g_H = g_0 \Big(\frac{U_1}{U}\Big)^4 + \mathcal{O}\Big(\frac{U_1}{U}\Big)^{12} \ , \tag{31}$$

where $g_0 = \sqrt{3}\chi_0/R^4\alpha$. As we see, g_{H} vanishes asymptotically, the signal of asymptotic freedom. It should be stressed, however, that the 't Hooft coupling approaches zero with a power law and not with the familiar perturbative logarithmic way. Note also that $\beta_g'(0) = -4$ in accordance with the universal behaviour for the first derivative of the

beta-function for the coupling that was found in [8]. In the supersymmetric solution in particular, one finds, by using (24), that the beta function is

$$\beta(g_H) = -4(g_H - g_H^*) , \tag{32}$$

where $g_H^{*2} = 4\pi N g_s$.

Next we turn to the quark-antiquark potential corresponding to the supersymmetric solution of section 2, by computing the Wilson loop. Using standard techniques [16, 17] we find that the potential is

$$E_{q\bar{q}} = \left(-U_0 + \frac{a}{U_0^3} \right) \frac{\eta_1 T}{\pi} , \qquad a = \frac{\chi_0 R^5}{4} , \tag{33}$$

where $\eta_1 = \frac{\pi^{1/2}\Gamma(3/4)}{\Gamma(1/4)} \simeq 0.599$, T is the total range of the Euclidean time-variable, and U_0 is the turning point of the trajectory given by

$$
\begin{aligned}
U_0 &= a^{1/4} \left(\frac{1}{6}\Delta^{1/3} - 2\Delta^{-1/3} \right)^{-1/2} , \\
\Delta &\equiv 108b^2 + 12\sqrt{12 + 81b^2} , \qquad b = \frac{a^{1/4}}{2\eta_1 R^2} L .
\end{aligned}
\tag{34}
$$

It turns out that, for $a^{1/4}/R^2 L \ll 1$, the first term in (33) is dominant, resulting in the usual Coulombic law behaviour [16, 17]. The first correction to it is $\mathcal{O}(L^3)$, as in [8, 10], which is also similar to the behaviour one finds [18] using near extremal D3-branes to describe finite temperature effects in the $\mathcal{N} = 4$ $SU(N)$ SYM theory at large N. However, in the opposite limit $a^{1/4}/R^2 L \gg 1$ the second term in (33) dominates giving

$$E_{q\bar{q}} \simeq \frac{a^{1/2}}{2\pi R^2} TL . \tag{35}$$

Hence, the quark-antiquark potential exhibits the typical confining behaviour and produces the area-law for the Wilson factor. The plot of the potential (33) is given below.

A second criterion [19] for confinement is the existence of a mass gap in the theory. In our case this is straightforward to check by examining the massless wave equation and realizing that the Einstein metric is just $AdS_5 \times S^5$.

At this point we stretch that since we go into the strong coupling regime for $U = 0$, we should introduce an infrared cutoff U_{IR} such that $U \geq U_{IR}$. This does not affect the behaviour in the UV, but it has consequences in the IR. It implies that there is a maximum length L_{IR}, up to which (35) can be trusted. Under this assumption, the range of validity of (35) is $a^{-1/4}R^2 \ll L \leq L_{IR}$.

Note Added

Just before submitting our paper in the hep-th we received [20] which overlaps with materials in section 3.
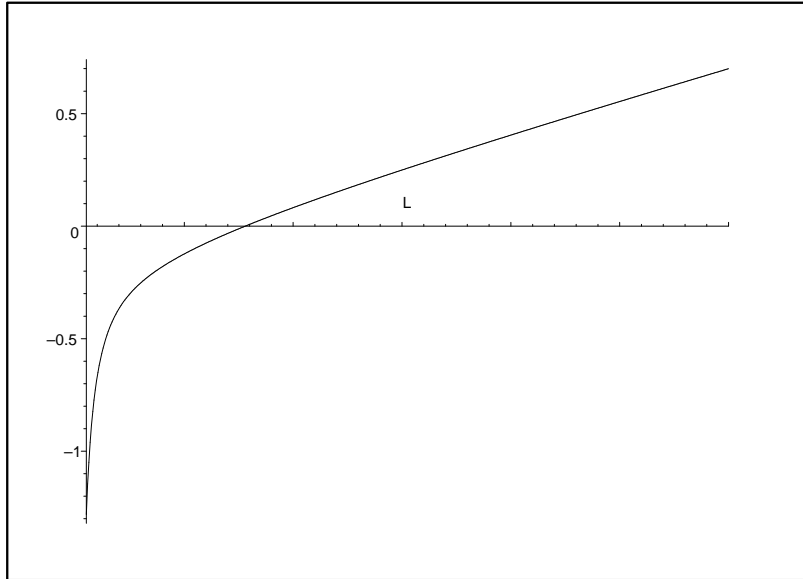
Figure 1: Plot of the quark-antiquark potential in (33) as a function of the separation distance L, in units where $a = (2\eta_1 R^2)^4$. For small L we have a Coulombic behaviour, whereas for large L, a linear one.

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