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On 'Graceful Exit' from inflationary phase in two-dimensional Liouville String Cosmology

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Abstract

Within the context of a super-critical (Liouville) string, we discuss (target-space) two-dimensional string cosmology. A numerical analysis indicates that the identification of time with the Liouville mode results in an expanding universe with matter which exhibits an inflationary phase, and 'graceful exit' from it, tending asymptotically to a flat-metric fixed point. This fixed point is characterized by a dilaton configuration which, depending on the initial conditions, either decreases linearly with the cosmic time, or is a finite constant. This implies that, in contrast to the critical string case, the string coupling remains bounded during the exit from the inflationary phase, and, thus, the pertinent dynamics can be reliably described in terms of a tree-level string effective action. The rôle of matter in inducing such phenomena is emphasized. It is also interesting to note that the asymptotic value of the vacuum energy, which in the σ-model framework is identified with the 'running' central charge deficit, depends crucially on the set of initial conditions. Thus, although preliminary, this toy model seems to share all the features expected to characterize a phenomenologically acceptable cosmological string model.

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1 Introduction

There is a vast number of inflationary models in the literature [1] which try to give a satisfactory answer to the traditional cosmological problems as the age,size and flatness of the universe.Almost all of these theories relay on a potential of a scalar field the so called "inflaton". The properties of this potential determine at a large extent the success of the model.The scalar potential can arise in various theories depending on the taste of the authors. An intriguing aspect of all inflationary theories to date is the way inflation ends, the so-called 'graceful exit' problem.There is currently a vast literature on the subject [1],dealing with this problem from both the theoretical and the phenomenological points of view.

In string theory there were a lot of attempts to develop a consistent theory of string cosmology,in which inflation plays a dominant rôle. There were found solutions to the non-linear σ-model equations for the graviton,dilaton and antisymmetric tensor in a Friedman-Robertson-Walker (FRW) background.Then the FRW spaces were interpreted as conformal field theories capable of being embedded in consistent string theories [2, 3]. The advandange of this string cosmological model is,that at tree level, does not depend on a scalar potential,so any inflationary solution will depend on the dynamics of the scalar field,and on its couplings to the other massless string modes. Unfortunately, these tree level β-functions of the background fields, do not have any inflationary solution.To get inflation, these equations must be modified by a non-trivial dilaton potential which is generated by string loop effects.

Veneziano and his collaborators [4] have suggested that the dilaton of (critical) string theory plays the rôle of the inflaton field of conventional inflationary theories [1]. In an implementation of this idea,they suggested a pre-big-bang cosmology in which the kinetic energy of a massless dilaton drives the inflation toward a singularity.Before the singularity is reached,stringy and/or nonperturbative effects bring an end to the inflationary phase and a transition occurs to the standard Friedman-Robertson-Walker (FRW) Universe. Considerable effort has been focused on the details of the graceful exit from the inflationary era.It is still not at all clear that this can be done.The reason is that such dilaton scenaria require knowledge of the precise underlying dynamics, in particular the dilaton potential $V(\Phi)$. The dilaton field is strictly massless in the critical-dimension string theories and one would naively expect that there is no dilaton potential.

However, compactification to four dimensions, as appropriate for any attempt to discuss realistic physics of the inflationary Universe, or inclusion of higher world-sheet topologies (string loops), may lead to all sorts of complications, and non-trivial interactions for the dilaton field, including the possibility of spontaneous breaking of scale symmetry, leading to a (small) mass for Φ. The inclusion of higher string topologies seems unavoidable in the framework of [4] due to the fact that the space-time configurations involved are characterized by a curvature growth and a dilaton

Φ increasing linearly with the cosmic time. Since the string coupling $g_s \sim e^\Phi$, it is evident that for large times $t \to \infty$ one enters the regime of strong coupling, which makes the framework of the lowest-order (tree level) effective action inadequate. However, even under the inclusion of higher loops, or the construction of exact conformal field theories to take into account non-perturbatively such higher topologies [5], one could not avoid such indefinetely-growing string coupling situations [6]. The system was not attracted by fixed points characterized by a constant, or decreasing with time, dilaton configurations corresponding to bounded string coupling, thereby making the computations based on perturbative string theory unreliable. Thus, the exit problem in such string cosmologies persists.

In addition to the lack of knowledge on the precise four-dimensional dilaton dynamics, the inflationary period is an *out of equilibrium* process. Indeed, in many recent attempts [7] to study inflation, the concept of a *stochastic* process governing the inflationary phase, began to emerge, which requires means of non-equilibrium field theories. In the context of critical-dimension string theory, where the various space-time field configurations obey world-sheet conformal-invariance conditions [8], equivalent to classical solutions of the low-energy field equations, it is rather hard to incorporate the non equilibrium dynamics of the "rolling-down-the-hill" phase of inflation.

In this work we will present a two-dimensional stringy cosmological toy model in which inflation relays on the dynamic evolution of scalar fields, and the transition from the inflationary phase to a FRW Universe occurs in a natural way as a result of the time evolution of the theory. To implement such a scenario, we modify the β-functions of the background fields of a critical string theory in such a way so as to include friction terms for all the fields involved. A consistent way to do that is to consider a non-critical (supercritical) string theory and idendify the time with the (time-like) Liouville mode [9, 10]. As emphasized in [10], such scenaria lead naturally to stochastic-type inflation [7]. In the context of the two-dimensional cosmological string model, it will be shown that the flow of (Liouville) time is such that the system is attracted by a fixed point characterized by a flat metric and a dilaton that, depending on the initial conditions, either decreases linearly with the cosmic time, or tends to a constant value. This situation implies that the string coupling remains bounded during the transition from the inflation to the post-inflation period, in contrast to the critical string situation [4, 6]. We note here that to get a non-critical string we have to include matter, the rôle of which is played here by the Tachyon field, which in two-dimensional target-space times is a massless matter field.

The structure of the article is as follows: We first review in brief the generic formalism of time as a Liouville mode [9], and we discuss the generalized conditions of Liouville - restored conformal invariance. These replace the critical-string conformal (Weyl) invariance conditions resulting in the vanishing of the world-sheet generalized β-functions (Weyl anomaly coefficients) for the set of σ-model background fields.

Next we examine the specific case of two-dimensional string cosmology. We solve the generalized set of Liouville-restored conformal invariance conditions for backgrounds corresponding to the matter and spacetime metric fields. The cosmological nature of the problem requires that such backgrounds exhibit solely Liouville (= time) dependence. We seek, and find, solutions for the metric field, which exhibit inflationary behaviour at a certain stage of Liouville evolution, and we pay particular attention to demonstrate a "graceful exit" from the inflationary phase. Asymptotically in time, the spacetime becomes an ordinary flat universe. There is also a linear expansion at a late stage of the Liouville evolution. As a byproduct of our analysis we note that the above cosmological model is characterized by a non-zero Liouville(= time)-dependent vacuum energy, which relaxes to an asymptotic time independent value. The asymptotic value of the cosmological constant depends on the initial conditions; there are cases where this asymptotic value vanishes.

Our solutions are at present numerical. However we consider it a non-trivial fact that consistent inflationary scenaria, with the right properties to account for a "graceful exit" from the exponential expansion phase, can be found, at least numerically, within the framework of non-critical Liouville strings. It is highly non-trivial that the Liouville-restored conformal invariance conditions yield this behaviour upon the identification of the Liouville field with time [9, 11]. The existence of such solutions in the two-dimensional toy cosmological model we consider here is encouraging for the extension of such analyses to higher-target-space-dimensional string theories. This is left for future investigations.

We also stress the fact that the Liouville-restored conformal invariance conditions differ (to lowest order $O(\alpha)'$ in the Regge-slope expansion) from Einstein's equations for the metric, encountered in analogous treatments of critical string models. From a "field-theoretic" point of view, this is to be expected from the fact that non critical Liouville dynamics is describing out-of-equilibrium (non-classical) physical processes [9], believed to characterize the inflationary Universe. This is an important generic feature of Liouville dynamics, in the picture where one identifies the Liouville mode with the target time [9, 11].

2   Liouville String formalism

After this introduction, we now proceed with the analysis of our pilot two-dimensional cosmological model. We commence with a brief description of the Liouville-dressing procedure for non-critical string, with the Liouville mode viewed as a local worldsheet renormalization group scale [9]. Consider a conformal σ-model, described by an action S^* on the world-sheet Σ, which is deformed by (non conformal) deformations $\int_\Sigma g^i V_i d^2\sigma$, with V_i appropriate vertex operators.

$$S_g = S^* + \int_\Sigma g^i V_i d^2\sigma \tag{1}$$

The non-conformal nature of the couplings g^i implies that their (flat)world-sheet renormalization-group β-functions, β^i, are non vanishing. The generic structure of such β-functions, close to a fixed point, $\{g^i = 0\}$ reads:

$$\beta^i = (h_i - 2)g^i + c^i_{jk}g^j g^k + o(g^3). \tag{2}$$

In the context of Liouville strings, world-sheet gravitational dressing is required. The "gravitationally"-dressed couplings, $\lambda^i(g, t)$, which from our point of view correspond to renormalized couplings in a curved space, read to $O(g^2)$ [12, 13]:

$$\lambda^i(g, t) = g^i e^{\alpha_i t} + \frac{\pi}{Q \pm 2\alpha_i}c^i_{jk}g^j g^k t e^{\alpha_i t} + O(g^3), \qquad Q^2 = c - 25 \tag{3}$$

where t is the (zero mode) of the Liouville mode, Q^2 is the central charge deficit, and α_i are the gravitational anomalous dimensions:

$$\alpha_i(\alpha_i + Q) = h_i - 2 \qquad \text{for} \qquad c \geq 25 \tag{4}$$

Below we shall concentrate exclusively to the supercritical string case, $Q^2 \geq 0$, which from the point of view of identifying the Liouville mode with target time, corresponds to a Minkowskian signature spacetime manifold [2].

Due to the renormalization (3), the critical-string conformal invariance conditions, amounting to the vanishing of flat-space β-functions, are now substituted by:

$$\ddot{\lambda}^i + Q\dot{\lambda}^i = -\beta^i(\lambda) \qquad \text{for } c \geq 25. \tag{5}$$

where the notation $\beta^i(\lambda)$ denotes flat-world-sheet β-functions but with the formal substitution $g^i \to \lambda^i(g, t)$. Note the minus sign in front of the flat-world-sheet β-functions β^i in (5), which is characteristic of the supercriticality of the string [12, 13]. As we see later, for our two-dimensional string cosmology, the sign will be crucial for the existence of acceptable inflationary solutions demonstrating "graceful exit" from the exponential expansion phase. Upon the identification of the Liouville mode with the target time the dot denotes temporal derivative.

An important comment we would like to make concerns the possibility of deriving the set of equations (5) from a target space action. This issue has been discussed in the affirmative in ref.[10], where it was shown that the set of equations (5) satisfies the Helmholtz conditions for the existence of an action in the 'space of couplings' $\{g^i\}$ of the non-critical string. Upon the identification of target time with the Liouville mode [9] this action becomes identical with the target space action describing the off-shell dynamics of the Liouville string. We should stress the fact that the action is off shell, in the sense that the on-shell conditions correspond to the vanishing of the β-functions β^i. In our case $\beta^i \neq 0$, and the identification of the Liouville mode with the target time implies that the space-time graviton β-function on the right-hand-side of (5), as well as other target-space tensorial structures, appearing inside the β^i functions for the various modes, contain temporal (Liouville) components as

well. In this respect, our non-equilibrium Liouville string approach to the temporal evolution [2] should be contrasted with the naive interpetation of a Liouville string as a critical equilibrium string living in a space time with one extra dimension. In that case the corresponding β functions of the Liouville-dressed theory would satisfy classical equations of motion. As mentioned above, in our approach the conditions describing the restoration of conformal invariance by means of Liouville dressing are *not* to be intepreted as classical equations of motion of a string living in a space-time with one extra target dimension. Thus our analysis below should be distinguished from previous analyses on Liouville cosmology [14].

A generic feature of Liouville dynamics, is that in terms of the world-sheet action, the normalization of the Liouville kinetic term can always be arranged (by chosing appropriate counterterms) to correspond to a target spacetime of Friedman-Robertson-Walker (FRW) type; i.e. the time-like metric component (under the assumption that the Liouville mode is time) is:

$$G_{00} = -1. \tag{6}$$

We remind the reader that the Minkowskian signature is due to supercriticality ($Q^2 = c - 25 \geq 0$) assumption. This will be understood in what follows.

Before proceeding to discuss our two-dimensional cosmology, we would like to make a final remark concerning the physical origin of deviations from criticality in string theories. This will hopefully shed more light in the physics underlying our scenario. Deviations from conformal invariance imply, from a target-space viewpoint, that the relevant background field is "off-shell", i.e. is not a classical solution of some equations of motion. As argued in [9], such a situation may be encountered in the context of effective low-energy theories of quantum gravity. Due to quantum fluctuations of the metric field, corresponding to microscopic event horizons (spacetime boundaries), there is the possibility for low-energy propagating matter to be trapped inside or on such boundaries. This can be explicitly demonstrated, for instance, in the case of a Dirichlet (D) brane representation of such defects on spacetime. A low energy closed string state can split into open strings with their ends attached to the D-brane. Such trapping processes cannot be measured by a low-energy observer who performs local scattering experiments "asymptotically far" from these microscopic horizons. From the observer's point of view, therefore, the appearance of singular metric fluctuations will result in making the low-energy matter system, consisting of propagating degrees of freedom only, an "open" quantum system. Information is "lost" into degrees of freedom pertaining to the quantum recoil of the D-brane horizon. In turn, such recoil degrees of freedom cause a sufficient distortion of spacetime, characterized by non-trivial particle creation, leading to decoherence of the low-energy matter system. A detailed analysis and formalism of such issues has been documented in [15], where we refer the interested reader for details.

For our purposes below we assume that, within the context of a two-dimensional cosmological model, such quantum processes in the early Universe have resulted in a matter deformation of the stringy σ-model, which does not satisfy classical equations of motion and hence, from a conformal point of view, departs from the critical string case. Back-reaction of such matter onto spacetime structure, prevents the metric from satisfying Einstein's equations (to lowest order in the α' expansion, where we concetrate ourselves throughout this work.)

With these in mind, our proposal for the two-dimensional string cosmology may now be formulated as follows: the low-energy (local, propagating) fields are the metric, G_{ij}, the dilaton, Φ, and "tachyon", T, fields [1]. In flat space times, the two-dimensional tachyon field is actualy massless and constitutes our low-energy matter. As we shall see the presence of such matter is crucial for the inflationary scenario.

The $\mathcal{O}(\alpha')$ β-functions, corresponding to these fields, read:

$$
\begin{aligned}
\text{Graviton} \quad \tilde{\beta}^G_{ij} &= \alpha'\left(R_{ij} + 2\nabla_i\nabla_j\Phi - \nabla_i T\nabla_j T\right) \\
\text{Dilaton} \quad \tilde{\beta}^\Phi &= -R + 4\nabla_i\Phi\nabla^i\Phi - 4\nabla^2\Phi + \nabla_i T\nabla^i T - 2T^2 + Q^2 \\
\text{Tachyon} \quad \tilde{\beta}^T &= -2\nabla^2 T + 4\nabla_i\Phi\nabla^i T - 4T.
\end{aligned}
\tag{7}
$$

In the above we have taken into account the freedom to fix the tachyon potential in string theory [16], by appropriate field redefinitions, such that it only incorporates T^2 terms ($V(T) = -2T^2$). The tilde denotes Weyl anomaly coefficients, which replace the ordinary renormalization-group β-functions in the case of stringy σ-models, as a result of target-space local diffeomorphisms [14, 17].

In the context of critical strings the vanishing of these $\tilde{\beta}$-functions can be interpreted as equations of motion from the action

$$
S = \int d^2x\sqrt{-g}\left\{e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - (\nabla T)^2 - V(T) - Q^2\right]\right\},
\tag{8}
$$

In this notation the string coupling is $g_s = e^\Phi$. As a general remark we stress that in two target-space-time dimensions there is no Einstein frame, i.e. a frame in which the conformal dilaton factor $e^{-2\Phi}$ in front of the curvature term in the action (8) can be removed by a field redefinition. Thus, in this case the σ-model frame is also the 'physical' frame. This will always be understood in the following.

---

[1]Due to the Abelian Gauge symmetry $B_{MN} \to B_{MN} + \partial_{[M}\Lambda_{N]}$, in two-dimensional space times the antisymmetric tensor field has no propagating modes, given that it may be eliminated from the low-energy action. The remaining discrete mode is thereby considered part of the unobservable gravitational environment.

In our non-critical string context the Weyl-anomaly coefficients $\tilde{\beta}$ are related off-shell with variations of the above action [18],

$$\tilde{\beta}^i \sim \mathcal{G}^{ij} \frac{\delta S}{\delta g^j} \tag{9}$$

where $\mathcal{G}^{ij}$ is related to the (inverse) Zamolodchikov metric in theory space [21], given by the world-sheet two-point function of vertex operators. As mentioned previously, the set of equations (5,9) satisfies the Helmholtz conditions for its being derived from an off-shell action in theory space of the non-critical string [10]. The purpose of this article is to point out that, upon the identification of the Liouville mode with the target time, this *specific set* of field equations will yield cosmological solutions, capable of describing inflationary phase of an expanding string universe and graceful exit from it.

To this end we combine (5) with (7), where now the indices $i,j = 1, 2$ span a two-dimensional target space time. The dots refer to (Liouville) time t derivatives, and the cosmological model is obtained by assuming that the various background fields exhibit only (Liouville) time t dependence. The metric is assumed to have the FRW form:

$$G_{ij} = \begin{pmatrix} -1 & 0 \\ 0 & e^{b(t)} \end{pmatrix} \tag{10}$$

The important comment we wish to make concerns the fact that, due to the renormalizability of the (non-critical) σ-model, there is an additional equation [19] which should supplement (5), the Curci-Paffutti relation, which relates the dilaton β function, and hence the effective running central charge of the theory, with the rest of the β functions:

$$-\nabla_i \tilde{\beta}^\Phi + 2G^{lj}\nabla_l\beta^G_{ij} - 4G^{lj}\nabla_l\Phi\beta^G_{ij} + \nabla_i\tilde{\beta}^T = 0 \tag{11}$$

Although this equation holds formally in the flat world-sheet case, however in our framework it should also hold for the $\beta^i(\lambda)$ functions, i.e. the flat-world-sheet β functions upon the substitution of the σ-model couplings with the Liouville dressed ones. It will provide a highly non-trivial constraint, which should be respected by the process of identifying the Liouville (world-sheet) renormalization scale with the target time [9].

We are now seeking solutions to the system of equations (5), (7),(11) exhibiting at a certain stage in Liouville time t inflationary behaviour (exponential expansion in the spatial volume). Given the choice for the metric (10), and the assumption that all the fields are time dependent only, the equations (5) take the form:

$$\frac{3}{2}\ddot{b} + \frac{5}{4}\dot{b}^2 + \dot{b}(Q - \dot{\Phi}) = 0$$

$$\frac{\ddot{b}}{2} + \frac{\dot{b}^2}{4} - 2\ddot{\Phi} + \dot{T}^2 = 0$$

$$5\ddot{\Phi} - \ddot{b} - \frac{1}{2}\dot{b}^2 - 4\dot{\Phi}^2 + \dot{\Phi}(Q + 2\dot{b}) - \dot{T}^2 - 2T^2 + Q^2 = 0$$

$$3\ddot{T} + \dot{T}(\dot{b} - 4\dot{\Phi} + Q) - 4T = 0$$

$$\dddot{b} - 4\dddot{\Phi} + \dot{b}\ddot{b} + 8\dot{\Phi}\ddot{\Phi} - 2\dot{b}\dot{\Phi} - 2\ddot{\Phi}\dot{b} + \dot{T}^2(\dot{b} - 4\dot{\Phi}) + 4\ddot{T}\dot{T} - \dot{Q}Q = 0, \qquad (12)$$

where the first four refer to the diagonal components of the metric, the dilaton and the tachyon fields and the last one is the time component of the Curci-Paffuti equation (11). Note that the equation for the non-diagonal component of the metric and the space component of the Curci-Paffuti equation are trivially satisfied.

It can be seen, for example, from the first of the equations in (12) that in principle there are both inflationary and non-inflationary solutions. In particular if we assume that the dilaton and the central charge are slowly varying with time, then negative values of the quantity $(Q(t) - \dot{\Phi}(t))$, gives an exponentially growing scale factor $(exp(b(t)))$, while positive values of the same quantity yields power-law scale factor. Thus we have to see if our system of equations gives the right relative magnitude to the dilaton and the central charge in order to ensure an inflationary era followed by a power-law expansion, leading asymptotically to flat space.

Using the first the second and the fourth equations in (12) we solve for the second derivatives of the fields b, Φ and T, and eliminate their higher derivatives from the last one, concluding to the following set of four equations:

$$\ddot{b} = -\frac{2Q\dot{b}}{3} + \frac{2\dot{\Phi}\,\dot{b}}{3} - \frac{5\dot{b}^2}{6}$$

$$\ddot{\Phi} = -\frac{Q\dot{b}}{6} + \frac{\dot{\Phi}\dot{b}}{6} - \frac{\dot{b}^2}{12} + \frac{\dot{T}^2}{2}$$

$$\ddot{T} = \frac{4T}{3} - \frac{Q\dot{T}}{3} + \frac{4\dot{\Phi}\dot{T}}{3} - \frac{\dot{b}\dot{T}}{3}$$

$$Q\,\dot{Q} = \frac{Q\dot{b}^2}{3} + \frac{\dot{b}^3}{6} + \frac{2}{3}\dot{b}^2\dot{\Phi}. \qquad (13)$$

The equation coming from the dilaton β-function (third equation in (12)) is left as a compatibility condition.

3 Inflationary Solutions in Liouville String Theory

In order to get solutions it seems necessary to distinguish two major cases: (i) solutions where the central charge deficit $Q^2(t) \to$ const $\neq 0$ as $t \to \infty$, and (ii) solutions where $Q^2(t) \to 0$, as $t \to \infty$. Although, as we shall see, from an effective

field theory view point, *both* cases seem plausible, however, at least at present, it is not clear to us whether both situations can be met in an actual string theory framework. Indeed, in case (i), the asymptotic string theory will be a standard non-critical string theory in two-dimensional target-space, with the constant central charge deficit compensating the non-critical dimensionality of the target space [12, 2]. Such string theories are known to exist as exact conformal field theory models. In case (ii), however, the resulting string theory will be a critical string theory. The asymptotic vanishing of the central charge, which plays the rôle of a target-space cosmological constant, may be understood in that case as a result of the effects of the higher-level string modes of the target-space two-dimensional string. At present, we do not have an exact conformal field theory description of such models, but in view of the existence of consistent non-trivial examples in the case of stringy black-holes [20] we conjecture that such a case can also represent consistent cosmological string backgrounds.

3.1   Solutions attracted by a linear-dilaton fixed point

We start our analysis from case (i). To obtain the solutions of (13) in this case we adopt the following "quasi-linear" method. We separate the fields in their asymptotic values plus fields which tend asymptotically to zero, namely:

$$
\begin{aligned}
\dot{\Phi} &\equiv d &+\phi_1 \\
Q &\equiv & Q_0 + Q_1 \\
\dot{b} &\equiv & b_1 \\
T &\equiv T_0 &, \qquad \dot{T} \equiv T_1,
\end{aligned}
\tag{14}
$$

where the constants d and Q_0 are related through the relation $Q_0 = -d(1+\sqrt{17})/2$, which results from the requirement that the dilaton equation is satisfied, and the fields $(\phi_1, b_1, T_1, T_0, Q_1)$ vanish asymptotically. We assume that the constant $d < 0$ in order to have weak gravity asymptotically in time. (The case $d = 0$ will be discussed separately).

The system can be written in the form

$$
\dot{\vec{x}} = \mathbf{A}\vec{x} + \vec{\mathbf{F}}(\vec{x})
\tag{15}
$$

where $\vec{x} = (\phi_1, b_1, T_1, T_0, Q_1)^\perp$, $\mathbf{A}$ is the 5×5 matrix determining the linear part of the system and $\vec{\mathbf{F}}(\vec{x})$ gives the nonlinear terms.
The solution of the system can be given in an iterative form:

$$
\vec{x}_{(n+1)}(t) = \vec{x}_{(n)}(t) + \int_{t_0}^{t} ds\, \mathbf{Y}(t)\mathbf{Y}^{-1}(s)\vec{\mathbf{F}}\left[\vec{x}_{(n)}(s)\right],
\tag{16}
$$

where the matrix $\mathbf{Y}$ satisfies the equation

$$
\dot{\mathbf{Y}} = \mathbf{A}\mathbf{Y}.
\tag{17}
$$

Note that the iteration in (16) converges to the full solution. The starting point of the iteration procedure is the solution of the linear system with the correct asymptotic behaviour, which reads:

$$
\begin{aligned}
b_1 &= C_1 exp\left[\frac{2(d-Q_0)t}{3}\right] \\
\phi_1 &= \frac{C_1}{4} exp\left[\frac{2(d-Q_0)t}{3}\right] \\
T_0 &= C_2 exp\left[(A-B)t\right] \\
T_1 &= C_2(A-B)exp\left[(A-B)t\right], \\
Q_1 &= 0
\end{aligned}
\tag{18}
$$

where $A = (4d-Q_0)/6$ and $B = \sqrt{(4d-Q_0)^2+48}/6$. Note that the constant of integration C_1 has to be positive. Note also that the matter field vanishes exponentially with time, a characteristic shared also by the full solution as we shall see later on.

Now at the first step in the iteration the solution is given by:

$$
\vec{x}(t) = \mathbf{Y}(t)\vec{C} + \mathbf{Y}(t)\int_{t_0}^t ds\mathbf{Y}(t)\mathbf{Y}^{-1}(s)\vec{\mathbf{F}}\left[\mathbf{Y}(s)\vec{C}\right],
\tag{19}
$$

where $\mathbf{Y}(t)\vec{C}$, is the solution of the linear part of the system while $\mathbf{Y}(t_0)\vec{C}$ is the set of initial conditions. Even from the first step of the iteration one can see that there are regions of the parameters (e.g $C_1 \gg C_2$) for which the dilaton field remains practically constant and the scale factor of the metric increases exponentially. We remind the reader that this is a common feature of all inflationary scenaria, where the rôle of the inflaton is played in this case by the dilaton field. Of course the solutions at this order although instructive are not full solutions of the system. So we proceed to present full numerical solutions. In order to get these solutions we use the iteration up to second order. In this way we take expressions for the fields which are almost exact asymptotically due to the convergence of the iteration. Then we use the values of these expressions at a certain point (in time) as initial conditions and we let the system evolve numerically.

One of the basic features of all solutions is the existence of an initial singularity $(t \to -\infty)$. Of course our consideration starts immediately after the singularity since a proper treatment of true initial conditions of the Universe is an issue which has to take into account the full quantum gravity effects. Indeed, we do not expect a proper string theoretic cosmological model to be described only in terms of an effective field theory based on the low-string-level fields. In the context of our non-critical string scenario, the existence of an initial singularity, describes quite naturally the effects of higher-string level degrees of freedom, including the quantum-mechanical (discrete) ones, responsible for the non-criticality of the effective field theory [9]. The effects of

such modes are expected to be strong at very early stages of the Universe. Moreover, higher curvature terms, as well as higher order string loop corrections should also be taken into account at such early stages, given that the string coupling g_s grows infinitely strong near the initial singularity.

We now note that the signature of the inflationary era is the sign of the difference $Q(t) - \dot{\Phi}(t)$ as is already mentioned. This difference becomes negative for a certain period provided that the density of the matter field (T) becomes weak immediately after the singularity. In particular in the inflationary era it is of the order of magnitude of the asymptotic value of Q, (Q_0). In the figures that follow we present a particular solution in which the abovementioned characteristics become more clear.

In figure 1 the scale factor $(exp[b(t)])$ is plotted. It is clear that it exhibits an exponential growth period. The exit from the inlfation comes by a power-law scale factor which settles down to a flat space-time metric. From figure 2 it is clear that the difference $Q(t) - \dot{\Phi}(t)$ being negative (certifying the inflation) at early times turns to positive values indicating thus the exit from inflation. In figure 3 the dilaton field is plotted. Note that it remains practically constant during inflation, whilst at later times it *decreases linearly* with the cosmic time. This latter feature is very important, as it implies that the string coupling $g_s \sim e^{\Phi} \to 0$, as $t \to \infty$, and therefore the tree-level effective action (8) is *sufficient* to describe the transition from the inflationary to post-inflationary era. This feature is *due to the supercriticality* of the initial stringy configurations, $C > 25$. It is the opposite situation from what is happening in critical strings [4, 6], thereby making our super-critical (Liouville) string model a viable model to describe 'graceful exit' from the inflationary era. As mentioned previously, of course, our tree-level effective-field theory description breaks down for very early stages, near the initial singularity.

In figure 4 we show the matter field (T). We see here that at the beginning of the inflation the value of the matter field is of the order of the asymptotic value of Q. Solutions with matter density one order of magnitude bigger than Q_0 do not have inflationary era. This can be understood since in a condensed enough Universe gravitational forces prevent exponential expansion. In figure 5 we show the evolution of Q. We see that during the inflationary period it is almost linerly dependent on time and it relaxes to a constant (non-zero value) asymptotically. We also note that $Q(t)$ *changes sign* at the end of inflation. Finally in figure 6 we show, for completeness, the time dependence of the 'running' central charge deficit $Q^2(t)$. Note that, in contrast to the inflationary phase where $Q^2(t)$ decreases rapidly with time, the post-inflationary period is characterized by an increasing with time $Q^2(t)$ before the latter settles to its final (equilibrium) constant value asymptotically. This is in agreement with the fact that the Liouville mode is a non-unitary world-sheet field for supercritical strings [2]. Thus, the conditions of ref. [21] for a monotonic decrease of the running central charge are not valid. However, as we see from figure

6, there is an overall decrease of the central charge deficit during the flow from its initial ('near the singularity') to the final (equilibrium) value. We point out that a similar situation, where the central charge decreases overall, but oscillates before settling to its non-trivial world-sheet fixed point value, also characterizes the dilaton cosmology of [14].

The overall decrease of $Q^2(t)$ in Liouville strings is expected on general grounds [22], given the connection of the irreversibility of the world-sheet renormalization group flow with 'loss of information' associated with stringy modes having world-sheet momenta beyond the ultraviolet cut-off of the effective theory [21, 9]. This world-sheet cut-off should not be confused with a space-time cut-off. However, one may find a proper mapping to a target-space ultraviolet scale, as a result of the embedding of the world sheet in a target space time. It is in this sense that the non-criticality of the effective string describes information loss due to stringy modes unobservable in the context of the low-energy effective field theory describing cosmological observations.

3.2 Solutions attracted by a constant dilaton fixed point

We next turn our attention to demonstrating the existence of solutions to (13) which are characterized by an asymptotic vanishing of the central charge deficit $Q^2(t \to \infty) \to 0$. Such solutions are known to characterize certain two-dimensional black hole models [20], and it may be the case that exact conformal field theories exist which also describe cosmological models. From the inflationary scenaria view point such solutions are important in that they are characterized by a relaxing to zero cosmological constant in target space [20], something which might be a phenomenological requirement when one extends such scenaria to four dimensional space times.

The method adopted previously is not convenient to find these solutions. The main reason is that if we set $d = Q_0 = 0$ in (14) the linear part of the system (13) becomes trivial and does not permit the iteration (16). Nevertheless we can see that there exist such solutions with the desired features (inflationary era and asymptotic flatness), using a different method which we describe shortly in the following.

To this end we first note that, upon using the first of equations (13), the relation

$$Q = -\dot{b} \tag{20}$$

solves the last of these equations. Hence, in the following we shall seek solutions satisfying the relation (20) between the central charge and the field b. From the solutions we have already obtained in our analysis above we see that this relation holds immediately after the (initial) singularity. If we assume this feature to characterize also the asymptotic region ($t \to \infty$), then we observe that it is possible to enforce

the central charge deficit to *vanish asymptotically*, $Q^2(t) \to 0$. The second fact that we infer from the solutions we have obtained above is that the field b is monotonic in time. If we assume this to hold in the case of asymptotically vanishing central charge deficit as well, then we can consider the fields Φ and T being functions of b. Anticipating flat space time solutions asymptotically $(t \to \infty)$, we shall concentrate on the case of weak $b \to 0$ field, for which a perturbative expansion of the solutions in powers of b is valid. Indeed, rewriting the system of equations (13) in terms of the new variable, b, we can find a series solution for these fields. We omit the details for brevity, and we only state the final result for the solution:

$$T(b) = c_2 \left\{ -b - \frac{b^2}{6} - \frac{55 + 12c_2^2}{216}b^3 - \frac{109 + 60c_2^2}{2592}b^4 \right\} + O(b^5)$$

$$\Phi(b) = \frac{b}{4} + \frac{1 + 4c_2^2}{16}b^2 + \frac{c_2^2}{18}b^3 + \frac{212c_2^2 - 3}{3456}b^4 + O(b^5). \tag{21}$$

This solution gives asymtotically vanishing matter density and asymptotically constant dilaton. This constant can be set equal to zero since our equations are insensitive in constant dilaton shifts. For the field b itself we find the equation:

$$\dot{b}^2(t) = b^2(t) + a_3(c_2)b^3(t) + a_4(c_2)b^4(t) \tag{22}$$

where the constants $a_3(c_2)$, $a_4(c_2)$ are complicated expressions of c_2 which can be found from the expression:

$$\dot{b}^2 = \frac{2T^2(b)}{\frac{13}{12} + \frac{7}{6}\phi'(b) - 4\phi'^2(b) + \frac{3}{2}T'^2(b)}. \tag{23}$$

Solving now the equation (22) for b as a function of time the solution (relevant for our purpose) is of the form:

$$b(t) = -\frac{4e^{t+c_0}}{a_3^2 - 4a_4 + 2a_3e^{t+c_0} + e^{2(t+c_0)}}. \tag{24}$$

We therefore see clearly that there are solutions which asymptotically in time $(t \to 0)$ lead to flat space with constant dilaton and vanishing central charge deficit. Already from the above approximate expression we can distinguish two major cases: (i) if we consider solutions with $a_4 > 0$ the expression in (24) yields an exponential factor of the form already presented in figure 1. (ii) On the other hand in the case of $a_4 < 0$ the expression in (24) indicates a scale factor leading to flat space in both asymptotic regions with a deep throat in the middle region. Although this latter class is interesting it lies beyond the scope of this article. Of course these expressions are approximate, but the two classes of solutions can be confirmed numerically, upon using initial conditions for the fields consistent with the above approximate solutions.

4 Instead of Conclusions

From the analysis presented in this article it seems that a simple two-dimensional cosmological model, based on super-critical (Liouville) strings, is characterized by phenomenologically acceptable features: exponential expansion (inflation), 'graceful exit' from it, and relaxation towards an asymptotically 'flat' string universe with a non-zero or zero constant vacuum energy (depending on the initial conditions). The most important feature is that the 'graceful exit' from the inflationary phase is achieved because of the fact that the Liouville σ-model is attracted by the linear- or constant- dilaton fixed points (depending on the initial conditions), in such a way that the string coupling always remains bounded during the inflationary and post-inflationary periods. Thus, higher world-sheet topologies do not play a rôle in the physics of the asymptotic time region $t \to \infty$. This should be contrasted with the critical-string situation.

The rôle of the non-criticality of the string, viewed as a non-equilibrium system was crucial. In this respect we note that in our approach there is non-trivial entropy production [9], determined by the overall decrease of the effective ('running') central charge of the theory, $Q^2(t) > 0$, during the flow towards a non-trivial fixed point. The entropy change expresses the amount of information carried by (string) modes whose world-sheet 'momenta' lie above the ultraviolet cut-off scale of the effective theory. The issue of precise estimates of the entropy production in the context of non-critical strings is left for future investigations.

An additional important feature of our approach was the rôle of matter in inducing the above-described temporal evolution of the non-critical string universe. It is not clear to us whether the existence of an initial singularity, which seems to characterize the solutions, is an inevitable feature of all such non-critical string scenaria, even in higher dimensions, or just a peculiarity of the two-dimensional toy model. It would be interesting to attempt to extend this analysis to higher dimensional theories, based on non-critical strings, including finite temperature considerations. This would allow for a study of more realistic inflationary scenaria, including the issue of reheating, a currently "hot"' subject.

It should be stressed once again that above we have worked in the so-called σ-model (string) frame, in which lengths have been measured in string 'rods'. Thus the inflationary scenaria we have found represent a true expansion of the stringy universe. In two-dimensions this is the only frame, and one is free from the ambiguities characterizing the four dimensional case, where a simple linear dilaton solution in the Einstein frame may be interpreted [23] as an *equilibrium* solution corresponding to a non-expanding constant universe, provided the measurement of distances is done in 'string rods'. In such a case one expects that inflationary, or in general expanding-universe, scenaria would be described by more complicated space time configurations.

It should also be born in mind that, at present, our results are preliminary, and one cannot make definite claims regarding the 'exit problem' from the inflationary phase of (non-critical) string theory. Many issues, such as the rôle of metric fluctuations on the 'exit' phase, entropy estimates and bounds in our non-critical string universe, reheating etc, are left open. These will hopefully constitute topics of future work. However, we believe that the current results are sufficiently interesting to encourage further studies of inflationary scenaria based on non-critical (non-equilibrium) Liouville strings.

Acknowledgements

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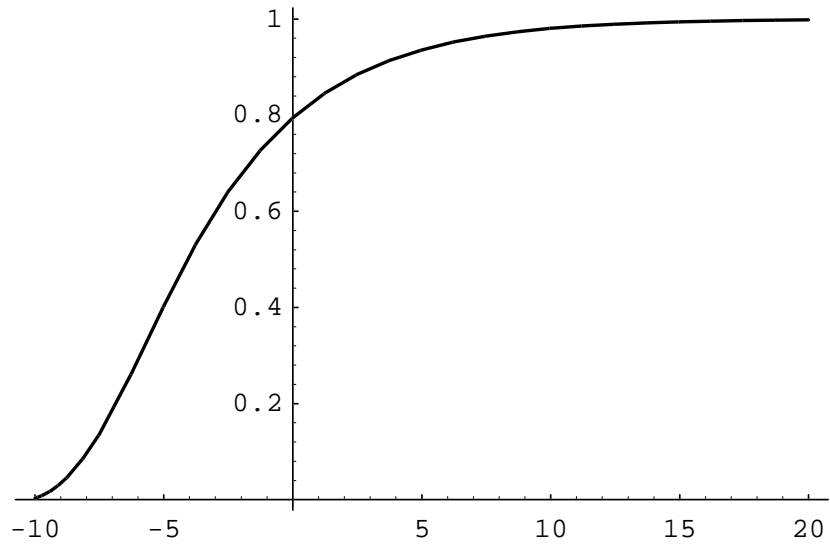[23] N. Sanchez and G. Veneziano, Nucl. Phys. B333 (1990), 253.

Figure 1: The scale factor $e^{b(t)}$ is plotted versus the cosmic time t, in our supercritical-string-inspired cosmological model. The system is characterized by a period of exponential growth (inflation), which in the units of the figure lies in the time interval between -10 and -5, succeeded by a period of power-law expansion, and, eventually, a stationary phase for $t > 10$.
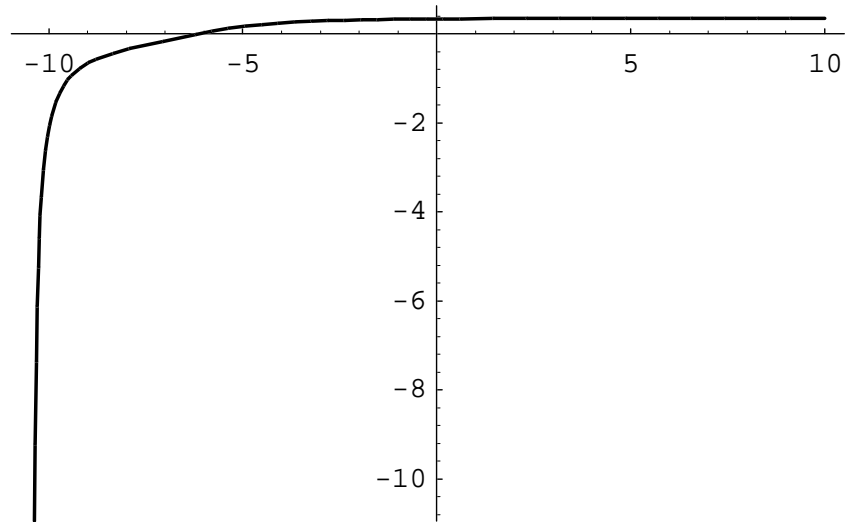
Figure 2: The difference $Q(t) - \dot{\Phi}$ versus the cosmic time t, whose sign is crucial for the existence of the inflationary period. During the inflationary period this quantity is negative. Then, its sign changes, signalling the exit from the inflationary era.
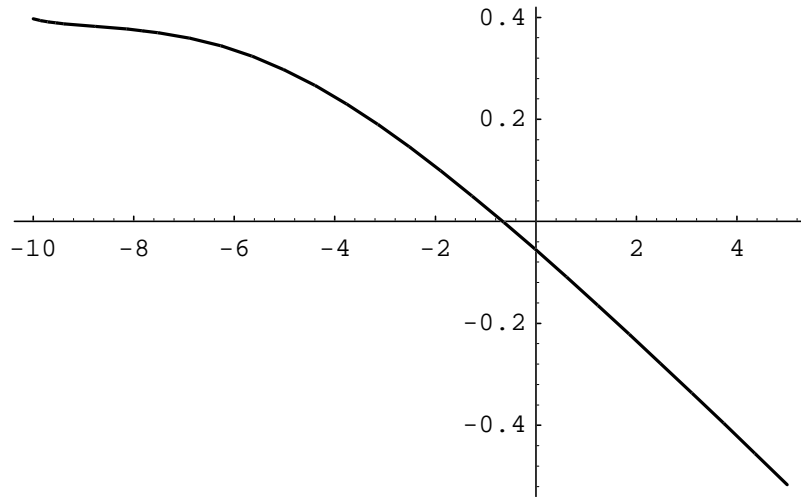
Figure 3: The dilaton configuration versus the cosmic time. The dilaton starts from positive values. Near the initial singularity (not exhibited in the figure) the dilaton approaches positive infinity. Its value drops sharply towards the inflationary era. During inflation the dilaton remains finite and almost constant (positive). it changes sign during the exit period, and becomes linearly decreasing with cosmic time asymptotically.
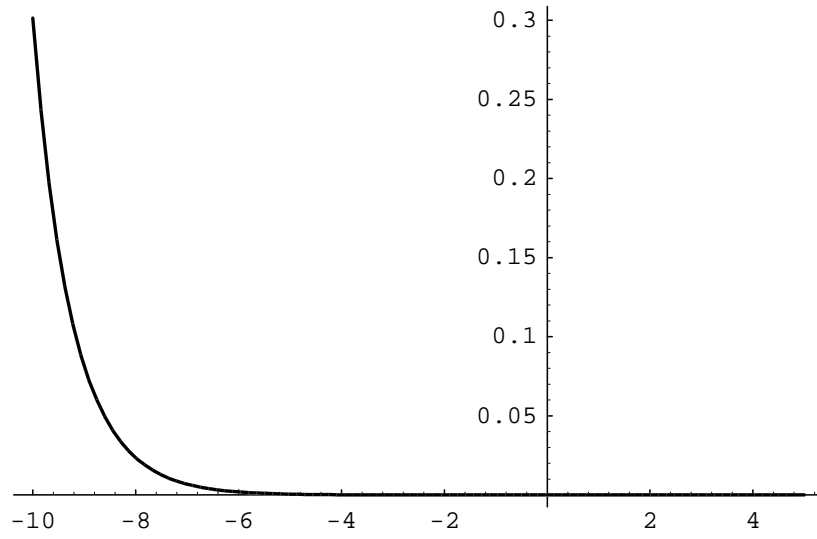
Figure 4: The matter field (Tachyon) as a function of the cosmic time. During the inflationary period it drops sharply from large values, of order of the central charge deficit Q, to practically zero value.
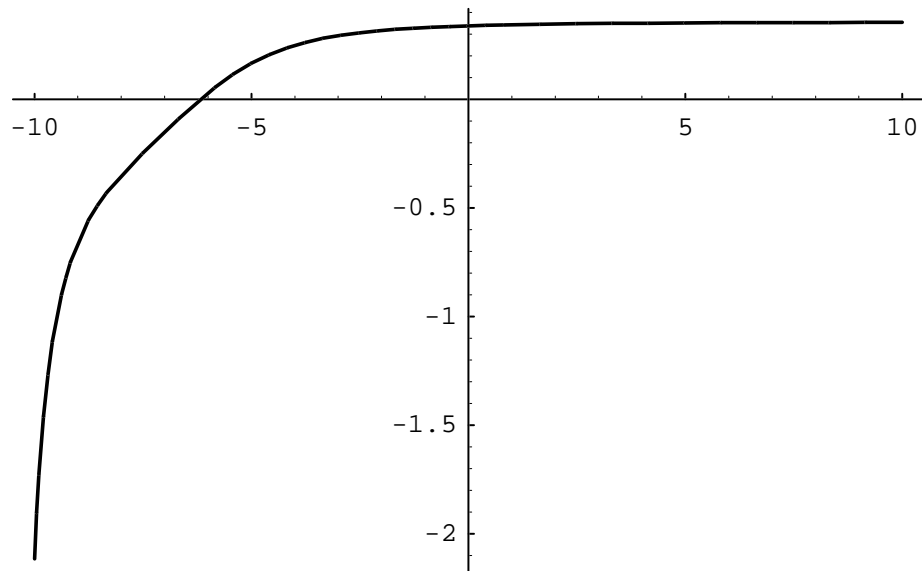
Figure 5: The evolution of $Q(t)$ versus the cosmic time t. Note that the inflationary period is characterized by an almost linear dependence on time.
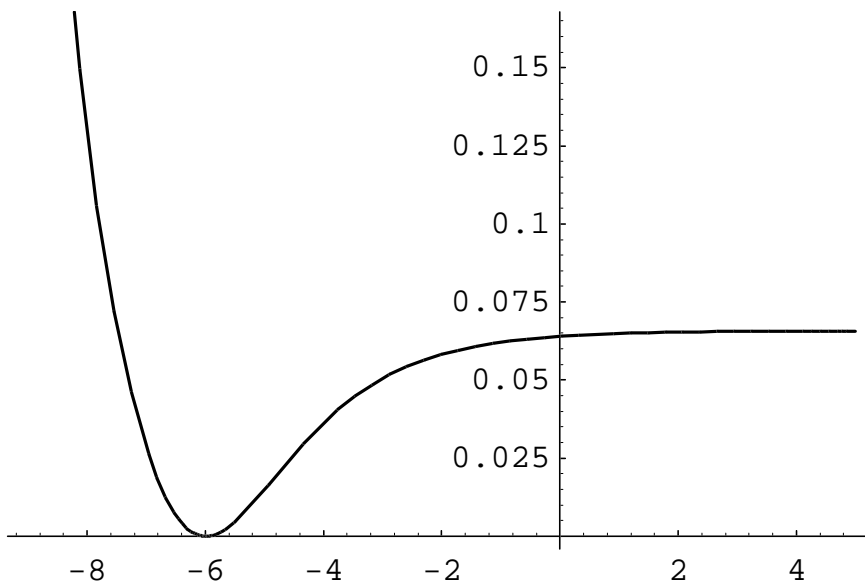
Figure 6: The evolution (versus the cosmic time t) of the central charge deficit $Q^2(t) \geq 0$ in our supercritical string model. Note that $Q^2(t)$ decreases rapidly with t during inflation. In contrast, the post-inflationary period is characterized by an increasing with time central charge deficit until the latter settles to its final (equilibrium) value. The increase of $Q^2(t)$ is in agreement with the fact that the Liouville mode is a non-unitary world-sheet field. However, there is an overall decrease of the central charge deficit during the flow from its initial ('near the singularity') to a final (equilibrium) value, as expected on general grounds.