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Holographic View of Causality and Locality via Branes in AdS/CFT Correspondence [1]

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abstract

We study dynamical aspects of holographic correspondence between $d = 5$ anti-de Sitter supergravity and $d = 4$ super Yang-Mills theory. We probe causality and locality of ambient spacetime from super Yang-Mills theory by studying transmission of low-energy brane waves via an open string stretched between two D3-branes in Coulomb branch. By analyzing two relevant physical threshold scales, we find that causality and locality is encoded in the super Yang-Mills theory provided infinite tower of long supermultiplet operators are added. Massive W-boson and dual magnetic monopole behave more properly as extended, bilocal objects. We also study causal time-delay of low-energy excitation on heavy quark or meson and find an excellent agreement between anti-de Sitter supergravity and super Yang-Mills theory descriptions. We observe that strong ‘t Hooft coupling dynamics and holographic scale-size relation thereof play a crucial role to the agreement of dynamical processes.

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1 Introduction

Initiated by important observation of Maldacena [1], it is now widely believed that an anti-de Sitter (AdS) supergravity is dual to large N limit of conformal field theory (CFT). One of the important features of this so-called AdS/CFT correspondence is that the duality demonstrates the holographic principle [15, 16, 17] – (d+1)-dimensional semi-classical gravity is holographically encoded into d-dimensional quantum field theory.

An immediate question has been how one can decode (d+1)-dimensional semi-classical gravity holographically out of d-dimensional quantum field theory. In addressing this question, all previous investigations on the AdS/CFT correspondence has focused exclusively on *static aspects* such as matching symmetries, excitation spectra and correlation functions. The holography, however, dictates much stronger correspondence than this. It implies that one can decode *dynamical aspects* of (d+1)-dimensional semi-classical gravity out of d-dimensional quantum field theory.

In this paper, we begin to investigate the *dynamical aspects* of the AdS/CFT correspondence [2]. More specifically, in the context of correspondence between semi-classical limit of anti-de Sitter Type IIB superstring in five dimensions and large-N limit of $SU(N)$ super Yang-Mills theory in four dimensions, we study whether and how *causality* and *locality* of the former can be decoded out of the latter. As a probe, we will consider a macroscopic string (either fundamental or Dirichlet string), whose appropriate configurations have been idenfied with heavy quark, meson or baryon [6, 7].

Our main conclusion is that, in order to encode semi-classical dynamics of Type IIB supergravity on AdS_5, neither conventional (keeping only dimension-four terms) nor Dirac-Born-Infeld form of the $d = 4, \mathcal{N} = 4$ super Yang-Mills theory is sufficient. Infinite towers of higher-dimensional operators encompassing short and long supermultiplets are to be included.

In due course of building up this conclusion, we also emphasize that, unlike flat spacetime,

[2] Being so, throughout this paper, we will reinstate c, the speed of light explicitly. The Planck's constant $\hbar$ is still set to unity.

it is quite dangerous and very often misleading to draw any conclusion on *dynamical* aspects based on *static* ones in anti-de Sitter spacetime. To illustrate this, in flat space, consider an open string stretched between two points of distance $R = |\mathbf{x}_1 - \mathbf{x}_2|$. Obviously, static mass of the string $M = R/\ell_s^2$ is proportional to the causal time-delay $\Delta t = R/c$ for a pulse to propagate from one end to the other. In anti-de Sitter spacetime, as we will see in later sections, static mass of an open string stretched radially equals to $M = |U_1 - U_2|$ is not relatd to the causal time-delay $\Delta t = \frac{1}{c}|\frac{1}{U_1} - \frac{1}{U_2}|$ in any simple way.

We begin with, in Section 2, dynamics of a macroscopic string that corresponds to a heavy quark or monopole in the Yang-Mills side. The causality and locality of semi-classical anti-de Sitter supergravity is then studied by adding a pulse on one end of the string and observing subsequent propagation of the pulse along the string. The pulse on the string corresponds, in the Yang-Mills side, to a spherical radiation wave-front of the heavy quark. We find that causal propagation of the pulse along the string is exactly encoded into causal propagation of the shperical wave-front of the heavy quark.

In Section 3, we study dynamics of a macroscopic string that corresponds to a heavy meson in the super Yang-Mills side. We again find that dynamics in the anti-de Sitter spacetime can be decoded correctly out of the heavy meson configuration in the super Yang-Mills side.

In Section 4, we consider Higgsing $SU(N)$ gauge group to $S[U(N-2) \times U(2)]$ and study low-energy dynamics of massive W-boson of $U(2) \to U(1) \times U(1)$. Via holography, the Yang-Mills theory ought to encode dynamics of two displaced D3-branes and an open string stretched between them in the bulk of anti-de Sitter spacetime. We then study (an extended version of) Thompson scattering of the open string by adding an incident low-energy plane wave on one of the two D3-branes . The plane wave will be scattered off by the open string and, after causal propagation along the string, part of the scattered wave will be excited on the other D3-brane. We estimate the transmission rate of the incident power to the other D3-brane via the string and show that the rate differs from the standard field theory result by the causal time-delay effect on the string.

In order to understand the causal time-delay phenomena better within the Yang-Mills theory context, we also derive an effective Lagrangian of the massive W-boson directly. The stretched open string, whose worldsheet dynamics is governed by $(1+1)$-dimensional field theory, couples two separate $(3+1)$-dimensional $U(1)$ super Yang-Mills theories. By integrating out the open string excitations, we find that dynamics of the massive W-boson in the $U(1) \times U(1)$ super Yang-Mills theory is described by a charged point particle whose Lagrangian contains proper-time derivatives of infinite order, similar essentially to the theory of Pais-Uhlenbeck [29]. This indicates that causality and locality of anti-de Sitter spacetime can be decoded out of super Yang-Mills theory only if infinite tower of short and long supermultiplets are included.

We end with further discussions on Section 5.

2 Causality and Locality in Flat Spacetime

Before dwelling into anti-de Sitter spacetime, we will first study causality and locality in flat spacetime. To make a direct comparison with anti-de Sitter spacetime later, as a probe, we will use a system consisting of two D3-branes and a open F- or D-string stretched between them.

Consider, in Type IIB string theory, two parallel D3-branes in flat spacetime $X = \mathbf{R}^{9,1}$. The D3-branes are oriented along $0, 1, 2, 3$ directions and are located along $4, \cdots, 9$ directions at $(r_1, 0, \cdots, 0)$ and $(r_2, 0, \cdots, 0)$, respectively. Low-energy dynamics on the D3-brane worldvolume is governed by $d = 4, \mathcal{N} = 4$ super Yang-Mills theory with gauge group $G = U(2)$. We will denote generators of $U(2)$ gauge group as T_a $(a = 1, 2, 3, 4)$, where $T_{1,2,3} = \sigma_{1,2,3}$ belong to the $SU(2)$ and $T_4 = id$ to the diagonal $U(1)$ subgroups. As the two D3-branes are separated by a distance $\Delta r = |r_1 - r_2|$, the $SU(2)$ gauge group is spontaneously broken to $U(1)$ generated by T_3 Cartan subalgebra. Together with the diagonal $U(1)$, the gauge group on the two D3-branes, which we label by 1 and 2, is then given by $H = U^{(1)}(1) \times U^{(2)}(1)$, generated by diagonal linear combinations, $(T_3 \pm T_4)/2$. A massive W-boson (or dual magnetic monopole) associated with the spontaneous symmetry breaking is represented by an open F-string (or D-string) stretched between the two D3-branes. The static mass of the W-boson and the dual magnetic monopole is given by

$$M_{\rm w} = T\Delta r, \qquad M_{\rm m} = \frac{T}{g_{\rm st}}\Delta r. \tag{2.1}$$

They are part of an isospin triplet under G (or its dual gauge group) and hence, under H, carry electric (or magnetic) charges $(Q, -Q)$ where $Q = \pm 1$ [3].

To investigate causality and locality of the flat Minkowski spacetime over the distance scale Δr, one will need to excite slightly, say, the first D3-brane and follow subsequent propagation of the excitation, which will eventually arrive at and excite the second D3-brane. In the limit $g_{\rm st} \rightarrow 0$, semi-classical dynamics of the Type IIB supergravity ought to obey both locality and causality. For low-energy excitation, the leading order process in this limit will be such that the excitation is transmitted from the first D3-brane to the second through an open F- or D-string stretched between them [4]. When exciting brane-wave on the first D3-brane, there are two channels available: massless Higgs or $U^{(1)}(1)$ gauge field waves. In what follows, we will consider exclusively adding a weak amplitude, plane-wave of the $U^{(1)}(1)$ gauge field on the first D3-brane [5]. The entire dynamical process is then viewed as the classic, Thomson scattering of the $U^{(1)}(1)$ radiation field off the open string.

The open string stretched between the two D3-branes is casually identified with a massive W-boson. This sounds paradoxical since the open string is an extended object whose characteristic scale is the string scale, ℓ_s, while the massive W-boson is a point-like object. An answer to this is well-known [8] ever since the advent of the D-branes. In this section, from the dynamical point of view, we will revisit this issue as it is intimately tied with understanding causality and locality of the ambient spacetime in which the D-branes are embedded. Specifically, we will

[3] Charge conjugation on the D3-brane worldvolume is generated by worldsheet parity reversal of the attached open strings.

[4] Processes involving more than one open strings or closed strings are at least $\mathcal{O}(g_{\rm st})$. At weak coupling, $g_{\rm st} \rightarrow 0$, their contribution is negligible.

[5] The Higgs field excitation can be deduced from that of gauge fields as they are related by underlying $\mathcal{N} = 4$ supersymmetry.

study the Thomson scattering process in two opposite limits of the massive W-boson, first in a point-particle limit and second in a stretched open string limit. We will find shortly that, over the distance scale Δr, super Yang-Mills theory will perceive nonlocality and acausality in the limit the W-boson may be treated as a point-particle. In order to recover locality and causality, the W-boson ought to behave more like an extended, bilocal object. Crossover between the two limits takes place, as anticipated, at $\Delta r \approx \mathcal{O}(\ell_s)$, the minimum distance scale of the ambient spacetime when probed by Type IIB F-string. Somewhat surprisingly, we will find that the same conclusion holds for a massive magnetic monopole, viz. an open D-string stretched between the two D3-branes.

2.1 Scattering of Flat Brane-Wave by Point-Like W-Boson

As is set up, a plane-wave of monochromatic radiation of $U^{(1)}(1)$ gauge field is incident on a free, massive W-boson of charge Q and mass $M_{\rm W}$. The W-boson will be accelerated and emit radiation and, if the energy of the incident radiation is low enough compared to the mass M, the emitted radiation will have the same frequency as the incident radiation. The whole process then can be described by conventional Thomson scattering process, for which the W-boson is treated as a point-particle. As the W-boson is charged under H, the emitted radiation will consist of both $U^{(1)}(1)$ and $U^{(2)}(1)$ gauge fields field. In terms of the D3-branes, this means that both D3-branes will be excited by dipole oscillation of the stretched open string (interpreted as the massive W-boson) after the incident $U^{(1)}(1)$ radiation on the first D3-brane scatters off it.

2.1.1 Scattering Equation of Motion

We will study dynamics of a massive W-boson, located at $\mathbf{X}(t)$, interacting with gauge fields of the gauge group H. From the D3-brane point of view, the W-boson is realized by an open F-string stretched between the two D3-branes. Denote spatial position (on the two D3-branes) of its endpoints as $\mathbf{X}_{1,2}(t)$. In the limit the W-boson is treated as a point-particle, the position of the W-boson $\mathbf{X}(t)$ ought to coincide with the two endpoint positions, $\mathbf{X}_{1,2}(t)$ all the time.

Consider an incident plane-wave radiation of the $U^{(1)}(1)$ gauge field, whose wave vector and frequency are denoted by $\mathbf{k}$ and ω, respectively. As the W-boson is charged with charge $(Q, -Q)$ under H, electric field of the incident radiation:

$$\mathcal{E}(\mathbf{x}, t) = \mathbf{E}^{(1)}(t)\, e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad \mathbf{E}^{(1)}(t) = \mathbf{E}_0^{(1)} e^{-i\omega t}, \tag{2.2}$$

according to the Lorentz force equation, will exert an *instantaneous* acceleration to the massive W-boson:

$$\ddot{\mathbf{X}}_1(t) = \ddot{\mathbf{X}}_2(t) = \frac{Q}{M_{\rm w}} \mathbf{E}^{(1)}(t) e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{2.3}$$

Throughout this paper, we will consider the weak field-strength limit. In this case, the W-boson is in nonrelativistic motion and, at leading order, the spatial dependence of the electric field on the right-hand side of Eq.(2.3) may be ignored.

The Eq.(2.3) shows that the two ends, even though separated by a distance Δr, accelerates in an identical manner, viz. the stretched open F-string behaves like a rigid rod. It indicates that, should the open F-string be treated as a point-particle, super Yang-Mills theory on the D3-brane worldvolume would entail inevitably nonlocality over the distance scale Δr in the direction perpendicular to the D3-brane. This is of course as it should be, in order for the perpendicular direction to be interpreted as color isospin direction.

2.1.2 Transmission Rate

To appreciate the significance of the Eq.(2.3), we will now calculate the rate of energy transmission T from the first D3-brane to the second through the stretched open string. From Eq.(2.3), one can determined the instantaneous power $P(t)$ radiated into a polarization state ε by the W-boson is [6]

$$\frac{dP}{d\Omega_2} = \frac{Q^2}{4\pi c^2} |\epsilon^* \cdot \ddot{\mathbf{X}}(t)|^2. \tag{2.4}$$

[6]For a weak field limit of the incident radiation, the W-boson may be treated as heavy enough. In this case, spin and charge degrees of freedom decouple each other. We will henceforth treat the W-boson as a spinless particle.

Averaging over a period $2\pi/\omega$, during which the charged particle moves a negligible fraction of one wavelength,

$$\left\langle \frac{dP}{d\Omega_2} \right\rangle = c \frac{\mathbf{E}_0^{(1)^2}}{8\pi} \left(\frac{Q^2}{M_{\rm w} c^2} \right)^2 |\epsilon^* \cdot \hat{\mathbf{E}}_0^{(1)}|^2. \tag{2.5}$$

The first factor $(\mathbf{E}_0^{(1)^2}/8\pi)c$ is nothing but the incident energy flux, viz. the time-averaged Poynting vector for the plane wave. Hence, differential transmission rate for an unpolarized incident radiation is given by

$$\left(\frac{dT}{d\Omega_2} \right)_{\rm classical} = \left(\frac{Q^2}{M_{\rm w} c^2} \right)^2 \cdot \frac{1}{2} \left(1 + \cos^2 \theta \right), \tag{2.6}$$

in which θ denotes the scattering angle in the laboratory frame. The total transmission rate T obtained thereof is

$$T_{\rm classical} = \frac{8\pi}{3} \left(\frac{Q^2}{M_{\rm w} c^2} \right)^2. \tag{2.7}$$

Characteristic feature of the classical Thomson scattering is that the cross-section is *independent* of the frequency of the scattered radiation. The result Eq.(2.6) is valid only at low frequency limit, $\omega \ll Mc^2$, for which the gauge field can be treated as a classical wave. As the energy of the incident radiation becomes comparable to the W-boson mass, the scattering process should be treated quantum mechanically, viz. treating the radiation as photons. Treated in the Coulomb gauge for which the transition matrix element is identical to the classial amplitude, the modification is from the phase space factor and hence is purely kinematical. The result is

$$\left(\frac{dT}{d\Omega_2} \right)_{\rm quantum} = \left(\frac{Q^2}{M_{\rm w} c^2} \right)^2 \left(\frac{\omega'}{\omega} \right)^2 \cdot \frac{1}{2} (1 + \cos^2 \theta), \tag{2.8}$$

where the ratio of the outgoing to the incident frequency is given by the well-known Compton formula:

$$\frac{\omega'}{\omega} = \left(1 + 2 \frac{\omega}{M_{\rm w} c^2} \sin^2 \frac{\theta}{2} \right)^{-1}. \tag{2.9}$$

For low frequency limit, $\omega \ll M_{\rm w} c^2$, one easily obtains the transmission rate as

$$T_{\rm quantum} = \frac{8\pi}{3} \left(\frac{Q^2}{M_{\rm w} c^2} \right)^2 \left(1 - 2 \frac{\omega}{M_{\rm w} c^2} + \cdots \right). \tag{2.10}$$

Had we have considered a point-like limit of the magnetic monopole dual to the W-boson, represented by an open D-string stretched between the two D3-branes, the transmission rate would be essentially the same as above except that M_w is to be replaced by the mass of the magnetic monopole M_m and that the charge Q is interpreted as the dual magnetic charge.

To recapitulate, in the limit the W-boson is point-like, viz. the stretched F-string moves rigidly, the transmission of radiation energy from the first D3-brane to the second through W-boson is *instantaneous* and is a consequence of the standard field theory result, Eq.(2.3). Based on this fact, we conclude that the D3-brane worldvolume dynamics, if treated in terms of conventional super Yang-Mills theory and point-like W-bosons, would perceive nonlocality and acausality over a distance scale Δr in the six-dimensional transverse space in X. From the super Yang-Mills theory point of view, this conclusion should be hardly surpring as the color isospin space is only an internal space and is not part of the four-dimensional spacetime (worldvolume of the D3-branes).

2.2 Scattering of Flat Brane-Wave by Charged Open String

From the underlying string theory point of view, nonlocality and acausality over the distance scale Δr are extremely bizzare. The transverse to the D3-branes is six-dimensional subspace in X and string theory ought to exhibit locality and causality, at least over a long distance limit. For example, energy transfer from the first D3-brane located at $r = r_1$ to the second at $r = r_2$ through the open string between them ought to take a causal time delay

$$\Delta t \equiv \frac{1}{c}(r_2 - r_1) = \frac{\Delta r}{c}. \tag{2.11}$$

Resolution of the puzzle is quite simple. As nonlocality and acausality has arisen from the point-particle limit of the W-boson, one expects that locality and causality will become transparent in the opposite limit, viz. the W-boson (realized as an open string stretched between the two D3-branes) is treated as a full-pledged string. This means that one will now need to study the Thomson scattering of the incident $U^{(1)}(1)$ radiation off a string-like W-boson. In fact, in this section, we will find that the W-boson dynamics is drastically modified from that in the point-particle limit and that the modified dynamics is precisely what allows to restore locality and causality of X.

2.2.1 Scattering Equations of Motion

Consider again the Thomson scattering of the $U^{(1)}(1)$ field radiation off the W-boson, but now in the limit the W-boson is treated as a full-fledged open string. Dynamics of the string is governed by Type IIB Green-Schwarz action, supplemented by an appropriate boundary action. In Nambu-Goto formulation, which is sufficient for the Thomson scattering process, the action is constructed in Appendix A. After fixing the κ-symmetry in a gauge compatible with the boundary conditions and worldsheet reparametrization symmetries in static gauge, whose steps are explained in detail in Appendix A, one finds that the action for the string transverse coordiinates $\mathbf{X}(r,t)$ is reduced to the form:

$$
\begin{aligned}
I_{\rm string} &= \int dt \left[-M_{\rm w} + \frac{T}{2}\left(\frac{1}{c^2}(\partial_t \mathbf{X})^2 - (\partial_r \mathbf{X})^2\right) + \cdots\cdots \right] \\
&+ \sum_{I=1,2} \int dt\, Q_I \left[A_0^{(1)}(\mathbf{r},t) + \frac{1}{c}\dot{\mathbf{X}}_1 \cdot \mathbf{A}^{(1)}(\mathbf{r},t) \right]_{r=\mathbf{X}_I(t)}, \qquad (Q_1 = -Q_2 = \pm 1).
\end{aligned} \tag{2.12}
$$

The first term in the expansion represents the static mass of the string $T\Delta r$, which equals to the mass of the W-boson $M_{\rm w}$ (see Eq.(2.1)). Consider, as posed above, Thomson scattering of a low-energy, monochromatic plane-wave of the $U^{(1)}(1)$ gauge field off the string endpoints. The boundary condition of the string coordinates $\mathbf{X}(t,r)$ is given by

$$
\begin{aligned}
T\partial_r \mathbf{X}(t,r)\Big|_{r=r_1} &= Q\mathbf{E}^{(1)}(t) \\
T\partial_r \mathbf{X}(t,r)\Big|_{r=r_2} &= 0 \ .
\end{aligned} \tag{2.13}
$$

According to the first boundary condition, the string endpoint $\mathbf{X}_1(t)$ (on the first D3-brane) will undergo a dipole oscillation and generate a pulse that will propagate subsequently along the string. The pulse may be decomposed into spectral components

$$
\mathbf{X}(t,r) = \int \frac{d\omega}{2\pi} \left[\, \mathbf{a}(\omega)\, e^{-i\omega(t-r/c)} \ + \ \tilde{\mathbf{a}}(\omega) e^{-i\omega(t+r/c)} \, \right], \tag{2.14}
$$

where the spectral amplitudes $\mathbf{a}(\omega), \tilde{\mathbf{a}}(\omega)$ are determined uniquely by the Thomson scattering boundary condition Eq.(2.13):

$$\mathbf{a}(\omega) \;=\; \tilde{\mathbf{a}}^*(\omega) \;=\; \frac{Q\mathbf{E}_0^{(1)}}{2\omega T}\left(\cot\omega\Delta t - i\right). \tag{2.15}$$

From Eqs.(2.14, 2.15), one immediately obtains an equation of motion for the string endpoint $\mathbf{X}_1(t)$ on the first D3-brane:

$$M_{\rm w}\left(\frac{\tanh\Delta t\partial_t}{\Delta t\partial_t}\right)\ddot{\mathbf{X}}_1(t) \;=\; Q\mathbf{E}^{(1)}(t). \tag{2.16}$$

One first finds that the inertia mass of the open string equals to the static mass, $M_{\rm w}$, as is dictated by the underlying Lorentz invariance. Compared to the point-particle limit of the W-boson, Eq.(2.3), equation of motion of the endpoint $\mathbf{X}_1(t)$ is changed to a one involving infinite-order kernel. As will be shown in section 2.3, the extra kernel, which has originated from taking into account of the string oscillations, is what enables low-energy dynamics of the D3-brane worldvolume to reproduce locality and causality of the ambient spacetime X. For now, note that, due to the kernel, one may interpret the endpoint $\mathbf{X}_1(t)$ to behave *heavier* than the point-like W-boson mass $M_{\rm w}$.

What about the dynamics of the second endpoint $\mathbf{X}_2(t)$ attached to the second D3-brane? For point-particle limit of the W-boson, we have seen that the two endpoints behaves identically, see Eq.(2.3). What one now finds, however, is that the dynamics of the second endpoint is governed by a different equation of motion:

$$M_{\rm w}\left(\frac{\sinh\Delta t\partial_t}{\Delta t\partial_t}\right)\ddot{\mathbf{X}}_2(t) \;=\; Q\mathbf{E}^{(1)}(t). \tag{2.17}$$

Comparison with Eq.(2.16) shows that the infinite-order kernel associated with the second endpoint is different from that of the first one. In particular, the kernel lets us to interpret that the seocond endpoint $\mathbf{X}_2(t)$ behaves *lighter* than the point-like W-boson mass $M_{\rm w}$.

The Eqs.(2.16, 2.17) constitute defining equations for the Thompson scattering off the open F-string. Compared to the Thomson scattering off the massive W-boson, new effects have entered through D3-brane-separation-dependent (see Eq.(2.11)), infinite-order kernels. As the two endpoints obey different equations of motion, at any moment, one would expect that $\mathbf{X}_1(t) \neq \mathbf{X}_2(t)$. Therefore, from the super Yang-Mills theory perspective, it suggests to view the W-boson more naturally as a sort of an extended, bi-local object.

2.2.2 Transmission Rate Across Open String

Following the same steps as in section 2.1.2, it is straightforward to calculate the transmission rate T (defined earlier in section 2.1.2) from the new equation of motion, Eq.(2.17) of the string endpoint on the second D3-brane.

Actually, one will need to take into account of another important corrections to the transmission rate: virtual W-boson pair and radiation reaction effects. They are entirely of field-theoretic origin and have nothing to do with string oscillation effects. Because of these effects, in field theory treatment, the W-boson might be viewed roughly as a particle with an effective size set by the W-boson Compton wavelength. Heuristically, the effects may be understood as follows. Low-energy dynamics of a single W-boson may be described by taking a non-relativistic limit of the W-boson gauge field W_μ:

$$\begin{aligned} W_0 &= \frac{1}{\sqrt{2M_\mathrm{w}}} \frac{i}{M_\mathrm{w} c} e^{-iM_\mathrm{w}c^2 t}\, \mathbf{D}\cdot\mathbf{\Phi}(\mathbf{r},t) \\ \mathbf{W} &= \frac{1}{\sqrt{2M_\mathrm{w}}} e^{-iM_\mathrm{w}c^2 t}\mathbf{\Phi}(\mathbf{r},t), \end{aligned} \tag{2.18}$$

in which charge-conjugate, anti-particle part is suppressed. Expanding the super Yang-Mills Lagrangian in the non-relativistic limit and taking the semi-classical limit, one obtains a Lagrangian for the W-boson particle:

$$\begin{aligned} L_\mathrm{w} &= \frac{1}{2} m_\mathrm{w} \dot{\mathbf{X}}\cdot\dot{\mathbf{X}} + Q\left(A_0 + \frac{1}{c}\dot{\mathbf{X}}\cdot\mathbf{A}\right) + \mathbf{S}\cdot\mathbf{B} + \cdots \\ &- \int d\mathbf{r}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right), \end{aligned} \tag{2.19}$$

where parts containing classical spin dynamics and interaction with Higgs field are suppressed for brevity. Following Abraham-Lorentz-type approach [9], it is straightforward to derive an equation of motion that takes into account of radiative reaction force. One obtains

$$M_{\rm w}\left(1-\frac{Q^2}{4\pi}\frac{\partial_t}{M_{\rm w}}+\cdots\right)\ddot{\mathbf{X}}(t)=Q\mathbf{E}_0(t). \tag{2.20}$$

If one takes into account of virtual W-boson pair corrections, one also obtains a term similar to the second on the left-hand side. At low frequency, the second term is suppressed compared to the first by $\mathcal{O}(\omega/M_{\rm w})$ and hence, according to Eq.(2.4), gives rise to $\mathcal{O}(\omega^2/M_{\rm w}^2)$ correction to the transmission rate.

Taking into account of the above effects, one obtains the transmission rate for F-string as:

$$T_{\rm F-string}=\frac{2\pi}{3}\left(\frac{Q^2}{M_{\rm w}c^2}\right)^2\left(1-2\frac{\omega}{M_{\rm w}c^2}+\cdots\right)\cdot\left(1-\left(\frac{\omega}{M_{\rm w}c^2}\right)^2+\cdots\right)\cdot\left[\frac{\omega\Delta t}{\sin\omega\Delta t}\right]^2. \tag{2.21}$$

The first two brackets represent the result derived from the limit the W-boson is treated as a point-particle. The third bracket is the correction due to virtual W-boson and radiation reaction effect estimated above. The last bracket comes from the string oscillation effects, viz. from the fact that the massive W-boson is actually a stretched open string.

It is instructive to consider the transmission rate for, instead of a F-string, a D-string is stretched between the two D3-branes. From super Yang-Mills point of view, the open D-string corresponds to a magnetic monopole dual to W-boson. In this case, scattering of the brane wave takes place from interaction of the open D-string with magnetic component of the radiation field. The corresponding transmission rate is obtained straighforwardly:

$$T_{\rm D-string}=\frac{2\pi}{3}\left(\frac{Q^2}{M_{\rm m}c^2}\right)^2\left(1-2\frac{\omega}{M_{\rm m}c^2}+\cdots\right)\cdot\left(1-\left(\frac{\omega}{M_{\rm w}c^2}\right)^2+\cdots\right)\cdot\left[\frac{\omega\Delta t}{\sin\omega\Delta t}\right]^2, \tag{2.22}$$

where Q should now be interpreted as magnetic charge of the dual W-boson. One again finds that, apart from the kinematical, Compton scattering correction, corrections of $\mathcal{O}(\omega^2/M_{\rm w}^2)$ arise from the classical finite-size of the magnetic monopole, set by the Compton wavelength of the W-boson, and radiation reactive force effects.

We emphasize that, for both W-boson and its dual magnetic monopole, the correction due to finite-size and radiation reactive force effects is set by the W-boson mass, $M_{\rm w}$. Given that it is the only low-energy scale in the system at weak coupling limit, the fact that the correction for both W-boson and magnetic monopole is governed by the same low-energy scale should not be surprising. Incidentally, the correction also entails an interesting deviation from S-duality of the $d = 4, \mathcal{N} = 4$ super Yang-Mills theory.

2.3 Locality and Causality in Flat Spacetime

From the super Yang-Mills theory point of view, the entire dynamical process is nothing but the classic, Thomson scattering of radiation off a massive W-boson.

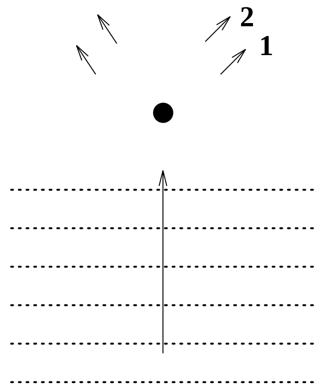

Figure 1: Thompson scattering of a massive W-boson. The $U^{(1)}(1)$ plane wave field, whose wave front is denoted by dashed lines, is scatted off the point-like massive W-boson. The scattered spherical wave fields of both $U^{(1)}(1)$ and $U^{(2)}(1)$ gauge groups are radiated away simultaneously.

2.3.1 Retarded Dynamics of String Endpoints

It is easy to see that the string endpoint on the second D3-brane experiences a retarded motion.

Consider now the same process as viewed from the stretched open string between the two D3-branes. As shown in Figure 2, the incident plane wave field of $U^{(1)}(1)$ gauge group is localized on the D3-brane at $x_4 = r_1$. Once the incident wave field shines the string endpoint on this D3-brane and induce oscillatory motion, spherical wave field of the $U^{(1)}(1)$ gauge group will be radiated off. At the same time, the oscillation pulse of the endpoint will propagate along the string and, after causal time-delay $\Delta t = \Delta r/c$, spherical wave field of the $U^{(2)}(1)$ gauge group will begin to be radiated off. Hence, unless the two endpoints of the open string reacts somehow non-locally, the radiation field of the two gauge groups will not be simultaneous.

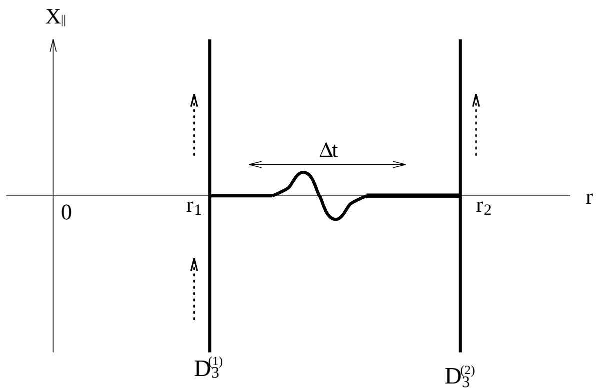

Figure 2: a stretched F- or D-string connecting two probe D3-branes in flat spacetime background. vibration of the stretched string causes retarded signal propagation between the two D3-branes only when the separation is bigger than the string scale.

When viewed from ten-dimensional flat spacetime, the D3-brane locations $x_5 = r_1, r_2$ are two distinct points between which the open string is stretched. Hence, it makes all sense to address causality and locality for processes between the two points $x_5 = r_1, r_2$. When viewed from Yang-Mills theory, however, the 5-th direction is interpreted as 'color isospin' direction defined at each point on $(3 + 1)$-dimensional D3-brane world-volume. It is unimaginable to conceive a possible causal time delay $\Delta r/c$ for a component of the color isospin to rotate to another.

2.3.2 Physical Thresholds: 'Winding' versus 'Momentum'

As is evident from the transmission rates, Eqs.(2.21, 2.22), frequency-dependent corrections are characterized by two distinct physical threshold scales inherent to the problem. The first is set by the W-boson mass, $M_{\rm w} = T\Delta r$, defining a physical threshold (of field theoretic origin) above which description in terms of abelian gauge theory with gauge group H breaks down. The second is set by energy gap of string oscillation, $1/\Delta t = c/\Delta r$, above which a point-particle description of the W-boson breaks down. Being proportional to Δr and inversely to Δr, respectively, we shall be referring the first threshold as 'winding threshold' $\Lambda_{\rm w}$ and the second as 'momentum threshold', $\Lambda_{\rm m}$:

$$\Lambda_{\rm w} = M_{\rm w} = T\Delta r, \qquad \Lambda_{\rm m} = \frac{1}{\Delta t} = \frac{\hbar c}{\Delta r}. \tag{2.23}$$

As there are two distinct physical thresholds, depending on which one sets the lower scale, low-energy dynamics of the open string will become quite different. Note that winding and momentum thresholds depend on the spatial separation Δr inversely. Therefore, one finds a version of string uncertainty relation

$$\Lambda_{\rm w}\Lambda_{\rm m} \quad \approx \quad \mathcal{O}\left(\hbar c T\right), \qquad\qquad (T = \frac{1}{\ell_s^2}) \tag{2.24}$$

where the right-hand-side is set by fundamental constants only.

Hence, if $\Delta r \ll \ell_s$, relevant scale for low-energy processes is set by the 'winding threshold'. As string oscillation excitations are completely irrelevant, the stretched open string behaves literally like a rigid-rod, to which the D3-branes serve as a guiderail. Thus, being a rigid body, the open string is best approximated as a point-particle, W-boson. This is the limit in which the D3-brane worldvolume dynamics is best approximated by super Yang-Mills theory, either in conventional or Dirac-Born-Infeld form.

we see that physics is non-local over the distance Δr. This is what we should have expected since this is a sub-stringy scale, for which conventional spacetime picture does not hold. If $\Delta r \gg \ell_s$, the Yang-Mills theory does not provide a good description and one has to take into account of full-fledged string oscillation threshold.

Normally, the massive W-boson mass $T\Delta r$ sets a physical threshold to the Yang-Mills theory. In the present case, the effective inertia masses of the string end-points have zeros at frequency ω harmonics of $1/\Delta r$. This resonance effect originates from oscillatory excitation of the open string and hence gives rise to a *new physical threshold*. The stretched open string will behave essentially as a point-particle (massive W-boson) only if the new threshold is much higher than the first. This yields the well-known condition for validity of the Yang-Mills theory description of the D3-branes:

$$T\Delta r \ll \frac{1}{\Delta r} \quad \rightarrow \quad \Delta r \ll \ell_s. \tag{2.25}$$

Exactly the same condition applies for the stretched string being a D-string. Stretched D-string is identified with magnetic monopole. Its classical size is given by the massive W-boson scale $T\Delta r$ [7] and defines the first threshold scale. Requirement that this threshold scale is much less than the oscillation threshold scale again yields Eq.(2.25).

2.3.3 Stretching Beyond Super Yang-Mills Theory

The above analysis tells us more. Suppose Δr becomes larger than the string scale. This is the regime where the conventional Yang-Mills theory description in terms of either renormalizable or Dirac-Born-Infeld action is no longer valid. Nevertheless, can one still use Yang-Mills theory description possibly with some modification? The above analysis, especially, the generalized point-particle description of the stretched open string Eqs.(2.16, 2.17) suggests that this is indeed so. The infinite-order time derivatives in the equations of motion imply that, in Yang-Mills theory, infinite tower of long super-multiplets (which involves higher-order derivatives) is potentially capable of describing the stretched string in terms of point-particle description. To see this, we now turn to derivation of an effective Lagrangian of the stretched open string.

3 Dynamical Aspect of Scale-Size Relation

In [6, 7], it has been shown that a macroscopic F-string (D-string) in the bulk corresponds a heavy quark (monopole) in Yang-Mills theory. Ground-state configuration of the string is straight, extending radially outward. Harmonic fluctuation of the string has been studied previously [6, 20]. Using the result, we now investigate the issue of causality and locality.

In this section, we will study dynamics of charged external probes. As a charged state forms a super-selection sectors, it will be necessary to control long-range Coulomb field produced by

[7] At weak coupling, classical size of the stretched D-string $T\Delta r$ is much larger than the Compton wavelength of the monopole $T\Delta r/g_{\rm st}$ and hence a semi-classical treatment of the monopole is justifiable.

the charged particle. This may be achieved, for example, by Higgsing, viz. separating some of the D3-branes and study charged state with respect to the gauge group of the separated D3-branes.

By the presence of large-N D3-branes, ambient spacetime is now turned into anti-de Sitter spacetime:

$$ds^2 = \ell_s^2 \left[\frac{U^2}{g_{\rm eff}} \left(-dt^2 + d\mathbf{x}_\parallel^2 \right) + g_{\rm eff} \frac{dU^2}{U^2} + g_{\rm eff} d\Omega_5^2 \right]. \tag{3.26}$$

The most important consequence out of the anti-de Sitter spacetime background is that the scale position U in the anti-de Sitter spacetime is related directly to the scale size R in the super Yang-Mills theory:

$$R \quad = \quad \frac{g_{\rm eff}}{U}. \tag{3.27}$$

This relation has been first derived from evaluation of the Wilson loop [6, 7] and later extended in generality [10].

3.1 Holography of Heavy Quark Dynamics

Consider a heavy quark of large-N super Yang-Mills theory at zero temperature. To control long-range Coulomb field produced by the quark, we will separate a single D3-brane to the Coulomb branch and consider a quark coupled to it. Denote the location of the displaced D3-brane as U_*. Then, a heavy quark (monopole) of the super Yang-Mills theory is realized by a macroscopic F- (D-) string ending on the displaced D3-brane at $U = U_*$ and stretched outward along U-direction. Dynamics of the macroscopic string may be studied using Nambu-Goto action in the anti-de Sitter space Eq.(3.26). Taking static gauge $(\tau, \sigma) = (t, U)$ and expanding string transverse coordinates $(\mathbf{X}_\parallel, \Omega_5)$ up to harmonic fluctuations, one finds [6]

$$L_{\rm NG} = -\int_{U_*}^{\infty} dU + \int_{U_*}^{\infty} dU \mathcal{L}^{(2)} + L_{\rm boundary} \tag{3.28}$$

where

$$\mathcal{L}^{(2)} = \frac{1}{2} \left[\left(\frac{1}{c^2} \dot{\mathbf{X}}_\parallel^2 - \frac{U^4}{g_{\rm eff}^2} {\mathbf{X}'_\parallel}^2 \right) + U^2 \left(\frac{g_{\rm eff}^2}{U^4} \frac{1}{c^2} \dot{\Omega}_5^2 - {\Omega'_5}^2 \right) \right] + \cdots \tag{3.29}$$

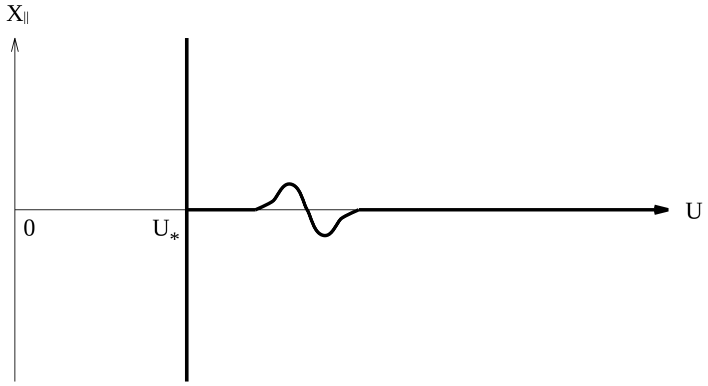

Figure 3: string attached to a probe D3-brane in anti-de Sitter space

and $L_{\rm boundary}$ specifies boundary conditions at $U = U_*$.

Let us begin with *static properties* of the macroscopic string. The first term in Eq.(3.28) represent the *static mass* of the string

$$M_{\rm string}(U_*) = \int_{U_*}^{\Lambda_{\rm UV}} dU, \qquad\qquad \Lambda_{UV} \to \infty. \tag{3.30}$$

For any finite U_*, the static mass diverges as the cutoff $\Lambda_{\rm UV} \to \infty$. As shown first in [6, 7] and discussed further in [10, 11], this is nothing but anti-de Sitter manifestataion of divergent self-energy of a heavy quark. A physically meaningful, finite quantity would be a *residual static mass* between strings ending on different locations of U_*, which we may define as

$$\begin{aligned} \Delta M_{\rm string}(U_*) &\equiv \int_0^{U_*} dU \, \frac{dM_{\rm string}}{dU} \\ &= U_*. \end{aligned} \tag{3.31}$$

Using the scale-size relation Eq.(3.27), one then finds that $\Delta M_{\rm string}(U_*) = g_{\rm eff}/R_*$ corresponds to the Coulomb field energy measured at radius R_*.

What does this mean? In order to understand this scale, let us go back to the interpretation of the macroscopic from Yang-Mills theory side. At the origin of the Coulomb branch (viz. $U_* = 0$), the macroscopic string creates not only a heavy quark (transforming as the defining representation of $SU(N)$) itself but also long-range color Coulomb field around it, as depicted in the left of Figure 2. Away from the origin of the Coulomb branch, one might expects that the long-range Coulomb field is cut-off at the Compton wavelength U_*^{-1} of the massive gauge bosons [8]. However, as pointed out in [6, 7], the strong ‘t Hooft coupling dynamics cuts off the long-range field at a larger scale, $g_{\text{eff}} U_*^{-1}$.

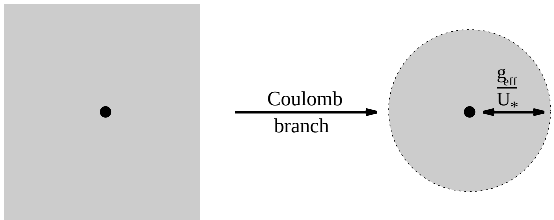

Figure 4: Coulomb branch view gauge field generated by a test quark (monopole)

Thus, we find that ΔM_{string} is nothing but the ‘field energy’, the energy associated with Coulomb field outside radius $R_* = g_{\text{eff}}/U_*$ in the Yang-Mills theory. Summarizing, static property (such as static mass) of the macroscopic string is determined by the quantity $\int dU$.

What about *dynamical* properties? Consider exciting a low-energy pulse at the outer end of the string. The pulse will then propagate at the speed of light (which has been explicitly checked in [6]) along the string toward U_*. According to the UV-IR relation, in super Yang-Mills theory, the pulse looks like a spherical Yang-Mills wave generated at the position of the heavy quark and then propagates outward to the outer edge of the Coulomb field, $R_* = g_{\text{eff}} U_*^{-1}$. From Yang-Mills point of view, for the outgoing spherical wave propagate a radial size R_*, it will take Yang-Mills time:

$$(\Delta t)_{\text{YM}} = \frac{1}{c} R_*. \tag{3.32}$$

If causality is implied by the holographic relation, then the time-delay for the pulse to propagate along the string should be equal to that for the spherical wave to propagate outward. Let us check this explicitly. The string pulse propagates along the string, viz. a null geodesic

[8] Recall that the massive gauge bosons are BPS particles

in (U, t) subspace of AdS_5:

$$\frac{1}{c}\frac{\partial \mathbf{X}_\parallel}{\partial t} = \pm \frac{U^2}{g_{\rm eff}}\frac{\partial \mathbf{X}_\parallel}{\partial U}; \qquad \frac{g_{\rm eff}}{U^2}\frac{1}{c}\frac{\partial \Omega_5}{\partial t} = \pm \frac{\partial \Omega_5}{\partial_U}. \tag{3.33}$$

Thus, measured in Yang-Mills units, the anti-de Sitter time-delay $(\Delta t)_{\rm AdS}$ for the pulse to travel from $U = \infty$ to $U = U_*$ along the string is given by

$$(\Delta t)_{\rm AdS} = -\frac{1}{c} g_{\rm eff} \int_\infty^{U_*} \frac{dU}{U^2} = \frac{1}{c}\frac{g_{\rm eff}}{U_*}. \tag{3.34}$$

Remarkably, the holography, more specifically, the UV-IR relation Eq.(3.27) shows that the bulk propagation time is exactly the same as that along the boundary!

The last quantity is precisely the time-delay for a spherical Yang-Mills wave to propagate over the scale size $[0, R_*]$. We thus find the holography works nicely as well for dynamical properties. In particular, we find that locality and causality in U-direction is encoded into causal propagation of spherical waves whose wave front varies in size.

The same analysis goes through for monopole. Here, an interesting issue is posed by the mechanism. As is well-known, the bare monopole has a semiclassical size, the mass of the massive W-boson.

3.2 Vibrating Quark (Monopole) at Finite Temperature

It is also possible to show that the causality and locality holds also for super Yang-Mills theory at nonzero temperature. The near-horizon geometry of near extremal D3-branes is described by the anti-de Sitter-Schwarzschild spacetime, whose (t, R) subspace is given by:

$$ds^2 = \ell_s^2 \left[-\left(1 - \frac{U_0^4}{U^4}\right)\frac{U^2}{g_{\rm eff}} dt^2 + g_{\rm eff}\frac{dU^2}{U^2}\left(1 - \frac{U_0^4}{U^4}\right)^{-1}\right] \tag{3.35}$$

From Nambu-Goto Lagrangian, one again finds that the static mass of the string is given by

$$M = \int_{U_0}^\infty dU. \tag{3.36}$$

Consider introducing a heavy quark to the Yang-Mills theory. Once again, the quark is described by a macroscopic F-string stretched along U-direction [20, 21], as depicted in Figure

3. In the present case, the string terminates at the Schwarzschild horizon, $U = U_0$, where it melts completely and dissipates the F-string flux into the inner horizon region.

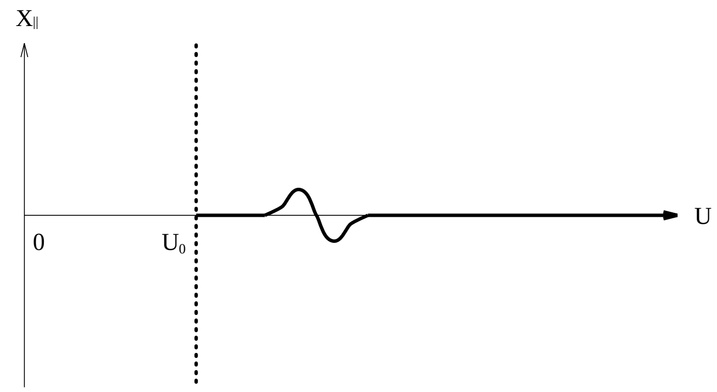

Figure 5: a vibrating semi-infinte string in anti-de Sitter Schwarzschild space

This phenomenon has a direct counterpart in super Yang-Mills theory. At zero temperature, Coulomb field of the static quark extends everywhere, as depicted at the left in Figure 4. At finite temperature, however, the Coulomb field becomes Debye-screened and is expected to extend to the scale of Debye wavelength $U_0^{-1} = (g_{\rm eff} T)^{-1}$. Using the holographic UV-IR relation, one now actually finds that the Coulomb field is cut off at a larger scale $g_{\rm eff} U_0^{-1}$. As in zero temperature case, one may interpret this as reflecting the fact that strong 't Hooft coupling dynamics gives rise to the cutoff at a larger scale $g_{\rm eff} U_0^{-1}$.

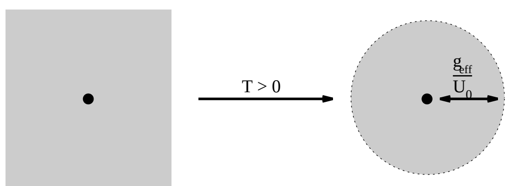

Figure 6: an isolated quark (monopole) at finite temperature $\mathcal{N} = 4$ super Yang-Mills theory

In fact, by demanding causality, one can understand the fact that string must end at $U = U_0$. Consider a weak pulse injected at $U = \infty$ and propagating inward. The time-delay for the pulse to reach the position $U(U_0 \leq U \leq \infty)$ is easily estimated:

$$(\Delta t)_{\rm AdS} = \frac{g_{\rm eff}}{U_0} \int_0^{U_0/U} \frac{dx}{1 - x^4}. \tag{3.37}$$

The time-delay becomes infinite when $U \to U_0$. The inner-horizon region $U < U_0$ is unphysical since the time-delay is negative, viz. acausal. As is explicit in Eq.(3.37), the time delay is measured in units of $g_{\rm eff} U_0^{-1}$ reflects the fact that it is a characteristic scale size in Yang-Mills theory at finite temperature.

In fact, the scale $R_0 = g_{\rm eff} U_0^{-1}$ corresponds to a *physical threshold* scale in Yang-Mills theory at finite temperature. At weak 't Hooft coupling limit, Matsubara energy scale associated with dimensionally reduced three-dimensional fermions (satisfying anti-periodic boundary condition around Euclidean time) is $\mathcal{O}(T)$, which equals to $U_0/g_{\rm eff}$. If this observation continues to hold somehow in super Yang-Mills theory at zero temperature, then we are prompted to expect that the scale $R_* \equiv g_{\rm eff} U_*^{-1}$ actually corresponds to a new *physical threshold* scale. Recall that this scale is completely different from either Compton wavelength of massive gauge bosons U_*^{-1}, which certainly corresponds to a physical threshold. Nevertheless, the fact that the scale R_* is $g_{\rm eff}$ times Compton wavelength of massive gauge bosons is tantalizingly similar to the observation of Shenker made in a different context [23].

3.3 Holography of Heavy Meson Dynamics

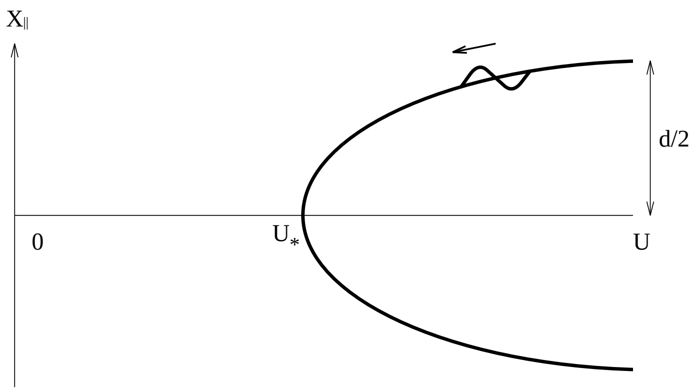

Figure 7: signal propagation along the Wilson loop in anti-de Sitter space

A more intricate bulk probe is the macroscopic string configuration corresponding to the heavy meson [6, 7]. In the static gauge, a first-integral of motion:

$$G^2 U'^2 + G = \frac{g_{\rm eff}^2}{U_*^4} \tag{3.38}$$

has been derived. Small vibration of the string propagates along the static configuration of string $U = U(\sigma)$. The time delay for a signal to travel between the quark and anti-quark (both 'located' at $U = \infty$) is again calculated straightforwardly by solving string equation of motion along the null geodesic:

$$\sqrt{G} dU^2 = \frac{1}{\sqrt{G}} \left(-dt^2 + d\sigma^2 \right) . \tag{3.39}$$

This yields

$$(\Delta t)_{\rm AdS} = \int_{-d/2}^{+d/2} d\sigma \sqrt{GU'^2 + 1}. \tag{3.40}$$

Using Eq.(3.38), one immediately finds that

$$\begin{aligned} (\Delta t)_{\rm AdS} &= 2\frac{g_{\rm eff}}{U_*^2} \int_{U_*}^{\infty} dU \frac{1}{\sqrt{U^4/U_*^4 - 1}} \\ &= \frac{2\sqrt{\pi}\Gamma(5/4)}{\Gamma(3/4)} \frac{g_{\rm eff}}{U_*}. \end{aligned} \tag{3.41}$$

From the super Yang-Mills theory perspective, natural scale of the meson is the inter-quark distance d. Hence, one expects that

$$(\Delta t)_{\rm YM} = A \frac{1}{c} d, \tag{3.42}$$

where A is a $\mathcal{O}(1)$ numerical coefficient, which should reflect the details of the dipole field configuration produced by the quark-antiquark pair. For mesons, precise form of the UV-IR relation has been derived [6, 7]:

$$\frac{d}{2} = \frac{\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)} \frac{g_{\rm eff}}{U_*}, \tag{3.43}$$

Hence, equating Δt of anti-de Sitter spacetime with that of Yang-Mills theory, one finds that

$$A = \frac{\Gamma(5/4)\Gamma(1/4)}{\Gamma^2(3/4)} = 2.188... \tag{3.44}$$

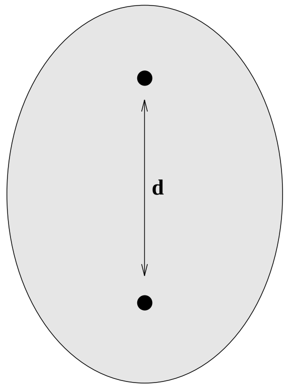

Figure 8: aerial view of quark anti-quark meson and field around them. The gapless signal propagates within the dashed region.

4 Causality and Locality in Anti-de Sitter Spacetime

Having understood dynamical aspect of the scale-size relation better, especially for semi-infinite string, we will now reconsider scattering of brane waves by an open string and consequent implication to causality and locality, but now all in anti-de Sitter spacetime.

We will now show that, throughout the moduli space of the Coulomb branch (except possibly at the origin), 'momentum' threshold of the open string is always lower than 'winding' threshold in the strong 't Hooft coupling limit. This is precisely what is needed in order for the super Yang-Mills theory to be able to encode causality and locality of the anti-de Sitter spacetime. Turning around, one now finds that, in large-N, strong coupling limit of super Yang-Mills theory, W-bosons ought to behave as a sort of bi-local object with infinite tower of light 'internal' excitations.

4.1 D3-Branes in the Coulomb Branch

The anti-de Sitter spacetime is generated as the near-horizon geometry of infinitely many ($N \to \infty$) D3-branes, coincident one another. Therefore, to explore causality and locality of the anti-de Sitter spacetime, from the viewpoint of super Yang-Mills theory, one will need to examine low-energy moduli dynamics over the Coulomb branch.

As in flat spacetime, we will consider displacing two out of N coincident D3-branes, at positions U_1, U_2 respectively, and a macroscopic open string stretched between them. We will again excite the displaced D3-branes with low-energy waves and study scattering of the waves by the open string. The configuration corresponds to the following sequential symmetry breaking:

$$SU(N) \to G = S\left[U(N-2) \times U(2)\right] \to H = S\left[U(N-2) \times (U(1) \times U(1))\right]. \qquad (4.45)$$

From super Yang-Mills theory perspectives, this appears quite a complicated problem, as the excitation of, say, first D3-brane will reach to the second D3-brane not only through the open string stretched between them but also indirectly through the coincident $(N-2)$ D3-branes, with which the two D3-branes interact. At the least, because of $\mathcal{O}(N^2)$ possible transmission channels through the 'background' $(N-2)$ D3-branes, one might be tempted to conclude that the transmission mechanism in large-N super Yang-Mills theory is not mediated by massive W-bosons but by complicated large-N, strong 't Hooft coupling dynamics.

This, however, should not be the case. Let us study super Yang-Mills theory with gauge group $G = S[U(N-2) \times U(2)]$. As we are interested in dynamics of the two displaced D3-branes, we will integrate out over the Higgs branch, viz. open strings transforming in $(\mathbf{N-2}, \overline{\mathbf{2}})$ and its complex-conjugate representations. In the limit $N \to \infty$ and large 't Hooft coupling, the integration amounts to resummation of large-N Feynman diagrams. As have shown in [18], the result is to produce anti-de Sitter spacetime, in which the two displaced D3-branes is embedded.

Intuitively, this can be understood by observing that root-mean-squared positions of $(N-2)$ D3-branes and displaced two D3-branes are $(g_{\rm st}N)^{1/4}$ and $(g_{\rm st})^{1/4}$, respectively. That is, in the large-N limit, position of the two displaced D3-branes is well-localized inside the anti-de Sitter spacetime. It should also become clear that, even after adding a stretched open string between the two displaced D3-branes (associated with the symmetry breaking $G \to H$), integration over the Higgs branch is essentially the same, viz. both the two D3-branes and the stretched open string between them will experience the ambient background as anti-de Sitter spacetime.

Henceforth, at large-N and 't Hooft coupling regime, in order to study causality and locality over the Coulomb branch, one will only need to study low-energy excitation of massive W-boson in (quantum-corrected) super Yang-Mills theory with symmetry breaking pattern $U(2) \to U(1) \times U(1)$.

4.2 Scattering of Brane Wave by Charged Open String

Dynamics of the open string stretched between the two D3-branes is governed by the Type IIB Green-Schwarz action in the anti-de Sitter spacetime. In Appendix B, for Nambu-Goto form of the action, details of gauge fixing of local κ- and reparametrization symmetries is explained. It turns out the gauge-fixing as well as analysis of string dynamics thereof become enormously simplified once one adopts a new coordinate, $R \equiv g_{\rm eff}/U$. According to the scale-size holography relation Eq.(3.27), the new coordinate R is nothing but 'size' variable of super Yang-Mills theory. It is interesting that, in analyzing physical process in anti-de Sitter spacetime, description in terms of super Yang-Mills theory is more suited and efficient. Indeed, in terms of the 'size' coordinate R, the metric of the anti-de Sitter spacetime is described in Poincaré coordinates:

$$ds^2_{\rm AdS} = \frac{g_{\rm eff}}{R^2}\Big[- dt^2 + d\mathbf{x}_\parallel^2 + dR^2 + R^2 d\Omega_5^2\Big]. \qquad (4.46)$$

As elaborated in Appendix B, this coordinate choice leads the Green-Schwarz action to an almost identical form, after gauge fixing, to the one in flat spacetime, for both bosonic and fermions parts. In fact, it has been observed already [6] that harmonic analysis of string fluctuation becomes considerably simplied in the Poincaré coordinate system. The gauge-fixed action of the open string is now given by

$$\begin{aligned} I_{\rm string} &= \int dt \left[-M_{\rm w} + \frac{g_{\rm eff}}{2}\int_{R_1}^{R_2} \frac{dR}{R^2}\left(\frac{1}{c^2}(\partial_t \mathbf{X})^2 - (\partial_R \mathbf{X})^2\right) + \cdots \right] && (4.47) \\ &+ \sum_{I=,2} Q_I \int dt \left[A_0^{(I)}(\mathbf{r}, t) + \frac{1}{c}\dot{\mathbf{X}}_1 \cdot \mathbf{A}^{(1)}(\mathbf{r}, t)\right]_{\mathbf{r}=\mathbf{X}_I(t)}. && (4.48) \end{aligned}$$

The first point to note is that static mass of the string in anti-de Sitter spacetime, the first term in Eq.(4.47), $g_{\rm eff} \int dR/R^2$ equals to $(U_2 - U_1)$ in terms of the original 'scale' coordinate.

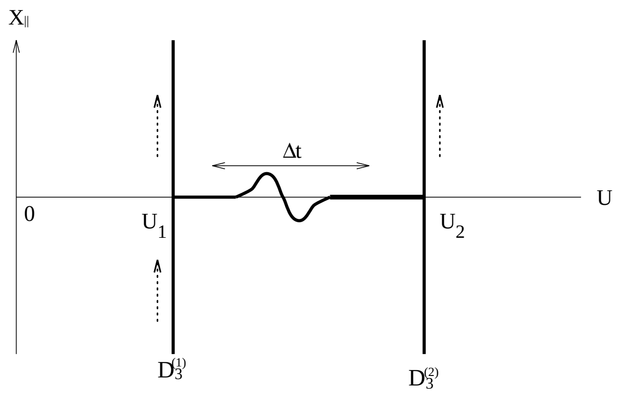

Figure 9: a stretched F- or D-string connecting two probe D3-branes in the Coulomb branch. vibration of the stretched string causes retarded radiation between the two D3-branes.

Similar result holds also for an open D-string stretched between the two D3-branes. Hence,

$$M_{\rm w} = \Delta U, \qquad\qquad M_{\rm m} = \frac{1}{g_{\rm st}} \Delta U. \tag{4.49}$$

Recalling that $\delta U = T\Delta r$ (cf. Eq.(3.26)), this means that static mass of the string in anti-de Sitter spacetime is the same as that in flat spacetime. This is, of course, an obvious fact from the super Yang-Mills theory perspective. Both W-boson and dual magnetic monopole are BPS-saturated states and hence their masses are not renormalized as one interpolates from weak to strong 't Hooft coupling regime. The weak and strong 't Hooft coupling regimes are, roughly speaking, dual to flat and anti-de Sitter spacetimes.

One might have expected that, as the mass of the open string is determined by the distance between the two D3-branes, the W-boson can serve as an excellent probe of whether the ambient spacetime is flat or curved. For example, from Yang-Mills theory perspective, Feynman diagrams involving virtual W-bosons depends on the W-boson mass and hence on the distance between the two D3-branes. Large mass expansion of the Feynman diagrams corresponds to small energy expansion.

For the Thomson scattering in flat spacetime, we have observed that the static mass of the W-boson and the causal time-delay are governed by one length scale, Δr, the separation between the two D3-branes. One might tempted to guess that it will be the same for anti-de Sitter spacetime, as the separation is the only relevant length scale in the problem. This, however, turns out not quite right. We will now show that the causal time-delay is governed not by the scale ΔU, but by

$$\Delta\tau \equiv \frac{1}{c}\Delta R = \frac{1}{c} g_{\rm eff} \Big| \frac{1}{U_1} - \frac{1}{U_2} \Big|. \tag{4.50}$$

It is clear that $\Delta\tau$ has no simple functional relation to ΔU, the scale that has set the W-boson mass for both flat and anti-de Sitter spacetime.

4.2.1 Scattering Equation of Motion

Consider excitation of a low-energy, monochromatic plane-wave of the $U^{(1)}(1)$ gauge field on the first D3-brane. In static gauge, the boundary condition of the transverse string coordinates $\mathbf{X}(t,R)$ is given by

$$\begin{aligned} \frac{g_{\rm eff}}{R^2} \partial_R \mathbf{X}(t,R) \Big|_{R=R_1} &= Q\mathbf{E}^{(1)}(t), \\ \frac{g_{\rm eff}}{R^2} \partial_R \mathbf{X}(t,R) \Big|_{R=R_2} &= 0 \, . \end{aligned} \tag{4.51}$$

From Eq.(4.47), one finds that a weak pulse propagating along the string is described by

$$\mathbf{X}(t,R) = \int \frac{d\omega}{2\pi} \Big[\mathbf{a}(\omega) \left(1 - i\omega\frac{R}{c}\right) e^{-i\omega(t-R/c)} + \tilde{\mathbf{a}}(\omega) \left(1 + i\omega\frac{R}{c}\right) e^{-i\omega(t+R/c)} \Big]. \tag{4.52}$$

Again, the boundary condition, Eq.(4.51), fixes the spectral amplitudes $\mathbf{a}(\omega)$, $\tilde{\mathbf{a}}(\omega)$ uniquely:

$$\mathbf{a}(\omega) = \tilde{\mathbf{a}}^*(\omega) = \left(\frac{R_1/g_{\rm eff}}{\omega}\right)\left(\frac{Q\mathbf{E}_0^{(1)}}{\omega}\right) \frac{e^{-i\omega R_2/c}}{2i\sin\omega\Delta\tau}. \tag{4.53}$$

Substituting Eq.(4.53) into Eq.(4.52), one obtains equation of motion for the string endpoint $\mathbf{X}_1(t)$ on the first D3-brane is given by:

$$M_{\rm w} \left(1 + \frac{R_1}{R_2}\left(\frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t} - 1\right)\right)^{-1} \ddot{\mathbf{X}}_1(t) = Q\mathbf{E}^{(1)}(t). \tag{4.54}$$

Comparison with the corresponding equation Eq.(equationofmotion) in the flat spacetime indicates that the infinite-order kernal has been changed completely for both functional form and dependence on the D3-brane locations, $U_{1,2}$. The inertia mass and the fact that the kernel makes the endpoint effectively heavier remains the same.

Proceeding in a similar manner, one also finds an equation of motion for the string endpoint $\mathbf{X}_2(t)$ on the second D3-brane:

$$M_{\rm w}\left(\frac{\sinh\Delta\tau\partial_t}{\Delta\tau\partial_t}\right)\ddot{\mathbf{X}}_2(t) = Q\mathbf{E}^{(1)}(t). \tag{4.55}$$

Interestingly, functional form of the infinite-order kernel is exactly the same as in the flat space case, Eq.(2.17). However, argument of the kernel (which defines the 'momentum threshold' scale) is completely changed: it was inverse of the W-boson mass $\Delta r = \ell_s^2 \Delta U$ in flat spacetime, while, in anti-de Sitter spacetime, it is $\Delta R = \Delta(g_{\rm eff}/U)$. The threshold scale provide an information on size of Yang-Mills field configuration corresponding to the pulse propagation. We have thus learned that causality and locality of the anti-de Sitter spacetime is encoded via pulse sizes in super Yang-Mills theory.

4.2.2 Transmission Rate Across Open String

It is straightforward to estimate transmission rate of the brane-wave on the first D3-brane to the second. As argued above, the transmission takes place effectively via the stretched open string in the anti-de Sitter spacetime background. The equation of motion Eq.(secondend) shows that, compared to the flat spacetime, the infinite-order kernel depends on Δz, separation between the two D3-branes measured in units of 'size' (following from scale-size relation Eq.(xxx)) in the super Yang-Mills theory. Henceforth, the transmission rate $T_{\rm F-string}$ for a F-string in anti-de Sitter spacetime is obtained straightfowardly as

$$T_{\rm F-string} = \frac{2\pi}{3}\left(\frac{Q^2}{M_{\rm w}c^2}\right)^2\left(1 - 2\frac{\omega}{M_{\rm w}c^2} + \cdots\right)\cdot\left(1 - \left(\frac{\omega}{M_{\rm w}c^2}\right)^2 + \cdots\right)\cdot\left[\frac{\omega\Delta\tau}{\sin\omega\Delta\tau}\right]^2. \tag{4.56}$$

Similarly, the transmission rate $T_{\rm D-string}$ for a D-string in anti-de Sitter spacetime is

$$T_{\rm D-string} = \frac{2\pi}{3}\left(\frac{Q^2}{M_{\rm m}c^2}\right)^2\left(1 - 2\frac{\omega}{M_{\rm m}c^2} + \cdots\right)\cdot\left(1 - \left(\frac{\omega}{M_{\rm w}c^2}\right)^2 + \cdots\right)\cdot\left[\frac{\omega\Delta\tau}{\sin\omega\Delta\tau}\right]^2. \tag{4.57}$$

In establishing the above result, we have use the fact that, first, both W-boson and its dual magnetic monopole are BPS-saturated and hence their masses are completely protected and that Thomson scattering process is governed solely by a perturbative expansion parameter,

$Q\mathbf{E}^{(1)}, \omega \ll M_{\rm w}$. As such, even if the super Yang-Mills theory should be studied at strong ‘t Hooft coupling limit, transmission rates derived above are consistent in so far as the brane waves are weak enough.

4.3 Locality and Causality in Anti-de Sitter Spacetime

explain why the first endpoint is modified so drastically while the second is not.

4.3.1 Physical Thresholds in Anti-de Sitter Spacetime

In the last subsection, we have derived two physical threshold scales relevant for probing causality and locality of anti-de Sitter spacetime: the ‘winding’ threshold scale defined by the W-boson mass and the ‘momentum’ threshold scale defined by the Yang-Mills scale size:

$$\Lambda_{\rm w} = M_{\rm w} = \Delta U\,; \qquad \Lambda_{\rm m} = \frac{1}{\Delta\tau} = \frac{U_1 U_2}{g_{\rm eff}} \frac{1}{\Delta U}. \tag{4.58}$$

It is then clear that in strong ‘t Hooft coupling limit $g_{\rm eff} \to \infty$, as ΔU is held fixed at a generic point in Coulomb branch (away from the origin), $\Lambda_{\rm m}$ is smaller than $\Lambda_{\rm w}$. One thus finds a new uncertainty relation for anti-de Sitter spacetime:

$$\Lambda_{\rm w}\,\Lambda_{\rm m} \quad \approx \quad \mathcal{O}(\frac{U_1 U_2}{g_{\rm eff}}). \tag{4.59}$$

Note that, in contrast to the flat spacetime situation, the uncertainty does depend on the location of the two D3-branes, $U_{1,2}$, viz. location in the moduli space of the Coulomb branch.

As proclaimed, the causal time delay in anti-de Sitter spacetime is governed by the difference in z-coordinate (which is identified with the size in super Yang-Mills theory by the holography principle) and is not related to that in flat spacetime in any simple manner.

It should now become clear that, in the strong ‘t Hooft coupling limit, the ‘momentum’ threshold $\Lambda_{\rm m}$ stays always lighter than the ‘winding’ threshold. As such, from super Yang-Mills theory perspective, the W-boson behaves more like a flexible bilocal object rather than a point-particle.

What about at the origin of the Coulomb branch? As the 'momentum' threshold depends on both $g_{\rm eff}$ and $U_{1,2}$. The answer clearly depends on the order one takes the limits $g_{\rm eff} \to \infty$ and $U_{1,2} \to 0$. If one takes $g_{\rm eff} \to 0$ first and then $U_{1,2} \to 0$, it is clear that the 'momentum' threshold stays light all the time. If one takes oppositely, for example, $\Delta U \to 0$ first and $U_{1,2} \to 0$ afterward, the 'winding' threshold always dominates.

4.3.2 Holographic Encoding in Super Yang-Mills Theory

It is then clear how the Thomson scattering looks like in the large-N super Yang-Mills theory. The open string is holographically projected to a shell of spherically symmetric Yang-Mills field configuration, as depicted in Figure 10, whose inner and outer radius are $g_{\rm eff}/U_1$ and $g_{\rm eff}/U_2$ respectively. Once the W-boson is hit by the incident plane wave, the scattered wave of the two isospin components will emanate from each of the two radii. Any observer located at asymptotic region will then perceive the radiation as originating from the same point in space but with a time delay $(\Delta t)_{\rm YM}$. This is how the causal time-delay in the bulk AdS_5 is encoded in the boundary super Yang-Mills theory.

It now becomes clear how the time delay, Eq.(4.50), is encoded in the super Yang-Mills theory with light 'momentum' threshold. The W-boson behaves as a flexible bilocal object. What it means is that the two radiation wave-front are separated in radial size by ΔR. To an observer sitting far away from the W-boson position, the two color isospin components (namely, $U^{(1)}(1) \times U^{(1)}(1)$'s) will arrive at the observer's detector with a time delay $\Delta\tau$!

In super Yang-Mills theory, this process is nothing but Thompson scattering — scattering of low-energy photon on a heavy charged particle.

Having concluded that super Yang-Mills theory defined either in renormalizable or in DBI Lagrangians is not capable of encoding causality and locality of anti-de Sitter spacetime, we now ask what modifications, if possible, can do so. The answer to this question can be drawn

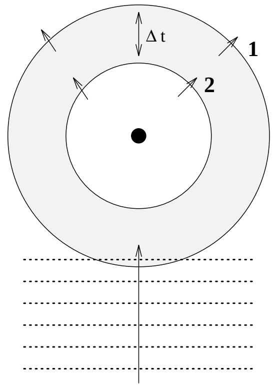

Figure 10: Yang-Mills theory view of the Thompson scattering

again from Eq.(4.55), expressed in the form of retardation relation:

$$\begin{aligned} M_{\rm w}\ddot{\mathbf{X}}(t) &= \int_{t\ge t'} dt' \Big\langle t \Big| \frac{\Delta\tau\partial_t}{\sinh \Delta\tau\partial_t} \Big| t' \Big\rangle Q\mathbf{E}^{(1)}(t') \\ &= Q\int_{t\ge t'} dt' \left\langle t \right| \sum_{n=-\infty}^{+\infty} (-)^n \frac{\partial_t^2}{\partial_t^2 + (n/\Delta\tau)^2} \left| t' \right\rangle \mathbf{E}^{(1)}(t') \end{aligned} \tag{4.1}$$

Expanding each term in the sum in power series of $\Delta\tau\partial_t$, one immediately finds that the interaction vertex of massive W-boson with gauge fields is governed by an infinite tower of higher-dimensional operators of the form $\partial_t^{2m}\mathbf{E}$. In $d = 4, \mathcal{N} = 4$ super Yang-Mills theory, operators of such a form are classified as *long supermultiplets*.

It should not be surpring that long supermultiplets are responsible for encoding causality and locality. We have argued that string oscillator excitations are needed to exhibit locality and causality. It was shown that the long supermultiplets arise precisely from the massive string excitations.

5 Discussions

In this paper, we have studied several dynamical issues in the holographic correspondence between Type IIB supergraivty on AdS_5 and super Yang-Mills theory. In particular, we have investigaged how locality and causality of the five-dimensional anti-de Sitter spacetime is encoded into four-dimensional dynamics of super Yang-Mills theory. The main conclusion is that, in order to encode causality and locality of anti-de Sitter spacetime, the super Yang-Mills theory ought to contain infinite tower of long supermultiplets (in addition to short supermultiplets summarized by non-abelian Born-Infeld Lagrangian).

Before closing, we would like to compare our result with earlier works, in which dynamical issues have been briefly mentioned.

Appendix: Effective Field Theory of String Endpoints

In this appendix, using covariant Green-Schwarz formalism, we will explain derivation of gauge-fixed, open string worldsheet action, Eqs.(2.12, 4.47). The derivation turns out somewhat nontrivial, as two Majorana-Weyl spinors of the covariant Green-Schwarz action are constrained both by κ-symmetry gauge-fixing and by open string boundary conditions. We will show that both requirements lead to so-called 'D-brane gauge' as a unique choice of the spinor projection. The 'D-brane gauge' has been proposed previously as a natural gauge-fixing condition of the κ symmetry. Interestingly, we find that exactly the same condition also arises from the analysis of open string boundary conditions.

We will also derive, after integrating out string excitations, an effective field theory describing low-energy dynamics of two endpoints of the string. As pointed out in sections 2 and 4, the dynamics is described by a Lagrangian with infinite-order kernels, which turns out exactly the same as (two particle generalization of) the Pais-Uhlenbeck's model. Remarkably, according to the analysis of Pais and Uhlenbeck, the particular infinite-order kernel is the one compatible with convergence, positive-definiteness and causality.

A Green-Schwarz Action in Flat Spacetime

Dynamics of an open F- or D-string stretched between the two parallel D3-branes, as depicted in Figure 2, is governed by the covariant Green-Schwarz action [31]. Denote bosonic and fermionic string coordinates as $(X^M; \Theta^A_\alpha)$ $(M = 0, \cdots, 9; A = 1, 2; \alpha = 1, \cdots, 32)$, which map worldsheet Σ (spanned by σ^i $(i = 0, 1)$) to coset superspace $Poincare(9, 1|2)/SO(9, 1)$. In Type IIB string theory, $\Theta^{1,2}$ are Majorana-Weyl spinors of same chirality. In the Nambu-Goto formulation, the worldsheet action is given by

$$\begin{aligned} I_{\rm string} &= I_{\rm GS} + I_{\rm D3-Q} \\ I_{\rm GS} &= -T\int_\Sigma d\tau d\sigma \left[\sqrt{-{\rm det} L^{\hat a}_i L^{\hat a}_j} + i\int_{M_3} L^{\hat a} \wedge \overline{L}^I \tau_3^{IJ} \Gamma^{\hat a} \wedge L^J . \right. \end{aligned} \qquad \text{(A.1)}$$

Here, $I_{\rm D3-Q}$ is a boundary action of the string endpoints, M denotes a three-dimensional manifold whose boundary equals to the string worldsheet, and

$$\begin{aligned} L^{\hat{a}} &= \delta^{\hat{a}}_M dX^M - i\overline{\Theta}^A \Gamma^{\hat{a}} d\Theta^A, \qquad (\hat{a} = 0, 1, \cdots, 9) \\ L^I = d\Theta^I. & \end{aligned} \tag{A.2}$$

are invariant Cartan one-forms of the coset superspace. The Wess-Zumino term in Eq.(A.1) is given by

$$L_{\rm WZ} = 2idX^{\hat{a}} \wedge \left(\overline{\Theta}^1 \Gamma^{\hat{a}} d\Theta^1 - \overline{\Theta}^2 \Gamma^{\hat{a}} d\Theta^2\right) - 2\left(\overline{\Theta}^1 \Gamma^{\hat{a}} d\Theta^1\right) \wedge \left(\overline{\Theta}^2 \Gamma^{\hat{a}} d\Theta^2\right). \tag{A.3}$$

Apart from the boundary Lagrangian, the covariant Green-Schwarz action Eq.(A.1) is invariant under $\mathcal{N} = 2$ global supersymmetry $(A = 1, 2)$:

$$\delta_\epsilon \Theta^A = \epsilon^A; \qquad \delta_\epsilon X^{\hat{a}} = i\overline{\epsilon}^A \Gamma^{\hat{a}} \Theta^A, \tag{A.4}$$

and local κ symmetry, when expressed in terms of worldsheet *scalar* spinors $\tilde{\kappa}$ [9],

$$\delta_{\tilde{\kappa}}\, \Theta^A = \left(1 + (-)^A \mathcal{P}\right) \tilde{\kappa}^A, \qquad \delta_\kappa\, X^{\hat{a}} = i\overline{\Theta}^A \Gamma^{\hat{a}} \delta_{\tilde{\kappa}} \Theta^A, \tag{A.5}$$

where

$$\mathcal{P} = \frac{1}{2} \frac{\Pi^{\hat{a}} \wedge \Pi^{\hat{b}}}{\sqrt{-h}} \Gamma^{\hat{a}\hat{b}}, \qquad \mathcal{P}^2 = 1. \tag{A.6}$$

A.1 Open String Boundary Conditions

The last term in Eq.(A.1) is the Lagrangian describing interaction of the open string endpoints with gauge and Higgs fields on the worldvolume of the two D3-branes. Denoting the $d = 4, \mathcal{N} = 4$ vector supermultiplet as (A, λ), the boundary Lagrangian in the lowest order in derivative expansion reads

$$L_{\rm D3-Q} = Q \oint_{\partial\Sigma} d\tau \dot{X}^{\hat{a}} \left(A_{\hat{a}}(X) - \overline{\lambda}(X)\Gamma_{\hat{a}}\theta\right). \tag{A.7}$$

[9] The worldsheet vector κ and worldsheet scalar $\tilde{\kappa}$ are related each other by $\kappa_i^{1,2} = -i\Gamma^{\hat{a}}\Pi_i^{\hat{a}}\tilde{\kappa}^{2,1}$.

The boundary action represents turning on gauge and Higgs fields on the D3-brane worldvolume. The boundary conditions, derived from variation of the Green-Schwarz action, are given by [10]

$$D \quad : \quad \delta X^{\perp} = 0, \qquad \overline{\Theta}^1 \Gamma^{\perp} \delta\Theta^1 + \overline{\Theta}^2 \Gamma^{\perp} \delta\Theta^2 = 0 \tag{A.8}$$

$$N \quad : \quad \Pi^{\parallel}_{\sigma} = 0, \qquad \overline{\Theta}^1 \Gamma^{\parallel} \delta\Theta^1 - \overline{\Theta}^2 \Gamma^{\parallel} \delta\Theta^2 = 0, \tag{A.9}$$

for Dirichlet and Neumann directions, respectively. To proceed further with gauge fixing, one will need to find linear boundary conditions for fermions that are consistent with Eqs.(A.8, A.9). Because of sign difference between $A = 1, 2$ terms in Eqs.(A.8, A.9), a unique choice of the linear boundary condition is of the form

$$\Theta^A = \Gamma_5 \mathcal{M}^A{}_B \Theta^B, \qquad \Gamma_5 \equiv \Gamma^0 \Gamma^1 \cdots \Gamma^3, \tag{A.10}$$

where (2×2) matrix $\mathcal{M}$ is determined to be $\mathcal{M}_{AB} = \epsilon_{AB}$ by the requirement that the boundary condition is compatible with Majorana and Weyl conditions and with invertibility of Eq.(A.10). Hence, for open Green-Schwarz superstring attached between the two D3-branes, the boundary conditions are:

$$\begin{aligned} \delta X^{\perp} &= 0, & \partial_\sigma X^{\parallel} &= 0 \\ (\delta_{AB} - \Gamma_5 \epsilon_{AB})\, \Theta^B &= 0 & (\delta_{AB} + \Gamma_5 \epsilon_{AB})\, \partial_\sigma \Theta^B &= 0. \end{aligned} \tag{A.11}$$

A.2 Gauge Fixing

Due to the local κ-symmetry, half of the degrees of freedom of $\Theta^{1,2}$'s are redundant. Since the spinors $\Theta^{1,2}$ have the same chirality, the κ-symmetry may be gauge fixed conveniently by setting $\Theta^1 = i\Theta^2 = \frac{1}{2}\Psi$. The gauge-fixed action then reads

$$I_{\rm GS} = T \iint d\tau d\sigma \left[\sqrt{-\det \partial_i X^{\hat{a}} \partial_j X^{\hat{a}}} + \frac{i}{2} dX^{\hat{a}} \wedge (\overline{\Psi} \Gamma^{\hat{a}} d\Psi) \right], \tag{A.12}$$

[10] Once, on the D3-brane worldvolume, Higgs and gauge fields are turned on, boundary conditions are expected to be modified. It is easy to show, however, that Higgs field background does not modify the Dirichlet boundary condition at all, while gauge field background affects the Neumann boundary condition. As we will be considering background fields that are weak enough, these modifications will be ignored throughout.

in which half of $\mathcal{N}=2$ supersymmetry is realized linearly

$$\delta_\epsilon X^{\hat{a}} = i\bar{\epsilon}\Gamma^{\hat{a}}\Psi, \qquad \delta_\epsilon \Psi = -\frac{1}{2}\frac{dX^{\hat{a}}\wedge dX^{\hat{b}}}{|dX\wedge dX|}\Gamma^{\hat{a}\hat{b}}\epsilon, \tag{A.13}$$

and the other half as a nonlinear fermionic symmetry

$$\delta_\xi X^{\hat{a}} = 0, \qquad \delta_\xi \Psi = \xi. \tag{A.14}$$

For the reparametrization invariance, we will choose the static gauge

Finally, after reinstating the speed of light c, one obtains the gauge-fixed action of the open string as:

$$I_{\rm GS} = I_{\rm string} + I_{\rm D3-Q} \tag{A.15}$$

where

$$\begin{aligned} I_{\rm string} &= \int dt \left[-M_{\rm w} + \frac{T}{2}\int_{r_1}^{r_2} dr \left(\frac{1}{c^2}(\partial_t \mathbf{X})^2 - (\partial_r \mathbf{X})^2 \right) + \cdots \right] \\ I_{\rm D3-Q} &= \sum_{I=1,2} Q_I \int dt \left[A_0^{(I)}(\mathbf{r},t) + \frac{1}{c}\dot{\mathbf{X}}_I(t)\cdot \mathbf{A}^{(I)}(\mathbf{r},t) \right]_{\mathbf{r}=\mathbf{X}_I(t)}, \qquad (Q_1 = -Q_2 = \pm 1). \end{aligned} \tag{A.16}$$

A.3 Low-Energy Effective Lagrangian of String Endpoints

We will begin with solving the equations of motion for $\mathbf{X}(t,r)$, subject to fixed but arbitrary location of the string endpoints, $\mathbf{X}_{1,2}(t)$. Consider a Fourier transformed, harmonic solution

$$\begin{aligned} \tilde{\mathbf{X}}(\omega, r) &= \int dt e^{+i\omega t}\mathbf{X}(t,r) \\ &= \mathbf{a}(\omega)e^{-i\omega(t-r/c)} + \tilde{\mathbf{a}}(\omega)e^{-i\omega(t+r/c)}. \end{aligned}$$

The boundary condition specified,

$$\tilde{\mathbf{X}}(\omega,r)\Big|_{r=r_1} = \tilde{\mathbf{X}}_1(\omega), \qquad \tilde{\mathbf{X}}(\omega,r)\Big|_{r=r_2} = \tilde{\mathbf{X}}_2(\omega), \tag{A.17}$$

relates the spectral amplitudes $\mathbf{a}(\omega), \tilde{\mathbf{a}}(\omega)$ uniquely to the (Fourier transformed) string endpoints $\tilde{\mathbf{X}}_{1,2}(\omega)$. Using the relation, it is possible to express normal derivatives at the string endpoints

$$\partial_r \tilde{\mathbf{X}}_1(\omega) \equiv \partial_r \tilde{\mathbf{X}}(\omega,r)\Big|_{r=r_1}, \qquad \tilde{\mathbf{X}}_2(\omega) \equiv \partial_r \tilde{\mathbf{X}}(\omega,r)\Big|_{r=r_2} \tag{A.18}$$

in terms of $\tilde{\mathbf{X}}_{1,2}(\omega)$. The result is ($\Delta t \equiv \Delta r/c$)

$$\partial_r \tilde{\mathbf{X}}_1(\omega) = -\frac{\omega \Delta t}{\sin \omega \Delta t} \left(\frac{\tilde{\mathbf{X}}_1(\omega) \cos \omega \Delta t - \tilde{\mathbf{X}}_2(\omega)}{c \Delta t} \right)$$
$$\partial_r \tilde{\mathbf{X}}_2(\omega) = -\frac{\omega \Delta t}{\sin \omega \Delta t} \left(\frac{\tilde{\mathbf{X}}_1(\omega) - \tilde{\mathbf{X}}_2(\omega) \cos \omega \Delta t}{c \Delta t} \right). \tag{A.19}$$

In order to obtain an effective action for the string endpoints, we will need to integrate out massive string excitations. As the string action, I_{string}, is quadratic in $\mathbf{X}(r,t)$ at leading order, it amounts to imposing the equation of motion of $\mathbf{X}(r,t)$ back to the action, Eq.(A.16). One then obtains, after Fourier-transform back to $\mathbf{X}_{1,2}(t)$,

$$I_{\text{string}} = \int dt \, \left[-M_{\text{w}} - \frac{T}{2} \Big(\mathbf{X}_2(t) \cdot \partial_r \mathbf{X}_2(t) - \mathbf{X}_1(t) \cdot \partial_r \mathbf{X}_1(t) \Big) + \cdots \right]. \tag{A.20}$$

The string action is now expressed entirely in terms of boundary data of the open string positions on the two D3-branes.

Inserting Eq.(A.19) to Eq.(A.20), one finally obtains the low-energy effective action of the string endpoints:

$$I_{\text{GS}} = I_{\text{string}} + I_{\text{D3-Q}} \tag{A.21}$$
$$I_{\text{string}} = \int dt \left[-M_{\text{w}} - \frac{T}{2} \sum_{I,J=1,2} \mathbf{X}_I(t) \mathcal{K}^{\text{flat}}_{IJ} \left(\Delta t \partial_t \right) \mathbf{X}_J(t) \right]$$
$$I_{\text{D3-Q}} = \sum_{I=1,2} \int dt \, Q_I \left[A_0^{(I)}(\mathbf{r},t) + \frac{1}{c} \dot{\mathbf{X}}_I(t) \cdot \mathbf{A}^{(I)}(\mathbf{r},t) \right]_{\mathbf{r}=\mathbf{X}_I(t)}, \qquad (Q_1 = -Q_2 = \pm 1).$$

In the action, I_{string}, the infinite-order kernels are defined by:

$$\mathcal{K}^{\text{flat}}_{11} = \mathcal{K}^{\text{flat}}_{22} = +\frac{1}{\Delta r} \left(\frac{\Delta t \partial_t}{\tanh \Delta t \partial_t} \right)$$
$$\mathcal{K}^{\text{flat}}_{12} = \mathcal{K}^{\text{flat}}_{21} = -\frac{1}{\Delta r} \left(\frac{\Delta t \partial_t}{\sinh \Delta t \partial_t} \right). \tag{A.22}$$

Let us now focus on the Thomson scattering by D3-brane waves, studied in section 2. Initially, the radiation field is present only for the first D3-brane, viz. $A_0^{(2)} = \mathbf{A}^{(2)} = 0$. In this case, the equation of motion for $\mathbf{X}_2(t)$ is reduced simply to

$$\left(\cosh \Delta t \partial_t \right) \, \mathbf{X}_2(t) = \mathbf{X}_1(t), \tag{A.23}$$

viz. a retardation relation

$$\mathbf{X}_2(t) = \int dt' \Big\langle t \Big| \frac{1}{\cosh \Delta t \partial_t} \Big| t' \Big\rangle \, \mathbf{X}_1(t'). \tag{A.24}$$

After $\mathbf{X}_2(t)$ is integrated out of I_{string} using Eq.(A.23), the low-energy effective Lagrangian for the Thomson scattering process is obtained:

$$\begin{aligned} L_{\text{Thomson}} = & - M_{\rm w} + \frac{M_{\rm w}}{2} \dot{\mathbf{X}}_1(t) \left(\frac{\tanh \Delta t \partial_t}{\Delta t \partial_t} \right) \dot{\mathbf{X}}_1(t) + \cdots \\ & + Q \left[A_0^{(1)}(\mathbf{r}, t) + \frac{1}{c} \dot{\mathbf{X}}_1 \cdot \mathbf{A}^{(1)}(\mathbf{r}, t) \right]_{\mathbf{r}=\mathbf{X}_1(t)} . \end{aligned} \tag{A.25}$$

The Eqs.(2.16, 2.17) follow immediately from the above effective Lagrangian and the retardation relation Eq.(A.23).

B Green-Schwarz Action in Anti-de Sitter Spacetime

The Green-Schwarz action in $AdS_5 \times S^5$ can be obtained in a closed form via super-coset space construction. In Nambu-Goto form, the action reads

$$I_{\text{AdS}} = T \iint d\tau d\sigma \, \left[\sqrt{-\text{det} L_i^{\hat{a}} L_j^{\hat{a}}} + 4i \int_0^1 ds L_i^{\hat{a}}(s) \overline{\Theta} \tau_3 \Gamma^{\hat{a}} L_j(s) \right] . \tag{B.26}$$

Here, $\hat{a} = (a, a') = (0, 1, \cdots, 4, 5, \cdots, 9)$, $I, J = 1, 2$ and $\tau_{1,2,3}$ are Pauli matrices. The invariant worldsheet one-forms $L^I(s)$ and $L^{\hat{a}}(s)$ are given by

$$\begin{aligned} L^I(s) &= \left(\frac{\sinh s\mathcal{M}}{\mathcal{M}} \mathcal{D}\Theta \right)^I \\ L^{\hat{a}}(x) &= e_M^{\hat{a}} dX^M - 4i \overline{\Theta}^I \Gamma^{\hat{a}} \left(\frac{\sinh^2 \frac{1}{2} s\mathcal{M}}{\mathcal{M}^2} \mathcal{D}\Theta \right)^I , \end{aligned} \tag{B.27}$$

where (X^M, Θ^I) are the bosonic and fermionic F-string coordinates and

$$\begin{aligned} \left(\mathcal{M}^2 \right)^{IJ} &= \left[\gamma^{a'} (i\tau_2 \Theta)^I \overline{\Theta}^J \gamma^{a'} - \gamma^a (i\tau_2 \Theta)^I \overline{\Theta}^J \gamma^a \right] + \frac{1}{2} \left[\gamma^{a'b'} \Theta^I (\overline{\Theta} i\tau_2)^J \gamma^{a'b'} - \gamma^{ab} \Theta^I (\overline{\Theta} i\tau_2)^J \gamma^{ab} \right] \\ (\mathcal{D}\Theta)^I &= \left(\partial + \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} \right) \Theta^I + \frac{1}{2} E^{\hat{a}} \Gamma_{\hat{a}} \left(\tau_2 \Theta \right)^I . \end{aligned} \tag{B.28}$$

Also, $L^I \equiv L^I(s = 1)$, $L^{\hat{a}} \equiv L^{\hat{a}}(s = 1)$.

The action is invariant under the κ-symmetry

B.1 Gauge Fixing

In order to fix the local κ-symmetry, it turns out most convenient to make a change of variable $R = g_{\rm eff}/U$ and use the Poincaré coordinates. Up to the conformal rescaling, the ten-dimensional spacetime is flat. To proceed in an analogous manner to the flat spacetime situation, we will follow the observation of Kallosh and take the gauge-fixing

$$\Theta^I_- = 0 \qquad \text{where} \qquad \Theta^I_\pm \equiv (\mathcal{P}_\pm \Theta)^I, \quad \mathcal{P}^{IJ}_\pm = \frac{1}{2}\left(\tau_0 \pm i\Gamma_5 \tau_2\right)^{IJ}. \tag{B.29}$$

The κ-gauge-fixed action is quartic in fermions. However, if one makes a change of variable $z = g_{\rm eff}/U$, as mentioned in seciton 4, the action is dramatically simplified, as the metric is conformal to ten-dimensional flat spacetime [11].

The final form of the gauge-fixed Green-Schwarz action of the open string is:

$$I_{\rm AdS} = \int dt \, \left(L_{\rm string} + L_{\rm D3-Q}\right) \tag{B.30}$$

where

$$L_{\rm string} = \left[-M_{\rm w} + \frac{g_{\rm eff}}{2}\int_{R_1}^{R_2} \frac{dR}{R^2}\left(\frac{1}{c^2}(\partial_t \mathbf{X})^2 - (\partial_R \mathbf{X})^2\right) + \cdots\right] \tag{B.31}$$

$$L_{\rm D3-Q} = \sum_{I=1,2} Q_I \left[A_0^{(I)}(\mathbf{r}, t) + \frac{1}{c}\dot{\mathbf{X}}_I(t)\cdot \mathbf{A}^{(1)}(\mathbf{r}, t)\right]_{\mathbf{r}=\mathbf{X}_I(t)}, \quad (Q_1 = -Q_2 = \pm 1). \tag{B.32}$$

and the boundary interaction action is exactly the same in the flat spacetime.

B.2 Low-Energy Effective Lagrangian of String Endpoints

One can obtain low-energy effective Lagrangian of the string endpoints repeating steps of flat spacetime case, section A.2. After Fourier transformation, a harmonic solution of the string coordinate $\tilde{\mathbf{X}}(\omega, R)$ now takes a form

$$\begin{aligned}\tilde{\mathbf{X}}(\omega, R) &= \int dt e^{+i\omega t}\mathbf{X}(t, R) \\ &= \mathbf{a}(\omega)\left(1 - i\omega\frac{R}{c}\right)e^{-i\omega(t-R/c)} + \tilde{\mathbf{a}}(\omega)\left(1 + i\omega\frac{R}{c}\right)e^{-i\omega(t+R/c)},\end{aligned}$$

[11] In [34], closely related observation has been made but was interpreted as the effect of T-duality along all directions of D3-brane worldvolume. As is clear from the present argument, this interpretation is totally unnecessary.

where the harmonic modes are expanded in terms of the Hankel functions, $H^{(\pm)}_{3/2}(\omega R)$. Eliminating $\mathbf{a}(\omega), \tilde{\mathbf{a}}(\omega)$ between $\tilde{\mathbf{X}}_{1,2}(\omega)$ and $\partial_z \tilde{\mathbf{X}}_{1,2}(\omega)$, one finds relations ($\Delta\tau \equiv \Delta z/c$)

$$\begin{aligned}
\frac{1}{R_1}\partial_R \tilde{\mathbf{X}}_1(\omega) &= \frac{\omega^2}{\mathcal{N}}\left[\left(\sin\omega\Delta\tau + \omega\frac{R_2}{c}\cos\omega\Delta\tau\right)\tilde{\mathbf{X}}_1(\omega) - \omega\frac{R_1}{c}\tilde{\mathbf{X}}_2(\omega)\right] \\
\frac{1}{R_2}\partial_R \tilde{\mathbf{X}}_2(\omega) &= \frac{\omega^2}{\mathcal{N}}\left[\omega\frac{R_2}{c}\tilde{\mathbf{X}}_1(\omega) + \left(\sin\omega\Delta\tau - \omega\frac{R_1}{c}\cos\omega\Delta\tau\right)\tilde{\mathbf{X}}_2(\omega)\right],
\end{aligned} \tag{B.33}$$

where

$$\mathcal{N} = \left(1 + \omega^2\frac{R_1 R_2}{c^2}\right)\sin\omega\Delta\tau - \omega\Delta\tau\cos\omega\Delta\tau. \tag{B.34}$$

Imposing the equation of motion for $\mathbf{X}(t,z)$ to the action Eq.(B.31), one obtains

$$I_{\rm string} = \int dt\,\left[-M_{\rm w} - \frac{g_{\rm eff}}{2}\left(\frac{1}{R_2^2}\mathbf{X}_2(t)\cdot\partial_R\mathbf{X}_2(t) - \frac{1}{R_1^2}\mathbf{X}_1(t)\cdot\partial_R\mathbf{X}_1(t)\right) + \cdots\right]. \tag{B.35}$$

Inserting Eq.(B.33) to the action Eq.(B.35), after Fourier-transforming back, one finally arrives at the low-energy effective action of the string endpoints in anti-de Sitter spacetime:

$$\begin{aligned}
I_{\rm AdS} &= \int dt\,\left(I_{\rm string} + I_{\rm D3-Q}\right) \qquad\qquad (B.36)\\
L_{\rm string} &= -M_{\rm w} - \frac{g_{\rm eff}}{2}\sum_{I,J=1,2}\mathbf{X}_I(t)\cdot\mathcal{K}^{\rm AdS}_{IJ}(\Delta\tau\partial_t)\cdot\mathbf{X}_J(t) \\
L_{\rm D3-Q} &= \sum_{I=1,2} Q_I\left[A_0^{(I)}(\mathbf{r},t) + \frac{1}{c}\dot{\mathbf{X}}_I(t)\cdot\mathbf{A}^{(I)}(\mathbf{r},t)\right]_{\mathbf{r}=\mathbf{X}_I(t)}, \qquad (Q_1 = -Q_2 = \pm 1).
\end{aligned}$$

The infinite-order kernels in the action $I_{\rm string}$ are now given by:

$$\begin{aligned}
\mathcal{K}^{\rm AdS}_{11} &= -\frac{1}{R_1}\left(1 + \frac{R_2/c}{\Delta\tau}\frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t}\right)\left(1 - \frac{R_1R_2}{c^2}\partial_t^2 - \frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t}\right)^{-1} \\
\mathcal{K}^{\rm AdS}_{22} &= +\frac{1}{R_2}\left(1 - \frac{R_1/c}{\Delta\tau}\frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t}\right)\left(1 - \frac{R_1R_2}{c^2}\partial_t^2 - \frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t}\right)^{-1} \\
\mathcal{K}^{\rm AdS}_{12} &= \mathcal{K}^{\rm AdS}_{21} = +\frac{1}{\Delta R}\left(\frac{\Delta\tau\partial_t}{\sinh\Delta\tau\partial_t}\right)\left(1 - \frac{R_1R_2}{c^2}\partial_t^2 - \frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t}\right)^{-1}.
\end{aligned} \tag{B.37}$$

Let us again consider Thomson scattering of D3-brane wave, studied in section 4, by setting $A_0^{(2)} = \mathbf{A}^{(2)} = 0$. From Eq.(B.36), equation of motion for $\mathbf{X}_2(t)$ yields a retardation relation

$$\left(1 + \frac{R_1}{R_2}\left(\frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t} - 1\right)\right)\left(\frac{\sinh\Delta\tau\partial_t}{\Delta\tau\partial_t}\right)\mathbf{X}_2(t) = \mathbf{X}_1(t). \tag{B.38}$$

Integrating out $\mathbf{X}_2(t)$ from the action $I_{\rm string}$, using Eq.(B.38), one finally obtains the low-energy effective Lagrangian for the Thomson scattering of D3-brane waves:

$$\begin{aligned} L_{\rm Thomson} = & - \quad M_{\rm w} + \frac{M_{\rm w}}{2}\dot{\mathbf{X}}_1(t)\left(1 + \frac{R_1}{R_2}\left(\frac{\Delta\tau\partial_t}{\tanh\Delta\tau\partial_t} - 1\right)\right)^{-1}\dot{\mathbf{X}}_1(t) + \cdots \\ & + \quad Q\left[A_0^{(1)}(\mathbf{r},t) + \frac{1}{c}\dot{\mathbf{X}}_1(t)\cdot\mathbf{A}^{(1)}(\mathbf{r},t)\right]_{\mathbf{r}=\mathbf{X}_1(t)}. \end{aligned} \tag{B.39}$$

From the effective Lagrangian Eq.(B.39) and the retardation relation Eq.(B.38), one gets easily the equations of motion of string endpoints, Eqs.(4.44, 4.45).

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