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Universal Thermal Instabilities and the High-Temperature Phase of the $N = 4$ Superstrings*

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Abstract

Using the properties of gauged $N = 4$ supergravity, we show that it is possible to derive a universal thermal effective potential that describes all possible high-temperature instabilities of the known $N = 4$ superstrings. These instabilities are due to non-perturbative dyonic modes, which become tachyonic in a region of the thermal moduli space $\mathcal{M} = \{s, t, u\}$; \mathcal{M} is common to all non-perturbative dual-equivalent $N = 4$ superstrings in five dimensions. We analyse the non-perturbative thermal potential and show the existence of a phase transition at high temperatures corresponding to a condensation of 5-branes. This phase is described in detail, using an effective non-critical string theory.

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1 Introduction

At finite temperatures the partition function $Z(\beta)$ and the mean energy $U(\beta)$ develop power pole singularities in $\beta \equiv T^{-1}$ if the density of states of a system grows exponentially with the energy:

$$\begin{aligned}\rho(E) &\sim E^{-k} e^{bE}, \\ Z(\beta) &= \int dE \rho(E) e^{-\beta E} \sim \frac{1}{(\beta - b)^{(k-1)}}, \\ U(\beta) &= -\frac{\partial}{\partial \beta} \ln Z \sim (k-1) \frac{1}{\beta - b} + \dots\end{aligned}\tag{1.1}$$

At the critical temperature, $T_H = b^{-1}$, various thermodynamical quantities diverge [1]. An alternative interpretation of the pole singularity in $U(\beta)$ follows from the identification of the temperature with the inverse radius of a compactified Euclidean time on S^1 : $2\pi T = 1/R$. In this representation, the partition function is given by the (super-)trace over the thermal spectrum of the theory in $(D-1)$ dimensions:

$$\ln Z(\beta) = \text{Str} \ln \mathcal{M}(\beta).\tag{1.2}$$

The pole singularity is then a manifestation of a thermal state becomes massless at the Hagedorn temperature T_H . Thus, the knowledge of the thermal spectrum $\mathcal{M}(\beta)$ as a function of the S^1 radius $R = \beta/2\pi$, determines T_H [2].

Perturbative string theories provide examples of an exponentially growing density of states, with $k = D$, the dimension of space-time, and $b^{-1} \sim \mathcal{O}(\alpha')^{-1/2}$ [6]–[8]. In superstrings the states that become tachyonic at T_H have necessarily a non-zero winding number n [3, 4, 5].

From the perturbative study of the $N = 4$ strings we can see that the states that become tachyonic above T_H correspond to the $N = 4$ BPS states that preserve half of the supersymmetries ($N = 2$) [9]. However, in the $N = 4$ theories the masses of the non-perturbative BPS states are known as well thanks to the $N = 4$ supersymmetry algebra with central extension [10]–[12]. We can therefore identify all perturbative and non-perturbative BPS states that are able to induce high- T instabilities using the string duality properties among the heterotic–type IIA–type IIB–type I strings with $N = 4$ supersymmetry [13]–[16].

In this talk I will summarize the results of Ref. [9] concerning non-perturbative $N = 4$ theories at finite temperatures.

2 Thermally modified $N = 4$ BPS masses

In order to obtain the thermal partition function one modifies the boundary conditions around the S^1 -Euclidean time by a spin-statistical phase. In perturbative string theories the consequence of this phase is to shift the Kaluza–Klein momenta of the S^1

$$P_{L,R} = \left(\frac{m + Q - \frac{n\delta}{2} \pm \frac{nR}{\alpha'}}{R} \right)^2, \quad (2.3)$$

and reverse the GSO projection in the n -odd winding sector; $Q = Q_L + Q_R$ is the helicity operator while $\delta = 1$ in the heterotic string and $\delta = 0$ in type II strings [3, 4, 5, 9].

The left- (and right-) supersymmetric GSO projection(s) implies that in the even winding sector all states have $M^2 \geq 0$. However, some of the states with odd winding number can become tachyonic by a reversion of the GSO-projection [3, 4, 5, 9]. The only states that can become tachyonic have $n = \pm 1$ and left-helicity $Q_L = \pm 1$ (right-helicity = $-Q_R$ for type II). They are scalars in $(D - 1)$ -dimensions (the longitudinal components of the D -dimensional gravitons). The Hagedorn temperature corresponds to the critical value of the S^1 radius at which the first tachyonic state appears when $2\pi R = T^{-1}$ decreases.

The appearance of tachyons can never arise in any perturbative supersymmetric field theory, since it behaves like the zero-winding sector of strings. In non-perturbative supersymmetric field theories, however, such an instability can arise due to thermal dyonic modes, which behave like the odd winding string states. Indeed, before the temperature modification, the heterotic–type II duality in five dimensions exchanges the winding number n with the magnetic charge ℓ :

$$\mathcal{M}^2 = \left(\frac{m}{R} + \frac{nR}{\alpha'_H} + \frac{\ell R}{\lambda_H^2 \alpha'_H} \right)^2, \quad (2.4)$$

where m and n are the S^1 momentum and winding numbers, and ℓ is the non-perturbative wrapping number for the heterotic 5-brane around $T^4 \times S^1$; λ_H is the string coupling in $D = 6$; the tension of the 5-brane $T_5 = 1/\lambda_H^2$ in α'_H units. Using the \mathcal{S} -duality relations:

$$\lambda_H = \frac{1}{\lambda_{IIA}}, \quad \lambda_H^2 \alpha'_H = \alpha'_{II}, \quad (2.5)$$

we can express the above mass formula in terms of type IIA parameters:

$$\mathcal{M}^2 = \left(\frac{m}{R} + \frac{nR}{\alpha'_{II}\lambda_{IIA}^2} + \frac{\ell R}{\alpha'_{II}} \right)^2 = \left(\frac{m}{R} + \frac{nR}{\alpha'_H} + \frac{\ell R}{\alpha'_{II}} \right)^2. \quad (2.6)$$

The momentum and winding numbers are now m and ℓ ; n is the wrapping number for the type IIA NS 5-brane around $K_3 \times S^1$. From the six-dimensional viewpoint, m/R is the Kaluza–Klein momentum, while the last two terms correspond to BPS strings with tension

$$T_{p,q} = \frac{p}{\alpha'_H} + \frac{q}{\alpha'_{II}}, \quad (2.7)$$

where p, q are relatively prime integers, $(n, l) = k(p, q)$. The common divisor k defines the wrapping of the $T_{p,q}$ string around S^1 ; q is the charge of the fundamental string and p the magnetic charge of the solitonic string obtained by wrapping the NS 5-brane around K_3 . The $T_{p,q}$ -strings cannot become tensionless since they never are associated to vanishing cycles of the internal manifold.

The five-dimensional thermal mass formula is obtained by the non-perturbative generalization of the temperature deformation using the (p, q) -string picture of the non-perturbative BPS spectrum by replacing $m \rightarrow m + Q' + \frac{n}{2}$ and reversing the GSO-projection in the k -odd sector of the (p, q) -strings (Q' is the helicity operator of the 5D-thermal theory):

$$\mathcal{M}_T^2 = \left(\frac{m + Q' + \frac{kp}{2}}{R} + k T_{p,q} R \right)^2 - 2 T_{p,q} \delta_{k,\pm 1} \delta_{Q',0}. \quad (2.8)$$

This thermal formula reproduces the perturbative result for both heterotic and type IIA theories. In the heterotic perturbative limit $\lambda_H \rightarrow 0$, only the $\ell = 0 = q$ states survive, while in the type IIA perturbative limit $\lambda_{II} \rightarrow 0$, only the $n = 0 = p$ states survive. Note that in the general case of a $T_{p,q}$ string with the temperature deformation, the condition $mk \geq 0$ becomes $mk \geq -1$, because of the inversion of the GSO-projection.

From Eq. (2.8), it follows that if the heterotic coupling λ_H is smaller than the critical value

$$\lambda_H < \lambda_H^c = \frac{\sqrt{2} + 1}{2}, \quad (2.9)$$

the first tachyon appears at $R = (\sqrt{2} + 1)\sqrt{\alpha'_H/2}$, which corresponds to the heterotic Hagedorn temperature. On the other hand, if the heterotic theory is strongly coupled, $\lambda_H > \lambda_H^c$, the first tachyon appears at $R = \sqrt{2\alpha'_H} \lambda_H = 2\sqrt{\alpha'_{II}/2}$; this corresponds to

the type IIA Hagedorn temperature. Besides the above two would-be tachyons, the mass formula (2.8) leads in general to two series of potentially tachyonic states with $m = -1$. However the critical temperature $(2\pi R)^{-1}$ for each of the states in both series is always higher than the lowest Hagedorn heterotic temperature while, as discussed above, the type IIA Hagedorn temperature first appears when the heterotic coupling exceeds the critical value λ_H^c (2.9).

In order to include the type IIB dual $N = 4$ string, we need to discuss five-dimensional theories at finite temperature, taking into account the compactification radius R_6 from six to five dimensions. Type IIA and IIB strings are then related by the inversion of R_6 . The extension to four dimensions of the mass formula (2.8) is straightforward. It depends on three parameters, the string coupling g_H , the temperature radii R and R_6 . It is convenient to introduce the three combinations

$$t = \frac{RR_6}{\alpha'_H}, \quad u = \frac{R}{R_6}, \quad s = g_H^{-2} = \frac{t}{\lambda_H^2}, \quad (2.10)$$

in terms of which the thermally shifted BPS mass formula reads [9]:

$$\mathcal{M}_T^2 = \left(\frac{m + Q' + \frac{kp}{2}}{R} + k T_{p,q,r} R \right)^2 - 2 T_{p,q,r} \delta_{|k|,1} \delta_{Q',0}, \quad (2.11)$$

where $T_{p,q,r}$ is then an effective string tension

$$T_{p,q,r} = \frac{p}{\alpha'_H} + \frac{q}{\lambda_H^2 \alpha'_H} + \frac{r R_6^2}{\lambda_H^2 (\alpha'_H)^2} = \frac{p}{\alpha'_H} + \frac{q}{\alpha'_{IIA}} + \frac{r}{\alpha'_{IIB}}; \quad (2.12)$$

here, the various α' and the radius R are expressed in terms of s, t, u and in terms of the four-dimensional Planck scale $\kappa = \sqrt{8\pi} M_P^{-1}$:

$$\alpha'_H = 2\kappa^2 s, \quad \alpha'_{IIA} = 2\kappa^2 t, \quad \alpha'_{IIB} = 2\kappa^2 u, \quad R^2 = \alpha'_H t u = 2\kappa^2 s t u \quad (2.13)$$

The temperature radius R is by construction identical in all three string theories. Note that $l = kq$ corresponds to the wrapping number of the heterotic 5-brane around $T^4 \times S^1_R$ as in five dimensions, while $l' = kr$ corresponds to the same wrapping number after performing a \mathcal{T} -duality along the $S^1_{R_6}$ direction, which is orthogonal to the 5-brane. All winding numbers, n, l, l' , correspond to magnetic charged states from the field theory point of view. Their masses are proportional to the temperature-radius R and are not thermally shifted; p, q, r are all non-negative relatively prime integers. This follows from the BPS conditions and the $s \leftrightarrow t \leftrightarrow u$ duality symmetry in the

undeformed supersymmetric theory. Furthermore, $mk \geq -1$ because of the inversion of the GSO projection in the temperature-deformed theory. Using these constraints, one can show that there are, in general, two potential tachyonic series with $m = -1$ and $p = 1, 2$ [9] generalizing the five-dimensional result. One of the perturbative heterotic, type IIA, or type IIB potential tachyons corresponds to a critical temperature that is always lower than those of the two series with $p = 1, 2$.

The above discussion shows that the temperature modification of the mass formula inferred from perturbative strings and applied to the non-perturbative BPS mass formula produces the appropriate instabilities in terms of the Hagedorn temperatures. In Ref. [9] we show that it is possible to go beyond the simple enumeration of Hagedorn temperatures and construct the full temperature-dependent effective potential associated with the would-be tachyonic states. This will allow a study of the nature of the non-perturbative instabilities and the dynamics of the various thermal phases.

3 Thermal effective potential

Our procedure to construct the thermal effective theory is as follows: we consider five-dimensional $N = 4$ theories at finite temperature. They can then effectively be described by four-dimensional theories, in which supersymmetry is spontaneously broken by thermal effects. Since we want to limit ourselves to the description of instabilities, it is sufficient to retain, in the full $N = 4$ spectrum, only the potentially massless and tachyonic states. This restriction will lead us to consider only spin 0 and $1/2$ states, the graviton and one of the gravitinos. This sub-spectrum is described by an $N = 1$ supergravity with six chiral multiplets: the three moduli S , T , U , and the three would-be tachyonic states Z_A , $A = 1, 2, 3$. Using the properties of the $N = 4$ (gauged) supergravities in four dimensions [17]–[19],[5], it is possible to derive the temperature modification associated to the Scherk-Schwarz temperature gauging [21, 22, 5, 4]. This is done in Ref. [9], where the Kähler manifold K and the superpotential W of the effective $N = 1$ supergravity [23] are derived:

$$K = -\log(S + S^*) - \log(T + T^*) - \log(U + U^*) - 2\log(1 - 2|Z_A|^2 + |Z_A^2|^2) ,$$

$$W = 2\sqrt{2} \left[\frac{1}{2}(1 - Z_A^2)^2 + (TU - 1)Z_1^2 + SUZ_2^2 + STZ_3^+ Z_3^2 \right] . \quad (3.14)$$

The resulting scalar potential has a complicated expression. Important simplifications occur, however, when we restrict ourselves to the would-be tachyonic states, $z_A = \text{Re } Z_A$ and in $s = \text{Re } S$, $t = \text{Re } T$, $u = \text{Re } U$. Following the analysis of Ref. [9] only these directions are relevant to the vacuum structure of the potential and to possible phase transitions. The resulting scalar potential V is:

$$\begin{aligned}
V &= V_1 + V_2 + V_3, \\
\kappa^4 V_1 &= \frac{4}{s} \left[(\xi_1 + \xi_1^{-1}) H_1^4 + \frac{1}{4} (\xi_1 - 6 + \xi_1^{-1}) H_1^2 \right], \\
\kappa^4 V_2 &= \frac{4}{t} \left[\xi_2 H_2^4 + \frac{1}{4} (\xi_2 - 4) H_2^2 \right], \\
\kappa^4 V_3 &= \frac{4}{u} \left[\xi_3 H_3^4 + \frac{1}{4} (\xi_3 - 4) H_3^2 \right],
\end{aligned} \tag{3.15}$$

where the moduli fields ξ_i are given in terms of R^2 and the various α' :

$$\xi_1 = tu = \frac{2R^2}{\alpha'_H}, \quad \xi_2 = su = \frac{2R^2}{\alpha'_{IIA}}, \quad \xi_3 = st = \frac{2R^2}{\alpha'_{IIB}}. \tag{3.16}$$

V is a simple fourth-order polynomial, when expressed in terms of new field variables H_A , $A = 1, 2, 3$,

$$H_A = \frac{z_A}{(1 - x^2)}, \quad x^2 = \left(1 - \sum_A Z_A^2 \right), \tag{3.17}$$

that take values on the entire real axis. At $H_i = 0$, the Kähler metric is $4\delta_B^A$, the scalar potential is normalized according to $V = 4\kappa^2 \sum_A m_A^2 H_A^2 + \dots$. The masses m_A^2 correspond to the mass formula for the heterotic, IIA and IIB tachyons.

Having the full effective thermal potential of the theory we are able to study the phase structure of the thermal effective theory. There are four different phases corresponding to specific regions of the s , t and u moduli space. Their boundaries are defined by critical values of the moduli s , t , and u (or of ξ_i , $i = 1, 2, 3$), or equivalently by critical values of the temperature, the (four-dimensional) string coupling and the compactification radius R_6 . These four phases are [9]:

1. The *low-temperature* phase:

$$T < (\sqrt{2} - 1)^{1/2} / (4\pi\kappa).$$

2. The *high-temperature heterotic* phase:

$$T > (\sqrt{2} - 1)^{1/2} / (4\pi\kappa) \quad \text{and} \quad g_H^2 < (2 + \sqrt{2})/4.$$

3. The *high-temperature type IIA* phase:

$$T > (\sqrt{2} - 1)^{1/2}/(4\pi\kappa), \quad g_H^2 > (2 + \sqrt{2})/4 \quad \text{and} \quad R_6 > \sqrt{\alpha'_H}.$$

4. The *high-temperature type IIB* phase:

$$T > (\sqrt{2} - 1)^{1/2}/(4\pi\kappa), \quad g_H^2 > (2 + \sqrt{2})/4 \quad \text{and} \quad R_6 < \sqrt{\alpha'_H}.$$

• The *low-temperature phase*, which is common to all three strings, is characterized by

$$H_1 = H_2 = H_3 = 0, \quad V_1 = V_2 = V_3 = 0. \quad (3.18)$$

The potential vanishes for all values of the moduli s , t and u , which are then only restricted by the stability of the phase, namely the absence of tachyons in the mass spectrum of the scalars H_A . Since the (four-dimensional) string couplings are

$$s = \sqrt{2}g_H^{-2}, \quad t = \sqrt{2}g_A^{-2}, \quad u = \sqrt{2}g_B^{-2},$$

this phase exists in the perturbative regime of all three strings. The relevant light thermal states are just the massless modes of the five-dimensional $N = 4$ supergravity, with thermal mass scaling like $1/R \sim T$.

• The *high-temperature heterotic phase* is defined by

$$\xi_H > \xi_1 > \frac{1}{\xi_H}, \quad \xi_2 > 4, \quad \xi_3 > 4, \quad (3.19)$$

with $\xi_H = (\sqrt{2} + 1)^2$. The inequalities on ξ_2 and ξ_3 eliminate type II instabilities. In this region of the moduli, the potential becomes, after minimization with respect to H_1 , H_2 and H_3 :

$$\kappa^4 V = -\frac{1}{s} \frac{(\xi_1 + \xi_1^{-1} - 6)^2}{16(\xi_1 + \xi_1^{-1})}.$$

It has a stable minimum for fixed s (for fixed α'_H) at the minimum of the self-dual quantity $\xi_1 + \xi_1^{-1}$:

$$\xi_1 = 1, \quad H_1 = \frac{1}{2}, \quad H_2 = H_3 = 0, \quad \kappa^4 V = -\frac{1}{2s}. \quad (3.20)$$

The transition from the low-temperature vacuum is due to a condensation of the heterotic thermal winding mode H_1 or, equivalently, to a condensation of type IIA NS 5-brane in the type IIA picture. At the level of the potential only, this phase exhibits a runaway behaviour in s . We will show in the next section that a stable solution to the effective action exists with non-trivial metric and/or dilaton.

- The *high-temperature type IIA and IIB phases* are defined by the inequalities

$$\xi_2 < 4 \quad \text{and/or} \quad \xi_3 < 4. \quad (3.21)$$

In this region of the parameter space, either H_2 or H_3 become tachyonic and acquire a vacuum value:

$$H_2^2 = \frac{4 - \xi_2}{8\xi_2}, \quad \kappa^4 V_2 = -\frac{1}{t} \frac{(4 - \xi_2)^2}{16\xi_2}, \quad (3.22)$$

and/or

$$H_3^2 = \frac{4 - \xi_3}{8\xi_3}, \quad \kappa^4 V_3 = -\frac{1}{u} \frac{(4 - \xi_3)^2}{16\xi_3}. \quad (3.23)$$

In contrast with the high-temperature heterotic phase, the potential does not possess stationary values of ξ_2 and/or ξ_3 , besides the critical points $\xi_{2,3} = 4$. Suppose for instance that $\xi_2 < 4$ and $\xi_3 > 4$. The resulting potential is then V_2 only and ξ_2 slides to zero. In this limit, $V = -1/(st\kappa^4)$, and the dynamics of $\phi \equiv -\log(stu)$ is described by the effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{II} = -\frac{e}{2\kappa^2} \left[R + \frac{1}{6}(\partial_\mu \phi)^2 - \frac{2}{\kappa^2} e^\phi \right].$$

The other scalar components $\log(t/u)$ and $\log(s/u)$ have only derivative couplings since the potential only depends on ϕ . They can be taken as constant and arbitrary. The dynamics only restricts the temperature radius $\kappa^{-2}R^2 = e^{-\phi}$, R_6 and the string coupling is not constrained. The ground state of this phase corresponds to a non-trivial gravitational and dilaton background satisfying the Einstein and ϕ equations of $\mathcal{L}_{\text{eff}}^{II}$. This background solution defines the high-temperature type II vacuum. We will not study this solution further. Instead, we will examine in more detail the high- T heterotic phase.

4 The high- T heterotic phase transition

Using the information contained in the effective theory, which is characterized by Eqs. (3.20), the equations of motion of all scalar fields are satisfied, with the exception of the dilaton $s = \text{Re } S$. The resulting bosonic effective Lagrangian that describes the dynamics of s and $g_{\mu\nu}$ is:

$$\mathcal{L}_{\text{bos}}^H = -\frac{1}{2\kappa^2} e R - \frac{e}{4\kappa^2} (\partial_\mu \ln s)^2 + \frac{e}{2\kappa^4 s}. \quad (4.24)$$

For all (fixed) values of s , the cosmological constant is negative, since $V = -(2\kappa^4 s)^{-1}$ and the apparent geometry is anti-de Sitter. But the effective theory (4.24) does not stabilize s . Rewriting $\mathcal{L}_{\text{bos}}^H$ in the heterotic string frame,

$$e^{-2\phi} = s \quad \text{and} \quad g_{\mu\nu}^{\text{str}} = \frac{2\kappa^2}{\alpha'_H} e^{-2\phi} g_{\mu\nu}, \quad (4.25)$$

we obtain

$$\mathcal{L}_{\text{str}}^H = \frac{e^{-2\phi}}{\alpha'_H} \left[-eR + 4e(\partial_\mu\phi)(\partial^\mu\phi) + \frac{2e}{\alpha'_H} \right]; \quad (4.26)$$

it is easy to show that the ϕ -equation of motion is that of a 2D-sigma-model β -function equation with $\beta_\phi = 0$ [25] and with central charge deficit

$$\delta\hat{c} = \frac{2}{3}\delta c = -4. \quad (4.27)$$

In the string frame, a background solution (4.26) has a flat (sigma-model) metric $\tilde{g}_{\mu\nu}^{\text{str}} = \eta_{\mu\nu}$ and a linear dilaton background on a spatial coordinate:

$$\tilde{\phi} = Q_\mu x^\mu, \quad \text{with} \quad Q^2 = \frac{\delta\hat{c}}{8\alpha'_H} = \frac{1}{2\alpha'_H}. \quad (4.28)$$

In this background there is a shift for all boson masses, $M_B^2 \rightarrow M_B^2 + Q^2 = M_B^2 + m_{3/2}^2$ because of the non-trivial dilaton. The fermionic masses are also shifted because of the temperature. As a result, the perturbative heterotic mass spectrum shows, fermion–boson mass degeneracy due to a residual supersymmetry existing in this background [5, 9]. At the non-perturbative level, however, this degeneracy is lost in the non-perturbative massive sector of the theory, although the ground state remains supersymmetric [9]. Thus, the high- T phase is expected to be stable in the weak coupling heterotic regime, because of the $N = 2$ residual supersymmetry. The reader can find more details at this point in Ref. [9].

5 The high- T non-critical string

As we discussed above, the high- T phase of $N = 4$ strings is described by a non-critical string with central charge deficit $\delta\hat{c} = -4$, provided the (six-dimensional) heterotic string is in the weakly coupled regime, $\lambda_H \leq (\sqrt{2} + 1)/2$. One possible description is in terms of the (5+1) super-Liouville theory compactified (at least) on S_R^1 , with radius fixed at the fermionic point $R = \sqrt{\alpha'_H/2}$. The perturbative stability of this ground

state is guaranteed when there is at least $\mathcal{N}_{sc} = 2$ superconformal symmetry on the world-sheet, implying at least $N = 1$ supersymmetry in space-time.

An explicit example with $\mathcal{N}_{sc} = 4$ superconformal was given in Refs. [26, 27, 9]. It is obtained when, together with the temperature S_R^1 , there is an additional compactified coordinate on $S_{R_6}^1$, with radius fixed at the fermionic point $R_6 = \sqrt{\alpha'_H/2}$. These two circles are equivalent to a compactification on $[SU(2) \times SU(2)]_k$ at the limiting value of level $k = 0$. Indeed, at $k = 0$, only the six world-sheet fermionic coordinates survive describing a $\hat{c} = 2$ system (with an $SO(4)_{k=1}$ current algebra) instead of $\hat{c} = 6$ of $k \rightarrow \infty$, consistently with the decoupling of four supercoordinates, $\delta\hat{c} = -4$. The central-charge deficit is compensated by the linear motion of the dilaton associated to the Liouville field, $\phi = Q^\mu x_\mu$ with $Q^2 = 1/(2\alpha'_H)$, so that $\delta\hat{c}_L = 8Q^2\alpha'_H = 4$.

Using the techniques developed in Ref. [28, 27], we derive in Refs. [9] the one-loop (perturbative) partition function in the high- T heterotic phase. Here I stress some of our results. More details will appear in Ref. [9].

- The initial $N = 4$ supersymmetry is reduced in the high- T heterotic phase to $N = 2$. This agrees with the effective field theory analysis of the high- T phase. The (perturbative) bosonic and fermionic mass fluctuations are degenerate because of the remaining $N = 2$ supersymmetry ($\mathcal{N}_{sc} = 4$ superconformal on the world-sheet).
- The spectrum of the theory contains two sectors, $h = 0$ and $h = 1$: the $h = 0$ sector has no massless fluctuations; the bosonic and fermionic masses (squared) are shifted by $m_{3/2}^2$ because of the dilaton background and the temperature coupling; all masses are larger than or equal to $m_{3/2}$. This is again in agreement with the effective theory analysis.
- In the $h = 1$ sector there are massless excitations, as expected from the (5+1) super-Liouville theory [29, 27].
- The 5 + 1 Liouville background can be regarded as an Euclidean 5-brane solution wrapped on $S^1 \times S^1$ preserving one-half ($N = 2$) of the initial $N = 4$ space-time supersymmetries.
- The massless space-time fermions coming from the $h = 1$ sector are six-dimensional space-time spinors; they are also spinors under the $SO(4)_{k=2}$ right-moving group

defined at the fermionic point of the $S_{R_1}^1 \times S_{R_6}^1$ compactification; they are also in the vector representation of an $SO(28)_{k=1}$ heterotic right-moving group.

- The massless space-time bosons are in the same right-moving representation, *e.g.* spinors under $SO(4)_{k=1}$ and vectors under $SO(28)_{k=1}$ right-moving groups. In addition, they are spinors under $SO(4)_{k=1}$ left-moving group. Together with the massless fermions, they form 28 $N = 2$ hypermultiplets sitting on the special quaternionic space:

$$\mathcal{H} = \frac{SO(4, 28)}{SO(4) \times SO(28)}.$$

The 28 massless hypermultiplets in the $h = 1$ sector are the only states that survive in the zero-slope limit. Their effective field theory is described by an $N = 2$ 4D-sigma-model on a hyper-Kähler manifold \mathcal{K} , which is obtained from \mathcal{H} in the decoupling limit of gravity. This topological theory corresponds to the infinite temperature limit of the $N = 4$ strings after the heterotic Hagedorn phase transition [9].

Although the $5 + 1$ Liouville background is perturbatively stable, owing to the $\mathcal{N}_{sc} = 4$ superconformal symmetry, its stability is not ensured at the non-perturbative level when the heterotic coupling is large:

$$g_H^2(x_\mu) = e^{2(\phi_0 - Q^\mu x_\mu)} > g_{\text{crt}}^2 = \frac{\sqrt{2} + 1}{2\sqrt{2}} \sim 0.8536. \quad (5.29)$$

As we explained above, the high- T heterotic phase exists only if $g_H^2(x_\mu)$ is lower than a critical value that separates the heterotic and Type II high- T phases. Thus one expects a domain-wall in space-time, at $Q^\mu x_\mu^0 = 0$, separating these two phases: $g_H^2(Q^\mu x_\mu^0) \sim g_{\text{crt}}^2$. The domain wall problem can be avoided by replacing the $(5 + 1)$ super-Liouville background by a more appropriate one having the same superconformal properties, $\mathcal{N}_{sc} = 4$, obeying however the additional perturbative constraint $g_H^2(x_\mu) \ll g_{\text{crt}}^2$ in the entire space-time.

Exact superstring solutions based on gauged WZW two-dimensional models with $\mathcal{N}_{sc} = 4$ superconformal symmetries have been studied in the literature [30, 26, 27, 31]. In Ref. [9], various candidate backgrounds with $\delta\hat{c} = -4$ are considered. The first one is that of $(5 + 1)$ super-Liouville ($\delta\hat{c}_L = 4$) already examined above. Another class of candidate backgrounds is based on the non-compact parafermionic spaces \mathcal{M}_p^4 that are described by gauged WZW models:

$$\mathcal{M}_p^4 = \left[\frac{SL(2, R)}{U(1)_{V,A}} \right]_{k=4} \times \left[\frac{SL(2, R)}{U(1)_{V,A}} \right]_{k=4} \times U(1)_{R^2=\alpha'_H/2} \times U(1)_{R_6^2=\alpha'_H/2}$$

$$\equiv \left[\frac{SL(2, R)}{U(1)_{V,A}} \right]_{k=4} \times \left[\frac{SL(2, R)}{U(1)_{V,A}} \right]_{k=4} \times SO(4)_{k=1}; \quad (5.30)$$

the indices A and V stand for the ‘‘axial’’ and ‘‘vector’’ WZW $U(1)$ gaugings. Then, many backgrounds can be obtained by marginal deformations of the above, which preserve at least $\mathcal{N}_{sc} = 2$, or also by performing \mathcal{S} - or \mathcal{T} -duality transformations on them.

As already explained, the appropriate background must verify the weak-coupling constraint (5.29). This weak-coupling limitation is achieved in the ‘‘axial’’ parafermionic space $\mathcal{M}_p^4(\text{axial})$. Indeed, in this background, $g_H^2(x_\mu)$ is bounded in the entire non-compact four-dimensional space, with coordinates $x_\mu = \{z, z^*, w, w^*\}$, provided the initial value of $g_0^2 = g_H^2(x_\mu = 0)$ is small:

$$\frac{1}{g_H^2(x_\mu)} = e^{-2\phi} = \frac{1}{g_0^2} (1 + zz^*) (1 + ww^*) \geq \frac{1}{g_0^2} \gg \frac{1}{g_{\text{crt}}^2}. \quad (5.31)$$

The metric of this background is everywhere regular,

$$ds^2 = \frac{4dzdz^*}{1 + zz^*} + \frac{4dwdw^*}{1 + ww^*}, \quad (5.32)$$

while the Ricci tensor and the scalar curvature

$$R_{zz^*} = \frac{1}{(1 + zz^*)^2}, \quad R_{ww^*} = \frac{1}{(1 + ww^*)^2}, \quad R = \frac{1}{4(1 + zz^*)} + \frac{1}{4(1 + ww^*)}$$

vanish for asymptotically large values of $|z|$ and $|w|$ (asymptotically flat space). This space has maximal curvature when $|z| = |w| = 0$. This solution has a behaviour similar to the one of the Liouville solution in the asymptotic regime $|z|, |w| \rightarrow \infty$. In this limit, the dilaton ϕ becomes linear when expressed in terms of the flat coordinates x_i :

$$\phi = -\text{Re}[\log z] - \text{Re}[\log w] = -Q^1|x_1| - Q^2|x_2|,$$

$$x_1 = -\text{Re}[\log z], \quad x_2 = -\text{Re}[\log w], \quad x_3 = \text{Im}[\log z], \quad x_4 = \text{Im}[\log w]. \quad (5.33)$$

In the large- $|z|$ and $|w|$ limit, \mathcal{M}_p^4 is flat with $ds^2 = 4(dx_i)^2$. The important point here is that for large values of $|x_1|$ and $|x_2|$, $\phi \ll 0$, in contrast to the Liouville background in which $\phi = Q^1x_1 + Q^2x_2$ and therefore the dilaton is becoming positive and arbitrarily large in one half of the space, thus violating the weak-coupling constraint (5.29).

We then conclude that the high- T heterotic phase is well described by the $\mathcal{M}_p^4(\text{axial})$. This space is $N = 2$ supersymmetric and stable when $g_0^2 \ll 1$, since it is everywhere

perturbative with degenerate massive bosonic and fermionic fluctuations. The non-perturbative states are superheavy and decouple in the limit of vanishing coupling $g_0^2 \rightarrow \infty$.

On the heterotic or type IIA side, the high-temperature limit after the high- T heterotic phase transition corresponds to a topological $N = 2$ supersymmetric theory described by a 4D-sigma-model on a non-trivial hyper-Kähler manifold. On the type IIB side, on the other hand, the high- T phase corresponds to a tensionless string defined by a limit that generalizes the large- N limit of Yang–Mills theory, $\alpha'_H \rightarrow \infty$, $\lambda_H \rightarrow 0$, with $\alpha'_H \lambda_H$ fixed [9]. It is very interesting to study further the properties of these theories that describe the high- T phase of string theory, in more general compactifications with lower number of supersymmetries, and to study their possible cosmological implications.

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