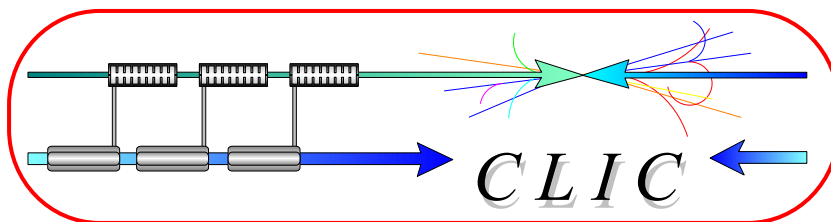


CERN - European Laboratory for Particle Physics



CLIC Note 366

Loss factor Dependence on Group Velocity in Disk-Loaded Travelling-Wave Structures

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Abstract

The loss factor, a quantity linked to the energy lost by a point-like charge when traversing an accelerating (or decelerating) structure, can be computed using programs which solve Maxwell's equations in time domain and provide the correct result within the limitations inherent to the numerical simulation process. An alternative method, commonly used, consists in the derivation of the loss factor from the parameter R/Q , which is computed using codes operating in frequency-domain. Recent calculations of the loss factors for disk-loaded structures performed with the two methods have produced diverging results. The discrepancy of the results is a function of the group velocity and can be eliminated by introducing a correction term in the formula linking the loss factor to the R/Q obtained from frequency-domain calculations.

Geneva, Switzerland
25 January 1999

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1. SOME BASIC FORMULAE

The longitudinal loss factor in any structure is by definition the ratio of the energy lost by a point-like charge q that traverses the structure over the square of the charge:

$$k_{\delta} = \frac{\Delta U}{q^2} \quad (1)$$

The point charge interacts primarily with the fundamental structure mode at angular frequency ω_0 and generates a longitudinal wake potential of the form:

$$W_{z0} = 2k_{\delta0} \cos\left(\frac{\omega_0 s}{c}\right) \quad (2)$$

The peak induced wake potential is then

$$\hat{V}_0 = q \hat{W}_{z0} = 2qk_{\delta0} \quad (3)$$

From (1) and (3) we derive

$$k_{\delta0} = \frac{\hat{V}_0^2}{4U_{0TW}} \quad (4)$$

U_{0TW} is the energy of the fundamental travelling-wave mode left in the structure by the point charge. We see that $k_{\delta0}$ is a property of the structure geometry independent of the driving charge.

It can be shown [1] that the loss factor for a gaussian bunch with standard deviation σ to the fundamental mode is:

$$k_{\sigma0} = k_{\delta0} e^{-\left(\frac{\omega_0 \sigma}{c}\right)^2} = k_{\delta0} F^2(\sigma) \quad (5)$$

For a resonator with high Q fundamental mode, the ratio R/Q can be written as:

$$\left(\frac{R}{Q}\right)_{0SW} = \frac{V_0^2}{2\omega_0 U_{0SW}} \quad (\text{circuit } \Omega) \quad (6a)$$

Expression (6a) gives the correct value for standing waves. For the travelling wave case we have used twice this value:

$$\left(\frac{R}{Q}\right)_{0TW} = \frac{V_0^2}{\omega_0 U_{0SW}} \quad (\text{circuit } \Omega) \quad (6b)$$

Substituting (6b) into (4) and assuming $U_{0SW} = 2U_{0TW}$, we obtain for the point charge loss factor:

$$k_{\delta 0} = \frac{\omega_0}{2} \left(\frac{R}{Q} \right)_{0TW} \quad (7)$$

We know how to compute the R/Q of a given structure in frequency domain using codes like URMEL [2] and MAFIA [3]. From (7) we can derive the point charge loss factor. Alternatively we can compute the bunch loss factor in time domain using codes like ABCI [4] and MAFIA. We can then obtain the point charge loss factor from (5).

2. RESULTS FROM COMPUTATIONS

We have computed the travelling wave loss factors of the fundamental mode for disk-loaded structures of simplified geometry (a=2. mm, b=4.23 mm, p=3.332 mm, t=0.6 mm, no iris rounding) using URMEL and expressions (6b) and (7) in frequency domain and ABCI in time domain. We have changed the iris radius and cavity radius of the structure to increase progressively the fundamental mode group velocity while keeping its frequency constant at 30 GHz. Table 1 shows the results from the frequency domain computations performed with URMEL, while Table 2 contains the results obtained from ABCI in time domain (24 cells, bunch with $\sigma=0.6$ mm). We have made use of the useful feature provided in ABCI, which allows the integration of the bunch loss factor spectrum, to determine the fundamental mode loss factors reported in Table 2.

TABLE 1: frequency domain results using URMEL.

a [mm]	b [mm]	$\beta_g = \frac{v_g}{c}$	$\left(\frac{R'}{Q} \right)_{0TW}$ [Ω/m]	$k'_{\delta 0}$ [V/pC/m]	$\frac{k'_{\delta 0}}{(1 - \beta_g)}$ [V/pC/m]
2.0	4.230	0.06	13783.	1296.	1378.
3.0	4.832	0.19	7404.	697.	860.
4.0	5.552	0.33	4087.	385.	575.
5.0	6.382	0.45	2342.	220.	400.
6.0	7.232	0.56	1415.	135.	302.

TABLE 2: time domain results using ABCI ($\sigma=0.6$ mm)

a [mm]	$k'_{\sigma 0}$ [V/pC/m]	$k'_{\delta 0}$ [V/pC/m]
2.0	1187.	1370.
3.0	762.	879.
4.0	512.	590.
5.0	362.	417.
6.0	270.	311.

When comparing the values in the fifth column of Table 1 with those in the last column in Table 2, we see that there is a large discrepancy between the loss factors computed in frequency and time domain using the same structures with cylindrical symmetry.

The discrepancy increases with the mode group velocity. However when we multiply the loss factor obtained in the frequency domain computations by the correction term

$\frac{1}{(1-\beta_g)}$, we obtain the last column of Table 1 which is in very good agreement with the

corresponding values in Table 2. We have plotted in Figure 1 the values of the loss factor as function of the group velocity. The lower curve shows the values obtained in frequency domain without the correcting factor. When this factor is introduced we obtain the upper curve which coincides with the values from the time domain computations.

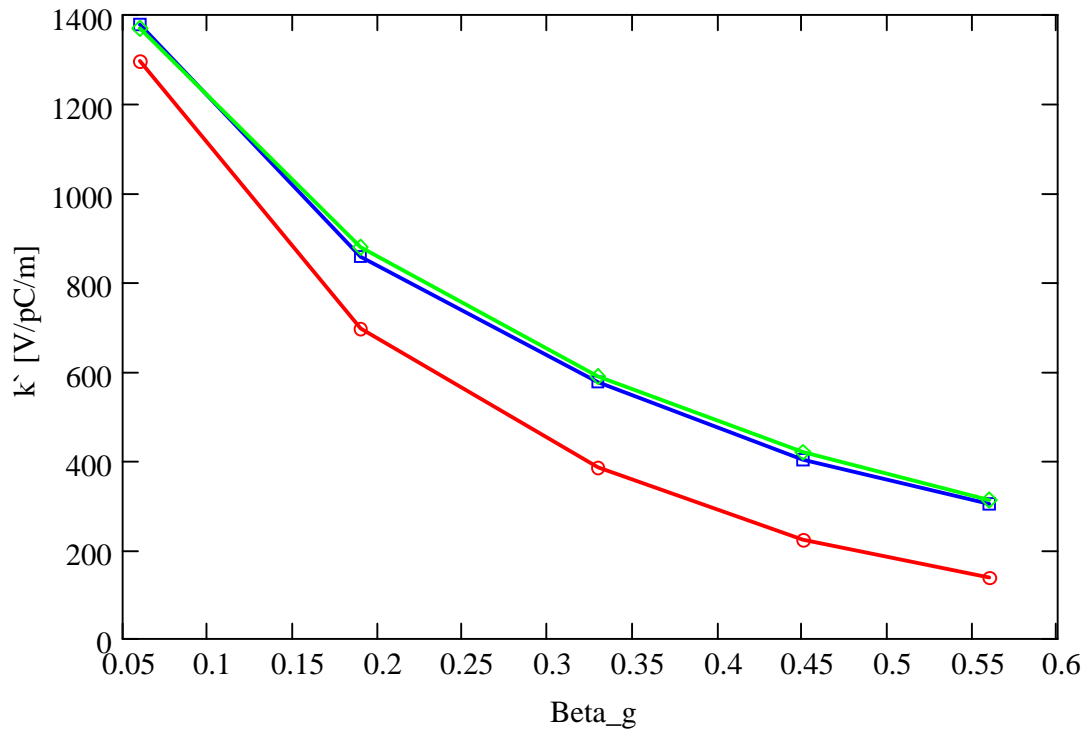


Figure 1 Loss factor as function of normalised group velocity

3. CONCLUSION:

In the comparison of the results obtained in frequency and time domain for the loss factor of the fundamental mode in travelling-wave disk-loaded structures of simple geometry, we have found a large discrepancy which is a function of the mode group velocity. The discrepancy is eliminated when the loss factor obtained in frequency domain is multiplied

by the correcting term $\frac{1}{(1-\beta_g)}$.

An intuitive justification for this correction may be found in the fact that the energy deposited by a single bunch exiting an initially empty travelling wave structure gets compressed as it travels at the group velocity in the same direction as the driving bunch.

The energy is concentrated in a fraction $(1 - \beta_g)$ of the structure length, which causes the local energy density and consequently the decelerating field to increase.

The energy compression is taken into account in the time domain computations, which solve directly Maxwell's equations, but it is not in equation (7) derived from frequency domain computations based on standing waves. We propose the correct expression for the loss factor as follows:

$$k_{\delta 0} = \frac{\omega_0}{2} \left(\frac{R}{Q} \right)_{OTW} \left(\frac{1}{1 - \beta_g} \right) \quad (8)$$

We have used simple disk-loaded structures to perform quick computations, however the conclusion reached is quite general and directly applicable to any type of structure. The correction term has a small impact upon the beam loading computations for low group velocity structures, but a large effect for the power output considerations of the CLIC Power Extraction and Transfer Structure (PETS) [5], which have group velocity of about 0.5 c.

4. ACKNOWLEDGMENT

The described investigations are a direct consequence of unexpectedly large bunch decelerations, observed in the PETS of the CERN Test Facility (CTF2).

The authors thank E. Jensen for constructive discussions and for his contribution in the formal presentation of the results.

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