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Universal continuum limit of non-perturbative lattice non-singlet moment evolution

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Abstract

We present evidence for the universality of the continuum limit of the scale dependence of the renormalization constant associated with the operator corresponding to the average momentum of non-singlet parton densities. The evidence is provided by a non-perturbative computation in quenched lattice QCD using the Schrödinger Functional scheme. In particular, we show that the continuum limit is independent of the form of the fermion action used, i.e. the Wilson action and the non-perturbatively improved clover action.

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We presented in ref. [1] a quenched non-perturbative calculation for the renormalization constant associated with the operator corresponding to the average momentum of non-singlet parton densities. The calculation was based on a finite size recursive scheme that allows us to reconstruct in the continuum the non-perturbative evolution of the renormalization constant in the Schrödinger Functional (SF) scheme [2, 3, 4, 5]. In ref. [1] we used a standard Wilson action where the continuum limit is approached with a rate that is linear in the lattice spacing a in leading order. The results for the step scaling functions presented in ref. [1] also showed sizeable quadratic corrections that had to be included in the fit and made the continuum extrapolation more hazardous.

We repeated the same calculation with a non-perturbatively improved clover action [4] that changes the form of the lattice artefacts. Linear corrections are still to be expected, because the $\mathrm{O}(a)$-improvement of the action should be complemented with the improvement of the operators and of the boundary counterterms in order to lead to a full cancellation of effects appearing linear in a. However, the continuum limit cannot depend upon the discretization chosen and should be universal. Our results do indeed show such a universality and therefore put our previous continuum extrapolations on a firmer basis. We find that the extrapolation of the step scaling function, computed with the $\mathrm{O}(a)$-improved action, to the contiuum need again quadratic terms in the fit.

We refer to ref. [1] for more of the details about the calculation which we here only shortly summarize as follows. We calculate the renormalization constant of the twist-two non-singlet operator defined by:

$$\mathcal{O}^{qNS}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\psi}(x)\gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \frac{\lambda^f}{2}\psi(x) \ + \ \text{trace terms} \ . \tag{1}$$

The basic ingredient for the reconstruction of the non-perturbative scale dependence of the renormalization constants of the above operator is the finite-size step scaling function σ_Z defined by:

$$Z(sL) = \sigma_Z(\bar{g}^2(L))Z(L) \ , \tag{2}$$

where L is the physical length that plays the role of the renormalization scale and Z the renormalization constant of the operator, which is defined by:

$$O^R(\mu) = Z(1/a\mu)^{-1}O^{\mathrm{bare}}(a/L) \ . \tag{3}$$

Z is obtained from the SF matrix element of the operator on a finite volume L^3T normalized to its tree level:

$$O^{\mathrm{bare}}(a/L) = Z(L/a)O^{\mathrm{tree}} \; . \tag{4}$$

The framework of the Schrödinger Functional, which describes the quantum time evolution between two fixed classical gauge and fermion configurations defined at time $t = 0$ and $t = T$, has been used extensively in the recent literature [6, 7] to calculate non-perturbative renormalization constants of local operators. Among the advantages of the method, we only quote the possibility of performing the computations at zero physical quark mass and of using non-local gauge invariant sources for the fermions without need of a gauge-fixing procedure. In our particular case, we exploit both features. Our observable is defined by [8]:

$$Z = \frac{f_2(x_0 = L/4)}{\sqrt{f_1}} \Bigg/ \left(\frac{f_2(x_0 = L/4)}{\sqrt{f_1}}\right)_{\mathrm{tree}} \; , \tag{5}$$

with f_2 given by

$$f_2(x_0) = -a^6 \sum_{\mathbf{y},\mathbf{z}} \mathrm{e}^{\mathbf{ip(y-z)}} \langle \frac{1}{4}\bar{\psi}(\mathrm{x})\gamma_{\{1} \overleftrightarrow{\mathrm{D}}_{2\}} \frac{1}{2}\tau^3\psi(\mathrm{x})\bar{\zeta}(\mathbf{y})\Gamma\frac{1}{2}\tau^3\zeta(\mathbf{z})\rangle \tag{6}$$

and f_1 by

$$f_1 = -a^{12} \sum_{\mathbf{y},\mathbf{z},\mathbf{v},\mathbf{w}} \langle \bar{\zeta}'(\mathbf{v})\frac{\boldsymbol{\tau^3}}{\mathbf{2}}\zeta'(\mathbf{w})\bar{\zeta}(\mathbf{x})\frac{\boldsymbol{\tau^3}}{\mathbf{2}}\zeta(\mathbf{y})\rangle \tag{7}$$

where $\zeta = \delta/\delta\bar{\psi}_c$ and $\bar{\zeta} = -\delta/\delta\psi_c$ are the derivatives with respect to the two-component classical fermion fields ($\bar{\psi}_c$ and ψ_c, respectively) at the boundary $x_0 = 0$, while ζ' and $\bar{\zeta}'$ are the corresponding derivatives at the boundary $x_0 = T$. The projection on the classical components is achieved by the projector $P_\pm$ defined by $\frac{1}{2}(1 \pm \gamma_0)$. On the boundaries the theory possesses only a *global* gauge invariance that is preserved by the quantities defined above. The values of x_0 (set to $T/4$) and of the non-zero component of the momentum p_x (set to $2\pi/L$) are both scaled in units of L, which therefore remains the only scale besides the lattice spacing a. The quantity f_1 serves as a normalization factor that removes the wave function renormalization constant of the ζ fields in order to isolate the running associated with the operator in eq. (1) only.

In order to compute the running of Z in eq. (5) its step scaling function $\sigma_Z \equiv \sigma_{\bar{Z}}/\sigma_{f_1}$ with

$$\sigma_{\bar{Z}} = \frac{f_2(x_0 = L/4)}{(f_2(x_0 = L/4))_{\rm tree}} \; ; \;\; \sigma_{f_1} = \frac{\sqrt{f_1}}{\left(\sqrt{f_1}\right)_{\rm tree}} \tag{8}$$

has to be evaluated in the continuum. To this end, the physical size L ($= T$ in our case) is kept fixed in physical units by keeping fixed the renormalized coupling constant in the SF scheme, renormalized at the physical volume scale. By increasing the number of lattice points while keeping the value of L constant, the lattice spacing in physical units is reduced and a continuum extrapolation of the step scaling function can be performed.

The presence of a large non-zero momentum induces potentially large lattice artefacts in the numerator of the renormalization constant in eq. (5). Indeed, in ref. [1], for this reason, we had to extrapolate independently the step scaling functions $\sigma_{\bar{Z}}$ and for σ_{f_1}, with a quadratic and a linear fit in the lattice spacing, respectively.

As already mentioned, the calculation done with a non-perturbatively improved action, without improving the boundary terms and the operator (whose matrix element in our case is tree-level-improved) does not eliminate all linear lattice artefacts and certainly does not eliminate the quadratic artefacts, if present with the Wilson action. Therefore, also with the clover action we have done independent extrapolations for $\sigma_{\bar{Z}}$ and for σ_{f_1}, which are compared with our previous results using the Wilson action in tables 1 and 2 for the different values of L (or equivalently $\bar{g}^2(L)$) considered in ref. [1]. Within the errors, there is full consistency between the two sets of results obtained with different fermion actions, showing the universality of the continuum limit. Given the consistency of the extrapolated values of the step scaling functions from both fermion actions, we performed a combined fit, using both data sets obtained with the two fermion actions. In the combined fit we demanded that the extrapolations go to the same continuum value of the step scaling function. In figures 1 and 2 we report results at fixed non-zero lattice spacing used for the extrapolation and the corresponding combined fit: while the linear dependence upon the lattice artefacts for σ_{f_1} is almost eliminated, the quadratic terms for $\sigma_{\bar{Z}}$ are still needed, as expected. In tables 1 and 2 we report the χ^2 per degree of freedom (d.o.f.) values of the combined fit with those of the original fit of ref. [1] and a fit for the case of the improved action alone. The χ^2 values are totally comparable, as are our final values for the continuum step scaling function. Note that the error compared with the ones given in ref. [1] gets decreased by about a factor $\sqrt{2}$.

We have shown that the continuum extrapolation of the evolution of the renormalization constant corresponding to the average momentum of non-singlet parton densities shows the expected independence upon the lattice action used. Over the range of scales explored, the evolution is adequately described with a three loop expression for the anomalous dimensions [1]. Further simulations that extend the running into the perturbative region, where a good description in terms of the known perturbative evolution can be made, are under way.

$\bar{g}^2(L)$	σ_{f_1}(Wilson)	χ^2	σ_{f_1}(impr)	χ^2	σ_{f_1}(combined)	χ^2
1.8811	0.912(7)	2.61	0.910(7)	0.06	0.911(5)	0.90
2.1000	0.897(9)	1.17	0.902(8)	3.22	0.900(6)	1.54
2.4484	0.886(9)	6.90	0.884(7)	0.17	0.885(6)	2.36
2.7700	0.866(10)	0.52	0.871(10)	1.58	0.869(7)	0.73
3.48	0.837(11)	0.39	0.815(10)	0.01	0.825(7)	0.86

Table 1: The values of the step scaling function σ_{f_1} are given extrapolated to the continuum. We compare the cases for Wilson fermions (2nd column), using the O(a)-improved action (4th column) and from the combined fit (6th column). The running coupling is computed in the SF scheme. We give also the χ^2/d.o.f.

$\bar{g}^2(L)$	$\sigma_{\bar{Z}}$(Wilson)	χ^2	$\sigma_{\bar{Z}}$(impr)	χ^2	$\sigma_{\bar{Z}}$(combined)	χ^2
1.8811	0.917(8)	0.19	0.890(7)	0.20	0.902(5)	2.55
2.1000	0.891(9)	0.20	0.891(9)	0.89	0.891(6)	0.36
2.4484	0.891(9)	1.01	0.884(9)	0.03	0.887(6)	0.43
2.7700	0.854(13)	0.01	0.858(13)	0.002	0.856(9)	0.02
3.48	0.855(12)	6.44	0.840(13)	0.04	0.848(9)	2.42

Table 2: The values of the step scaling function $\sigma_{\bar{Z}}$ are given extrapolated to the continuum. We compare the cases for Wilson fermions (2nd column), using the O(a)-improved action (4th column) and from the combined fit (6th column).

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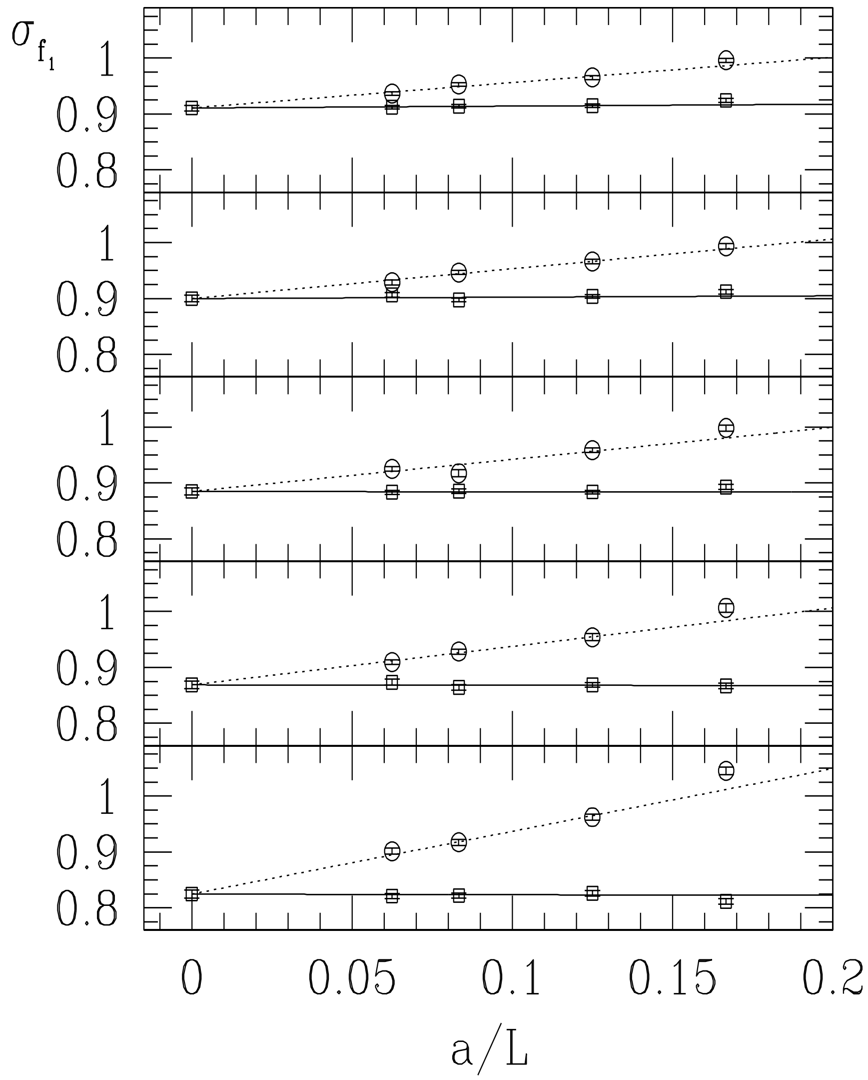

Figure 1: Continuum extrapolation of σ_{f_1} using a combined fit at all values of $\bar{g}^2$ of table 1, with $\bar{g}^2$ increasing from top to bottom. Circles and the dashed line correspond to using Wilson fermions. Squares and the solid line correspond to using the O(a)-improved action. The fit for σ_{f_1} is linear in both cases using the three points with smallest a/L.

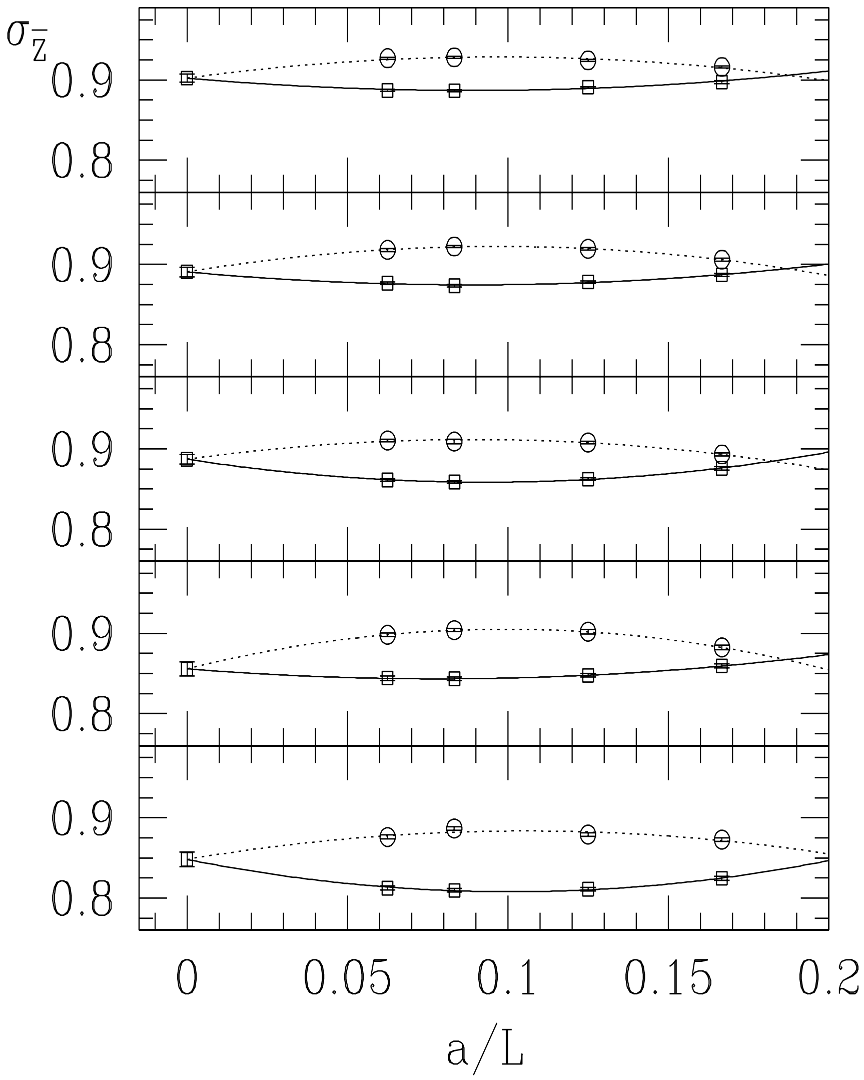

Figure 2: Continuum extrapolation of $\sigma_{\bar{Z}}$ using a combined fit at all values of $\bar{g}^2$ of table 2, with $\bar{g}^2$ increasing from top to bottom. The fit for $\sigma_{\bar{Z}}$ is quadratic using all data points. Notation is as in fig. 1.