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$N = 1$ supersymmetric $SU(4) \times SU(2)_L \times SU(2)_R$ effective theory from the weakly coupled heterotic superstring

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Abstract

In the context of the free–fermionic formulation of the heterotic superstring, we construct a three generation $N = 1$ supersymmetric $SU(4) \times SU(2)_L \times SU(2)_R$ model supplemented by an $SU(8)$ hidden gauge symmetry and five Abelian factors. The symmetry breaking to the standard model is achieved using vacuum expectation values of a Higgs pair in $(\mathbf{4}, \mathbf{2}_R)+ (\bar{\mathbf{4}}, \mathbf{2}_R)$ at a high scale. One linear combination of the Abelian symmetries is anomalous and is broken by vacuum expectation values of singlet fields along the flat directions of the superpotential. All consistent string vacua of the model are completely classified by solving the corresponding system of $F-$ and $D-$flatness equations including non–renormalizable terms up to sixth order. The requirement of existence of electroweak massless doublets further restricts the phenomenologically viable vacua. The third generation fermions receive masses from the tree–level superpotential. Further, a complete calculation of all non–renormalizable fermion mass terms up to fifth order shows that in certain string vacua the hierarchy of the fermion families is naturally obtained in the model as the second and third generation fermions earn their mass from fourth and fifth order terms. Along certain flat directions it is shown that the ratio of the $SU(4)$ breaking scale and the reduced Planck mass is equal to the up quark ratio m_c/m_t at the string scale. An additional prediction of the model, is the existence of a $U(1)$ symmetry carried by the fields of the hidden sector, ensuring thus the stability of the lightest hidden state. It is proposed that the hidden states may account for the invisible matter of the universe.

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1 Introduction

During the last decade, a lot of work has been devoted in the construction of effective low energy models of elementary particles from the heterotic superstring. Several old successful $N = 1$ supersymmetric grand unified theories (GUTs) have been recovered through the string approach [1, 2, 3, 4, 5, 6], however only few of them were able to rederive a number of successful predictions of their predecessors. Yet, new avenues and radical ideas that were previously not considered or only poorly explored, have now been painstakingly investigated in the context of string derived or even string inspired effective theories. Among them, the issue of the additional $U(1)$–symmetries which naturally appear in string models and the systematic derivation of non–renormalizable terms boosted our understanding of the observed mass hierarchies and impelled people to systematically classify all possible textures consistent with the low energy phenomenology. Further, new and astonishingly simpler mechanisms of GUT symmetry breaking were introduced due to the absence of large Higgs representations, at least in the simplest Kac–Moody level ($k = 1$) string constructions.

In addition to the above good omen, some embarrassing difficulties have also appeared, such as the existence of unconfined fractionally charged states – which belong to representations not incorporated in the usual GUTs – and the very high unification scale. The new representations come as a result of the breaking of the large string symmetry via the GSO projections. The appearance of such states are not necessarily an ominous warning for a particular model, although a mechanism should be invented to make them disappear from the light spectrum. The real major difficulty however, was the generic property of the high string scale in contrast to the usual supersymmetric GUTs which unify at about two orders of magnitude below the string mass. In the weakly coupled heterotic string theory, this problem can find a solution in specific models, when extra matter multiplets exist to properly modify the running of the gauge couplings, or possible intermediate symmetries and string threshold effects [7] can help gauge couplings converge to their experimentally determined values at low energies.

In this paper, we derive an improved version of a string model proposed in [5], based on the observable gauge symmetry $SO(6) \times O(4)$ (isomorphic to $SU(4) \times SU(2)_L \times SU(2)_R$ Pati–Salam (PS) gauge group [8]) in the context of the free–fermionic formulation of the four dimensional superstring. As shown in [4] this gauge symmetry breaks down to the standard model without the use of the adjoint or any higher representation thus it can be built directly at the $k = 1$ Kac–Moody level. (Higher Kac–Moody level models are also possible to build, however, they imply small unification scale values of $\sin^2 \theta_W$ [9].)

The models based on the PS gauge symmetry have also certain phenomenological advantages. Among them, is the absence of coloured gauge fields mediating proton decay. This fact allows for the possibility of having a low $SU(4)$ breaking scale compared to that of other GUTs, provided that the Higgs coloured fields do not have dangerous Yukawa couplings with ordinary matter. Possible ways to avoid fast proton decay have been discussed also recently in the literature [10].

Moreover, specific GUT relations among the Yukawa couplings, like the bottom–tau equality, give successful predictions at low energies, while at the same time such relations reduce the number of arbitrary Yukawa couplings even in the field–theory version.

The above nice features are exhibited in the present string construction. Using a variant of the original string basis and the GSO projection coefficients [5], we obtain an effective field theory model with three generations and exactly one Higgs pair to break the $SU(4) \times SU(2)_R$ gauge symmetry. The effective low energy theory is the $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)$ electroweak standard gauge symmetry broken to $SU(3) \times U(1)_{em}$ by the two Higgs doublet fields. We derive the complete massless spectrum of the model and the Yukawa interactions including non–renormalizable terms up to sixth order. Among the massless states, a mirror (half)–family is also obtained which acquires mass at a very large scale. There are also singlet fields and exotic doublet representations with a sufficient number of Yukawa couplings. All the observable and hidden fields appear with charges under five surplus $U(1)$ factors where one linear combination of them is anomalous. The anomalous $U(1)$ symmetry generates a D–term contribution, which can be cancelled if some of the singlet fields acquire non–zero vacuum expectation values (vevs). To find the true vacua, we solve the $F-$ and $D-$flatness conditions and classify all possible solutions involving observable fields with non–zero vevs. We analyze in detail three characteristic cases where superpotential contributions up to sixth order suffice to provide fermion mass terms for all generations. Phenomenologically interesting alternative solutions are also proposed in the case where some of the hidden fields develop vevs too.

Among the novel features of the present model is the existence of a $U(1)$ symmetry – carried by hidden and exotic fields – which remains unbroken. As a consequence, the lightest hidden state is stable. These states form various potential mass terms in the superpotential of the model. In our analysis, we show the existence of proper flat directions where the lightest state obtains a mass at an intermediate scale, leading to interesting cosmological implications.

The paper is organized as follows: In Section 2 we give a brief description of the supersymmetric version of the model and discuss various phenomenological features, including the economical Higgs mechanism, the mass spectrum and the renormalization group. In particular we show how the Pati–Salam symmetry dispenses with the use of Higgs fields in the adjoint of $SU(4)$ to break down to the standard model. In Section 3, we propose the string basis as well as the GSO projections which yield the desired gauge symmetry and the massless spectrum. In Section 4 the gauge symmetry breaking of the string version is analyzed. Moreover, due to the existence of additional $U(1)$ symmetries, the issue of new (non–standard) hypercharge embeddings is discussed. Particular embeddings where all fractionally charged states obtain integral charges are discussed in some detail. In Section 5 we derive the superpotential couplings and present a preliminary phenomenological analysis to set the low energy constraints and reduce the number of phenomenologically acceptable string vacua. In Section 6 we classify all solutions of the $F-$ and D–flatness equations including non–renormalizable superpotential contributions up to sixth order. A detailed phenomenological analysis of the promising string vacua in connection with their low energy predictions is presented in Section 7. Particular attention is given in the doublet higgs mass matrix, the fermion mass hierarchy and the colour triplet mass matrix. In Section 8, we present a brief discussion on the role of the hidden sector and extend the solutions of string vacua including hidden field vevs. Finally, in the Appendices A–D we present tables with the complete string spectrum, details about the derivation of the higher order non–renormalizable superpotential terms, the $D-$ and $F-$ flatness equations with non-renormalizable contributions and the complete list of their tree–level solutions.

2 The Supersymmetric $SU(4) \times SO(4)$ Model

There is a minimal supersymmetric $SU(4) \times O(4)$ Model which can be considered as a surrogate effective GUT of the possible viable string versions, incorporating all the basic features of a phenomenologically viable string model. The Yukawa couplings are determined by the Pati–Salam (PS) gauge symmetry and possible additional $U(1)$–family symmetries which are usually added (as in any other GUT) by phenomenological requirements, (i.e., fermion mass hierarchy, proton stability etc). This GUT version, however, provides us with insight in constructing the fully realistic string version. Therefore, here we briefly summarize the parts of the model relevant for our analysis [4]. The gauge group is $SU(4) \times O(4)$, or equivalently the PS gauge symmetry [8]

$$\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R. \tag{1}$$

The left–handed quarks and leptons are accommodated in the following representations,

$$F^{i\alpha a}_{L} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) = \begin{pmatrix} u^\alpha & \nu \\ d^\alpha & e \end{pmatrix}^i \tag{2}$$

$$\bar{F}^i_{x\alpha R} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} d^{c\alpha} & e^c \\ u^{c\alpha} & \nu^c \end{pmatrix}^i \tag{3}$$

where $\alpha = 1, \ldots, 4$ is an $SU(4)$ index, $a, x = 1, 2$ are $\mathrm{SU}(2)_{L,R}$ indices, and $i = 1, 2, 3$ is a family index. The Higgs fields are contained in the following representations,

$$h^x_a = (\mathbf{1}, \mathbf{2}, \mathbf{2}) = \begin{pmatrix} {h_+}^u & {h_0}^d \\ {h_0}^u & {h_-}^d \end{pmatrix} \tag{4}$$

where h^d and h^u are the low energy Higgs superfields associated with the minimal supersymmetric standard model (MSSM). The two 'GUT' breaking higgs representations are

$$H^{\alpha b} = (\mathbf{4}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} u^\alpha_H & \nu_H \\ d^\alpha_H & e_H \end{pmatrix} \tag{5}$$

and

$$\bar{H}_{\alpha x} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} u^{c\alpha}_H & e^c_H \\ u^{c\alpha}_H & \nu^c_H \end{pmatrix}. \tag{6}$$

Fermion generation multiplets transform to each other under the changes $\mathbf{4} \to \bar{\mathbf{4}}$ and $\mathbf{2}_L \to \mathbf{2}_R$ while the bidoublet higgs multiplet transforms to itself. However, the pair of fourplet–higgs fields does not have this property, discriminating $\mathbf{2}_L$ and $\mathbf{2}_R$. Thus, when they develop vevs along their neutral components $\tilde{\nu}_H, \tilde{\nu}^c_H$,

$$\langle H \rangle = \langle \tilde{\nu}_H \rangle \sim M_{GUT}, \quad \langle \bar{H} \rangle = \langle \tilde{\nu}^c_H \rangle \sim M_{GUT} \tag{7}$$

they break the $SU(4) \times SU(2)_R$ part of the gauge group, leading to the standard model symmetry at M_{GUT}

$$\mathrm{SU(4)} \times \mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \longrightarrow \mathrm{SU(3)}_C \times \mathrm{SU(2)}_L \times \mathrm{U(1)}_Y. \tag{8}$$

Under the symmetry breaking in Eq. (8), the bidoublet Higgs field h in Eq. (4) splits into two Higgs doublets h^u, h^d whose neutral components subsequently develop weak scale vevs,

$$\langle h^d_0 \rangle = v_1, \quad \langle h^u_0 \rangle = v_2 \tag{9}$$

with $\tan\beta \equiv v_2/v_1$.

In addition to the Higgs fields in Eqs. (5),(6) the model also involves an $SU(4)$ sextet field $D = (\mathbf{6}, \mathbf{1}, \mathbf{1})$ and four singlets ϕ_0 and φ_i, $i = 1, 2, 3$. ϕ_0 is going to acquire a vev of the order of the electroweak scale in order to realize the Higgs doublet mixing, while φ_i will participate in an extended 'see–saw' mechanism to obtain light majorana masses for the left–handed neutrinos. Under the symmetry property $\varphi_{1,2,3} \rightarrow (-1) \times \varphi_{1,2,3}$ and $H(\bar{H}) \rightarrow (-1) \times H(\bar{H})$ the tree–level mass terms of the superpotential of the model read [4]:

$$W = \lambda_1^{ij} F_{iL} \bar{F}_{jR} h + \lambda_2 H H D + \lambda_3 \bar{H} \bar{H} D + \lambda_4^{ij} H \bar{F}_{jR} \varphi_i + \mu \varphi_i \varphi_j + \mu h h \tag{10}$$

where $\mu = \langle \phi_0 \rangle \sim \mathcal{O}(m_W)$. The last term generates the higgs mixing between the two SM Higgs doublets in order to prevent the appearance of a massless electroweak axion. The following decompositions take place under the symmetry breaking (8):

$$\begin{aligned}
F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) &\rightarrow Q(\mathbf{3}, \mathbf{2}, -\frac{1}{6}) + \ell(\mathbf{1}, \mathbf{2}, \frac{1}{2}) \\
\bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) &\rightarrow u^c(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + \nu^c(\mathbf{1}, \mathbf{1}, 0) + e^c(\mathbf{1}, \mathbf{1}, -1) \\
\bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) &\rightarrow u^c_H(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}) + d^c_H(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) + \nu^c_H(\mathbf{1}, \mathbf{1}, 0) + e^c_H(\mathbf{1}, \mathbf{1}, -1) \\
H(\mathbf{4}, \mathbf{1}, \mathbf{2}) &\rightarrow u_H(\mathbf{3}, \mathbf{1}, -\frac{2}{3}) + d_H(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \nu_H(\mathbf{1}, \mathbf{1}, 0) + e_H(\mathbf{1}, \mathbf{1}, 1) \\
D(\mathbf{6}, \mathbf{1}, 1) &\rightarrow D_3(\mathbf{3}, \mathbf{1}, \frac{1}{3}) + \bar{D}_3(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}) \\
h(\mathbf{1}, \mathbf{2}, \mathbf{2}) &\rightarrow h^d(\mathbf{1}, \mathbf{2}, \frac{1}{2}) + h^u(\mathbf{1}, \mathbf{2}, -\frac{1}{2})
\end{aligned}$$

where the fields on the left appear with their quantum numbers under the PS gauge symmetry, while the fields on the right are shown with their quantum numbers under the SM symmetry.

The superpotential Eq. (10) leads to the following neutrino mass matrix [4]

$$\mathcal{M}_{\nu,\nu^c,\varphi} = \begin{pmatrix} 0 & m_u^{ij} & 0 \\ m_u^{ji} & 0 & M_{GUT} \\ 0 & M_{GUT} & \mu \end{pmatrix} \tag{11}$$

in the basis $(\nu_i, \nu^c_j, \varphi_k)$. Diagonalization of the above gives three light neutrinos with masses of the order $\mu(m_u^{ij}/M_{GUT})^2$ as required by the low energy data, and leaves right–handed majorana neutrinos with masses of the order M_{GUT}. Additional terms not included in Eq. (10) may be forbidden by imposing suitable discrete or continuous symmetries [11, 12] which, in fact, mimic the role of various $U(1)$ factors and string selection rules appearing in realistic string models. The sextet field $D(\mathbf{6},\mathbf{1},\mathbf{1})$ carries colour, while after the symmetry breaking it decomposes in a triplet/triplet–bar pair with the same quantum numbers of the down quarks. Now, the terms in Eq. (10) HHD and $\bar{H}\bar{H}D$ combine the uneaten (down quark–type) colour triplet parts of H, $\bar{H}$ with those of the sextet D into acceptable GUT–scale mass terms [4]. When the H fields attain their vevs at $M_{GUT} \sim 10^{16}$ GeV, the superpotential of Eq. (10) reduces to that of the MSSM augmented by right-handed neutrinos. Below M_{GUT} the part of the superpotential involving matter superfields is just

$$W = \lambda_U^{ij} Q_i {u^c}_j h_2 + \lambda_D^{ij} Q_i {d^c}_j h_1 n + \lambda_E^{ij} \ell_i {e^c}_j h_1 + \lambda_N^{ij} L_i \nu^c_j h_2 + \cdots \tag{12}$$

The Yukawa couplings in Eq. (12) satisfy the boundary conditions

$$\lambda_1^{ij}(M_{GUT}) \equiv \lambda_U^{ij}(M_{GUT}) = \lambda_D^{ij}(M_{GUT}) = \lambda_E^{ij}(M_{GUT}) = \lambda_N^{ij}(M_{GUT}). \tag{13}$$

Thus, Eq. (13) retains the successful relation $m_\tau = m_b$ at M_{GUT}. Moreover from the relation $\lambda_U^{ij}(M_{GUT}) = \lambda_N^{ij}(M_{GUT})$, and the fourth term in Eq. (10), through the see–saw mechanism we obtain light neutrino masses which satisfy the experimental limits. The $U(1)$ symmetries imposed by hand in this simple construction play the role of family symmetries $U(1)_A$, broken at a scale $M_A > M_{GUT}$ by the vevs of two $SU(4)\times O(4)$ singlets $\theta, \bar{\theta}$, carrying charge under the family symmetries and leading to operators of the form

$$O_{ij} \sim (F_i \bar{F}_j) h \left(\frac{H\bar{H}}{M^2}\right)^r \left(\frac{\theta^n \bar{\theta}^m}{M'^{n+m}}\right) + \mathrm{h.}c. \tag{14}$$

obtained from non–renormalizable (NR) contributions to the superpotential. Here, M' represents a high scale $M' > M_{GUT}$ which may be identified either with the $U(1)_A$ breaking scale M_A or with the string scale M_{string}. Such terms have the task of filling in the entries of fermion mass matrices, creating textures with a hierarchical mass spectrum and mixing effects between the fermion generations.

Before we proceed to the construction of a particular string model let us examine how a three–generation $SU(4) \times SU(2)_L \times SU(2)_R$ model can be realized. As we have already explained the fermion generations are accommodated in $F_L(\mathbf{4},\mathbf{2},\mathbf{1}) + \bar{F}_R(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ while the Higgs fields are accommodated in $F_R(\mathbf{4},\mathbf{1},\mathbf{2}) + \bar{F}_R(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ representations. In the free–fermionic formulation the $SU(2)_L \times SU(2)_R$ is realized as $O(4)$ and the $\mathbf{2}_L$ and $\mathbf{2}_R$ representations are the two spinor representations $(\mathbf{2}^\pm)$ of $O(4)$. Calling $n_+, n_-, \bar{n}_+, \bar{n}_-$ the number of $(\mathbf{4},\mathbf{2}^+), (\mathbf{4},\mathbf{2}^-)$,$(\bar{\mathbf{4}},\mathbf{2}^+)$ and $(\bar{\mathbf{4}},\mathbf{2}^-)$ representations we come to the conclusion that one minimal three generation model is obtained for

$$\vec{n}_{min} = (n_+, n_-, \bar{n}_+, \bar{n}_-) = (3, 1, 0, 4)$$

where $\mathbf{2}_L$ is identified with $\mathbf{2}^+$. A mirror minimal model can be obtained by interchanging the two $SU(2)$'s, i.e. $L \leftrightarrow R$, and identifying $\mathbf{2}^+ \sim \mathbf{2}_R$

$$\vec{n}_{min} = (n_+, n_-, \bar{n}_+, \bar{n}_-) = (1, 3, 4, 0).$$

Furthermore, one can consider the existence of vector–like states that do not affect the net number of generations since, in principle they can obtain superheavy masses. Thus a general three–generation $SU(4)\times SU(2)_L\times SU(2)_R$ model corresponds to one the following vectors

$$\vec{n}_{rs}=(n_+,n_-,\bar{n}_+,\bar{n}_-)=(3+r,1+s,r,4+s)\ ,\ r,s=0,1,\ldots$$

or

$$\vec{n}_{rs}=(n_+,n_-,\bar{n}_+,\bar{n}_-)=(1+s,3+r,4+s,r)\ ,\ r,s=0,1,\ldots$$

We can rewrite the above relations in a more compact form

$$\begin{aligned} n_++n_- &= \bar{n}_++\bar{n}_-=4+p,\ p=0,1,2,\ldots \\ n_+-\bar{n}_+ &= \bar{n}_--n_-=\pm 3 \end{aligned} \tag{15}$$

Thus, there exist an infinity of three–generation $SU(4)\times O(4)\sim SU(4)\times SU(2)_L\times SU(2)_R$ models each of them uniquely characterized by an integer (p) related to the differences (15) and a sign ($\pm$). We will therefore refer to a particular model using the notation $k^\pm$ that is the two minimal models will be referred as 0^+ and 0^-.

As stressed in the introduction, one severe problem that has to be resolved in a candidate string model is the discrepancy between the unification scale as this is found when the minimal supersymmetric spectrum is considered, and the two orders higher string scale implied by theoretical calculations. In previous works, it was shown that this difficulty may be overcome in several ways [13, 14]. In particular, the class of string models as that of Ref. [5] predict additional matter fields which can help the couplings merge at the high string scale without disturbing the low energy values of $\sin^2\theta_W$ and α_s. Perhaps the most elegant way to achieve this, is to make the couplings run closely from the string to the phenomenological unification scale $M_U\sim 10^{16}$GeV. As a first step one may add the mirror fields [14]

$$M=(\mathbf{4},\mathbf{2},\mathbf{1});\quad \bar{M}=(\bar{\mathbf{4}},\mathbf{2},\mathbf{1}) \tag{16}$$

which guarantee the equality of the $SU(2)_L$ and $SU(2)_R$ gauge couplings $g_L=g_R$ between the two scales. According to the classification proposed to the previous paragraph, this model is classified as 1^+ or 1^-. The running of the $SU(4)$ coupling can be adjusted by an additional number of extra colour sextets which are in general available in the string versions of the present model. Indeed, with three generations and denoting collectively the number of fourplet sets with n_4 the beta functions now become

$$b_{2L}\equiv b_{2R}=-1-4n_4;\quad b_4=6-n_6-4n_4 \tag{17}$$

which show that a sufficient number of sextet fields may guarantee a g_4 running almost identical with that of $g_{L,R}$. The string model we are proposing in the next section has exactly one mirror pair and four sextet fields, whereas additional exotic states may also contribute to the beta functions if they remain in the light spectrum.

The introduction of the mirror representations (16) leads to the existence of another symmetry in the model: we observe that the whole spectrum now is completely symmetric with respect to the two $SU(2)$'s in the sense that under the simultaneous change $\mathbf{2}_L \leftrightarrow \mathbf{2}_R$ and the $\mathbf{4} \leftrightarrow \bar{\mathbf{4}}$ of $SU(4)$, it remains invariant. More precisely, under this symmetry the representations of the model are mapped as follows

$$\begin{aligned}
\bar{F}_R &\leftrightarrow F_L \\
H, \bar{H} &\leftrightarrow \bar{M}, M \\
D, h, \phi_i &\leftrightarrow D, h, \phi_i
\end{aligned} \tag{18}$$

This symmetry persists also in the present string model, while tree–level as well as higher order Yukawa interactions are also invariant under these changes. As we will see, this symmetry is broken by the vacuum which will be determined by the specific solutions of the flatness conditions.

After the above short description, we are ready to present the string derived model where most of the above features appear naturally. In addition, novel predictions will emerge such as the appearance of exotic states with charges which are fractions of those of ordinary quarks and leptons, a hidden 'world' and a low energy $U(1)$ symmetry.

3 The String Model

In the four dimensional free–fermionic formulation of the heterotic superstring, fermionic degrees of freedom on the world sheet are introduced to cancel the conformal anomaly. The right–moving non–supersymmetric sector in the light–cone gauge contains the two transverse space–time bosonic coordinates $\bar{X}^\mu$ and 44 free fermions. The supersymmetric left moving sector, in addition to the space–time bosons X^μ and their fermionic superpartners ψ^μ includes also 18 real free fermions χ^I, y^I, ω^I $(I = 1, ..., 6)$ among which supersymmetry is non–linearly realized. The world–sheet supercurrent is

$$T_F = \psi^\mu \partial X_\mu + \sum_I \chi^I \, y^I \omega^I \tag{19}$$

Then, the theory is invariant under infinitesimal super–reparametrizations of the world–sheet as the conformal anomaly cancels separately in each sector. Each world–sheet fermion f_i is allowed to pick up a phase $\alpha_{f_i} \varepsilon (-1, 1]$ under parallel transport around a non–contractible loop of the world–sheet

$$f_i \to -e^{\imath \pi \alpha_{f_i}} f_i \tag{20}$$

A spin structure is then defined as a specific set of phases for all world–sheet fermions,

$$\alpha \;=\; [\alpha_{f_1^r}, \alpha_{f_2^r}, \ldots, \alpha_{f_k^r}; \alpha_{f_1^c}, \alpha_{f_2^c}, \ldots, \alpha_{f_l^c}] \tag{21}$$

where r stands for real, c for complex and $k + 2l = 64$. For real fermions the phases $\alpha_{f_i^r}$ have to be integers while α_{ψ^μ} is independent of the space–time index μ.

The partition function is then defined as a sum over a set of spin structures (Ξ)

$$Z(\tau) \;\propto\; \sum_{\alpha,\beta \in \Xi} c \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mathrm{Z} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \;, \tag{22}$$

where $Z\binom{\alpha}{\beta}$ is the contribution of the sector with boundary conditions α, β along the two non–contractible circles of the torus and $c\binom{\alpha}{\beta}$ a phase related to the GSO projection. Both Ξ and $c\binom{\alpha}{\beta}$ are subject to string constraints which guarantee the consistency of the theory.

A string model in the context of free fermionic formulation of the four–dimensional superstring is constructed by specifying a set of n basis vectors [1] $(b_0 = \mathbf{1}, b_1, b_2, \ldots, b_{n-1})$ of the form (21) (which generate $\Xi = \sum_i m_i b_i$) and a set of $\frac{n(n-1)}{2} + 1$ independent phases $c\binom{b_i}{b_j}$. Once a consistent set of basis vectors and a choice of projection coefficients is made, the gauge symmetry, the massless spectrum and the superpotential of the theory are completely determined. In particular, the massless states of a certain sector $\alpha = (\alpha_L; \alpha_R) \in \Xi$ are obtained by acting on the vacuum $|0\rangle_\alpha$ with the bosonic and fermionic mode operators. The massless states ($M_L^2 = M_R^2 = 0$) are found by the Virassoro mass formula

$$\begin{aligned} M_L^2 &= -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + \sum_f frequencies \\ M_R^2 &= -1 + \frac{\alpha_R \cdot \alpha_R}{8} + \sum_f frequencies \end{aligned}$$

where the sum is over the oscillator frequencies

$$\nu_f = \frac{1 + \alpha_{f_i}}{2} + \text{integer}, \quad \nu_{f^*} = \frac{1 - \alpha_{f_i}}{2} + \text{integer} \tag{23}$$

The physical states are obtained after the application of the GSO projections demanding

$$\left(e^{\imath \pi b_i F_\alpha} - \delta_\alpha c^* \begin{pmatrix} \alpha \\ b_i \end{pmatrix} \right) |\text{physical state}\rangle_\alpha = 0 \tag{24}$$

where $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α and $\delta_\alpha = +1$ when ψ^μ is antiperiodic. The operator $b_i F_\alpha$ is

$$b_i F_\alpha = \left(\sum_{f \in \text{left}} - \sum_{f \in \text{right}} \right) b_i(f) F_\alpha(f) \tag{25}$$

where $F_\alpha(f)$ is the fermion number operator counting each fermion mode f once and its complex conjugate f^* minus once. It should be remarked that in the sector where all the fermions are antiperiodic there is always a state $|\mu, \nu\rangle = \Psi^\mu_{-1/2} (\overline{\partial X})^\nu_{-1} |0\rangle_0$ which survives all projections and includes the graviton, the dilaton and the two–index antisymmetric tensor.

The present string model is defined in terms of nine basis vectors $\{S, b_1, b_2, b_3, b_4, b_5, b_6, \alpha, \zeta\}$ and a suitable choice of the GSO projection coefficient matrix. The resulting gauge group has a Pati–Salam ($SU(4) \times SU(2)_L \times SU(2)_R$) non–Abelian observable part, accompanied

[1]By **1** we denote the vector where all fermions are periodic.

by four Abelian ($U(1)$) factors and a hidden $SU(8) \times U(1)'$ symmetry. The nine basis vectors are the following

$$
\begin{array}{lcll}
\zeta & = & \{ & ; \;\; \bar{\Phi}^{1\ldots 8}\} \\
S & = & \{\psi^\mu, \;\; \chi^{1\ldots 6}, & ; \;\; \} \\
b_1 & = & \{\psi^\mu, \;\; \chi^{12}, (y\bar{y})^{3456}, & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^1\} \\
b_2 & = & \{\psi^\mu, \;\; \chi^{34}, (y\bar{y})^{12}, (\omega\bar{\omega})^{56} & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^2\} \\
b_3 & = & \{\psi^\mu, \;\; \chi^{56}, (\omega\bar{\omega})^{1234} & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^3\} \\
b_4 & = & \{\psi^\mu, \;\; \chi^{12}, (y\bar{y})^{36}, (\omega\bar{\omega})^{45} & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^1\} \\
b_5 & = & \{\psi^\mu, \;\; \chi^{34}, (y\bar{y})^{26}, (\omega\bar{\omega})^{15} & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^2\} \\
b_6 & = & \{ & ; \;\; \bar{\Psi}^{1\ldots 5}, \bar{\eta}^{123}, \bar{\Phi}^{1\ldots 4}\} \\
\alpha & = & \{ \;\; (y\bar{y})^{36}, (\omega\bar{\omega})^{36}\bar{\omega}^{24} & ; \;\; \bar{\Psi}^{123}, \bar{\eta}^{23}, \bar{\Phi}^{45}\}
\end{array}
\tag{26}
$$

The specific projection coefficients we are using are given in terms of the exponent coefficients c_{ij} in the following matrix

$$
c_{ij} = \begin{array}{c} \\ z \\ S \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ \alpha \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} z & S & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & \alpha \end{array} \\
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1
\end{pmatrix}
\end{array}
\tag{27}
$$

where the relation of c_{ij} with $c(b_i, b_j)$ is

$$
c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = e^{\imath\pi c_{ij}}
$$

All world–sheet fermions appearing in the vectors of the above basis are assumed to have *periodic* boundary conditions. Those not appearing in each vector are taken with *antiperiodic* ones. We follow the standard notation used in references [3, 5]. Thus, $\psi^\mu, \chi^{1\ldots 6}, (y/\omega)^{1\ldots 6}$ are real left, $(\bar{y}/\bar{\omega})^{1\ldots 6}$ are real right, and $\bar{\Psi}^{1\ldots 5}\bar{\eta}^{123}\bar{\Phi}^{1\ldots 8}$ are complex right world sheet fermions. In the above, $\mathbf{1} = b_1 + b_2 + b_3 + \zeta$ and the basis element S plays the role of the supersymmetry generator as it includes exactly eight left movers. Further, $b_{1,2,3}$ elements reduce the $N = 4$ supersymmetries into $N = 1$, while the initial $O(44)$ symmetry of the right–moving sector results to an observable $SO(10) \times SO(6)$ gauge group at this stage. The $SO(10)$ part corresponds to the five $\bar{\Psi}^{1\ldots 5}$ complex world sheet fermions while all chiral families at this stage belong to the **16** representation of the $SO(10)$. Vectors $b_{4,5}$ reduce further the symmetry of the left moving sector, while the introduction of the vector b_6 deals with the hidden part of the symmetry. Finally, the choice of the vector α determines the final gauge symmetry (observable and hidden sector) of the model which is

$$
SU(4) \times O(4) \times U(1)^4 \times \{U(1)' \times SU(8)\}_{hidden}. \tag{28}
$$

The observable gauge group consists of the non–Abelian $SO(6) \times O(4)$ symmetry which is isomorphic to the left–right Pati–Salam symmetry [8]. There are also four $U(1)_{i=1,\ldots,4}$ factors related to $\bar{\eta}_{1,2,3}$-complex and the $\bar{\omega}^{24}$-real pair of world–sheet fermions of the right–moving sector. All the superfields of the observable sector carry non–zero charges under these four $U(1)$ symmetries. Therefore, the latter are expected to play a very important role in the determination of the Yukawa couplings, the fermion mass textures, R-parity violation and in general in all types of Yukawa interactions of the model. We note here that the observable fields do not carry charges under $U(1)'$.

The Abelian part of the group deserves a separate treatment since this class of models in general possess $U(1)$ symmetries which are anomalous. Indeed, while we find two of the $U(1)$ factors to be traceless $\mathrm{Tr}U(1)_1 = \mathrm{Tr}U(1)' = 0$, the other three are traceful, with $\mathrm{Tr}U(1)_2 = \mathrm{Tr}U(1)_3 = \mathrm{Tr}U(1)_4 = 24$. However, the $U(1)$ charges can be defined in such a way that only one combination is anomalous. Indeed, the linear combination

$$U(1)_A = U(1)_2 + U(1)_3 + U(1)_4, \tag{29}$$

has $\mathrm{Tr}\tilde{U}(1)_A = 72$, while there are other three combinations orthogonal to the one above, which are free of gauge and gravitational anomalies. These are,

$$\begin{aligned} \tilde{U}(1)_1 &= U(1)_1 \\ \tilde{U}(1)_2 &= U(1)_2 - U(1)_3 \\ \tilde{U}(1)_2 &= U(1)_2 + U(1)_3 - 2U(1)_4. \end{aligned} \tag{30}$$

The choice of the projection coefficients shown in (27) has led to the desired three generation model as well as some refinements of the previously proposed theory [5] which are phenomenologically appealing and deserve some discussion. The most important are, the new Yukawa couplings which give fermions masses, the mirror symmetry of the massless spectrum and the number of $SU(4)$ Higgs multiplets.

We start with the enumeration of representations candidates for families and $SU(4) \times SU(2)_R$ breaking Higgs fields as they appear in Appendix A. We first note that due to the presence of the various $U(1)$-factors, there is an arbitrariness in the embedding of the electromagnetic charge operator. We will discuss this in detail in the end of this section, however, to start with, we assume first the simplest case where $U(1)_{em}$ is defined in the standard way, (as in the original PS–symmetry), i.e:

$$Y = \frac{1}{\sqrt{6}}T_4 + \frac{1}{2}T_L + \frac{1}{2}T_R \tag{31}$$

where T_6, T_L, T_R are the diagonal $SU(4), SU(2)_L$ and $SU(2)_R$ generators respectively. Then, the massless spectrum is classified with respect to its group properties as follows:

- There are three copies of $[(\mathbf{4},\mathbf{2},\mathbf{1}) + (\bar{\mathbf{4}},\mathbf{1},\mathbf{2})]$ representations, available to accommodate the three generations.

- There is one $[(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) + (\mathbf{4},\mathbf{1},\mathbf{2})]$ pair which is interpreted as the Higgs pair triggering the $SU(4) \times SU(2)_R$ breaking.

- One pair $[(\mathbf{4},\mathbf{2},\mathbf{1})+(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})]$, (mirror to each other) replaces the second Higgs pair of the old string version [5]. Clearly, since there are no mirror families observed in the light spectrum, they should decouple at some high scale by forming a heavy mass state.

- There are a large number of singlet fields with zero electric charge, while carrying quantum numbers under the four $U(1)$ factors. In the determination of the flat directions of the model, their vevs have to be chosen in such a way so as to cancel the D–term. These singlets couple to ordinary matter via superpotential terms. When they develop vevs along certain flat directions they may create a hierarchical fermion mass spectrum through non–renormalizable couplings.

- There are eight hidden $SU(8)$-octet and octet–bar superfields (charged under the $U(1)^4 \times U(1)'$), which are also neutral under the usual charge definition. They can also acquire non–zero vevs leading to additional mass terms for ordinary or exotic matter fields.

- The exotic states of the model fall into two categories:
  i) There are two $SU(4)$ fourplets $H_4 = (\mathbf{4},\mathbf{1},\mathbf{1})$, $\bar{H}_4 = (\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$. After the symmetry breaking, they result to a $\mathbf{3}$ and $\bar{\mathbf{3}}$ pair with charges $\pm\frac{1}{6}$ respectively and two singlets with charges $\pm\frac{1}{2}$.
  ii) The second kind of exotic fields includes ten left–handed doublets X_{iL} and an equal number of right–handed ones X_{iR} with charges $\pm\frac{1}{2}$. The presence of exotic particles in the massless spectrum of the theory is a generic feature of level $k=1$ string constructions. Such states in general, are regarded as string models' "Achilles heel" since it is likely that they remain in the light spectrum down to the electroweak scale. There are mainly three solutions to this problem : first, one may choose a suitable flat direction where all of them become massive at a relatively large scale; as a second possibility, one can properly modify the string boundary conditions on the basis vectors, so that these states appear with non–trivial transformation properties under a hidden non–Abelian group. In this latter case the exotic states are confined at the scale where the gauge coupling of the hidden group becomes strong [15]. Clearly, for a given number of matter representations, the higher the rank of the group, the larger the confinement scale. As a third possibility, one may consider the modification of the charge operator (31) by the inclusion of additional $U(1)$ factors. In the present construction, we will discuss in some detail the last two possibilities. Later, we will give a brief account for their possible relevance on recent cosmological observations.

Let us point out here that the observable spectrum of the model respects the symmetry discussed in Section 2 with respect to the simultaneous interchanges $\mathbf{2}_L \leftrightarrow \mathbf{2}_R$ and $\mathbf{4} \leftrightarrow \bar{\mathbf{4}}$. In particular, left handed generation superfields are interchanged with right handed ones, while there is a similar change of roles of the $SU(4)$ higgs and mirrors. We will also see in the following sections that the tree–level and higher order Yukawa interactions remain also unaltered under the above interchanges. The above symmetry is broken however, by the vacuum when consistent $F-$ and D–flatness solutions are found. We will discuss this when the superpotential of the theory is presented and the corresponding flatness conditions are derived in the next sections.

4 Symmetry breaking and hypercharge embedding in the string model.

We will discuss now two related issues, the gauge symmetry breaking pattern and the various consistent embeddings of the weak hypercharge. After defining the consistent set of boundary conditions (26) described previously, one is left with an effective theory based on the symmetry (28) with the following general characteristics. There is an effective unification scale, namely the string scale M_{string}, where all couplings – up to threshold corrections – attain a common value. At this point one is left with an effective $N = 1$ supergravity theory while the gauge group structure is of the form $G = \prod_n G_n$, containing an 'observable' and a 'hidden' part. The two worlds are not completely decoupled. Hidden and observable fields are charged under five Abelian factors. The first symmetry breaking occurs when some of the singlet fields acquire vevs to cancel the D–term. Depending of the choice of the singlet vevs several (at most four) of the above $U(1)$'s break, the natural breaking scale M_A being of the order of the $D-$term, i.e,

$$\begin{aligned} M_A &\sim \sqrt{\xi} \\ &= \sqrt{\frac{TrQ_A}{192}\frac{g_{string}}{\pi}}M_{Pl} = \frac{\sqrt{3}}{2\pi}g_{string}M_{Pl} \end{aligned} \tag{32}$$

where $TrQ_A = 72$ is the trace of the anomalous $U(1)_A$ and $M_{Pl} \approx 4.2 \times 10^{18}$GeV is the reduced Planck mass. We note here that, if only the singlets were allowed to obtain a non–vanishing vev, at most four of the $U(1)$'s break; no singlet is charged under the Abelian symmetry $U(1)'$ which remains unbroken at this stage. The observable part $SU(4) \times SU(2)_L \times SU(2)_R$ has a rank larger than that of the MSSM symmetry, which breaks down to the SM–gauge group at an intermediate scale M_{GUT}, usually some two orders of magnitude below the string scale. The breaking occurs in the way described in Section 2. The necessity of the $SU(4) \times SU(2)_R$ symmetry breaking together with the $D-$ and F–flatness conditions require at least two of the $U(1)$ factors to break at a high scale.

There is finally the hidden $SU(8)$ part. This symmetry stays intact, as long as the $\mathbf{8}$ and $\overline{\mathbf{8}}$ fields do not acquire vevs. Note also that the octets are charged under $U(1)'$. In many flat directions which will be discussed subsequently, phenomenological requirements force some of the octets to obtain vevs and the symmetry $SU(8)\times U(1)'$ breaks to a smaller one. Now, a crucial observation (see the relevant Table in the Appendix A) is that all $\mathbf{8}$'s come with $U(1)'$ positive charge $(+1)$ whilst all $\overline{\mathbf{8}}$'s appear with the opposite (-1) charge. It is easy to show then, that, no matter how many of the available octet fields receive a non–zero vev, there is always a $U(1)''$ unbroken which is a linear combination of the $U(1)'$ and one of the generators of the $SU(8)$. Therefore, the hidden matter conserves a new $U(1)''$ symmetry which stays unbroken down to low energies. Its cosmological implications will be discussed in a subsequent section.

ω	$\sin^2\theta_W$
0	$\frac{3}{8}$
1	$\frac{3}{22}$

Table 1: The values of the weak mixing angle at the Unification scale, for two definitions of the weak hypercharge. In the second case, an additional $U(1)$-factor is assumed.

We turn now our discussion to the hypercharge embedding. As mentioned above, due to the appearance of extra $U(1)$–factors, the hypercharge generator is not uniquely determined. It can be any linear combination of the $U(1)_{B-L}$ the five available $U(1)$'s of the model and possible unbroken generators of the hidden gauge symmetry, provided the minimal supersymmetric low energy particle spectrum is generated. The standard weak hypercharge assignment – as this is defined in the original PS–symmetry – does not involve any of the surplus $U(1)$ factors discussed above. It is solely determined in the usual sense from the diagonal generators of the PS–symmetry as in (31). Under this definition in addition to the representations accommodating the MSSM–fields, the states found in $(\mathbf{4},\mathbf{1},\mathbf{1})+(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{2},\mathbf{1})$ representations obtain the exotic charges discussed above, while they are rather unusual in the grand unified models.

We will discuss here in some detail another possible definition of the hypercharge operator which is obtained by including the $U(1)'$-generator:

$$Y' = \frac{1}{\sqrt{6}}T_4 + \frac{1}{2}T_L + \frac{1}{2}T_R - \frac{\omega}{2}Q' \tag{33}$$

where Q' is related to the $U(1)'$ charge of the particular massless state and ω is the appropriate normalization constant. Choosing for example $\omega = 1$, all extra doublets $X_{L,R}$ obtain integral charges $(\pm 1, 0)$. On the other hand, this new embedding leads to the normalization of the hypercharge generator

$$k = \frac{5+12\omega^2}{3}. \tag{34}$$

The value of the weak mixing angle at M_{string} is $\sin^2\theta_W(M_{\text{string}}) = \frac{1}{1+k}$. Its values for the two lower ω's are given in Table 1. For $\omega = 0$, we obtain the standard GUT $\sin^2\theta_W$ prediction but the exotic states have fractional charges, whereas for $\omega = 1$ the $X_{L,R}$ doublets as well as the $(\mathbf{4},\mathbf{1},\mathbf{1})$ and $(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$ representations, obtain charges like those of the ordinary down quarks and leptons. It should not escape our attention that in this new hypercharge definition Y' the octet fields now appear with fractional charges $\pm 1/2$. This is not however a real problem. The coupling of the $SU(8)$ group becomes strong at a high scale, leading to a confinement (in close analogy with QCD), and forcing the octets to form bound states with the corresponding octet–bar fields. We should note here that this situation opens up the possibility of giving vevs to these condensates at a smaller scale and create new mass terms for the ordinary matter through their superpotential couplings. The new hypercharge definition predicts a low value for the weak mixing angle which is essentially the value obtained in a Kac–Moody level $k = 2$ string construction [9]. Starting however, from such a small initial value for $\sin^2\theta_W$ at M_{string}, there is no obvious way how the larger low energy value can be obtained in this case.

Let us close this section with a short comment on one more possibility of symmetry breaking. One can give vevs directly to the exotic $SU(2)_R$-doublet fields X_{iR}. (Both their components are charged ($Q_{em} = \pm 1/2$) under the standard hypercharge assignment (31)). The vev should be taken along the neutral direction defined by the proper linear combination. This will essentially lead to a definition of the hypercharge as in the case of $\omega = 1$. However, this approach has the advantage that the small initial value of $\sin^2\theta_W$ is defined now at the $SU(2)_R$-breaking scale which can be taken to be much lower that the string scale. This case requires a separate treatment since there are new fields entering the flatness conditions while new mass terms appear in the superpotential. Moreover, exotic doublets now look like the ordinary electron doublets while far reaching phenomenological implications appear.

5 The Superpotential of the string version.

We proceed now to the calculation of the perturbative superpotential. Clearly, the number of fields in the string version is significantly larger than those of its surrogate GUT discussed in Section 2. In fact, in the model of Section 2, the construction of the superpotential was rather easy since only gauge symmetries had to be respected. Here, however, not all gauge invariant terms are allowed; additional restrictions from world–sheet symmetries have to be taken into account since they eliminate a large portion of the gauge invariant superpotential terms. A short description of the calculation [16, 17] of the renormalizable as well as the non–renormalizable superpotential terms is given in the Appendix B.

The tree–level superpotential is

$$
\begin{aligned}
\frac{W_3}{g\sqrt{2}} \;=\;& \\
& \overline{F}_{5R}F_{4L}h_4 + \overline{F}_{3R}F_{3L}h_2 + \frac{1}{\sqrt{2}}\overline{F}_{5L}F_{4L}\zeta_2 + \frac{1}{\sqrt{2}}\overline{F}_{5R}F_{4R}\overline{\zeta}_3 + \\
& + \overline{F}_{5L}\overline{F}_{5L}D_4 + F_{4L}F_{4L}D_1 + F_{2L}F_{2L}D_2 + F_{1L}F_{1L}D_1 + F_{1L}\overline{H}_4X_{7L} + \\
& + \overline{F}_{1R}\overline{F}_{1R}D_1 + F_{4R}F_{4R}D_3 + \overline{F}_{2R}\overline{F}_{2R}D_2 + \overline{F}_{5R}\overline{F}_{5R}D_2 + \overline{F}_{2R}H_4X_{3R} + \\
& + \frac{1}{2}\Phi_2\left(\zeta_i\overline{\zeta}_i + \xi_i\overline{\xi}_i + h_3h_4 + \overline{H}_4H_4\right) \\
& + \Phi_4\left(\zeta_1\overline{\zeta}_3 + \zeta_3\overline{\zeta}_1\right) + \Phi_5\left(\zeta_2\overline{\zeta}_4 + \zeta_4\overline{\zeta}_2\right) \\
& + D_1D_2\Phi_{12} + D_1D_4\Phi^-_{12} + D_2D_3\overline{\Phi}^-_{12} + D_3D_4\overline{\Phi}_{12} \\
& + h_2\overline{\xi}_1h_4 + h_2\overline{\xi}_4h_3 + h_2X_{10L}X_{10R} + h_1\xi_1h_3 + h_1\xi_4h_4 + h_1X_{9L}X_{9R} \\
& + \overline{\Phi}_{12}\left(\xi_4\overline{\xi}_1 + h_3h_3 + Z_3\overline{Z}_3\right) + \overline{\Phi}^-_{12}\left(\overline{\zeta}_i\overline{\zeta}_i + \xi_3\overline{\xi}_2 + X_{9R}X_{10R}\right) \\
& + \Phi^-_{12}\left(\zeta_i\zeta_i + \xi_2\overline{\xi}_3 + X_{9L}X_{10L}\right) + \Phi_{12}\xi_1\overline{\xi}_4 + \Phi_{12}h_4h_4 + \\
& + \frac{1}{\sqrt{2}}\left(\zeta_1X_{1R}X_{6R} + \zeta_3Z_4\overline{Z}_5 + \zeta_4X_{2R}X_{5R} + \overline{\zeta}_2Z_5\overline{Z}_4 + \overline{\zeta}_4X_{1L}X_{6L} + \overline{\zeta}_1X_{2L}X_{5L}\right)
\end{aligned}
$$

$$+ \xi_1 Z_5 \overline{Z}_5 + \xi_2 X_{2R} X_{6R} + \overline{\xi}_2 X_{1L} X_{5L} + h_3 X_{2R} X_{5L} + h_4 X_{1L} X_{6R} \ . \qquad (35)$$

The fourth order superpotential terms are

$$\begin{aligned} w_4 \ = \ & \overline{F}_{5L} F_{4L} X_{1L} X_{6L} + \overline{F}_{5L} F_{3L} Z_3 \overline{Z}_4 + \overline{F}_{5L} F_{1L} X_{4L} X_{6L} + \overline{F}_{5R} F_{4R} X_{1R} X_{6R} + \\ & F_{4R} \overline{F}_{3R} \overline{Z}_3 Z_4 + F_{4R} \overline{F}_{2R} X_{1R} X_{8R} + \overline{F}_{2R} F_{2L} \overline{\zeta}_4 h_4 + \overline{F}_{1R} F_{1L} \zeta_1 h_4 + \\ & \overline{\zeta}_1 \xi_1 \overline{Z}_1 Z_1 + \zeta_2 h_4 X_{8L} X_{8R} + \zeta_2 \xi_3 X_{7R} X_{8R} + \zeta_2 \overline{\xi}_2 X_{7L} X_{8L} + \\ & \overline{\zeta}_2 h_3 X_{7L} X_{7R} + \overline{\zeta}_2 \xi_2 X_{7R} X_{8R} + \overline{\zeta}_2 \overline{\xi}_3 X_{7L} X_{8L} + \zeta_3 h_3 X_{3L} X_{3R} + \\ & \zeta_3 \xi_3 X_{3R} X_{4R} + \zeta_3 \overline{\xi}_2 X_{3L} X_{4L} + \overline{\zeta}_3 h_4 X_{4L} X_{4R} + \overline{\zeta}_3 \xi_2 X_{3R} X_{4R} + \\ & \overline{\zeta}_3 \overline{\xi}_3 X_{3L} X_{4L} + \zeta_4 \xi_1 \overline{Z}_2 Z_2 + Z_1 \overline{Z}_4 X_{3R} X_{5R} + \overline{Z}_2 Z_4 X_{2L} X_{7L} + \\ & Z_4 \overline{Z}_5 X_{2L} X_{5L} + Z_5 \overline{Z}_4 X_{2R} X_{5R} + X_{3L} X_{4R} X_{10L} X_{9R} + X_{4L} X_{3R} X_{9L} X_{10R} + \\ & X_{1L} X_{2R} X_{9L} X_{10R} + X_{2L} X_{1R} X_{9L} X_{10R} + X_{5L} X_{6R} X_{10L} X_{9R} + \\ & X_{6L} X_{5R} X_{10L} X_{9R} + X_{7L} X_{8R} X_{10L} X_{9R} + X_{8L} X_{7R} X_{9L} X_{10R} \ , \end{aligned} \qquad (36)$$

where in each terms an $\mathcal{O}(1) g/M_{Pl}$ multiplicative factor. Higher order terms up to sixth order have been also calculated and are presented in the Appendix B.

Having obtained the spectrum of the model, as well as the available superpotential terms, we need to determine the vacuum of our theory, by making an appropriate choice of the vacuum expectation values of the Higgs fields (fourplets, bidoublets and a sufficient number of singlets) and possibly some of the hidden $SU(8)$ multiplets. All these choices should be consistent with the $D-$ and $F-$ flatness conditions. A complete account of all possible solutions of these conditions will be given subsequently, however, not all of those solutions are satisfactory from the phenomenological point of view. A final conclusion about the viability of a certain flat direction however cannot be drawn before adequately high order NR–terms are taken into account. There are two main reasons for this: first, it is possible that a particular viable flat direction at a certain order, is destroyed when higher order NR–terms are included in the calculation. Second, even if a phenomenologically promising flat direction can be proven to persist at higher orders, it is possible that the new NR superpotential terms create undesired mass terms. For example, a usual phenomenon is that they fill in many entries in the Higgs mass matrix, so it is possible that there is no massless higgs left to break the symmetry. As a consequence, one may have further constraints on the particular flat direction by forcing some additional fields to obtain a zero vev. This will be discussed in a subsequent section.

From the above remarks, it is evident that the right choice of the vacuum of the model is not an easy task. In the next section our endeavor will be concentrated in the classification of all flat directions and their relevance to the low energy phenomenological expectations. It is useful therefore, in order to pin down the few promising vacua from the hundreds of available solutions, to summarize the basic observations which will help us to complete this task. This will enable us to determine the right flat direction and choose those singlet (and possibly hidden) field vevs that guarantee a successful description of the low energy phenomenological theory.

- We start with the Higgs mechanism; we first observe that there is only one pair of Higgs fields available to break $SU(4)$ symmetry, namely the fourplet F_{4R} and in general one linear combination of the fields $\bar{F}_{1,2,3,5}$. Thus, in order to keep F_{4R} massless and prevent a mass term through the tree–level coupling $F_{4R}\bar{F}_{5R}\bar{\zeta}_3$, we need to impose $\langle\bar{\zeta}_3\rangle = 0$.

- In addition to the three generations expected to appear at low energies, the model predicts also the existence of one additional $(\mathbf{4},\mathbf{2},\mathbf{1})$ representation plus its mirror $\bar{F}_{5L} = (\bar{4},2,1)$. Since no mirror families appear in the low energy spectrum, we need a mass term of the form $\langle\chi_i\rangle F_{iL}\bar{F}_{5L}$ (where χ_i some of the singlets with non–zero vevs) to give a heavy mass to the mirror $\bar{F}_{5L}$. A candidate term could be $\zeta_2\bar{F}_{5L}F_{4L}$ which exists already at the tree–level. At fourth order there is also the term $\bar{F}_{5L}F_{3L}Z_3\bar{Z}_5$. Thus, up to fourth order, we obtain

$$\sum_i \langle\chi_i\rangle F_{iL}\bar{F}_{5L} = (\zeta_2 F_{4L} + F_{3L}Z_3\bar{Z}_4 + \cdots)\bar{F}_{5L} \tag{37}$$

where $\{\cdots\}$ stand for possible higher order NR–terms involving fields that may acquire vevs. Clearly, if we wish to make the mirror multiplets heavy with superpotential terms up to fourth order, we should demand from flatness conditions either $\zeta_2 \neq 0$, or $Z_3\bar{Z}_5 \neq 0$. Solutions with higher order NR–terms are also possible as it will be clear later.

- A number of sextet fields, D_i, $i = 1, ..., 4$ containing colour triplets as well as triplets surviving the Higgs mechanism appear also in the spectrum. In order to avoid possible proton decay problems we need also mass terms for those coloured fields. As far as the sextet fields are concerned, the sextet matrix at tree–level is

$$D_1D_2\Phi_{12} + D_1D_4\Phi_{12}^- + D_2D_3\overline{\Phi}_{12}^- + D_3D_4\bar{\Phi}_{12} \tag{38}$$

Their eigenmasses in terms of the scalar components of the singlet fields are

$$m_{D_i} \;=\; \pm\frac{1}{2}\left(\Sigma_{\Phi^2} \pm \sqrt{(\Sigma_{\Phi^2})^2 - (\Phi_{12}\bar{\Phi}_{12} - \Phi_{12}^-\bar{\Phi}_{12}^-)^2}\right) \tag{39}$$

with

$$\Sigma_{\Phi^2} \;=\; \Phi_{12}^2 + \bar{\Phi}_{12}^2 + {\Phi_{12}^-}^2 + {\overline{\Phi}_{12}^-}^2 \tag{40}$$

The above eigenvalues are all non–zero whenever the condition

$$\Phi_{12}\bar{\Phi}_{12} - \Phi_{12}^-\bar{\Phi}_{12}^- \neq 0$$

is fulfilled. Therefore, a satisfactory flat direction should keep the appropriate singlets with non–zero vevs. It will be clear later that in most of the phenomenologically viable string vacua, higher order NR–contributions will prove necessary to make all sextet fields massive.

In forming the mass matrices for triplets, we should also take into account the fact that there are also coloured triplets in the Higgs pair $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) + (\mathbf{4},\mathbf{1},\mathbf{2})$. Thus, recalling in mind the sextet decomposition $D_4 \to D_4^{\bar{3}} + D_4^3$, the term $F_{4R}F_{4R}D_4$ gives a heavy mass to $\bar{d}^c_{4R}D_4^{\bar{3}}$ and the terms

$$\bar{F}_{5R}\bar{F}_{5R}D_2 + \bar{F}_{2R}\bar{F}_{2R}D_2 + \bar{F}_{1R}\bar{F}_{1R}D_1$$

make another $\bar{3} - 3$ combination massive. This linear combination depends on the choice of the fields which are going to accommodate the families and higgses. This will be precisely determined as long as a specific flat direction is chosen.

- There is a new interesting feature of this version; There are two candidate terms at fourth order to provide masses to the second generation

$$\bar{F}_{1R}F_{1L}h_4\zeta_1 + \bar{F}_{2R}F_{2L}h_4\bar{\zeta}_4 \tag{41}$$

They are expected to be of the right order, provided that at least one of the singlets $\zeta_1, \bar{\zeta}_4$ gets a non–zero vev.

After this preliminary analysis, we are ready now to explore other important aspects of the model. In the next section we will find all tree–level and higher order solutions to the flatness conditions which determine the consistent string vacua.

6 The solutions of the F– and D–flatness conditions

One of the main concerns in constructing effective supersymmetric models from superstrings, is to find the flat directions along which the scalar potential vanishes. String models in general contain several flat directions which are lifted by higher order corrections to the superpotential and supersymmetry breaking effects. The latter set also the scale of the scalar potential. Another interesting fact in string model building concerning these flat directions, is the existence of a D–term contribution [18, 19]. As has been discussed in the previous section, there is a linear combination of the four surplus $U(1)$ factors accompanying the observable gauge group of the model, which remains anomalous. The standard anomaly cancellation mechanism [19] results to a shift of the vacuum where several scalar components of the singlet (and possibly hidden) superfields develop non–zero vevs. Their magnitude is determined by the solution(s) of the complete system of the F- and D-flatness constraints.

Derivation of the flatness constraints

The F-flatness equations are easily derived from the superpotential. They are the set of equations resulting from the differentiation of $\mathcal{W}$ with respect to the fields of the massless spectrum f_i,

$$\frac{\partial}{\partial f_i}\mathcal{W} = 0$$

In this paper we will mainly concentrate on an analysis of the flatness conditions involving fields only from the observable sector. For completeness, we also give in the Appendix B the relevant contributions to the flatness conditions taking into account hidden field vevs as well as higher order corrections from NR–terms.

Taking the derivatives of the renormalizable superpotential $\mathcal{W}$ with respect to the observable fields, we obtain

$$\Phi_2 : \quad \zeta_i\bar{\zeta}_i = -\xi_i\bar{\xi}_i \tag{42}$$
$$\Phi_4 : \quad \zeta_1\bar{\zeta}_3 = -\bar{\zeta}_1\zeta_3 \tag{43}$$
$$\Phi_5 : \quad \zeta_2\bar{\zeta}_4 = -\bar{\zeta}_2\zeta_4 \tag{44}$$
$$\Phi_{12} : \quad \xi_1\bar{\xi}_4 = 0 \tag{45}$$
$$\bar{\Phi}_{12} : \quad \xi_4\bar{\xi}_1 = 0 \tag{46}$$
$$\Phi_{12}^- : \quad \zeta_i\zeta_i = -\xi_2\bar{\xi}_3 \tag{47}$$
$$\bar{\Phi}_{12}^- : \quad \bar{\zeta}_i\bar{\zeta}_i = -\xi_3\bar{\xi}_2 \tag{48}$$
$$\xi_1 : \quad \phi_2\bar{\xi}_1 = -\Phi_{12}\bar{\xi}_4 \tag{49}$$
$$\bar{\xi}_1 : \quad \phi_2\xi_1 = -\bar{\Phi}_{12}\xi_4 \tag{50}$$
$$\xi_2 : \quad \phi_2\bar{\xi}_2 = -\Phi_{12}^-\bar{\xi}_3 \tag{51}$$
$$\bar{\xi}_2 : \quad \phi_2\xi_2 = -\bar{\Phi}_{12}^-\xi_3 \tag{52}$$
$$\xi_3 : \quad \phi_2\bar{\xi}_3 = -\bar{\Phi}_{12}^-\bar{\xi}_2 \tag{53}$$
$$\bar{\xi}_3 : \quad \phi_2\xi_3 = -\Phi_{12}^-\xi_2 \tag{54}$$
$$\xi_4 : \quad \phi_2\bar{\xi}_4 = -\bar{\Phi}_{12}\bar{\xi}_1 \tag{55}$$
$$\bar{\xi}_4 : \quad \phi_2\xi_4 = -\Phi_{12}\xi_1 \tag{56}$$
$$\zeta_1 : \quad \phi_2\bar{\zeta}_1 + \Phi_4\bar{\zeta}_3 + 2\Phi_{12}^-\zeta_1 = 0 \tag{57}$$
$$\bar{\zeta}_1 : \quad \phi_2\zeta_1 + \Phi_4\zeta_3 + 2\bar{\Phi}_{12}^-\bar{\zeta}_1 = 0 \tag{58}$$
$$\zeta_2 : \quad \phi_2\bar{\zeta}_2 + \Phi_5\bar{\zeta}_4 + 2\Phi_{12}^-\zeta_2 + \bar{F}_{5L}F_{4L}/\sqrt{2} = 0 \tag{59}$$
$$\bar{\zeta}_2 : \quad \phi_2\zeta_2 + \Phi_5\zeta_4 + 2\bar{\Phi}_{12}^-\bar{\zeta}_2 = 0 \tag{60}$$
$$\zeta_3 : \quad \phi_2\bar{\zeta}_3 + \Phi_4\bar{\zeta}_1 + 2\Phi_{12}^-\zeta_3 = 0 \tag{61}$$
$$\bar{\zeta}_3 : \quad \phi_2\zeta_3 + \Phi_4\zeta_1 + 2\bar{\Phi}_{12}^-\bar{\zeta}_3 + \bar{F}_{5R}F_{4R}/\sqrt{2} = 0 \tag{62}$$
$$\zeta_4 : \quad \phi_2\bar{\zeta}_4 + \Phi_5\bar{\zeta}_2 + 2\Phi_{12}^-\zeta_4 = 0 \tag{63}$$
$$\bar{\zeta}_4 : \quad \phi_2\zeta_4 + \Phi_5\zeta_2 + 2\bar{\Phi}_{12}^-\bar{\zeta}_4 = 0 \tag{64}$$

On the left of the above equations we show the field with respect to which the superpotential is differentiated . In equations (59,62) both $SU(2)_L$ and $SU(2)_R$ fourplet fields have been included to exhibit the invariance of the equations under a straightforward generalization of the transformations (19). Indeed, it can be observed now that the F–flatness equations as well as the Yukawa interactions remain unaltered under the interchanges mentioned in previous sections. In particular, when $\bar{\mathbf{4}}$ and $\mathbf{2}_R$ are interchanged with $\mathbf{4}$ and $\mathbf{2}_L$ respectively, it can be seen that the superpotential remains invariant under the following renaming of the fields

$$F_5, \bar{F}_5 \leftrightarrow F_4, \bar{F}_4 \quad F_1, \bar{F}_1 \leftrightarrow F_2, \bar{F}_2 \quad D_1, D_3 \leftrightarrow D_2, D_4 \tag{65}$$
$$\zeta_1, \zeta_2 \leftrightarrow \bar{\zeta}_4, \bar{\zeta}_3 \quad \bar{\zeta}_1, \bar{\zeta}_2 \leftrightarrow \zeta_4, \zeta_3 \quad \Phi_1, \Phi_4 \leftrightarrow \Phi_3, \Phi_5 \tag{66}$$
$$\xi_2, \xi_3 \leftrightarrow \bar{\xi}_2, \bar{\xi}_3 \quad \Phi_{12}^- \leftrightarrow \bar{\Phi}_{12}^- \quad \Phi_2 \leftrightarrow \Phi_2 \tag{67}$$

This symmetry is also preserved by higher order Yukawa terms as can be easily checked from the terms presented in the Appendix. Nevertheless, the D–equations are not invariant. Clearly, any solution of them defines a vacuum which does not preserve the symmetry. In the subsequent, we make a definite choice with regard to this symmetry putting all $\langle F_{iL}\rangle = 0$.

The D–flatness equations for anomalous or non–anomalous $U(1)$ factors with hidden field contributions are also derived in the Appendix C. In the absence of non–zero hidden field vevs, they can be written in the following compact form

$$(\mathcal{D}_1): \quad f_3 = x_2 + x_3 \tag{68}$$

$$(\mathcal{D}_2): \quad f_4 - f_1 = x_2 + \frac{\zeta}{2} - \bar{\phi} \tag{69}$$

$$(\mathcal{D}_3): \quad \textstyle\sum_i x_i = -\frac{\xi}{3} \tag{70}$$

$$(\mathcal{D}_4): \quad \phi = x_1 + \frac{\xi}{2} \tag{71}$$

$$(\mathcal{D}_5): \quad f_4 - f_1 = f_2 + f_3 + f_5 \tag{72}$$

where, we have defined,

$$x_i = |\bar{\xi}_i|^2 - |\xi_i|^2; \qquad \zeta = \textstyle\sum_i (|\bar{\zeta}_i|^2 - |\zeta_i|^2) \tag{73}$$

$$\phi = |\bar{\Phi}_{12}|^2 - |\Phi_{12}|^2; \qquad \bar{\phi} = |\bar{\Phi}^-_{12}|^2 - |\Phi^-_{12}|^2 \tag{74}$$

$$f_4 = \frac{1}{2}|F_{4R}|^2; \quad f_i = \tfrac{1}{2}|\bar{F}_{iR}|^2, i = 1, 2, 3, 5 \tag{75}$$

Tree and higher level solutions of $F-$ and D–flatness constraints

A consistent phenomenological analysis of the model requires a complete knowledge of all vacua, therefore, a systematic approach to classify all $D-$ and F–flat directions is needed. When this is done, we will be able to know which fields acquire non–zero vevs in any specific flat direction[2]. These vevs will determine completely the masses of fermion and scalar fields through their superpotential couplings.

In the remaining of this section we present a systematic analysis of the above constraints, taking into account basic phenomenological requirements. This will limit considerably the number of possible solutions. Thus, for example, as already has been pointed out, it is necessary to impose the constraint $\langle\bar{\zeta}_3\rangle = 0$, in order to prevent a mass term for the Higgs field F_{4R} at tree–level. This, by no means ensures the existence of a consistent solution. We postpone the complete presentation after we obtain the set of mathematically consistent cases. In the present paper we restrict the analysis of the flatness conditions in the case where only observable fields acquire non–zero vevs. Solutions with hidden field vevs are much more involved and may result to interesting new vacua. Although a detailed investigation of the latter will not considered here, a brief discussion of their role is given later in this paper.

[2]For similar systematic analyzes in other models, see [20, 21, 22, 23].

We find it convenient to start our analysis from the F–flatness conditions (44,45) which imply four distinct cases:

$$\begin{array}{ll} (i) \quad \bar{\xi}_1 = \xi_1 = 0 \ , \ \xi_4 \neq 0; & (ii) \ \xi_1 = \xi_4 = 0; \\ (iii) \ \xi_4 = \bar{\xi}_1 = 0; & (iv) \ \bar{\xi}_4 = \bar{\xi}_1 = 0 \ , \xi_1, \xi_4 \neq 0 \ . \end{array}$$

From the above cases, only (iii, iv) have consistent solutions. Let's first explain why cases (i) and (ii) are rejected.

Cases (i) and (ii)

From Eqs. (55,56) we deduce $\phi_2 = 0$, while $\xi_4 \neq 0$ in Eq. (50) imposes $\bar{\Phi}_{12} = 0$. This however leads to inconsistency with equation $\mathcal{D}_4$, since the left side of the equation is negative while the right side is positive and non–zero. Similarly case (ii) where $\xi_1 = \xi_4 = 0$, is not soluble as can be easily seen from the equation $(\mathcal{D}_3)$+$(\mathcal{D}_1)$. We consider the two remaining cases (iii) and (iv)separately.

Case (iii)

From Eq. (50), we find $\phi_2 = 0$. Further, $\mathcal{D}_4$–flatness tells us that ξ_1 and $\bar{\Phi}_{12}$ cannot be simultaneously zero. Then, combining this with conditions (54) and (55) we conclude that

$$\Phi_{12} = 0, \ \bar{\xi}_4 = \xi_4 = 0, \ \xi_1 \neq 0, \ \phi_2 = 0 \tag{76}$$

while $\bar{\xi}_1 \cdot \bar{\Phi}_{12} = 0$.

Proceeding further, we classify all solutions in this case according to their number of free parameters and fields with zero vevs. At the tree–level, there are 17 solutions consistent with the $F-$ and D–flatness conditions. These are cases 1–17 of Table 9 of the Appendix D. Several of these flat directions are lifted when higher order NR–terms are included. On the other hand, other tree–level flat directions remain flat when additional constraints have are imposed on the field vevs. There are cases where a single tree–level flat direction results to more than one distinct cases at a higher level since the solution of the constraints may be satisfied for various choices of field vevs.

When NR contributions to flatness conditions up 6^{th} order are taken into account, the above tree–level solutions reduce to the first thirteen cases presented in Table 2. The first column numbers the solutions, while the last one denotes the number of free (complex) parameters left. The five columns in the middle show the fields with zero vevs, where for presentation purposes abbreviations in the field notation have been used. Thus, in the second row, the numbers $12, \overline{12}, 12^-, \overline{12}^-$, denote the fields $\Phi_{12}, \overline{\Phi}_{12}, \Phi_{12}^-, \overline{\Phi}_{12}^-$, and so on. The fields which are forced to obtain zero vevs due to higher order NR–contributions in the Yukawa potential, are included in curly brackets. Thus, for example, in the fourth column of the first case, the symbol $\{\bar{2}\}$ means that $\langle\bar{\xi}_2\rangle$ has a zero vev due to the inclusion of NR–terms. Further, for the same reason in the fifth column we also use the notation $\{\bar{1}\}, \{\bar{2}\}$ which should be translated to the conditions $\bar{F}_{1R} = \bar{F}_{2R} = 0$ imposed by NR–terms. In this notation, one can see the effect of NR–terms in the tree–level solutions presented in Appendix D. For example, the first solution in Table 9 (in the appendix) results to the first two distinct cases of Table 2 and so on.

Note that in Table 2 we present only the vanishing vev's of each particular solution. Substitution in the $F-$ and D–flatness conditions, results to a number of constraints characterizing each solution. These constraints are not presented in Table 2 but they have been taken into account in the calculation of free parameters. Specific examples will be presented later in Section 7. Due to the existence of free parameters, each solution of Table 2 can in principle generate a number of phenomenologically distinct cases. We will see in a subsequent section how some of these free parameters are forced to obtain zero vevs following the requirements of low energy phenomenology.

Case (iv)

In a similar manner, we proceed also in this case where $\bar{\xi}_1 = \bar{\xi}_4 = 0$. Eqs. (50), (54) lead to two sub–cases depending on whether ϕ_2 is zero or not.

- $(iv)_a$ When $\phi_2 = 0$, the analysis proceeds in analogy with case (iii). Thus, we find eight solutions at the tree–level which cases 18-25 of Table 9.
- (iv_b) For the case $\phi_2 \neq 0$, a tedious analysis leads to the unique tree–level solution

$$\xi_{2,3} = \bar{\xi}_i = \zeta_i = \bar{\zeta}_i = \bar{F}_{3R} = \bar{F}_{5R} = 0 \tag{77}$$

with $\Phi_2 = 4\,\Phi_{12}\,\bar{\Phi}_{12} \neq 0$

This is also included as case 26 in the complete list of the tree–level solutions of Table 9 in Appendix D. When NR–contributions are taken into account various flat directions are lifted and the total number of solutions is reduced to 4 which are shown in Table 2 (cases (14)-(17)).

Having obtained all consistent solutions, let us try to apply the preliminary phenomenological discussion of the previous section. We first point out that in eight of the cases above, all four Φ_{12}'s fields in the second row have zero vevs. Although nothing can be definitely said until a complete analysis with higher NR–terms is done, we consider them as less favored since they leave all four sextet fields massless at tree–level. Another two solutions on the other hand, has all $\zeta_i = \bar{\zeta}_i = 0$. Again, according to our previous analysis, it would be desirable to obtain a mass term for the second generation at fourth order where a natural fermion mass hierarchy is obtained. Such a solution should admit at least $\zeta_1 \neq 0$ and $\langle \bar{F}_{1R} \rangle = 0$, or $\bar{\zeta}_4 \neq 0$ and $\langle \bar{F}_{2R} \rangle = 0$. From this point of view, the cases admitting non–zero vevs for some of the $\zeta_i, \bar{\zeta}_i$ are more preferable. Few of them leave only the fourplet $\bar{F}_{1R} \neq 0$ to be interpreted as the second $SU(4) \times SU(2)_R$ breaking higgs, (the other being definitely F_{4R}), while there are several cases with $\langle \bar{F}_{3R} \rangle \neq 0$. Moreover, since in most of the cases $\langle \bar{F}_{5R} \rangle = 0$, this latter field together with F_{4L}, are suitable to accommodate the third generation fermions who may receive a tree–level mass term via the Yukawa coupling $\bar{F}_{5R} F_{4L} h_4$.

7 Higgs fields and fermion mass textures

We start our phenomenological analysis of the string model with the discussion on the Higgs sector. Clearly, all the consistent solutions of the flat directions considered in the previous

	$\Phi_{12}s$	Φ_i	$\xi_i, \bar{\xi}_i$	$\zeta_i, \bar{\zeta}_i$	$\bar{F}_i$	f.p.
1	$12, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, \bar{1}, \{\bar{2}\}, \bar{4}$	$3, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \bar{5}$	6
2	$12, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, \bar{1}, \bar{4}$	$3, \bar{3}$	$\{\bar{1}\}, \{\bar{3}\}, \bar{5}$	7
3	$12, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, \bar{1}, \{\bar{2}\}, \bar{4}$	$\bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \bar{5}$	6
4	$12, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, \bar{1}, \bar{4}$	$\bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{3}\}, \bar{5}$	7
5	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, [\bar{1}], \{\bar{2}\}, \bar{4}$	$3, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \bar{5}$	5
6	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,4,5$	$4, [\bar{1}], \{\bar{2}\}, \bar{4}$	$\bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \bar{5}$	6
7	$12, 12^{-}, \overline{12}^{-}$	$2,5$	$4, \bar{1}, \bar{4}$	$3, \bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{3}\}$	7
8	$12, 12^{-}, \overline{12}^{-}$	$2,5$	$4, \bar{1}, \{\bar{2}\}, \bar{4}$	$1, 3, \bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \{\bar{5}\}$	5
9	$12, 12^{-}, \overline{12}^{-}$	$2,5$	$4, \bar{1}, \bar{4}$	$\{1\}, 3, \bar{1}, \bar{3}$	$\{\bar{2}\}, \{\bar{3}\}, \{\bar{5}\}$	6
10	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,5$	$\{2\}, 4, [\bar{1}], \bar{4}$	$\{1\}, 3, \bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \{\bar{5}\}$	4
11	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,5$	$4, [\bar{1}], \{\bar{3}\}, \bar{4}$	$1, 3, \bar{1}, \bar{3}$	$\{\bar{1}\}, \{\bar{2}\}, \{\bar{5}\}$	4
12	$12, 12^{-}, \overline{12}^{-}$	$2,4$	$4, \bar{1}, \bar{4}$	$2, 3, 4, \bar{2}, \bar{3}, \bar{4}$	$\{\bar{1}\}, \{\bar{3}\}, \bar{5}$	5
13	12	2	$2, 3, 4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$1, 2, 3, 4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\{\bar{2}\}, \{\bar{3}\}, \bar{5}$	6
14	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,4,5$	$\bar{1}, \{\bar{2}\}, \bar{4}$	$3, \bar{3}$	$\{\bar{1}\}, \bar{5}$	9
15	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,4,5$	$\bar{1}, \{\bar{2}\}, \bar{4}$	$\bar{1}, \bar{3}$	$\{\bar{1}\}, \bar{5}$	9
16	$12, \overline{12}, 12^{-}, \overline{12}^{-}$	$2,5$	$\bar{1}, \{\bar{2}\}, \bar{4}$	$3, \bar{1}, \bar{3}$	$\{\bar{1}\}$	9
17			$2, 3, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$1, 2, 3, 4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\{\bar{2}\}, \bar{3}, \bar{5}$	7

Table 2: The solutions to the $F-$ and D–flatness equations with contributions of NR–terms up to sixth order. The fields appearing in the table have zero vevs. Those appearing in curly brackets {} are forced to have zero vevs form NR–contributions, while those in square brackets [] are set to zero to ensure the existence of at least one massless Higgs doublet. In the last column f.p. stands for the number of free parameters.

section automatically ensure the existence of one Higgs pair in $(\mathbf{4},\mathbf{1},\mathbf{2})+(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ to break the $SU(4)\times SU(2)_R$ symmetry. Our next task is the securing of a massless pair of $SU(2)_L$ Higgs doublets in order to break the electroweak symmetry. It suffices the existence of only one massless Higgs bidoublet $(1,2,2)$, since after the first stage of symmetry breaking two electroweak doublets with the correct quantum numbers arise

$$h(\mathbf{1},\mathbf{2},\mathbf{2})\rightarrow h^u(\mathbf{1},\mathbf{2},\frac{1}{2})+h^d(\mathbf{1},\mathbf{2},-\frac{1}{2}) \tag{78}$$

The Higgs matrix receives the following contributions from the available tree–level superpotential couplings

$$m_h=\left(\begin{array}{cccc} 0 & 0 & \xi_1 & \xi_4 \\ 0 & 0 & \bar{\xi}_4 & \bar{\xi}_1 \\ \xi_1 & \bar{\xi}_4 & \bar{\Phi}_{12} & \phi_2 \\ \xi_4 & \bar{\xi}_1 & \phi_2 & \Phi_{12} \end{array}\right), \tag{79}$$

No further contributions to the Higgs matrix exist up to sixth order. We will explore the eigenvalues of the above matrix in conjunction with the flatness solutions discussed in the previous section. In order to have at least one zero eigenvalue, the determinant of m_h should be zero

$$\mathrm{Det}(m_h) \;=\; (\xi_1\bar{\xi}_1-\xi_4\bar{\xi}_4)^2=0 \tag{80}$$

We notice that the determinant of the Higgs matrix does not depend on the fields vevs $\Phi_{12},\bar{\Phi}_{12}$ and ϕ_2.

We now come to the particular flat directions of Table 2. We observe that 13 solutions arising from cases $(iii),(iv)$ have automatically $\bar{\xi}_1=\bar{\xi}_4=0$. In the remaining 4 solutions the additional constraint $\bar{\xi}_1=0$ has to be imposed in order to ensure the existence of at least one massless Higgs doublet. These are the cases (5,6,10,11) where the symbol $[\bar{1}]$ in the third column is used to declare the Higgs matrix constraint on the singlet vev $\bar{\xi}_1$.

By inspection of the D–flatness equations (68)-(72) we infer that two pairs of bidoublets are always massive. Going to specific cases we find that the Higgs matrix in solutions (1-13) has exactly two zero eigenvalues corresponding to the pure states h_2,h_4, while rest are massive. Solutions (14-16) have two massless bidoublets. These are h_2 and the combination $h'\propto-\xi_4h_3+\xi_1h_4$ The remaining solution (17) has only one massless bidoublet and more particularly h_2.

Each one of the above cases leads to a distinct phenomenological model. It is convenient to classify them with respect to the $(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ multiplet available for the Higgs mechanism.

- There are seven cases, namely (1,3,5,6,8,10,11), where the only available field of this type is $\bar{F}_{3R}$ since, as can be seen from the table, $\langle\bar{F}_{(1,2,5)R}\rangle=0$.
- In one single case (7) the $(\bar{4},1,2)$–Higgs in general can be a linear combination of $\bar{F}_{2R},\bar{F}_{5R}$
- Only three solutions admit $\langle\bar{F}_{1R}\rangle\neq0$. These are (9,13,17).

- There are three solutions where the higgs may be a linear combination of $\bar{F}_{(2,3)R}$ (14,15) or $\bar{F}_{(2,3,5)R}$, $SU(4)$ (16) multiplets.
- Finally, solutions (2,4,12) admit only $\langle\bar{F}_{2R}\rangle \neq 0$.

Let us emphasize at this point that the above distinction between the solutions together with the massless electroweak Higgs field classification, is in accordance with common phenomenological characteristics. For example, solutions of the first kind above which impose $\langle\bar{F}_{3R}\rangle \neq 0$, have a rather larger number of Yukawa couplings available for fermion mass generation, making them more appealing. Also, from a further inspection of the superpotential terms, the second class of solutions with $\langle\bar{F}_{5R}\rangle \neq 0$ implies a mass for the h_4^u higgs via the tree–level term $\langle\bar{F}_{5R}\rangle F_{4L}h_4$. This fact leaves only one Yukawa coupling available for the up quarks, up to fifth order making these solutions less interesting. In what follows, we will work out in some detail some representative cases from Table 2.

CASE 1:
Let us start with the first solution in Table 2. Along this flat direction, the following 15 fields are required to have zero vevs

$$\Phi_{12} = \Phi_{12}^{-} = \bar{\Phi}_{12}^{-} = \Phi_{2,4,5} = \xi_4 = \bar{\xi}_{1,2,4} = \zeta_3 = \bar{\zeta}_3 = \bar{F}_{1,2,5} = 0\ . \tag{81}$$

Among them, $\bar{\xi}_2$ and $\bar{F}_{5R}$ are constrained to have zero vevs from sixth order contributions. Substituting the above condition to the full system of $D-$ and $F-$ flatness equations we obtain a reduced system of 9 equations for the remaining fields. These are

$$\begin{aligned}
\bar{\xi}_2\,\bar{\xi}_3 + \zeta_1^2 + \zeta_2^2 + \zeta_4^2 &= 0 && (82)\\
\bar{\zeta}_1^2 + \bar{\zeta}_2^2 + \bar{\zeta}_4^2 &= 0 && (83)\\
\xi_3\bar{\xi}_3\ + \zeta_1\bar{\zeta}_1 + \zeta_2\bar{\zeta}_2 + \zeta_4\bar{\zeta}_4 &= 0 && (84)\\
\zeta_2\bar{\zeta}_4 + \zeta_4\bar{\zeta}_2 &= 0 && (85)\\
\frac{1}{2}|\bar{F}_{3R}|^2 + |\xi_2|^2 + |\xi_3|^2 - |\bar{\xi}_3|^2 &= 0 && (86)\\
|\bar{F}_{3R}|^2 + 2\,|\xi_2|^2 + |\zeta_1|^2 + |\zeta_2|^2 + |\zeta_4|^2 - |\bar{\zeta}_1|^2 - |\bar{\zeta}_2|^2 - |\bar{\zeta}_4|^2 &= 0 && (87)\\
\frac{1}{2}|\bar{F}_{3R}|^2 - |\xi_1|^2 &= -\frac{\xi}{3} && (88)\\
|\xi_1|^2 + |\bar{\Phi}_{12}|^2 &= \frac{\xi}{2} && (89)\\
|F_{4R}|^2 - |\bar{F}_{3R}|^2 &= 0 && (90)
\end{aligned}$$

Taking into account that the total number of fields available to obtain vevs are 30 (assuming that hidden sector fields do not develop vevs), we end to a 6 parameter solution. This is the number of free parameters (f.p.) presented in the last column of Table 2. As seen from the above equations consistency of the solution requires a minimum number of the remaining 15 fields

$$\bar{F}_{3R},\ F_{4R},\ \Phi_{1,3},\ \xi_{1,2,3},\ \bar{\xi}_{1,2,3},\ \zeta_{1,2,4},\ \bar{\zeta}_{1,2,4},\ \bar{\Phi}_{12} \neq 0 \tag{91}$$

to be non–zero. These are $\xi_1 , \bar{\xi}_3 , \bar{F}_{3R} , F_{4R}$ and at least one of $\zeta_1, \zeta_2, \zeta_4$. The F_{4R} and $\bar{F}_{3R}$ vevs are not imposed by flatness but are required in order to obtain $SU(4) \times SU(2)_R$ breaking. Thus the higgses in this case, in the notation of Section 2, are

$$F_{4R} \equiv H(\mathbf{4}, \mathbf{1}, \mathbf{2}); \quad \bar{F}_{3R} \equiv \bar{H}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \tag{92}$$

To explore the hierarchy of the fermion mass spectrum, we note first that since $\langle \bar{F}_{3R} \rangle \neq 0$, the tree–level Yukawa coupling $\bar{F}_{3R} F_{3L} h_2$ cannot be used for a fermion mass term. Clearly, F_{3L} is more appropriate for a mirror partner for $\bar{F}_{5L}$. Therefore $\bar{F}_{5R}$ and F_{4L} are suitable to accommodate a family and more particularly the heaviest one as indicated by the tree–level superpotential term $\bar{F}_{5R} F_{4L} h_4$. In this case we need to impose $\langle \zeta_2 \rangle = 0$, to avoid a mass term of the form $\langle \zeta_2 \rangle F_{4L} \bar{F}_{5L}$.

Then, condition (85) results to two distinct cases, either $\langle \zeta_4 \rangle = 0$, or $\langle \bar{\zeta}_2 \rangle = 0$, each of them leading to a different phenomenological model. Although at this level of calculation it cannot be decided which of the two cases is appropriate, we consider the case $\langle \zeta_4 \rangle \neq 0$ as more favorable since it gives a tree–level mass term to a pair of exotic states. Thus we choose to explore the case $\langle \bar{\zeta}_2 \rangle = 0$.

To determine further the low energy parameters, we investigate the $SU(4)$ breaking scale constraints as well as the singlet vevs entering the mass operators. From (88),(89) it follows that the $SU(4)$ breaking scale has a well defined upper limit, determined exclusively from the D-term

$$|\bar{F}_{3R}| \leq \sqrt{\frac{\xi}{3}} = \frac{g_{string}}{2\pi} M_{Pl} \tag{93}$$

For perturbative values of g_{string} (93) gives a bound around the mass scale 10^{17}GeV. Further, from (87) we also conclude that $|\zeta_1| < |\bar{\zeta}_4|$. Up to fifth order, we find the following Yukawa couplings suitable for charged fermion masses

$$\bar{F}_{5R} F_{4L} h_4 + \frac{\langle \bar{\zeta}_4 \rangle}{M_{Pl}} \bar{F}_{2R} F_{2L} h_4 + \frac{\langle \zeta_1 \rangle}{M_{Pl}} \bar{F}_{1R} F_{1L} h_4 \tag{94}$$

The last two terms appear at the fourth order, thus an additional mass parameter in their denominators appears. (In the Appendix B the mass parameters in the denominators are omitted in order to simplify the notation.) These two terms are obviously hierarchically smaller than the first term which gives masses to the third generation; further, taking into account the flatness constraints, we infer that the second and first generations are accommodated in $\bar{F}_{2R}, F_{2L}$ and $\bar{F}_{1R}, F_{1L}$ respectively. From the constraints above, we are able to choose $\zeta_1 \ll \bar{\zeta}_4$, so that we satisfy the mass hierarchies. Moreover, this implies that $\bar{F}_{3R} \sim \bar{\zeta}_4$. Recalling that $\bar{F}_{3R}$ plays the role of the $SU(4)$–breaking higgs at the scale M_{GUT}, we find that the top–charm relation will determine further these vevs to be of the order

$$\frac{M_{GUT}}{M_{Pl}} \equiv \frac{\langle \bar{F}_{3R} \rangle}{M_{Pl}} \approx \frac{m_c^0}{m_t^0} \tag{95}$$

It is worth noticing that this relation which correlates the $SU(4)$ breaking scale with that of the scale $M \sim M_{Pl}$ through the charm–top ratio at M_{string}, is in excellent agreement with both, the flatness condition (93) as well as the unification scale of the minimal unification scenario. We thus conclude that in the flat direction under consideration the flavor assignments of the light standard model quarks and lepton fields are as follows

$$\begin{array}{rl} F_{1L}: \ (u,d),(e,\nu_e)\ ; & \bar{F}_{1R}: \ u^c, d^c, e^c, \nu_e^c \\ F_{2L}: \ (c,s),(\mu,\nu_\mu)\ ; & \bar{F}_{2R}: \ c^c, s^c, \mu^c, \nu_\mu^c \\ F_{4L}: \ (t,b),(\tau,\nu_\tau)\ ; & \bar{F}_{5R}: \ t^c, b^c, \tau^c, \nu_\tau^c \end{array} \tag{96}$$

Up to now, we have a rather successful picture of the fermion mass spectrum which is also in agreement with the string constraints. The above accommodation of the fermion generations and Higgs fields leaves no arbitrariness as far as the extra vector–like states are concerned: these are F_{3L} and $\bar{F}_{5L}$. The only mass term available at tree–level using fields of the observable sector, is proportional to the singlet vev $\langle\zeta_2\rangle$ however this is zero in the present case. Nevertheless, we observe that there are terms involving hidden fields which may acquire non–zero vevs and give a heavy mass to the mirror particles. For example, this can be obtained with a non–zero vev of the combination $\langle Z_3\bar{Z}_4\rangle \neq 0$ while they are constrained by the D–flatness to have equal vevs $|\langle Z_3\rangle| = |\langle\bar{Z}_4\rangle|$ [3].

We turn now to the neutrino sector. The three terms in (94) imply also Dirac masses for the corresponding neutrinos with initial conditions at M_{string} being the same as those for the up–quarks. Therefore a see–saw mechanism is necessary to bring them down to experimentally acceptable scales. An available term exists already in the tree–level superpotential, which couples the right handed neutrino $\nu_5^c \sim \bar{F}_{5R}^c$ with the singlet field $\bar{\zeta}_3$ via the vev $\langle F_{4R}\rangle$. This leads to a see–saw mechanism of the type discussed in Section 2. If we wish to find a final solution within the observable sector field vevs, however, a complete account of the neutrino mass problem needs the calculation of even higher non–renormalizable terms. Restricting ourselves to contributions of NR–terms up to fifth order, the see–saw mechanism, in principle, can be effective for all neutrino species only when additional hidden fields acquire vevs. In this case it is easily checked that the following additional terms are generated

$$A\nu_5^c\phi_2 + B\nu_2^c\phi_2 + C\nu_2^c\bar{\zeta}_3 + D\nu_1^c\bar{\zeta}_2 \tag{97}$$

arising from the hidden sector non–renormalizable contributions:

$$A = \langle F_{4R}Z_4\bar{Z}_5\rangle,\ B = \langle F_{4R}Z_4\bar{Z}_2\rangle,\ C = \langle F_{4R}Z_2\bar{Z}_4\rangle,\ D = \langle F_{4R}Z_1\bar{Z}_4\rangle. \tag{98}$$

The terms (97) complete the mechanism for all three flavours of neutrinos and lead to an extended see–saw of the type (11). We note however that the inclusion of the above hidden vevs requires a re–examination of the flatness conditions.

We come now to the fields having fractional charges. Since no $\pm 1/2$-charge particle has been observed, the doublet states $X_{L,R}$ should also receive masses at some point, presumably higher than the electroweak scale. If we write the doublet $X_{iR} = (\chi_i^+, \chi_i^-)$,

[3] A similar mechanism has also been used in the flipped–$SU(5)$ case [21]

then a possible mass term would be of the form:

$$\begin{aligned} W_X &= \langle\phi\rangle\epsilon_{ab}X^a_{iR}X^b_{jR} \\ &= \langle\phi\rangle(\chi_i^+\chi_j^- - \chi_i^-\chi_j^+) \end{aligned} \tag{99}$$

where ϵ_{ab} is the $SU(2)$ antisymmetric tensor. ϕ can be any combination of fields acquiring vevs resulting to an effective singlet along the neutral direction. A similar term can also exist for the left doublets X_L. Terms mixing left with right doublets are also possible, however, they lead to masses of the order of the electroweak scale and are not of interest here. At the tree–level, in the present flat direction we have the following mass terms

$$W_X^3 = \langle\zeta_4\rangle X_{2R}X_{5R} + \langle\xi_2\rangle X_{2R}X_{6R} + \langle\zeta_1\rangle X_{1R}X_{6R} \tag{100}$$

and similarly there are two terms for the left doublet fields. Notice that we have now made used of the non–zero vev $\langle\zeta_4\rangle \neq 0$ which, according to the flatness conditions (85) implies $\langle\bar\zeta_2\rangle = 0$. There are still three and four pairs of right and left doublets respectively needed to take masses at some scale well above m_W. All possible terms up to fifth order, have been collected in Appendix B. By an inspection of the relevant (to this flat direction) terms up to this order, it follows that few of the octet fields $Z_i, \bar Z_i$ of the hidden sector with non–zero vevs, are adequate to make all of them massive. As has already been pointed out, however, this will require a re–examination of the flatness conditions [24]. All the same, the observable singlet vevs may prove to be sufficient if higher order contributions are calculated. A similar term may also appear for the two $SU(4)$ fourplets $H_4 = (\mathbf{4},\mathbf{1},\mathbf{1})$, $\bar H_4 = (\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$.

There is finally the rather important issue concerning the triplet fields related to the stability of the proton. Recall first from the detailed analysis in Section 2 that the triplets live only in the sextets and the fourplet Higgs fields. There are two types of terms here to render them massive. In the present case, there is only one mass term available for the sextet fields at three level, namely $\bar\Phi_{12}D_3D_4$, while the terms $F_{4R}^2D_3$ and $\bar\zeta_{1,4}^2F_{4R}^2D_4$ offer additional couplings with the uneaten Higgs triplet of F_{4R}. Thus, up to fifth order, three triplet pairs remain light. Higher order NR–terms will make them massive. In particular, an inspection of the related seventh order non–renormalizable superpotential mass terms shows that there are plenty of available couplings rendering all but one pair massive

$$\begin{aligned} W_D^{\leq 7} &= D_1D_2\xi_1Z_3\bar Z_3Z_5\bar Z_5 + D_1D_3\left[\zeta_4\xi_1Z_2\bar Z_2(1+\Phi_1) + \xi_1Z_2\bar Z_2Z_5\bar Z_4\right] \\ &+ D_1D_4\left[\zeta_1^2Z_5\bar Z_5\xi_1 + \bar\zeta_4^2\xi_1Z_5\bar Z_5\right]. \end{aligned} \tag{101}$$

It can be checked that the only coloured field which remained uncoupled is the triplet d_3^c of the Higgs field $\bar H \equiv \bar F_{3R}$ leading to a pair of massless triplets. This has to do with the fact that fields arising from the second b_3 and being charged under peculiar $U(1)_4$ factor make only few non–zero Yukawa couplings with other fields. For the same reason, quarks and lepton fields do not also have dangerous couplings with this triplet field up to this order. At higher orders, singlet fields with non–zero $U(1)_4$ charge are expected to form NR–term mass terms for d_3^c and its partner so that proton decay could be avoided.

CASE 7

Here we have the following zero vevs

$$\Phi^-_{12}, \bar\Phi^-_{12}, \Phi_{2,5}, \xi_4, \bar\xi_{1,4}, \zeta_3, \bar\zeta_{1,3}, \bar F_{1R}, \bar F_{3R} = 0 \tag{102}$$

Following the steps of the analysis of the previous case we find that flatness reduces to 10 equations and thus the number of free parameters is 7. The $SU(4)$–breaking Higgs fields are F_{4R} and a linear combination of the fourplets $\bar F_{2R}$ and $\bar F_{5R}$. We now restrict to use the case $\langle \bar F_{2R}\rangle = 0$. Thus, the higgses are

$$F_{4R} \equiv H(\mathbf{4},\mathbf{1},\mathbf{2}); \bar F_{5R} \equiv \bar H(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) \tag{103}$$

Since $\langle\zeta_2\rangle \neq 0$, the trilinear term $\bar F_{5L}F_{4L}\zeta_2$ makes massive the extra vector–like states. On the other hand the terms

$$\bar F_{5R}F_{4L}h_4 + \bar F_{5L}F_{4L}\zeta_2 \to (\langle\nu^c_5\rangle h^u_4 + \langle\zeta_2\rangle\bar\ell_5)\ell_4 \tag{104}$$

mix a combination of the Higgs doublet in h_4 with $\bar\ell_5$ in $\bar F_{5L}$ leaving massless the combination

$$h^u = \cos\phi h^u_4 - \sin\phi\bar\ell_5,\ \tan\phi = \frac{\langle\zeta_2\rangle}{\langle\nu^c_5\rangle} \tag{105}$$

Once we have determined the electroweak Higgs eigenstates we are in a position to examine the available fermion mass terms. As previously, we will analyze Yukawa couplings up to fifth order. It is natural to accommodate the third generation in the representations arising from the sector b_3; due to the existing fermion hierarchy the heavy fermions are expected to obtain their mass through the only available tree–level term

$$\bar F_{3R}F_{3L}h_2 \quad\to\quad \langle h^u_2\rangle(tt^c + \nu_\tau\nu^c_\tau) + \langle h^d_2\rangle(bb^c + \tau\tau^c)$$

Then the lighter generations receive masses from non–renormalizable terms,

$$\langle h_4\bar\zeta_4(1+\Phi_1)\rangle\bar F_{2R}F_{2L} + \langle h_4\rangle\langle(\zeta_1(1+\Phi_3) + \bar F_{5R}F_{4R})\rangle\bar F_{1R}F_{1L} \tag{106}$$

where denominators of proper powers of M_{string} in the various NR–contributions are omitted. Taking into account (105), the first term of (106) becomes

$$\bar F_{2R}F_{2L}h_4\bar\zeta_4 \quad\to\quad \cos\phi\langle\bar\zeta_4 h^{u\prime}\rangle(Q_2u^c_2 + \nu^c_2\ell_2) + \langle h^d_4\rangle(Q_2d^c_2 + \ell_2e^c_2) \tag{107}$$

and similarly for the other terms. Additional contributions may arise when higher order NR–terms are taken into account.

The triplet mass matrix in the present case, receives contributions from terms involving the above non–zero vevs. Assuming the sextet decompositions $D_i = D^3_i + \bar D^3_i$ the triplet matrix takes the following form in the basis $D_1, D_2, D_3, D_4, \bar d^c_4$,

$$\begin{array}{c|ccccc}
 & D^3_1 & D^3_2 & D^3_3 & D^3_4 & \bar d^c_4(F_{4R}) \\
\hline
\bar D^3_1 & 0 & x & x & x & 0 \\
\bar D^3_2 & x & 0 & 0 & F_{4R}\bar\zeta^2_i & F_{4R}\bar\zeta^2_i \\
\bar D^3_3 & x & 0 & 0 & \bar\Phi_{12} & F_{4R} \\
\bar D^3_4 & x & 0 & \bar\Phi_{12} & 0 & 0 \\
\bar d^c{}_{5R}\,(\bar F_{5R}) & \bar F_{5R}\bar\zeta^2_i & \bar F_{5R} & 0 & 0 & 0
\end{array} \tag{108}$$

where $\bar{\zeta}_i$ stands for the non–vanishing vevs $\bar{\zeta}_2, \bar{\zeta}_4$. The symbol x in the first row and column of the above matrix represents possible contributions from NR–terms involving fields from the hidden sector. These are

$$\begin{aligned} W_D^{\mathcal{H}} &= D_1 D_2(\bar{\zeta}_2 Z_3 \bar{Z}_3 Z_5 \bar{Z}_4 + \xi_1 Z_3 \bar{Z}_3 Z_5 \bar{Z}_5) + D_1 D_3 \zeta_4 \xi_1 Z_2 \bar{Z}_2 \\ &+ D_1 D_4(\bar{\zeta}_2 \bar{\zeta}_4^2 Z_5 \bar{Z}_4 + \bar{\zeta}_4^2 \xi_1 Z_5 \bar{Z}_5 + \bar{\zeta}_4^2 \xi_1 Z_5 \bar{Z}_4) \end{aligned}$$

As can be seen from the mass–matrix, observable sector contributions up to seventh order make all but one pair of the coloured triplets massive. If hidden fields also acquire vevs then all triplets could become massive.

CASE 13

We briefly comment now on another characteristic case of Table 2, namely solution (13). This case is distinguished by two remarkable properties which are worth mentioning:
(i) First we observe that *all coloured sextets* become massive at tree–level. This can be seen from the mass formula (39) and the fact that only one of the four singlets involved in the tree–level mass matrix is required to have a zero vev (namely $\langle \Phi_{12} \rangle = 0$).
(ii) Second, we point out that the two lighter generations are not pure states, since they appear to mixing appears already at tree–level. To see this, we check first from Table 2 that the solution requires $\langle \bar{F}_{2,3,5} \rangle = 0$, thus the $SU(4) \times SU(2)_R$ Higgs fields are now F_{4R} and $\bar{F}_{1R}$. A possible mass term for the mirror states may appear now at a higher order (unless – as previously – hidden fields obtain non–zero vevs). The fermion mass terms are in this case

$$\bar{F}_{3R} F_{3L} \langle h_2 \rangle + \bar{F}_{5R} \left(F_{4L} \langle h_4 \rangle + F_{1L} h_2 \langle \bar{F}_{1R} F_{4R} h_4 \rangle \right) \tag{109}$$

Clearly, the right–handed fields leaving in $\bar{F}_{5R}$ mix with both F_{1L} and F_{4L}. The flavor assignments are now

$$\begin{array}{rl} F_{1L}: \ (u', d'), (e', \nu_e') \ ; & \bar{F}_{2R}: \ u^{c'}, d^{c'}, e^{c'}, \nu_{e'}^c \\ F_{2L}: \ (c', s'), (\mu', \nu_\mu') \ ; & \bar{F}_{2R}: \ c^{c'}, s^{c'}, \mu^{c'}, \nu_\mu^{c'} \\ F_{3L}: \ (t, b), (\tau, \nu_\tau) \ ; & \bar{F}_{3R}: \ t^c, b^c, \tau^c, \nu_\tau^c \end{array} \tag{110}$$

where primes are used to denote that there is mixing in the two lighter generations. We should point out here that the third family remains essentially decoupled due to the peculiar properties of the fourth $U(1)$. Only very high order NR–terms are possible to mix this family with the lighter ones. This fact of course predicts smaller mixing angles between the third family with the rest of the fermion spectrum in consistency with the phenomenological expectations.

8 A brief discussion on the role of the hidden sector fields

Up to now, we dealt with solutions of the flatness conditions considering only non–zero vevs for observable fields. In the phenomenological analysis of the previous section, however, we have seen that couplings involving only observable–sector field vevs are not adequate to make all exotic particles massive. There are mainly two important issues to be further investigated before this model is confronted with the low energy physics world. First, higher order NR–terms have to be calculated in order to find all possible contributions to the mass matrices of fermions, triplets and other fields presented in the previous section. On the other hand, hidden fields can also play a very important role on the determination of the true vacuum of the model. Since they carry no charge, they can also develop non–zero vevs and contribute to the masses of the light fields through their Yukawa interactions. This fact has been clear already in the three examples used in the previous section. Neutrino masses, exotic states, and few of the triplet fields become massive only when hidden fields are included. At the same time, in both cases, the additional terms contribute also to the flatness conditions, thus the new non–zero vevs have to be carefully chosen so that they define a consistent $F-$ and D–flat direction. For such an investigation, one has to modify the flatness solutions, starting again from the tree level cases which are included in Appendix D. A systematic analysis of this general case is possible, however, this goes beyond the scope of the present work [24]. For completeness, the $D-$flatness conditions, in the presence of non–zero hidden field vevs is given in Appendix C. Moreover, the $F-$flatness conditions are written with the hidden fields contributions up to fourth order in the same Appendix. Higher order NR–corrections with hidden as well as observable field vevs are easily extracted from the terms presented in Appendix B. In the following, we give a brief account of the possible solutions the hidden fields may give to some of the unanswered questions of the present string construction.

Several constraints have to be carefully derived before some of the hidden representations acquire vevs. An important constraint arises from the demand of existence of massless electroweak Higgs doublets. Although up to sixth order we have found no extra contributions to the Higgs doublet matrix, such terms may well exist in higher order, in particular when hidden fields are allowed to obtain non–zero vevs. It should be noted that due to the existence of high vevs associated with the $U(1)_A$ breaking scale $M_A \sim 10^{-1} M_{Pl}$ securing the existence of massless electroweak doublets is not an easy task for any superstring model. To be more specific, assume a generic form of doublet mass term

$$g \left(\frac{\Phi}{M_{Pl}}\right)^n \Phi\, h\, h$$

with Φ representing a typical singlet field obtaining a vev of the order $\langle\Phi\rangle \sim M_A$. It is easy to see then that even a $n = 14$ order NR–term would in principle produce higgs masses above the electroweak scale. Of course the existence of superfluous doublet fields –as is the case of the model under consideration– provides the hope that even at this high order of calculation there exist flat directions that preserve at least one pair of doublets massless.

Another severe constraint arises from the necessity to keep the large $SU(4) \times SU(2)_R$ breaking Higgs field F_{4R} massless. Up to sixth order, this can be ensured if the following combinations of vevs are zero

$$\begin{array}{rrr} \bar{\zeta}_3, & \bar{Z}_3 Z_4, & \phi_2 Z_4 \bar{Z}_5, \\ \phi_2 \bar{Z}_2 Z_4, & \bar{\zeta}_2 Z_1 \bar{Z}_4, & \xi_1 Z_1 \bar{Z}_5, \end{array}$$

$$\Phi_3\Phi_3\bar{Z}_3Z_4, \quad \bar{\zeta}_1\zeta_3\bar{Z}_1Z_4, \quad \bar{\zeta}_1\xi_1\bar{Z}_1Z_5. \tag{111}$$

One might think that the above constraints demand most of the hidden fields Z_i, $\bar{Z}_j$ to obtain zero vevs. We would like to point out however that it is possible to have a condition of the form $\langle Z_i\bar{Z}_j\rangle = 0$, while at the same time both fields may have non–zero vevs, $\langle Z_i\rangle \neq 0$ and $\langle\bar{Z}_j\rangle \neq 0$. This happens whenever the fields Z_i and $\bar{Z}_j$ obtain their non–zero vevs in orthogonal to each–other directions.

When some of the $SU(8)$ hidden fields acquire non–zero vevs, the $SU(8)\times U(1)'$ symmetry is broken to a smaller group. However, independently of the number of the hidden states which develop non–zero vevs, there is always at least one unbroken $U(1)''$ generator left, which is in general a linear combination of the $U(1)'$ and one of the generators of $SU(8)$. On the other hand, we note that the maximum number of $U(1)$ factors which may remain unbroken in this model is two. Indeed, it can be checked that the breaking of the $SU(4)\times SU(2)_R$ symmetry on one hand and the consistency of the flatness conditions on the other require at least the fields $F_{4R}, \bar{F}_{1R}$ and $\bar{\Phi}_{12}, \xi_1$ to develop non–zero vevs. These vevs break three of the five Abelian factors.

The survival of $U(1)$ symmetries in lower energies would imply the stability of lightest observable and/or hidden fields being charged under these symmetries. In all flat directions which were previously analyzed, when the various singlet fields obtain their vevs, they break four out of five $U(1)$ factors. Thus, only the aforementioned $U(1)''$ remains at low energies whilst, as a consequence the lightest hidden state will be stable. This fact has important cosmological implications which we now briefly discuss:

The last few years there is accumulating evidence from astronomical observations that the universe is dominated by invisible non–baryonic matter. According to a recent proposal [25, 26] the dark matter of the universe –which is expected to be ten times more that the luminous one– might be composed from non–thermal superheavy states produced in the early universe provided that the following two conditions are met: *i*) candidate particles Y should have a lifetime longer that the age of the universe, $\tau_Y \geq 10^{10}y$, and *ii*) they should not reach local thermal equilibrium with the primordial plasma. To avoid this constraint while having the correct number of Y to form the cold dark matter of the universe, it was suggested that these particles are created through the interaction of the vacuum with the gravitational field. Their mass is found to be around $m_Y \sim 10^{13}GeV$[4].

In the string vacua found in the previous sections, a number of the hidden states $Z_i, \bar{Z}_i$ in the present string construction receive masses at scales which are of the order of the string mass. There are few of $Z_i, \bar{Z}_i$ states however, which remain in the massless spectrum to lower scales. It is possible that in certain string vacua the lightest hidden state has a mass in the range $M_Y \sim 10^{13}$GeV as required in the above scenario. As an example, we construct here the octet mass matrix for the solution 1 of Table 2. In the basis $Z_{1,\ldots,5}$, the contributions up to sixth order involving only the non–zero vev observable fields give the

[4]For a similar discussion on the role of the hidden matter fields in other string models see also [27].

following texture

$$\begin{pmatrix} \xi_1\bar{\zeta}_1 & 0 & 0 & 0 & 0 \\ 0 & \xi_1\bar{\zeta}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{4R}\bar{F}_{3R} & 0 \\ 0 & 0 & 0 & \bar{\zeta}_2 & 0 \\ 0 & 0 & 0 & 0 & \xi_1 \end{pmatrix} \tag{112}$$

In this case, four out of five hidden octet/octet–bar pairs receive masses of the order of the $U(1)_A$ breaking mass scale $M_A \geq M_{GUT}$. If hidden fields are also allowed to obtain vevs, then, $Z_{4,5}$ are further mixed via the mass terms $\langle \xi_1 Z_1 \bar{Z}_1 \rangle \bar{Z}_4 Z_5 + \langle \Phi_3 Z_1 \bar{Z}_1 \rangle \bar{Z}_5 Z_4$. There is only one massless state (namely Z_3) up to this order. It is expected that higher order terms will provide a higher order NR–contribution and make the remaining lightest hidden pair massive at the right scale, which is of course much lower that the mass scale M_A of the other Z_i-fields, as required by the above cosmological scenario.

9 Conclusions

In this paper, we have worked out an $SU(4) \times SU(2)_L \times SU(2)_R$ model derived in the context of the four dimensional free fermionic formulation of the heterotic superstring. Choosing a set of nine vectors of boundary conditions on the world–sheet fermion phases and appropriate GSO projection coefficients, we derived a three–generation model supplemented by a mirror family and just the necessary Higgs representations to break the symmetry down to the standard model. In addition to the observable gauge symmetry, the string model possesses also five $U(1)$'s as well as a hidden $SU(8)$ gauge group. The model predicts the existence of new states beyond those of the minimal supersymmetric standard model massless spectrum. These involve a large number of neutral singlet fields, coloured $SU(4)$–sextets, $SU(8)$-octet hidden fields and exotic states with fractional charges under the standard hypercharge definition.

The superpotential of the model has been derived taken into account string selection rules. All fermion mass terms have been worked out in detail up to fifth order and the fermion and Higgs mass matrix textures have been assiduously analyzed. The model is found to possess an anomalous $U(1)$-symmetry implying the generation of a D–term which is canceled by vacuum expectation values of singlet fields along D– and F–flat directions of the superpotential.

To work out the phenomenological implications, we have performed a detailed analysis of all D– and F– flat directions including contributions of non–renormalizable superpotential terms up to sixth order. At tree–level, 26 solutions to the flatness conditions were found and were classified with respect to the fields which are demanded to have zero vevs in each particular case. It was further shown that, when sixth order NR–terms are included the solutions reduce to seventeen. Each solution is characterized by a the number of free parameters which are essentially the field vevs left undetermined by the particular solution. Particular attention has been paid in the determination of those conditions necessary to ensure the existence in the massless spectrum of the $SU(4)$ breaking higgses and at least two Higgs electroweak doublets in order to break the GUT and SM gauge symmetries respectively. These conditions have been imposed as additional constraints on the consistent D– and F–flat directions and all phenomenologically acceptable string vacua have been determined.

Three distinct flat directions, characterized by their $SU(4)$-higgs properties are investigated in detail and the predictions of the corresponding field theory models are discussed.
a). The first of these predicts that the M_{GUT}/M_{string}–ratio is related to the up quark mass ratio of the second and third generations. The choice of the GUT breaking Higgs representations leaves a sufficient number of Yukawa couplings which produce naturally a hierarchical fermion mass spectrum for all three generations through tree–level and fourth order non–renormalizable superpotential terms. Further, an analysis of the superpotential NR–terms up to sixth order shows that all but one of the colour triplet fields become massive. It is worth noting that there are no dangerous proton decay operators up to this order of calculation since the massless triplet pair does not couple to ordinary matter fields up to the sixth order. The absence of Yukawa couplings between this triplet and ordinary matter fields may be attributed to the properties of peculiar $U(1)$-symmetry of the specific string basis–vector generating this particular state. It is likely however that higher order terms may provide a heavy mass to the remaining colour triplet pair.
b). In a second case analyzed in this work, a similar hierarchical fermion mass pattern is found, while all triplets become massive if in addition hidden fields are allowed to acquire non–zero vevs. On the other hand, in contrast to the first model analyzed in Section 7, here the GUT scale has only an upper bound determined by the $U(1)_A$ breaking scale.
c). Finally, a third effective field theory model is analyzed where all colour sextet fields become massive at the tree–level. This model has fewer Yukawa couplings available for masses, however, additional fermion mass terms may arise from higher order non–renormalizable terms.

A novel feature of the effective field theory is the existence of an additional $U(1)$-symmetry which survives down to low energies, and it is possessed by exotic states and the hidden sector fields. It is argued that if the lightest of these states receives mass at some intermediate scale, may play a role in the dark matter of the universe.

In the present paper our phenomenological explorations have been restricted mainly with respect to the following two issues: First, while there exist various ways to define the electric charge operator of the model (due to the existence of surplus U(1) factors), only the standard hypercharge embedding has been considered in the phenomenological analysis. We believe that it is worth exploring also different types of embedding although one has to face difficulties mainly with low initial $\sin^2\theta_W$ values. Second, the investigation of flat directions has been limited in the cases where only 'observable' fields are allowed to obtain non–zero vevs. Certainly, the inclusion of the hidden states in the analysis will lead to a large number of new mass terms, the breaking of the hidden symmetry and modifications of the flat directions found in this work. Yet, such a possibility has to be compared with analogous investigations of higher order NR–terms will may or may not prove sufficient to obtain realist low energy effective theory.

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10 Appendix A: The spectrum

We collect here the massless observable and hidden superfield spectrum of the model. Fermionic string models contain always an untwisted –usually called Neveu–Schwarz (NS)– sector where all world–sheet fermions are antiperiodic. In this sector, the GSO projections leave always in the massless spectrum the multiplet which contains the graviton, the dilaton and the two–index antisymmetric tensor. The NS–sector includes also the gauge bosons and other Higgs and singlet fields. Twisted (R) sectors provide the generations and other matter fields.

The states are classified in four separate tables according to their transformation properties under the various parts of the gauge symmetry. In the first column of each table we give the symbol of the representation as this is used in the text. In the last column we show the relevant sector of the string basis. In all other columns we exhibit the gauge group properties of the states. Thus, Table 3 contains the observable fields, which have non–trivial transformation properties under the PS–symmetry. These are obtained from the sectors $b_{1,2,3,4,5}$ and $S+b_4+b_5$. They include the three generations, the higgses and other fields. Table 4 includes the PS singlets with their charges under the four $U(1)$ symmetries. In Table 5 we present the hidden $SU(8)$ fields with the corresponding charges under the five $U(1)$s. Finally, in Table 6 we collect all exotic states with fractional charges under the standard hypercharge assignment.

field	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	sector
$\overline{F}_{5L}$	$(\bar{\mathbf{4}},\mathbf{2},\mathbf{1})$	0	0	$-\frac{1}{2}$	0	b_5
$\overline{F}_{5R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	0	$+\frac{1}{2}$	0	
F_{4L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	$+\frac{1}{2}$	0	0	b_4
F_{4R}	$(\mathbf{4},\mathbf{1},\mathbf{2})$	0	$-\frac{1}{2}$	0	0	
$\overline{F}_{3R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$	0	0	$+\frac{1}{2}$	b_3
F_{3L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$+\frac{1}{2}$	0	0	$+\frac{1}{2}$	
$\overline{F}_{2R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	0	$+\frac{1}{2}$	0	b_2
F_{2L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	0	$+\frac{1}{2}$	0	
$\overline{F}_{1R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	0	$+\frac{1}{2}$	0	0	b_1
F_{1L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	$+\frac{1}{2}$	0	0	
D_1	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	-1	0	0	S
D_2	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	0	-1	0	
D_3	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	$+1$	0	0	
D_4	$(\mathbf{6},\mathbf{1},\mathbf{1})$	0	0	$+1$	0	
h_1	$(\mathbf{1},\mathbf{2},\mathbf{2})$	0	0	0	$+1$	
h_2	$(\mathbf{1},\mathbf{2},\mathbf{2})$	0	0	0	-1	
h_3	$(\mathbf{1},\mathbf{2},\mathbf{2})$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$S+b_4+b_5$
h_4	$(\mathbf{1},\mathbf{2},\mathbf{2})$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	

Table 3: Observable sector spectrum of the $SU(4)\times SU(2)_L\times SU(2)_R$ model.

field	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	sector
Φ_A , $A = 1, \ldots, 5$	0	0	0	0	S
Φ_{12}	0	$+1$	$+1$	0	
Φ_{12}^-	0	$+1$	-1	0	
$\overline{\Phi}_{12}$	0	-1	-1	0	
$\overline{\Phi}_{12}^-$	0	-1	$+1$	0	
ζ_i , $i = 1, \ldots, 4$	0	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$S + b_4 + b_5$
$\overline{\zeta}_i$, $i = 1, \ldots, 4$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0	
ξ_1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	
ξ_2	-1	$-\frac{1}{2}$	$+\frac{1}{2}$	0	
ξ_3	-1	$+\frac{1}{2}$	$-\frac{1}{2}$	0	
ξ_4	0	$+\frac{1}{2}$	$+\frac{1}{2}$	-1	
$\overline{\xi}_1$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$	
$\overline{\xi}_2$	$+1$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	
$\overline{\xi}_3$	$+1$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	
$\overline{\xi}_4$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$+1$	

Table 4: Non–Abelian singlet fields and their $U(1)^4$ quantum numbers (all these fields have zero $U(1)'$- charge).

field	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)'$	$SU(8)$	sector
Z_1	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$\mathbf{8}$	$b_1 + b_6(+\zeta)$
$\bar{Z}_1$	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$\bar{\mathbf{8}}$	
Z_2	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$\mathbf{8}$	$b_2 + b_6(+\zeta)$
$\bar{Z}_2$	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	$\bar{\mathbf{8}}$	
Z_3	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$\mathbf{8}$	$b_3 + b_6(+\zeta)$
$\bar{Z}_3$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	$\bar{\mathbf{8}}$	
Z_4	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$\mathbf{8}$	$b_4 + b_6(+\zeta)$
Z_5	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$\mathbf{8}$	
$\bar{Z}_4$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\bar{\mathbf{8}}$	$b_5 + b_6(+\zeta)$
$\bar{Z}_5$	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	$\bar{\mathbf{8}}$	

Table 5: Hidden sector states and their $U(1)^4 \times U(1)' \times SU(8)$ quantum numbers.

field	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)'$	sector
X_{1L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	0	$+\frac{1}{2}$	0	-1	$b_1+\alpha$
X_{2L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	0	-1	
X_{1R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	-1	
X_{2R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	-1	
X_{3L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$+1$	$b_4+\alpha$
X_{4L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	0	$+\frac{1}{2}$	0	-1	
X_{3R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	-1	
X_{4R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	0	$+1$	
X_{5L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+1$	$b_1+b_4+b_5+\alpha$
X_{6L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+1$	
X_{5R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+1$	
X_{6R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+1$	
X_{7L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$+1$	$b_1+b_2+b_4+\alpha$
X_{8L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	-1	
X_{7R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	
X_{8R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+1$	
X_{9L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+1$	$b_2+b_3+b_5+\alpha$
X_{10L}	$(\mathbf{1},\mathbf{2},\mathbf{1})$	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	-1	
X_{9R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	
X_{10R}	$(\mathbf{1},\mathbf{1},\mathbf{2})$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+1$	
H_4	$(\mathbf{4},\mathbf{1},\mathbf{1})$	$-\frac{1}{2}$	0	0	0	$+1$	$S+b_2+b_4+\alpha$
$\overline{H}_4$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$	$+\frac{1}{2}$	0	0	0	-1	

Table 6: Exotic fractionally charged states and their $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)^4\times U(1)'$ quantum numbers.

11 Appendix B: Non–renormalizable contributions

In the first part of this Appendix we give a brief description of the techniques used to calculate the tree–level and higher order NR–superpotential terms of the model. In the second part we give a list of the non–renormalizable superpotential terms involving mass terms (up to fith order) and F–flatness conditions (up to sixth order).

1) The calculation of non–renormalizable contributions to the superpotential in the context of free–fermionic formulation is a straightforward but rather tedious task. The rules for calculation of NR terms have been presented in [17] while explicit calculation for various models have been presented in [20, 28, 29]. In general, a superpotential term involving the chiral superfields $\Phi_1, \Phi_2, \ldots, \Phi_N$ is proportional to the correlator

$$\Phi_1\,\Phi_2\ldots\Phi_N \sim \left\langle V^f_{\Phi_1}\,V^f_{\Phi_2}V^b_{\Phi_3}\ldots V^b_{\Phi_N}\right\rangle$$

where V^f_Φ stands for the fermionic part of the vertex operator corresponding to the field Φ and V^b_Φ for the bosonic part. The correlators can be calculated using conformal field theory techniques developed in [30, 32, 31]. An important subtlety is that in order to guarantee conformal invariance the bosonic vertex operators $V^b_{\Phi_4}, \ldots, V^b_{\Phi_N}$ need to be pictured changed to the zeroth picture.

A superpotential term vanishes if the corresponding correlator vanishes otherwise it leads to an $\mathcal{O}(1)$ coupling. There are two systematic sources of zeros in the superpotential. The first is group invariance, the second are the internal symmetries associated with the fermionized compactified coordinates. The former is obvious while latter has been explored in [16] where a set of selection rules has been derived. Since these selection rules help significantly to the reduction of candidate superpotential terms we summarize here the basic results.

The fermions $\chi^1, \chi^2, \ldots, \chi^6$ corresponding to the compactified coordinates can be bosonized as follows

$$\begin{aligned}(\chi^1 \pm \imath\chi^2)/\sqrt{2} &= \exp\{\pm\imath S_{12}\}\\ (\chi^3 \pm \imath\chi^4)/\sqrt{2} &= \exp\{\pm\imath S_{34}\}\\ (\chi^5 \pm \imath\chi^6)/\sqrt{2} &= \exp\{\pm\imath S_{56}\}\end{aligned}$$

$N = 2$ world–sheet superconformal symmetry implies the existence of an extra current, which is expressed in terms of S_{ij} as follows

$$J(q) = \imath\partial_q(S_{12} + S_{34} + S_{56}) \tag{113}$$

and which is promoted to three $U(1)$'s generated by S_{12}, S_{34}, S_{56}. The relevant part of the vertex operators has the form

$$\begin{aligned}V^f_{-\frac{1}{2}} &\propto e^{(\alpha-\frac{1}{2})S_{12}}e^{(\beta-\frac{1}{2})S_{34}}e^{(\gamma-\frac{1}{2})S_{56}}\\ V^b_{-1} &\propto e^{\alpha S_{12}}e^{\beta S_{34}}e^{\gamma S_{56}}\end{aligned} \tag{114}$$

N	total	selection rule invariants	group invariants	final
3	73150	11719	372	66
4	595665	128928	339	34
5	16559487	2268256	10886	339

Table 7: The total number of candidate superpotential terms (for N=3,4,5) and their number after application of the selection rules, group invariance and the final number after complete evaluation of the correlators for the model under consideration.

where $-\frac{1}{2}, (-1)$ are the ghost numbers for fermions and bosons respectively. Physical states can now separated in two types NS (untwisted) and R(twisted). Each type can be further divided in three categories (the three orbifold planes in the orbifold language). In the notation of Eq. (114) the three categories of NS–fields have charges (α,β,γ)=$\{(1,0,0),(0,1,0),(0,0,1)\}$ while for R–fields (α,β,γ)=$\{(0,\frac{1}{2},\frac{1}{2}),(\frac{1}{2},0,\frac{1}{2}),(\frac{1}{2},\frac{1}{2},0)\}$ respectively.

Using the terminology explained above we can derive a set of selection rules based on the conservation of the three $U(1)$ charges S_{12}, S_{34}, S_{56}. These selection rules are presented in Table 8 for NR–terms up to ninth order. The notation we use is to write in square brackets the allowed partition of fields in each of the categories for given order N. The allowed field type (NS or R) appears as a subscript. As an example let us explain the allowed fifth order couplings. From the table we read $[3_R, 2_R, 0]$ when the number of NS–fields is zero and $[2_R, 2_R, 1_{NS}]$ when the number of NS–fields is one. The first selection rule means that in any non–vanishing coupling between twisted fields the three of them have to belong to a common plane while the other two should both reside in one of the other planes. In the case that one untwisted field participates in the coupling, the twisted ones should reside in the other two planes and there should be exactly two of them in each one. As seen from the table, all 5^{th} order couplings which contain more that one NS field, vanish.

In order to see the effect of the above selection rules we present in Table 7 the number of couplings that are eliminated (for $N \leq 5$) from this source in the model under consideration. We also present the number of couplings surviving group invariance and the final number of non–vanishing superpotential couplings. Going further to the evaluation of correlators one finds another source of zeros. These are the Ising type correlators arising due to the existence of non–trivial left–right paired world–sheet fermions. For tree–level couplings, the non–vanishing Ising correlators are

$$\langle\sigma_+\sigma_+\rangle \ \ ,\langle\sigma_-\sigma_-\rangle \ \ ,\langle\sigma_+\sigma_- f\rangle \ \ ,\langle\sigma_+\sigma_-\bar{f}\rangle \tag{115}$$

for higher order terms one can follow the rules of [32].

The whole problem of deriving the superpotential terms can be automated using a computer program [33]. The selection rules are initially used to reduce the number of candidate couplings, then group invariance is checked and finally all Ising type correlators are evaluated. The whole calculation takes a few seconds on a personal computer for $N = 5$ and comparable time for selected $N = 6$ couplings.

Using this program we have calculated non–vanishing superpotential couplings.

N	0 NS	1 NS	2 NS	3 NS	4 NS
3	$[1_R,1_R,1_R]$	$[2_R+1_{NS},0,0]$	-	$[1_{NS},1_{NS},1_{NS}]$	·
4	$[2_R,2_R,0]$	-	-	-	-
5	$[3_R,1_R,1_R]$	$[2_R,2_R,1_{NS}]$	-	-	-
6	$[4_R,2_R,0]$ $[2_R,2_R,2_R]$	$[3_R,1_R+1_{NS},1_R]$	$[2_R,2_R,2_{NS}]$	·	·
7	$[5_R,1_R,1_R]$ $[3_R,3_R,1_R]$	$[4_R,2_R,1_{NS}]$ $[4_R,2_R+1_{NS},0]$ $[2_R,2_R,2_R+1_{NS}]$	$[3_R,1_R+2_{NS},1_R]$ $[3_R,1_R+1_{NS},1_R+1_{NS}]$	$[2_R,2_R,3_{NS}]$	-
8	$[6_R,2_R,0]$ $[4_R,4_R,0]$ $[4_R,2_R,2_R]$	$[5_R,1_R+1_{NS},1_R]$ $[3_R,3_R,1_R+1_{NS}]$ $[3_R,3_R+1_{NS},1_R]$	$[4_R,2_R,2_{NS}]$ $[4_R,2_R+1_{NS},1_{NS}]$ $[4_R,2_R+2_{NS},0]$ $[2_R+2_{NS},2_R,2_R]$ $[2_R+1_{NS},2_R+1_{NS},2_R]$	$[3_R,1_R,1_R+3_{NS}]$ $[3_R,1_R+1_{NS},1_R+2_{NS}]$	$[2_R,2_R,4_{NS}]$

Table 8: Non–vanishing superpotential couplings up to 8^{th} order.

2)Here we present fifth and sixth order NR–contributions to the superpotential. For finiteness we list only terms related to fermion masses and flatness conditions.

a)The 5th order superpotential terms involving masses for observable fields are:

$$
\begin{aligned}
w_5 &= \bar{F}_{2R}F_{2L}h_4\bar{\zeta}_4\Phi_1 + \bar{F}_{1R}F_{1L}\zeta_1h_4\Phi_3 + \bar{F}_{5R}F_{4R}\bar{F}_{1R}F_{1L}h_4 \\
&+ \bar{F}_{5L}^2\Big((F_{4L}^2 + F_{1L}^2 + \bar{F}_{1R}^2)\bar{\Phi}_{12}^- + F_{4R}^2\Phi_{12} + D_1(h_3^2 + \bar{\xi}_1\xi_4) + D_3(\zeta_i^2 + \xi_2\bar{\xi}_3)\Big) \\
&+ \bar{F}_{5L}\Big(\bar{F}_{5R}\Big(D_1\bar{\zeta}_2h_3 + D_3\zeta_2h_4\Big) + F_{4L}\bar{F}_{2R}F_{2L}h_4 + F_{4R}(\bar{F}_{2R}F_{2L}\bar{\zeta}_3 + \bar{F}_{1R}F_{1L}\zeta_2)\Big) \\
&+ \bar{F}_{5R}^2\Big((F_{4L}^2 + F_{1L}^2 + \bar{F}_{1R}^2)\bar{\Phi}_{12} + F_{4R}^2\Phi_{12}^- + D_1(\bar{\zeta}_i^2 + \bar{\xi}_2\xi_3) + D_3(h_4^2 + \xi_1\bar{\xi}_4)\Big) \\
&+ \bar{F}_{5R}F_{4L}\bar{F}_{3R}F_{3L}\xi_1 + F_{4L}F_{4R}\Big(D_2\zeta_3h_3 + D_4\bar{\zeta}_3h_4\Big) \\
&+ F_{4L}^2\Big((\bar{F}_{2R}^2 + F_{2L}^2)\bar{\Phi}_{12} + D_2(\zeta_i^2 + \xi_2\bar{\xi}_3) + D_4(h_4^2 + \xi_1\bar{\xi}_4)\Big) \\
&+ F_{2R}^2\Big((\bar{F}_{1R}^2 + F_{1L}^2)\bar{\Phi}_{12} + D_1(\bar{\zeta}_i^2 + \bar{\xi}_2\xi_3) + D_3(h_4^2 + \xi_1\bar{\xi}_4)\Big) \\
&+ F_{2L}^2\Big((\bar{F}_{1R}^2 + F_{1L}^2)\bar{\Phi}_{12} + D_1(\bar{\zeta}_i^2 + \bar{\xi}_2\xi_3) + D_3(h_4^2 + \xi_1\bar{\xi}_4)\Big) \\
&+ F_{4R}^2\Big((\bar{F}_{2R}^2 + F_{2L}^2)\Phi_{12}^- + D_4(\bar{\zeta}_i^2 + \bar{\xi}_2\xi_3) + D_2(h_3^2 + \bar{\xi}_1\xi_4)\Big) \\
&+ \Big(F_{1L}^2 + \bar{F}_{1R}^2\Big)\Big(D_2(\zeta_i^2 + \xi_2\bar{\xi}_3) + D_4(h_4^2 + \xi_1\bar{\xi}_4)\Big)
\end{aligned}
\tag{116}
$$

b)The 5th order superpotential terms involving masses for exotic fields are:

$$
\begin{aligned}
w_5' &= \overline{F}_{5R}F_{4R}\Phi_4X_{1R}X_{6R} + \overline{F}_{5R}F_{4R}\Phi_5X_{1R}X_{6R} + \overline{F}_{5R}F_{4R}\zeta_3X_{9R}X_{10R} + \overline{\xi}_2\overline{Z}_1Z_4X_{4L}X_{5L} \\
&+ F_{4R}\overline{F}_{2R}\Phi_4X_{1R}X_{8R} + F_{4R}\overline{F}_{2R}\Phi_5X_{1R}X_{8R} + F_{4R}\overline{F}_{2R}\overline{\zeta}_3X_{6L}X_{8L} + \overline{\xi}_2Z_2\overline{Z}_4X_{1L}X_{7L} \\
&+ F_{4R}\overline{F}_{2R}\overline{\zeta}_3X_{5R}X_{7R} + F_{4R}\overline{F}_{1R}\zeta_4X_{3R}X_{5R} + F_{4R}\overline{F}_{1R}\overline{\zeta}_4X_{4L}X_{6L} + F_{4R}\overline{F}_{1R}\xi_2X_{3R}X_{6R} \\
&+ F_{4R}\overline{F}_{1R}\overline{\xi}_2X_{4L}X_{5L} + \Phi_1\zeta_2\overline{\xi}_2X_{7L}X_{8L} + \Phi_1\overline{\zeta}_2\xi_2X_{7R}X_{8R} + \Phi_1\overline{\zeta}_2\overline{\xi}_3X_{7L}X_{8L} \\
&+ \Phi_3\overline{\zeta}_3\xi_2X_{3R}X_{4R} + \Phi_3\zeta_3\xi_3X_{3R}X_{4R} + \Phi_3\zeta_3\overline{\xi}_2X_{3L}X_{4L} + \Phi_3\overline{\zeta}_3\overline{\xi}_3X_{3L}X_{4L} \\
&+ \Phi_4Z_1\overline{Z}_4X_{3R}X_{5R} + \Phi_4\overline{Z}_2Z_4X_{2L}X_{7L} + \Phi_4Z_4\overline{Z}_5X_{2L}X_{5L} + \Phi_4Z_5\overline{Z}_4X_{2R}X_{5R} \\
&+ \Phi_5Z_1\overline{Z}_4X_{3R}X_{5R} + \Phi_5\overline{Z}_2Z_4X_{2L}X_{7L} + \Phi_5Z_4\overline{Z}_5X_{2L}X_{5L} + \Phi_5Z_5\overline{Z}_4X_{2R}X_{5R} \\
&+ \zeta_1Z_2\overline{Z}_4X_{1R}X_{8R} + \overline{\zeta}_1Z_2\overline{Z}_4X_{2L}X_{7L} + \zeta_2Z_5\overline{Z}_4X_{9R}X_{10R} + \overline{\zeta}_2Z_1\overline{Z}_4X_{3L}X_{2L} \\
&+ \overline{\zeta}_2Z_1\overline{Z}_4X_{4R}X_{1R} + \zeta_3\overline{Z}_2Z_4X_{6L}X_{8L} + \zeta_3\overline{Z}_2Z_4X_{5R}X_{7R} + \overline{\zeta}_3Z_4\overline{Z}_5X_{9L}X_{10L} \\
&+ \zeta_4\overline{Z}_1Z_4X_{3R}X_{5R} + \overline{\zeta}_4\overline{Z}_1Z_4X_{4L}X_{6L} + \xi_2\overline{Z}_1Z_4X_{3R}X_{6R} + \xi_2Z_2\overline{Z}_4X_{2R}X_{8R} \\
&+ \xi_1Z_1\overline{Z}_5X_{3L}X_{2L} + \xi_1Z_1\overline{Z}_5X_{4R}X_{1R} + \xi_1\overline{Z}_2Z_5X_{6L}X_{8L} + \xi_1\overline{Z}_2Z_5X_{5R}X_{7R}
\end{aligned}
\tag{117}
$$

c) 5th and 6th order contributions to the $F-$flatness are [5]:

$$
\begin{aligned}
w_5'' = {} & \Phi_1\zeta_4\xi_1\overline{Z}_2Z_2 + \Phi_2\overline{F}_{5R}F_{4R}Z_4\overline{Z}_5 + \Phi_2F_{4R}\overline{F}_{2R}\overline{Z}_2Z_4 + \Phi_3\overline{\zeta}_1\xi_1\overline{Z}_1Z_1 \\
& + \overline{\Phi}_{12}\overline{F}_{5R}\overline{F}_{5R}\overline{F}_{1R}\overline{F}_{1R} + \overline{\Phi}_{12}\overline{F}_{2R}\overline{F}_{2R}\overline{F}_{1R}\overline{F}_{1R} + \overline{\Phi}_{12}^-\overline{Z}_2\overline{Z}_2Z_4Z_4 + \overline{\Phi}_{12}^-Z_4Z_4\overline{Z}_5\overline{Z}_5 \\
& + \Phi_{12}^-\overline{F}_{5R}\overline{F}_{5R}F_{4R}F_{4R} + \Phi_{12}^-F_{4R}F_{4R}\overline{F}_{2R}\overline{F}_{2R} + \Phi_{12}^-Z_1Z_1\overline{Z}_4\overline{Z}_4 + \Phi_{12}^-Z_5Z_5\overline{Z}_4\overline{Z}_4 \\
& + F_{4R}\overline{F}_{2R}\overline{\zeta}_3Z_2\overline{Z}_4 + F_{4R}\overline{F}_{1R}\overline{\zeta}_2Z_1\overline{Z}_4 + F_{4R}\overline{F}_{1R}\xi_1Z_1\overline{Z}_5 + \overline{\zeta}_2\overline{Z}_1Z_1Z_4\overline{Z}_4 \\
& + \zeta_3\overline{Z}_2Z_2Z_4\overline{Z}_4 + \xi_1\overline{Z}_1Z_1Z_4\overline{Z}_5 + \xi_1\overline{Z}_2Z_2Z_5\overline{Z}_4
\end{aligned}
\tag{118}
$$

[5]For convenience, we include here some of the terms listed above since they can contribute in both categories depending on the generation assignment.

$$
\begin{aligned}
w_6 \quad = \quad & \Phi_1\Phi_1\zeta_4\xi_1\overline{Z}_2 Z_2 + \Phi_1 F_{4R}\overline{F}_{2R}\overline{\zeta}_3 Z_2\overline{Z}_4 + \Phi_1\zeta_3\overline{Z}_2 Z_2 Z_4\overline{Z}_4 + \Phi_1\xi_1\overline{Z}_2 Z_2 Z_5\overline{Z}_4 \\
& + \Phi_2\Phi_5 F_{4R}\overline{F}_{2R}\overline{Z}_2 Z_4 + \Phi_3\Phi_3 F_{4R}\overline{F}_{3R}\overline{Z}_3 Z_4 + \Phi_3\Phi_3\overline{\zeta}_1\xi_1\overline{Z}_1 Z_1 + \Phi_3 F_{4R}\overline{F}_{1R}\overline{\zeta}_2 Z_1\overline{Z}_4 \\
& + \Phi_3 F_{4R}\overline{F}_{1R}\xi_1 Z_1\overline{Z}_5 + \Phi_3\overline{\zeta}_2\overline{Z}_1 Z_1 Z_4\overline{Z}_4 + \Phi_3\xi_1\overline{Z}_1 Z_1 Z_4\overline{Z}_5 + \Phi_4\overline{\Phi}_{12}\overline{F}_{5R}\overline{F}_{5R}\overline{F}_{1R}\overline{F}_{1R} \\
& + \Phi_4\overline{\Phi}_{12}\overline{F}_{2R}\overline{F}_{2R}\overline{F}_{1R}\overline{F}_{1R} + \Phi_4\Phi^{-}_{12} Z_1 Z_1\overline{Z}_4\overline{Z}_4 + \Phi_4 F_{4R}\overline{F}_{1R}\overline{\zeta}_2 Z_1\overline{Z}_4 + \Phi_4 F_{4R}\overline{F}_{1R}\xi_1 Z_1\overline{Z}_5 \\
& + \Phi_4\overline{\zeta}_2\overline{Z}_1 Z_1 Z_4\overline{Z}_4 + \Phi_4\xi_1\overline{Z}_1 Z_1 Z_4\overline{Z}_5 + \Phi_5\overline{\Phi}_{12}\overline{F}_{2R}\overline{F}_{2R}\overline{F}_{1R}\overline{F}_{1R} + \Phi_5\overline{\Phi}^{-}_{12}\overline{Z}_2\overline{Z}_2 Z_4 Z_4 \\
& + \Phi_5\overline{\Phi}^{-}_{12}\overline{Z}_2\overline{Z}_2 Z_4 Z_4 + \Phi_5\Phi^{-}_{12} F_{4R}F_{4R}\overline{F}_{2R}\overline{F}_{2R} + \Phi_5 F_{4R}\overline{F}_{2R}\overline{\zeta}_3 Z_2\overline{Z}_4 + \Phi_5\zeta_3\overline{Z}_2 Z_2 Z_4\overline{Z}_4 \\
& + \Phi_5\overline{\Phi}^{-}_{12}\overline{Z}_2\overline{Z}_2 Z_4 Z_4 + \Phi_5\Phi^{-}_{12} F_{4R}F_{4R}\overline{F}_{2R}\overline{F}_{2R} + \Phi_5 F_{4R}\overline{F}_{2R}\overline{\zeta}_3 Z_2\overline{Z}_4 + \Phi_5\zeta_3\overline{Z}_2 Z_2 Z_4\overline{Z}_4 \\
& + \Phi_5\overline{\Phi}^{-}_{12}\overline{Z}_2\overline{Z}_2 Z_4 Z_4 + \Phi_5\Phi^{-}_{12} F_{4R}F_{4R}\overline{F}_{2R}\overline{F}_{2R} + \Phi_5 F_{4R}\overline{F}_{2R}\overline{\zeta}_3 Z_2\overline{Z}_4 + \Phi_5\zeta_3\overline{Z}_2 Z_2 Z_4\overline{Z}_4 \\
& + \Phi_5\overline{\Phi}^{-}_{12}\overline{Z}_2\overline{Z}_2 Z_4 Z_4 + \Phi_5\Phi^{-}_{12} F_{4R}F_{4R}\overline{F}_{2R}\overline{F}_{2R} + \Phi_5 F_{4R}\overline{F}_{2R}\overline{\zeta}_3 Z_2\overline{Z}_4 + \Phi_5\zeta_3\overline{Z}_2 Z_2 Z_4\overline{Z}_4 \\
& + \Phi_5\xi_1\overline{Z}_2 Z_2 Z_5\overline{Z}_4 + \overline{F}_{5R}\overline{F}_{5R}\overline{F}_{3R}\overline{F}_{3R}\xi_1\overline{\xi}_2 + F_{4R}F_{4R}\overline{F}_{3R}\overline{F}_{3R}\xi_4\overline{\xi}_2 + F_{4R}F_{4R}\overline{F}_{1R}\overline{F}_{1R}\zeta_1\overline{\zeta}_3 \\
& + F_{4R}F_{4R}\overline{F}_{1R}\overline{F}_{1R}\overline{\zeta}_1\zeta_3 + F_{4R}\overline{F}_{3R}\zeta_1\overline{\zeta}_1\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{3R}\zeta_2\overline{\zeta}_2\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{3R}\zeta_3\overline{\zeta}_3\overline{Z}_3 Z_4 \\
& + \; F_{4R}\overline{F}_{3R}\overline{\zeta}_3\xi_1\overline{Z}_3 Z_5 + F_{4R}\overline{F}_{3R}\zeta_4\overline{\zeta}_4\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{3R}\xi_2\overline{\xi}_2\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{3R}\xi_3\overline{\xi}_3\overline{Z}_3 Z_4 \\
& + \; F_{4R}\overline{F}_{3R}\xi_1\overline{\xi}_1\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{3R}\xi_4\overline{\xi}_4\overline{Z}_3 Z_4 + F_{4R}\overline{F}_{1R}\zeta_1\overline{\zeta}_3\overline{Z}_1 Z_4 + F_{4R}\overline{F}_{1R}\overline{\zeta}_1\zeta_3\overline{Z}_1 Z_4 \\
& + \; F_{4R}\overline{F}_{1R}\overline{\zeta}_1\xi_1\overline{Z}_1 Z_5 + \overline{F}_{3R}\overline{F}_{3R}\overline{F}_{2R}\overline{F}_{2R}\xi_1\overline{\xi}_2 + \overline{F}_{3R}\overline{F}_{3R}\overline{F}_{1R}\overline{F}_{1R}\xi_1\overline{\xi}_3 + \zeta_1\overline{\zeta}_3\overline{Z}_1\overline{Z}_1 Z_4 Z_4 \\
& + \; \overline{\zeta}_1\zeta_3\overline{Z}_1\overline{Z}_1 Z_4 Z_4 + \overline{\zeta}_1\xi_1\overline{Z}_1\overline{Z}_1 Z_4 Z_5 + \zeta_2\overline{\zeta}_4 Z_2 Z_2\overline{Z}_4\overline{Z}_4 + \overline{\zeta}_2\zeta_4 Z_2 Z_2\overline{Z}_4\overline{Z}_4 \\
& + \; \zeta_4\xi_1 Z_2 Z_2\overline{Z}_4\overline{Z}_5 + \xi_2\overline{\xi}_4\overline{Z}_3\overline{Z}_3 Z_4 Z_4 + \overline{\xi}_2\overline{\xi}_4 Z_3 Z_3\overline{Z}_4\overline{Z}_4
\end{aligned}
\tag{119}
$$

12 Appendix C: F– and D–flatness equations

The identification of the flat directions in the scalar potential requires the vanishing of the $F-$ and $D-$terms. In Section 6, the complete $F-$flatness conditions with tree–level superpotential contributions were presented. Hidden field contributions are also easily calculated from the superpotential (35). Fourth order contributions from both hidden and observable sectors can also be found from the superpotential terms (36) presented in the same section. Higher order terms have also been calculated and are given in Appendix B. For convenience, the contributing fifth and sixth order NR superpotential terms are written separately in the w_5'' and w_6 pieces of the NR superpotential.

D–flatness

The D–flatness equations for the non-Anomalous $U(1)_i$ factors are given by

$$
(D_i) : \sum_{\phi_j} Q^i_j |\phi_j|^2 \;\; = \;\; 0 \;,\; i = 1, 2, 3 \tag{120}
$$

On the other hand, the Green–Schwarz anomaly cancellation mechanism in string theory generates a constant Fayet-Iliopoulos contribution to the D–term of the anomalous $U(1)_A$. This is proportional to the trace of the anomalous charge over all fields. To preserve supersymmetry the following D–flatness condition should be satisfied,

$$
(D_A) : \sum_{\phi_j} Q^A_j |\phi_j|^2 \;\; = \;\; -\xi \tag{121}
$$

where the sum extends over all singlet fields (including the $SU(4)\times O(4)$ breaking ones) and $\xi = \frac{3}{8\pi^2}e^{\Phi_D}$. If hidden fields acquire vevs, they should also be included in the above expressions.

Taking the combinations (D_1), $\frac{1}{2}((D_1)+(D_2))$, $\frac{1}{2}((D_3)-(D_4))$, $\frac{1}{2}(D_4)$ we obtain

$$\frac{1}{2}\left|\overline{F}_{3R}\right|^2 + |\xi_2|^2 - \left|\overline{\xi}_2\right|^2 + |\xi_3|^2 - \left|\overline{\xi}_3\right|^2 + \mathcal{H}_1 = 0 \tag{122}$$

$$\frac{1}{4}\left(\left|\overline{F}_{2R}\right|^2 - \left|\overline{F}_{1R}\right|^2 + \left|\overline{F}_{3R}\right|^2 + |F_{4R}|^2\right) + \tag{123}$$

$$+ |\xi_2|^2 - \left|\overline{\xi}_2\right|^2 + \left|\bar{\Phi}_{12}^-\right|^2 - \left|\Phi_{12}^-\right|^2 + \frac{1}{2}\sum_{i=1}^{4}\left(|\zeta_i|^2 - \left|\overline{\zeta}_i\right|^2\right) + \mathcal{H}_2 = 0 \tag{124}$$

$$\frac{1}{2}\left|\overline{F}_{3R}\right|^2 + \left|\overline{\xi}_1\right|^2 - |\xi_1|^2 + \left|\overline{\xi}_4\right|^2 - |\xi_4|^2 + \mathcal{H}_3 = -\frac{\xi}{3} \tag{125}$$

$$|\xi_1|^2 - \left|\overline{\xi}_1\right|^2 + \left|\bar{\Phi}_{12}\right|^2 - |\Phi_{12}|^2 + \mathcal{H}_4 = \frac{\xi}{2} \tag{126}$$

$\mathcal{H}_{1,2,3,4}$ stand for hidden vev contributions. These are

$$\begin{aligned}
\mathcal{H}_1 &= \frac{1}{2}\left(|\bar{Z}_3|^2 - |Z_3|^2\right)\\
\mathcal{H}_2 &= \frac{1}{4}\left(|\bar{Z}_2|^2 + |Z_2|^2 + 2\,|\bar{Z}_3|^2 + |\bar{Z}_5|^2 - |\bar{Z}_4|^2\right)\\
\mathcal{H}_3 &= \frac{1}{4}\left(-|\bar{Z}_2|^2 - |Z_2|^2 + |\bar{Z}_3|^2 + |Z_3|^2 + |\bar{Z}_4|^2 - |\bar{Z}_5|^2\right)\\
\mathcal{H}_4 &= \frac{1}{4}\left(|\bar{Z}_1|^2 + |Z_1|^2 + |\bar{Z}_2|^2 + |Z_2|^2 - |\bar{Z}_4|^2 - |Z_4|^2 + |\bar{Z}_5|^2 + |Z_5|^2\right)
\end{aligned}$$

We finally have the D–flatness conditions for the non-Abelian part of the gauge symmetry. For the $SU(4)\times O(4)$,

$$\left|\overline{F}_{1R}\right|^2 + \left|\overline{F}_{2R}\right|^2 + \left|\overline{F}_{3R}\right|^2 - |F_{4R}|^2 = 0 \tag{127}$$

while in the presence of the hidden non-zero vevs, the $SU(8)$ and $U(1)'$ $D-$flatness conditions should also be satisfied

$$(D_{U(8)}):\quad \textstyle\sum_{i=1}^{5}\left(|\bar{Z}_i|^2 - |Z_i|^2\right) = 0, \tag{128}$$

$$(D_{U(1)'}):\quad \textstyle\sum_{\chi_j} Q'_j|\chi_j|^2 = 0\ . \tag{129}$$

where χ_j stand for all fields carrying $U(1)'$ charge.

F–flatness

For completeness, we also write here the F-flatness conditions including superpotential contributions up to fourth order and hidden fields. Fifth and sixth order contributions are easily calculated from the NR–terms presented in Appendix B.

$$\Phi_2:\quad \zeta_i\bar{\zeta}_i + \xi_i\bar{\xi}_i = 0 \tag{130}$$

$$\Phi_4:\quad \zeta_1\bar{\zeta}_3 + \bar{\zeta}_1\zeta_3 = 0 \tag{131}$$

$$\Phi_5 : \quad \zeta_2\bar{\zeta}_4 + \bar{\zeta}_2\zeta_4 = 0 \tag{132}$$

$$\Phi_{12} : \quad \xi_1\bar{\xi}_4 = 0 \tag{133}$$

$$\bar{\Phi}_{12} : \quad \xi_4\bar{\xi}_1 + Z_3\bar{Z}_3 = 0 \tag{134}$$

$$\Phi_{12}^- : \quad \zeta_i\zeta_i + \xi_2\bar{\xi}_3 = 0 \tag{135}$$

$$\bar{\Phi}_{12}^- : \quad \bar{\zeta}_i\bar{\zeta}_i + \xi_3\bar{\xi}_2 = 0 \tag{136}$$

$$\xi_1 : \quad \phi_2\bar{\xi}_1 + \Phi_{12}\bar{\xi}_4 + Z_5\bar{Z}_5 + \bar{\zeta}_1 Z_1\bar{Z}_1 + \zeta_4 Z_2\bar{Z}_2 = 0 \tag{137}$$

$$\bar{\xi}_1 : \quad \phi_2\xi_1 + \bar{\Phi}_{12}\xi_4 = 0 \tag{138}$$

$$\xi_2 : \quad \phi_2\bar{\xi}_2 + \Phi_{12}^-\bar{\xi}_3 = 0 \tag{139}$$

$$\bar{\xi}_2 : \quad \phi_2\xi_2 + \bar{\Phi}_{12}^-\xi_3 = 0 \tag{140}$$

$$\xi_3 : \quad \phi_2\bar{\xi}_3 + \bar{\Phi}_{12}^-\bar{\xi}_2 = 0 \tag{141}$$

$$\bar{\xi}_3 : \quad \phi_2\xi_3 + \Phi_{12}^-\xi_2 = 0 \tag{142}$$

$$\xi_4 : \quad \phi_2\bar{\xi}_+\bar{\Phi}_{12}\bar{\xi}_1 = 0 \tag{143}$$

$$\bar{\xi}_4 : \quad \phi_2\xi_4 + \Phi_{12}\xi_1 = 0 \tag{144}$$

$$\zeta_1 : \quad \phi_2\bar{\zeta}_1 + \Phi_4\bar{\zeta}_3 + 2\Phi_{12}^-\zeta_1 = 0 \tag{145}$$

$$\bar{\zeta}_1 : \quad \phi_2\zeta_1 + \Phi_4\zeta_3 + 2\bar{\Phi}_{12}^-\bar{\zeta}_1 + \xi_1 Z_1\bar{Z}_1 = 0 \tag{146}$$

$$\zeta_2 : \quad \phi_2\bar{\zeta}_2 + \Phi_5\bar{\zeta}_4 + 2\Phi_{12}^-\zeta_2 + \frac{1}{\sqrt{2}}\bar{F}_{5L}F_{4L} = 0 \tag{147}$$

$$\bar{\zeta}_2 : \quad \phi_2\zeta_2 + \Phi_5\zeta_4 + 2\bar{\Phi}_{12}^-\bar{\zeta}_2 + Z_5\bar{Z}_4/\sqrt{2} = 0 \tag{148}$$

$$\zeta_3 : \quad \phi_2\bar{\zeta}_3 + \Phi_4\bar{\zeta}_1 + 2\Phi_{12}^-\zeta_3 + Z_4\bar{Z}_5/\sqrt{2} = 0 \tag{149}$$

$$\bar{\zeta}_3 : \quad \phi_2\zeta_3 + \Phi_4\zeta_1 + 2\bar{\Phi}_{12}^-\bar{\zeta}_3 + \bar{F}_{5R}F_{4R}/\sqrt{2} = 0 \tag{150}$$

$$\zeta_4 : \quad \phi_2\bar{\zeta}_4 + \Phi_5\bar{\zeta}_2 + 2\Phi_{12}^-\zeta_4 + \xi_1 Z_2\bar{Z}_2 = 0 \tag{151}$$

$$\bar{\zeta}_4 : \quad \phi_2\zeta_4 + \Phi_5\zeta_2 + 2\bar{\Phi}_{12}^-\bar{\zeta}_4 = 0 \tag{152}$$

$$Z_1 : \quad \bar{\zeta}_1\xi_1\bar{Z}_1 = 0 \tag{153}$$

$$\bar{Z}_1 : \quad \bar{\zeta}_1\xi_1 Z_1 = 0 \tag{154}$$

$$Z_2 : \quad \zeta_4\xi_1\bar{Z}_2 = 0 \tag{155}$$

$$\bar{Z}_2 : \quad \zeta_4\xi_1 Z_2 = 0 \tag{156}$$

$$Z_3 : \quad \bar{\Phi}_{12}\bar{Z}_3 + \bar{F}_{5L}F_{3L}\bar{Z}_4 = 0 \tag{157}$$

$$\bar{Z}_3 : \quad \bar{\Phi}_{12}Z_3 + F_{4R}\bar{F}_{3R}Z_4 = 0 \tag{158}$$

$$Z_4 : \quad \zeta_3\bar{Z}_5/\sqrt{2} + F_{4R}\bar{F}_{3R}\bar{Z}_3 = 0 \tag{159}$$

$$Z_5 : \quad \bar{\zeta}_2\bar{Z}_4/\sqrt{2} + \xi_1\bar{Z}_5 = 0 \tag{160}$$

$$\bar{Z}_4 : \quad \bar{\zeta}_2 Z_5/\sqrt{2} = 0 \tag{161}$$

$$\bar{Z}_5 : \quad \zeta_3 Z_4/\sqrt{2} + \xi_1 Z_5 = 0 \tag{162}$$

$$\overline{F}_{5R} : \quad F_{4R}\overline{\zeta}_3 = 0 \tag{163}$$

13 Appendix D: Tree–level flat directions.

We present here the tree–level flat directions of the model. As has been explained in Section 6, the solutions are classified in four distinct cases according to whether the singlet vevs $\xi_{1,4}, \bar{\xi}_{1,4}$ are zero or non–zero. It was shown there that only the cases $\xi_1 = \xi_4 = 0$ (assigned as case (iii) in Section 6) and $\bar{\xi}_1 = \bar{\xi}_4 = 0$ (with $\xi_1, = \xi_4 \neq 0$, referred as case (iv) in the same section) have solutions consistent with $F-$ and $D-$ flatness constraints.

Our analysis proceeded as follows: First we solved the constraints taking into account contributions only from the tree–level Yukawa superpotential. An exhaustive analysis shows that at tree–level there are 17 solutions for case (iii) and 9 solutions for case (iv). These solutions are presented in Tables 9. The five columns in the middle show the fields with zero vevs and the last column the number of free parameters. For further details in the notation, see explanation in section 6. Higher order NR–contributions up to sixth order, reject several of these cases, resulting to those presented in Section 6.

The complete list of the tree–level solutions given in Table 9 is related to flatness constraints involving fields only from the observable sector. These are easily extended to solutions involving hidden fields by using the flatness conditions of Appendix C. Solutions involving hidden field contributions of higher NR superpotential terms are more involved and need a separate treatment.

	$\Phi_{12}s$	Φ_i	$\xi_i, \bar{\xi}_i$	$\zeta_i, \bar{\zeta}_i$	$\bar{F}_i$	f.p.
1	$12, 12^-, \overline{12}^-$	$2,4,5$	$4, \bar{1}, \bar{4}$	$3, \bar{3}$	$\bar{5}$	9
2	$12, 12^-, \overline{12}^-$	$2,4,5$	$4, \bar{1}, \bar{4}$	$\bar{1}, \bar{3}$	$\bar{5}$	9
3	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4,5$	$4, \bar{4}$	$3, \bar{3}$	$\bar{5}$	9
4	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4,5$	$4, \bar{4}$	$\bar{1}, \bar{3}$	$\bar{5}$	9
5	$12, 12^-, \overline{12}^-$	$2,5$	$4, \bar{1}, \bar{4}$	$3, \bar{1}, \bar{3}$		9
6	$12, \overline{12}, 12^-, \overline{12}^-$	$2,5$	$4, \bar{4}$	$3, \bar{1}, \bar{3}$		9
7	$12, 12^-, \overline{12}^-$	$2,5$	$4, \bar{1}, \bar{4}$	$2,3, \bar{1}, \bar{2}, \bar{3}$		8
8	$12, 12^-, \overline{12}^-$	$2,4$	$4, \bar{1}, \bar{4}$	$2,3,4, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
9	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4$	$4, \bar{4}$	$2,3,4, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
10	$12, 12^-, \overline{12}^-$	2	$2,3,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}, \bar{2}$	6
11	$12, \overline{12}^-$	$2,4,5$	$2,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{3}$	$\bar{5}$	8
12	$12, \overline{12}^-$	$2,5$	$2,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{3}$	$\bar{5}$	8
13	$12, \overline{12}^-$	$2,4$	$2,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
14	$12, \overline{12}^-$	2	$2,3,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
15	$12, \overline{12}, \overline{12}^-$	2	$2,3,4, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	6
16	12	2	$2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}, \bar{3}$	7
17	$12, \overline{12}, 12^-, \overline{12}^-$	2	$3,4, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}, \bar{2}$	8
18	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4,5$	$\bar{1}, \bar{4}$	$3, \bar{3}$	$\bar{5}$	9
19	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4,5$	$\bar{1}, \bar{4}$	$\bar{1}, \bar{3}$	$\bar{5}$	9
20	$12, \overline{12}, 12^-, \overline{12}^-$	$2,5$	$\bar{1}, \bar{4}$	$3, \bar{1}, \bar{3}$		9
21	$12, \overline{12}, 12^-, \overline{12}^-$	$2,4$	$\bar{1}, \bar{4}$	$2,3,4, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
22	$12, \overline{12}, \overline{12}^-$	$2,4,5$	$2, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{3}$	$\bar{5}$	8
23	$12, \overline{12}, \overline{12}^-$	$2,5$	$2, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{3}$	$\bar{5}$	8
24	$12, \overline{12}, \overline{12}^-$	$2,4$	$2, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
25	$12, \overline{12}, \overline{12}^-$	2	$2,3, \bar{1}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{5}$	7
26			$2,3, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$1,2,3,4, \bar{1}, \bar{2}, \bar{3}, \bar{4}$	$\bar{3}, \bar{5}$	7

Table 9: The tree–level solutions to the $F-$ and $D-$flatness equations. The fields appearing in the table have zero vevs. In the last column f.p. stands for the number of free parameters.