

## Stabilization of Large Scale Structure by Adhesive Gravitational Clustering

T. Buchert<sup>1</sup>

*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

**Abstract.** The interplay between gravitational and dispersive forces in a multi-streamed medium leads to an effect which is exposed in the present note as the genuine driving force of stabilization of large-scale structure. The conception of ‘adhesive gravitational clustering’ is advanced to interlock the fairly well-understood epoch of formation of large-scale structure and the onset of virialization into objects that are dynamically in equilibrium with their large-scale structure environment. The classical ‘adhesion model’ is opposed to a class of more general models traced from the physical origin of adhesion in kinetic theory.

### 1. Previrialization and Adhesion.

‘Adhesive gravitational clustering’ touches on the basis of why structures stabilize after their formation. On the one hand, analytical understanding of structure *formation* is well advanced (see, e.g., the review by Sahni & Coles 1995). However, models that evolve inhomogeneities into the nonlinear regime also predict structure decay after their formation. On the other hand, the understanding of *virialization* is hosted in stellar systems theory of classically isolated bound objects. What are the equilibrium structures that could be called ‘relaxed’ while still interacting gravitationally with their large-scale structure environment? The fact that the transition to ‘virialized’ systems is not immediate is mirrored in expressions such as ‘previrialization’, invoked by Peebles and collaborators (Davis & Peebles 1977, Peebles 1990, Lokas et al. 1996). The ‘adhesion approximation’ as invented by Gurbatov et al. (1989) takes its title from the phenomenological observation that structures ‘stick together’ after shells of cold matter cross; numerical simulations predict multiple shell-crossings due to dragging forces that prevent the particles from escaping high-density regions. We shall identify these forces in the framework of kinetic theory. It is here where an important period of ‘adhesive clustering’ sets in, which eventually leads to the type of bound objects that we observe in the Universe and that may finally populate *fundamental planes* in spaces spanned by integral properties of these objects (Fritsch & Buchert 1999).

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<sup>1</sup>on leave from: Theoretische Physik, Ludwig-Maximilians-Universität, Theresienstr. 37, D-80333 München, Germany; email: buchert@theorie.physik.uni-muenchen.de

## 2. The formation of large-scale structure.

The understanding of large-scale structure formation may be centred on Zel'dovich's idea: just move the particles under inertia and follow them until they cross. For suitably scaled variables the exact solution of a force-free continuum reproduces the linear theory of gravitational instability in the linear limit (Zel'dovich 1970, Zel'dovich & Myshkis 1973, Shandarin & Zel'dovich 1989, Buchert 1989).

Taking Zel'dovich's simple view at face value, gravity is not important at all to form structures like curved sheets, filaments and clusters, their formation is merely an effect of focussing of trajectories we are familiar with in geometrical optics of light rays. The resulting network of *caustics* is unstable: just increase the depth (advance the time) of a water basin (the Universe) on the bottom of which (at the time of shell-crossing) you see a network of high-intensity (high-density) structures, and you have this phenomenon. Indeed, Zel'dovich's model applied to generic initial data, say powerlaw spectra with slopes in the range  $n < -3$  at the high-frequency end, reproduces astonishingly well the density fields predicted by N-body runs of the same initial data (Melott et al. 1995). 'Pancake models', which don't have structure on small scales in the initial conditions, fall in this category (Buchert et al. 1994). The spectral index  $n = -3$  distinguishes between two different scenarios of structure formation: either structures form starting from large scales (top-down), or they form starting hierarchically from small scales (bottom-up); the total power in some wave number interval has logarithmic dependence in the case  $n = -3$  implying that structures on every scale form at about the same instant, since all wave numbers were given similar initial amplitudes. In the other cases  $n > -3, n < -3$  the integrated power has powerlaw shape; hence,  $n = -3$  defines clearly distinct regimes.

Truncation of high-frequency information in the sense of cutting down the amplitude of short-wavelength perturbations to the 'critical spectrum'  $n = -3$  results in optimized schemes which can compete with N-body runs down to the scales of galaxy clusters (Coles et al. 1993, Melott et al. 1994). Refining Zel'dovich's step is possible in the framework of the Lagrangian perturbation theory, where it can be re-evaluated within the full gravitational context as a first-order solution (Buchert 1989; 1996 and ref. therein). As a consequence the performance can be improved (Melott et al. 1995); 'optimized Lagrangian schemes' work even for CDM-type initial data (Weiß et al. 1996). Also, other characteristics of the matter distribution are matching the expected (Bouchet et al. 1995).

Figure 1 shows a comparison of N-body density fields with those predicted by various analytical schemes (some of them will be described below): the 'adhesion approximation' works in the right direction and shows better performance than the Lagrangian schemes. Still, if the Lagrangian schemes are optimized, the 'adhesion approximation' falls short indicating that the model is not satisfactorily tailored for the gravitational multi-stream effect it was launched to describe. The spectral index of a powerlaw spectrum was  $n = -1$  which is an extreme test of the approximations: much power on small scales even results in the superiority of the first-order over the second-order Lagrangian approximation, since the latter accelerates structure decay after caustics have formed.

In the following we shall encounter a broader range of possible models that show how to consolidate and generalize the adhesion idea.

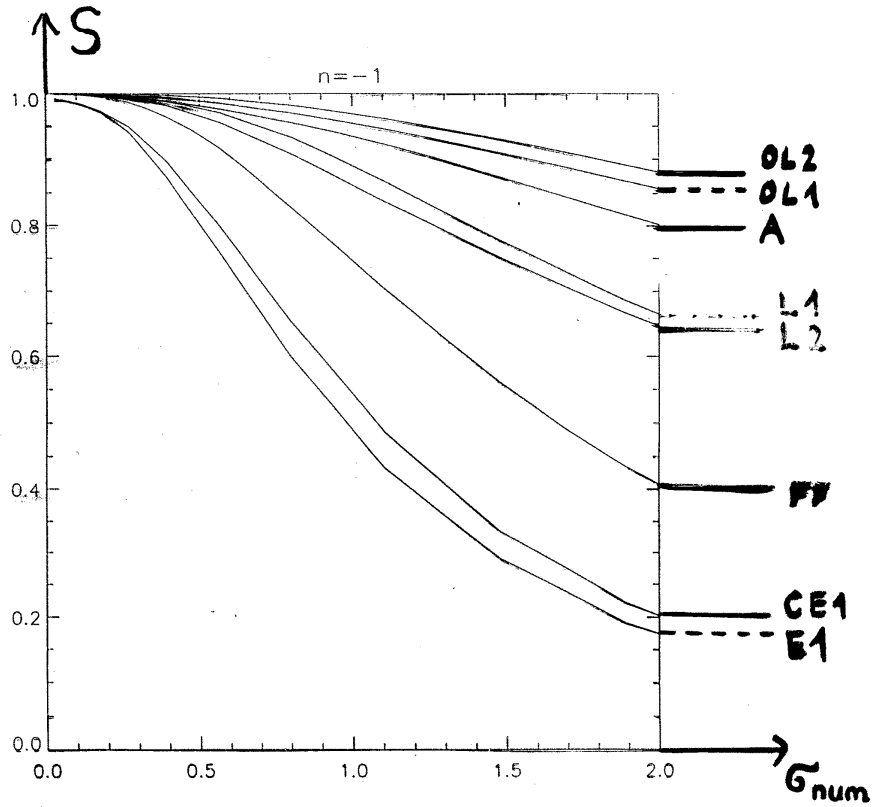


Figure 1. Comparison of analytical approximation schemes: Displayed is the cross-correlation coefficient  $S = \langle \frac{\delta_{\text{num}} \delta_{\text{approx}}}{\sigma_{\text{num}} \sigma_{\text{approx}}} \rangle$  that compares the density contrast fields  $\delta$  of a variety of analytical schemes (normalized to their r.m.s. values  $\sigma$ ) and averaged over some scale as a function of wavenumber; from top to bottom: Optimized Lagrangian approximations (second-order (Melott et al. 1995) and first-order (Coles et al. 1993, Melott et al. 1994)), Adhesion model (Gurbatov et al. 1989, Weinberg & Gunn 1990), Zel'dovich approximation (Zel'dovich 1970), Second-order Lagrangian approximation (Buchert & Ehlers 1993), Frozen-Potential approximation (Bagla & Padmanabhan 1994, Brainerd et al. 1994), Linear theory ('chopped', i.e. with adapted average, and unchopped).  $S$  measures whether mass is moved to the right place,  $S = 1$  means ideal congruence of both fields.

### 3. Adhesion: effective dynamics of multi-stream systems.

Working towards a model that continues to be valid after shell-crossing singularities develop may be grounded on multi-stream hydrodynamics of interpenetrating streams of a cold medium. This way to go, although being logically the most adequate one, may not have the broad range of expression that we gain by moving to a kinetic description. The conception to be presented below, however, is a phenomenological one. We keep the notion of a continuous (coarse-grained) fluid element, but we consider its motion in phase space. Thus we are dealing with a one-particle distribution function, or simply a phase space density, neglecting terms that would arise by coarse-graining the  $N$ -particle distribution function. The basic system of equations therefore consists of the so-called Vlasov-Poisson system. By averaging out the velocity information we end up with a set of equations that describe an effective dynamics of a possibly multi-streamed medium. (For details see: Buchert & Domínguez 1998). This effective dynamics will in general entail a hierarchy of evolution equations for the velocity moments of the phase space density. Here is what I shall call the “phenomenological closure” condition: imagine the shape of a fluid element in velocity space that crosses a caustic in real space. The so-called velocity ellipsoid, centered on a fluid element with a bulk velocity that originally coincides with the velocities of individual particles, will expand in a highly anisotropic fashion; as soon as a three-stream system develops, two of the streams will head off in almost opposite directions and the third (decaying) stream will have small velocity, so that overall the ellipsoid starts out to be a flattened disk. Only later, when more and more streams develop, the ellipsoid has a chance to become spherical (“virialized”). We are going to mask this ellipsoid with a sphere for all times and look at the effective multi-stream force that is exerted on the bulk motion  $\bar{\mathbf{v}} = \langle \mathbf{v} \rangle$  of the coarse-grained fluid element. Mathematically, we require that the velocity dispersion tensor be idealized by an isotropic tensor:

$$\Pi_{ij} = \varrho(\langle \mathbf{v}\mathbf{v} \rangle - \bar{\mathbf{v}}\bar{\mathbf{v}}) = p\delta_{ij} \quad ; \quad p = \alpha(\varrho) > 0 \quad . \quad (1)$$

The last equation requires in addition that the multi-stream pressure should be given as a function of the density only. This is the ‘poor man’s way’ to close the hierarchy.

In cosmology we study the evolution equations for the two dynamical fields peculiar-velocity  $\mathbf{u}$  and peculiar-acceleration  $\mathbf{w}$ , or the density  $\varrho$ , respectively. The corresponding effective evolution equations for these fields with the assumptions sketched above are ( $H := \frac{\dot{a}}{a}$ ):

$$\partial_t \varrho + 3H\varrho + \frac{1}{a}(\varrho u_i)_{,i} = 0 \quad , \quad \partial_t u_i + \frac{1}{a}u_j u_{i,j} + H u_i = w_i - \frac{\alpha'}{\varrho a} \varrho_{,i} \quad , \quad (2)$$

subjected to the Newtonian field equations  $w_{i,i} = -4\pi G a(\varrho - \varrho_H)$ ,  $w_{i,j} = w_{j,i}$ . Note that  $\mathbf{u}$  is the bulk velocity of a fluid element in coordinates that are comoving with the Hubble flow (the overbar is omitted for simplicity, and a comma means spatial derivative with respect to these coordinates). Figure 2 illustrates how this bulk flow will give away its kinetic energy into internal kinetic energy of the fluid element; the elements acquire a “temperature” when they move into the multi-stream region, while the amplitude of the bulk flow decays.

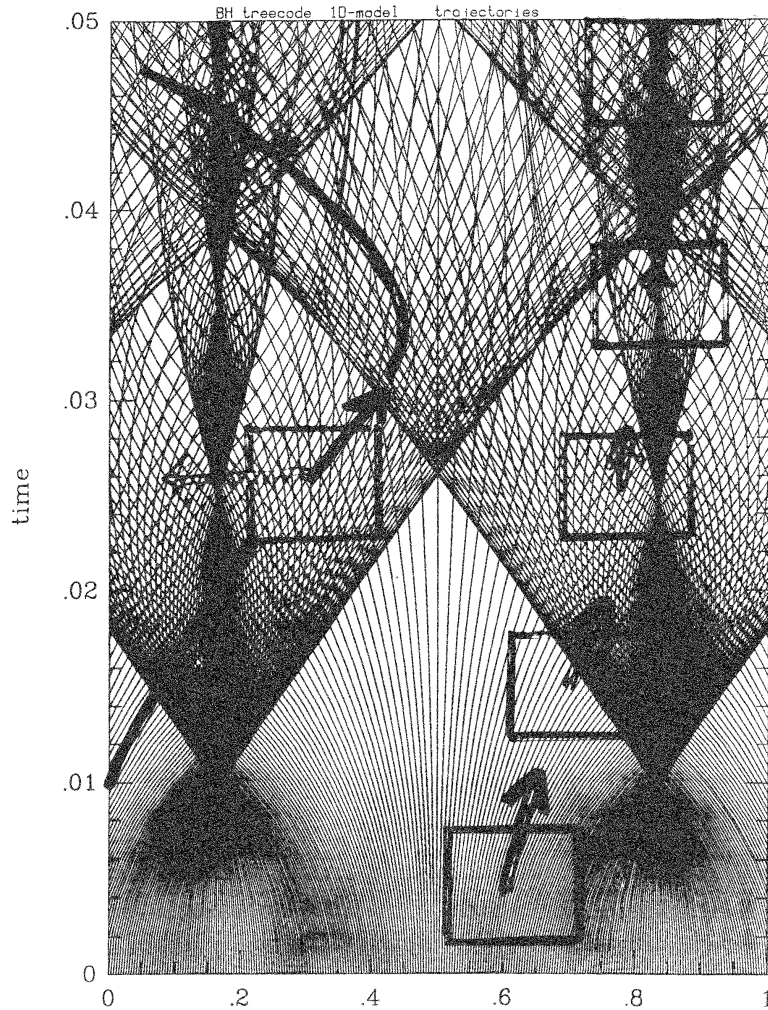


Figure 2. The coarse-graining idea is exemplified for a family of crossing trajectories from a 2D tree-code simulation: we appreciate the oscillatory behavior of innermost trajectories that are kept back from escaping the high-density region due to multi-stream forces; within the multi-stream regions there are many streams (velocities) at a given Eulerian position. Two possible situations are highlighted for a coarse-grained volume element with oscillatory bulk velocity (left) and smoothly decaying bulk velocity (right): the kinetic energy of the bulk motion is transformed into internal kinetic energy of the coarse-grained element as the element moves into the beast.

### 3.1. Old Adhesion.

One of the most committed of all approaches to multi-streaming is furnished by Burgers' equation. An exact solution in 3D is available. Still, numerical techniques await a highly resolved insight into the multi-stream regime. Instead, the “burgerlencing” effect, being quintessential for holding fluid elements together inside high-density peaks, is roughhewn into the phenomenological language of ‘adhesion’. A smart way to employ Burgers' equation on the cosmological stage was proposed by Gurbatov et al. (loc.cit.). Below, we give their formal arguments leading to the ‘adhesion approximation’, which we right after that shall derive from kinetic theory following Buchert & Domínguez (loc.cit.).

Certainly one of the simplest ways of stating the formal structure of the ‘Zel’dovich approximation’ (Zel’dovich 1970, 1973) is to postulate a law of motion of the form

$$\mathbf{w} = F(t)\mathbf{u} \quad . \quad (3)$$

If gravity drags into the direction of the peculiar-velocity field, then the gravitational field equations are not needed to close the system of equations (2): the peculiar-velocity solves

$$\dot{\mathbf{u}} + (H - F(t))\mathbf{u} = \mathbf{0} \quad . \quad (4)$$

If the field is appropriately scaled and a new time-variable is introduced (see below), Zel’dovich’s approximation manifests itself as an essentially force-free description of the continuum.

Gurbatov et al. (loc.cit.) proposed that one should add a forcing that is directly proportional to the Laplacian of the peculiar-velocity field to this equation which, in the scaled variables, attains the form of Burgers' equation. To derive the ‘adhesion approximation’ from the effective kinetic equations (2) we just need to insert the law of motion (3) into the second of the equations (2), which involves the multi-stream “pressure”:

$$\dot{\mathbf{u}} + (H - F(t))\mathbf{u} = \zeta F(t)\Delta_{\mathbf{q}}\mathbf{u} \quad ; \quad \zeta := \frac{\alpha'(\varrho)}{a^2} \frac{1}{4\pi G\varrho} \quad . \quad (5)$$

In the ‘adhesion approximation’ the function  $F(t)$  is determined as in Zel’dovich’s approximation by the requirement of reproducing the linear solution of gravitational instability:

$$F(t) = 4\pi G\varrho_H \frac{b(t)}{\dot{b}(t)} \quad , \quad (6)$$

where  $b(t)$  is identical to the growing density contrast mode solution of the Eulerian linear theory of gravitational instability for “dust” (i.e., it solves the equation  $\dot{b} + 2H\dot{b} - 4\pi G\varrho_H b = 0$ ).

Changing the temporal variable from  $t$  to  $b$  and defining a rescaled velocity field  $\tilde{\mathbf{u}} := \mathbf{u}/a\dot{b}$ , Equation (5) becomes the well-known key equation of the ‘adhesion approximation’ where  $\mu$  is assumed constant (Gurbatov et al. loc.cit.):

$$\frac{d\tilde{\mathbf{u}}}{db} = \mu\Delta_{\mathbf{q}}\tilde{\mathbf{u}} \quad , \quad \frac{d}{db} := \frac{\partial}{\partial b} + \tilde{\mathbf{u}} \cdot \nabla_{\mathbf{q}} \quad , \quad \mu := \frac{\zeta F(t)}{\dot{b}} = \frac{\alpha'(\varrho)}{a^2} \frac{\varrho_H}{\varrho} \frac{b}{\dot{b}^2} \quad . \quad (7)$$

### 3.2. The Lagrangian Linear Regime.

The ‘adhesion approximation’, as we saw above, can be derived from kinetic routes. Although this derivation is not a contrived experimenting way of formally getting the Laplacian forcing, we may criticize it for its limited range of validity in the kinetic framework. Buchert & Domínguez (loc.cit.) have shown that the description has to be limited to small velocity dispersion which is a necessary condition to still follow Zel’dovich’s trajectories for the bulk flow (the law of motion (3)). Starting out from the full system of effective kinetic equations we may pursue a systematic way of constructing models of ‘adhesion’ by using the Lagrangian perturbation theory. To the first order in the displacements from a homogeneous–isotropic reference cosmology Adler & Buchert (1999) obtain for the longitudinal part of the (Lagrangian) displacement field  $\mathbf{P}(\mathbf{X}, t)$ :

$$\ddot{\mathbf{P}} + 2H\dot{\mathbf{P}} - 4\pi G_{\rho_H}\mathbf{P} = \frac{C_\alpha}{a^2}\Delta_{\mathbf{X}}\mathbf{P} \ ; \ C_\alpha := \alpha' = \text{const.} \ , \ \dot{\mathbf{X}} = \mathbf{0} \ . \quad (8)$$

The markedly familiar differential operator in this equation helps to construct solutions of this Lagrangian linear equation from known solutions of the Eulerian linear theory. The so constructed solutions may be employed as models for adhesive gravitational clustering in the weakly nonlinear regime.

### 3.3. New Adhesion.

The apparent disparity between the standard ‘adhesion model’ and the Lagrangian perturbation approach can be made more transparent by reorganizing the general equations into a single equation for the gravitational peculiar–acceleration:

$$\ddot{\mathbf{w}} + 6H\dot{\mathbf{w}} + (2\dot{H} + 8H^2 - 4\pi G_{\rho_H})\mathbf{w} = 4\pi G_{\rho_H}\zeta\Delta_{\mathbf{q}}\mathbf{w} + \dot{\mathcal{R}} + 4H\mathcal{R} \ . \quad (9)$$

$\mathcal{R}$  represents nonlinear residuals in the equations which are touched a little further in a paper in preparation (Buchert 1999).

Hence, we are led to suggesting the following model equation for adhesive gravitational clustering in the nonlinear regime, neglecting the residuals:

$$\ddot{\mathbf{w}} + 6H\dot{\mathbf{w}} + (2\dot{H} + 8H^2 - 4\pi G_{\rho_H})\mathbf{w} = 4\pi G_{\rho_H}\zeta\Delta_{\mathbf{q}}\mathbf{w} \ . \quad (10)$$

A beautiful feature of this equation is that it embodies two limiting cases: 1. the standard ‘adhesion approximation’ in the limit of small velocity dispersion, and 2. the Lagrangian linear model (8) being a solution of

$$\ddot{\mathbf{w}} + 6H\dot{\mathbf{w}} + (2\dot{H} + 8H^2 - 4\pi G_{\rho_H})\mathbf{w} = \frac{C_\alpha}{a^2}\Delta_{\mathbf{X}}\mathbf{w} \ . \quad (11)$$

The proposed new models will have, by construction, an improved performance for the modeling of large–scale structure, but they will hopefully also give more insight into the clustering properties at the stages of stabilization of large–scale structure, previrialization and the onset to “virialized” systems. A further clue might be inferred with regard to the possible emergence of N–soliton states (compare Götz (1988) for a one–dimensional example).

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