# FOUR ISSUES IN CORRELATIONS AND FLUCTUATIONS 

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#### Abstract

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## 1 Introduction

The purpose of this talk, as I understand it, is to introduce briefly the subject of the Session to the participants who are not experts in the field. In trying to do this I shall heavily borrow from my recent summary of the Matrahaza workshop [1]. Since I am now much less constrained by the program of the meeting, however, this account shall reflect more adequately my personal views on the subject. I restrict myself to the four issues:
(i) Bose-Einstein interference;
(ii) Intermittency;
(iii) QCD and multiparticle correlations;
(iv) Event-by-Event fluctuations.

## 2 Bose-Einstein interference

The discussion of correlations in multiparticle production is at present largely dominated by effects of the Bose-Einstein interference. Let me thus start by a brief reminder what is this all about ${ }^{1}$.

The practical problem we face can be formulated as follows: given a calculation (or a model) which ignores identity of particles, the question is how to "correct" it in order to take into account the effects of quantum interference (which is the consequence of identity ${ }^{2}$ ). Let us thus suppose that we have an amplitude for production of N particles $M_{N}^{(0)}(q)\left(q=q_{1}, \ldots q_{N}\right)$, calculated with the identity of particles being ignored. The rules of quantum mechanics tell us that, to take the identity of particles into account, we have to replace $M_{N}^{(0)}(q)$ by a new amplitude $M_{N}(q)$ which is a sum over all permutations of the momenta $\left(q_{1}, \ldots q_{N}\right)$

$$
\begin{equation*}
M_{N}^{(0)}(q) \rightarrow M_{N}(q) \equiv \sum_{P} M_{N}^{(0)}\left(q_{P}\right) . \tag{1}
\end{equation*}
$$

This would be the end of the story if particle production was described by a single matrix element. In general, however, we have to average over parameters which are not measured and therefore the correct description of the multiparticle final state is achieved in terms of the density matrix

$$
\begin{equation*}
\rho_{N}^{(0)}\left(q, q^{\prime}\right)=\sum_{\omega} M_{N}^{(0)}(q, \omega) M_{N}^{(0) *}\left(q^{\prime}, \omega\right), \tag{2}
\end{equation*}
$$

rather than in terms of a single production amplitude. The sum in (2) runs over all quantum numbers $\omega$ which are not measured in a given situation. $\rho^{(0)}\left(q, q^{\prime}\right)$ gives all available information about the system in question. At this point it is useful to note that, when tranformed into (mathematically equivalent) Wigner representation

$$
\begin{equation*}
W_{N}(\bar{q}, x)=\int d(\Delta q) e^{i x \Delta q} \rho_{N}^{(0)}(\bar{q}, \Delta q) \tag{3}
\end{equation*}
$$

[^0]$\left(\bar{q}=\left(q+q^{\prime}\right) / 2 ; \quad \Delta q=q-q^{\prime}\right)$ it gives information about the distribution of momenta and positions of the particles (see, e.g., [3] for a discussion of this point).

Using (1) and (2) one easily arrives at the formula for the corrected (i.e., with identity of particles taken into account) density matrix $\rho_{N}\left(q, q^{\prime}\right)$ and one finally obtains the observed multiparticle density

$$
\begin{equation*}
\Omega_{N}(q)=\frac{1}{N!} \sum_{P, P^{\prime}} \rho^{(0)}\left(q_{P}, q_{P^{\prime}}\right) \tag{4}
\end{equation*}
$$

where the sum runs over all permulations $P$ and $P^{\prime}$ of the momenta $\left(q_{1}, \ldots q_{N}\right)$. The factor $\frac{1}{N!}$ appears because the phase space for $N$ identical particles is $N$ ! times smaller than the phase space for $N$ non-identical particles. The formula (4) is in common use ${ }^{3}$ and is the basis of our further discussion.

### 2.1 A theoretical laboratory: independent particle production

The case of independent particle production is an attractive theoretical laboratory which, although not expected to describe all details of the data, reveals -nevertheless- some generic features of the problem. This was first recognized by Pratt [4]. In terms of the density matrix, the independent production means that the density matrix factorizes into a product of singleparticle density matrices

$$
\begin{equation*}
\rho_{N}^{(0)}\left(q, q^{\prime}\right)=\rho^{(0)}\left(q_{1}, q_{1}^{\prime}\right) \rho^{(0)}\left(q_{2}, q_{2}^{\prime}\right) \ldots . \rho^{(0)}\left(q_{N}, q_{N}^{\prime}\right) \tag{5}
\end{equation*}
$$

and that the multiplicity distribution is the Poisson one

$$
\begin{equation*}
P^{(0)}(N)=e^{-\nu} \frac{\nu^{N}}{N!} . \tag{6}
\end{equation*}
$$

It turns out $[5,6]$ that in the case of a Gaussian density matrix the problem can be solved analytically. The main results (valid also in the general case of an arbitrary density matrix [7]) can be listed as follows.

[^1](a) All correlation functions $K_{p}\left(q_{1}, \ldots, q_{p}\right)$ and the single particle distribution $\Omega(q)$ can be expressed in terms of one (hermitian) function $L\left(q, q^{\prime}\right)=$ $L^{*}\left(q^{\prime}, q\right)$ of two momenta:
\[

$$
\begin{array}{r}
\Omega(q)=L(q, q) ; \quad K_{2}\left(q_{1}, q_{2}\right)=L\left(q_{1}, q_{2}\right) L\left(q_{2}, q_{1}\right) ; \\
K_{3}\left(q_{1}, q_{2}, q_{3}\right)=L\left(q_{1}, q_{2}\right) L\left(q_{2}, q_{3}\right) L\left(q_{3}, q_{1}\right)+L\left(q_{1}, q_{3}\right) L\left(q_{3}, q_{2}\right) L\left(q_{2}, q_{1}\right), \tag{7}
\end{array}
$$
\]

and analogous formulae for higher correlation functions.
(b) At very large phase-space density of particles, the distribution approaches a singular point representing the phenomenon of Bose-Einstein condensation: almost all particles populate the eigenstate of $\rho^{(0)}\left(q, q^{\prime}\right)$ corresponding to the largest eigenvalue. The resulting multiplicity distribution is very broad (almost flat) so that, e.g., the probability of an event with no single $\pi^{0}$ produced is non-negligible ${ }^{4}$.

It should be not surprizing if the very restrictive condition of independent production, as expressed by $(5,6)$ is not realized in nature. Nevertheless the comparison of the relations (7) with data is interesting, since they are a sort of reference point allowing to judge if the observed multiparticle correlations are "large" or "small" with respect to the observed two-body correlations. The existing evidence leads to rather interesting, although controversial, conclusions. At the Matrahaza meeting, Lorstad [10] demonstrated that there are practically no genuine three-particle correlations ${ }^{5}$ in $\mathrm{S}-\mathrm{Pb}$ collisions at CERN SPS. Since the two-particle correlations are clearly visible, this observation is not easy to reconcile with Eq.(7). It was earlier shown by Eggers et al [12] that the UA1 data are also in contradiction with (7), although in this case the 3 -body correlations seem to be too large to satisfy (7). On the other hand, it was shown recently by Arbex et al [13] that the NA22 data agree well with (7). This striking difference between the behaviour of heavy ion and "elementary" collisions is certainly very interesting and deserves further attention.

We cannot thus consider the results obtained from $(5,6)$ to be a realistic description of the data. Nevertheless, the main conclusion about the possi-

[^2]bility of Bose-Einstein condensation remains an interesting option which is worth a serious consideration $[14,15]$.

### 2.2 Monte Carlo simulations

In this situation, the practical method to study the effects of BE symmetrization on particle spectra is to implement it into the Monte Carlo codes. A "minimal" method of performing this task was suggested some time ago [3]. The idea is to take an existing code (which reproduces the distribution of particle momenta, i.e. the diagonal elements of the density matrix) and to introduce an ansatz for the off-diagonal elements of the multiparticle density matrix $(2)^{6}$. Each event generated by the MC code is then given a weight which is calculated as the ratio of symmetrized distribution [Eq.(4)], and the unsymmetrized one. In this way the modification of the original spectra is kept at the minimum.

A practical realization of this idea (in its simplest version) has been developped by the Cracow group [16] and shall be presented by Fialkowski at this meeting. They propose the unsymmetrized density matrix in the form

$$
\begin{equation*}
\rho_{N}^{(0)}\left(q, q^{\prime}\right)=P_{N}(\bar{q}) \prod_{i=1}^{N} w\left(q_{i}-q_{i}^{\prime}\right) \tag{8}
\end{equation*}
$$

where $P_{N}(q)$ is the probability of a given configuration obtained in JETSET and $w$ is a Gaussian. This prescription does not modify the diagonal elements of the unsymmetrized density matrix $(w(0)=1)$ and, moreover, does not introduce any new correlations between emission points of the produced particles (when transformed into Wigner representation, Eq.(4), the product $\Pi w\left(q_{i}-q_{i}^{\prime}\right)$ becomes the product $\left.\Pi w\left(x_{i}\right)\right)$. Thus (8) can indeed be considered as a minimal modification of the existing code. The autors find that this prescription represents well the existing data on two-particle correlations and that they can recover the experimental multiplicity distribution by a simple rescaling with the formula $P(N) \rightarrow P(N) c V^{N}$, without the necessity of refitting the JETSET parameters.

They also studied production of pair of $W$ bosons at LEP II [16] and found fairly strong effects of quantum interferrence. One may note, however,

[^3]that in present version of the model the position of particle emission point is not correlated with its momentum, whereas such correlation is likely to be present in reality. Consequently, the obtained results may be overestimating the effect. This deficciency is easy to repair [3] but on the prize of introducing more parameters.

A more fundamental approach has been pursued since some time by Andersson and Ringner [17] and shall be presented here by Todorova-Nova. It is based on the paper by Andersson and Hoffman [18]. In this case the "uncorrected" matrix element represents the decay of one Lund string which is then symmetrized according to the procedure explained in introduction. Two particle correlations are well described and several interesting effects are predicted. Among them: (a) the longitudinal and transverse correlations are expected to be different because they are controlled by two different physical mechanisms; (b) Three particle correlations are predicted non-vanishing and were actually calculated; (c) $W W$ production was studied and no significant mass shift is expected; (d) No multiplicity shift in the $W$ decay is predicted.

This last conclusion is a consequence of the fact that, in case of more than one string present in the final state, no symmetrization between particles stemming from different strings is performed. This corresponds to the assumption that the strings are created at a very large distance from each other. One thus may expect that in a more realistic treatment some multiplicity shift should be present ${ }^{7}$

Finally, let me add that in both [16] and [17] the "interconnection effect" [20] (which has tendency to reduce the multiplicity) is neglected. The full phenomenological analysis of the data is therefore certainly more complicated, as we shall hear from de Jong.

### 2.3 Probing the space-time structure

Much attention is also devoted to the information one may obtain from the data on quantum interference about the space-time structure of the multiparticle system created in the collision [c.f.Eq.(4)]. Although such analyses have a somewhat limited scope, as they (i) provide only information about the system at the freeze-out and (ii) require several additional assumptions - they give nevertheless a unique opportunity to investigate this problem.

[^4]Most of the caveats are thus usually postponed to the future (and better data) and the analysis is carried on.

The recently presented investigations were based on the hydrodynamic approach. Some of them were discussed here already during the Session on Heavy Ion Interactions. The main features ${ }^{8}$ are: (i) The shape of the particle emission region is consistent with the in-out scenario of Bjorken [21]; (ii) The longitudinal size of the "fireball" from which a bulk of particles are emitted is several times larger in heavy ion collisions than in hadronhadron interactions; (iii) Particle emission process starts rather late in heavy ion collisions (after about 4 fm in $\mathrm{S}-\mathrm{Pb}$ interactions []), as compared to the elementary collision where it happens immediately after collision []; (iv) the emission process, once started, does not last for a long time: less than 2 fm for elementary and about 3 fm for $\mathrm{S}-\mathrm{Pb}$ interactions.

These features clearly indicate that a heavy ion collision is indeed followed by creation of an longitudinally expansing "fireball" in which some kind of matter is "boiling" for a considerable time. Once it is sufficiently cooled, however, its decay is rather fast. It is presumably too early to claim that it is formed from the quark-gluon plasma but nevertheless this behaviour is rather suggestive.

## 3 Intermittency

Intermittency [22], postulates scaling of the multiparticle spectra. A rather complete review of the subject is now available [23], therefore I shall restrict myself to few remarks expressing my personal view on the progress achieved in the last decade.
(i) The scaling hypothesis can be formulated in many ways and, indeed, much work was devoted to improvements and generalizations of the original proposal [22] expressed in terms of the (normalized) factorial moments

$$
\begin{equation*}
F_{q} \equiv<n(n-1) \ldots(n-q+1)>/<n>^{q} \sim(\text { bin size })^{-f_{q}} . \tag{9}
\end{equation*}
$$

The result was an impressive progress in the developpment of more sophisticated tools which are much better suited for investigation of many, sometimes very detailed, aspects of the problem. Let me particularly emphasize

[^5]the importance of the correlation integrals, first introduced in [24]. Their use was decisive in improving accuracy of the data and thus to substantiate the evidence for the effect.
(ii) On the experimental side, the analysis of high precision data (particularly those of NA22 and UA1 experiments) allowed to establish -beyond a reasonable doubt- a close connection between intermittency and Bose-Einstein correlations, as suggested $[25,26]$ almost immediately after first experimental evidence for increasing factorial moments. This observation allowed to understand the scaling of the momentum spectra as a reflection of the -more fundamental- scaling in configuration space [27]. I have impression that the importance of this fact is not yet fully appreciated.
(iii) Recently, a general solution of the model of multiplicative cascade was obtained [28]. One can thus hope for a significant progress in understanding the scaling phenomenon.
(iv) Finally, let me also mention another interesting development, namely the generalization of the notion of scaling by the idea of self-affinity [29]. I think it is an interesting direction to pursue and I hope that more data on the subject shall be soon available. We shall hear more about this from Liu.

The major disappointment I see after all these years is that -in fact- no convincing theoretical basis was found for the phenomenon of intermittency, although it seems to be indeed an universal feature of particle spectra $[23,30]$ ${ }^{9}$. Does it mean that the simple effect one observes is a purely accidental result of summation of much more complex contributions? Perhaps. Nevertheless, I am convinced that the search for a more fundamental reason of the apparent scaling in particle spectra is worth to continue.

## 4 QCD and multiparticle correlations

It is now rather well established that the average multiplicity and single particle spectra are well described by perturbative QCD supplemented with the principle of parton-hadron duality [32].

In my opinion, at present, the real challenge to the idea of parton-hadron duality is to explain the data on differential correlation functions [27]. Indeed,

[^6]it is hard to understand how the momenta of the produced hadrons can follow so closely the momenta of the created partons that the correlations between them are not washed out ${ }^{10}$. Therefore a non-trivial extension of the principle of parton-hadron duality must be formulated in order to give quantitative meaning to perturbative calculations of multiple production. It was therefore rather recomforting to learn that indeed the predictions of perturbative QCD formulated some time ago [33], are badly violated by the L3 data [34]. On the other hand, the same data are well described by the JETSET code. The conclusion is that the hadronization part is not correctly taken into account by the simple (naive?) parton-hadron duality. More about that later in this Session by Mandl.

This is not to say that the subject is closed: Ochs [35] pointed out that the tested QCD calculations included several simplifying assumptions (the most important among them seems the neglect of energy-momentum consevation) and thus it is not obvious which part of the result is actually responsible for the failure. It is clear, nevertheless, that further work on these lines must seriously address the problem of parton-hadron duality and its range of application.

## 5 Event-by-event analysis

Event-by-event analysis clearly emerges as a next logical step in studies of multiparticle fluctuations. The subject is not yet well developped, however, and neither the physics nor methods sufficiently understood to define precisely what we are really searching for. Therefore, I can only list a few ideas of potential interest. I am fully aware that some of them may not work and that others, more interesting, may well be proposed in the near future.

There are two basic reasons why event-by event fluctuations attract attention. The first one, more spectacular, is to look for large deviations of some events from the average, with the hope of finding a hitherto unobserved effect. The second, more pragmatic, is to measure the distribution of a quantity defined for a single event and thus obtain additional information, helping

[^7]to understand the physics of the process. This is well illustrated by multiplicity distribution which is the simplest event-by-event analysis one may think of. It was studied since long time ${ }^{11}$ and was of great help in understanding the physics of multiparticle production.

Let me now go to my list:
(i) Recently, Stodolsky [36] proposed to study fluctuations in transverse momentum (see also [15]). The idea is that, if the transverse momentum distribution in an event can be related to its "temperature", one obtains in this way the distribution of "temperatures" of the events. Now, if thermodynamics is a correct decription of the process in question, the fluctuations of temperature can be related to to heat capacity $C_{V}$ of the system [37]:

$$
\begin{equation*}
\frac{(\Delta T)^{2}}{T^{2}}=\frac{k}{C_{V}} \tag{10}
\end{equation*}
$$

where $k$ is the Boltzmann constant. This obviously may be very helpful in searching for phase transition. Even far from phase transition, however, a measurement of this kind can provide a lot of information on (a) the properties of the system in question and (b) whether it is indeed close to thermodynamic equlibrium. To take a simple example: In case of ideal gas one has $E=C_{V} T$, where $E$ is the energy of the system. We thus obtain

$$
\begin{equation*}
\frac{(\Delta T)^{2}}{T^{2}}=\frac{k T}{E} . \tag{11}
\end{equation*}
$$

The point is that both L.H.S. and R.H.S. of this equation can be measured and thus one may hope to estimate the deviation of the system from the ideal gas approximation. ${ }^{12}$
(ii) Another important issue was raised by Hwa [39]. He pointed out the essential difference between the determination of fractal parameters in case of dynamical systems and in case of systems of many particles. In the dynamical system one can generate the time sequence and thus estimate how fast the different trajectories diverge. In case of multiparticle systems we do not have a time sequence and thus we have to rely on patterns. The question in this

[^8]case is: how different are the patterns of different events. Hwa proposed to measure the pattern of an event by the factorial moment associated with it. One can then ask the question how this measure fluctuates from event to event. Studying moments of this distribution provides a measure of event-to-event fluctuation ${ }^{13}$. When they are considered as function of bin size, it is possible to define appropriate fractal dimensions which conveniently summarize the information. For the details the reader is referred to the original paper [39]. I personally feel that this is an important conceptual step in our thinking about the problem, although I am not fully convinced that the proposed measure cannot be improved.
(iii) The studies of possible phase transitions in the multiparticle systems produced in high-energy collisions [41] suggest that the fractal behaviour may strongly fluctuate from one event to another. It follows that it is essential to be able to study the fractal behaviour in event-by-event analysis. The feasibility of this program was investigated recently [43]. It seems to be rather promising.
(iv) The factorial moments can also be considered as a very sensitive signature for clustering of particles in small bins of momentum phase-space. Indeed, a factorial moment of order $q$ is sensitive only to the clusters containig at least $q$ particles. This obviously eliminates very effectively any background. This point was recently illustrated by KLM collaboration analysing the collisions of $160 \mathrm{GeV} / \mathrm{A} \mathrm{Pb}$ nucleons in emulsion [44]. They found that events may differ drastically in the behaviour of their factorial moments, while no great difference is seen when they are analysed by other methods.
(v) The distribution of the HBT radii obtained from individual events was also recognized since some time as a very interesting object to study. Recently, first data on this subject were presented by NA49 collaboration $[45,1]$. Although statistics is still limited (and the authors themselves do not attach too much meaning to the details of the plot) the results clearly show that the measurement is feasible and we may well hope for some exciting news in not-too-distant future.

[^9]
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[^0]:    ${ }^{1}$ The physics of Bose-Einstein correlations was recently extensively reviewed by G.Baym [2]
    ${ }^{2}$ It should be understood that this problem is very common in quantum mechanical calculations, as illustrated, e.g., by evaluation of Feynman diagrams. I would like to thank J.Pisut and K.Zalewski for discussions of this question.

[^1]:    ${ }^{3}$ Using the symmetry properties of the density matrix, the double sum in (4) can be reduced to a single sum. The factor $\frac{1}{N!}$ is then absent.

[^2]:    ${ }^{4}$ This effect was also considered in connection of the possible production of the Disoriented Chiral Condensate [8, 9]. The present argument adds another obstacle on the difficult road to observation of DCC.
    ${ }^{5}$ The importance of the absence of 3-particle correlations in heavy ion collisions was emphasized already some time ago [11].

[^3]:    ${ }^{6}$ As seen from (3) this corresponds to introducing an - a priori arbitrary - distribution of particle emision points in configuration space.

[^4]:    ${ }^{7}$ A contribution to this problem was presented recently by B.Buschbeck et al. [19, 1].

[^5]:    ${ }^{8} \mathrm{~A}$ more detailed description is given in [1].

[^6]:    ${ }^{9}$ The second order phase transition was invoked by many authors [31] as a possible explanation. This is certainly a valid idea but it does not explain universality of the phenomenon.

[^7]:    ${ }^{10}$ This problem is much less serious if one considers only the integrated correlation functions. In this connection, see the discussion at the recent meeting on Correlations and Fluctuations, Matrahaza, June 1998 and contibutions to this session by Metzger and Chekanov.

[^8]:    ${ }^{11}$ Contributions related to multiplicity distributions shall be presented by Hegyi, Ploszajczak and Blazek.
    ${ }^{12}$ Recently, the first results on temperature fluctuations were presented by NA49 coll. [38].

[^9]:    ${ }^{13}$ To study the moments of the factorial moments was suggested already some time ago [40].

