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On (multi-)center branes and exact string vacua*

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Abstract

Multicenter supergravity solutions corresponding to Higgs phases of supersymmetric Yang–Mills theories are considered. For NS5 branes we identify three cases where there is a description in terms of exact conformal field theories. Other supergravity solutions, such as D3-branes with angular momentum, are understood as special limits of multi-center ones. Within our context we also consider 4-dim gravitational multi-instantons.

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1 Introduction

It has been conjectured that strong coupling issues in supersymmetric Yang–Mills (SYM) theories can be investigated using string theory on backgrounds containing AdS spaces [1] (see also [2]). In this note, based to an extend on [3], we present some supergravity solutions that describe the strong coupling regime of (SYM) theories for $SU(N)$ in the Higgs phase, where the gauge group is broken. We show that in the case of NS5-branes, where R–R fields are absent, there exist exact conformal field theory (CFT) descriptions in a number of interesting cases. The well known example corresponds to the $SU(2)\times U(1)$ WZW model CFT [4], representing the throat of a semi-wormhole, as the near horizon geometry of NS5 branes [5] at one center. A less known example corresponds to a specific limit of non-extremal NS5 branes (at one center), where the string coupling and non-extremality parameter both go to zero with their ratios held fixed. In this case the exact CFT description is in terms of $SL(2,\mathbb{R})/SO(1,1) \times SU(2)$ [6]. The first factor corresponds to the famous 2-dim black hole solution [7]. A third example [3] is that of the coset CFT $SU(2)/U(1) \times SL(2,\mathbb{R})/U(1)$ describing branes uniformly distributed on a circle whose radius dictates the Higgs expectation value for the scalars in the SYM theory side. We also present a solution that interpolates between the first and third cases. In addition, we give a new interpretation for the BPS limits of rotating D3-brane solutions as special limits of multicenter ones. We have also written an appendix discussing (multi)-Eguchi–Hanson and (multi)-Taub–NUT metrics with removable NUT singularities at points uniformly distributed on a circle.

2 Branes on a circle

Consider a d-dim supergravity solution corresponding to Nk parallel p-branes, which are separated into N groups, with k branes each, and have centers at $\vec{x} = \vec{x}_i$, $i = 1, 2, \cdots, N$. It is characterized by a harmonic function with respect to the $(n + 2)$-dim space E^{n+2}, which is transverse to the branes

$$H_n = 1 + \sum_{i=1}^{N} \frac{akC(\vec{x}_i)}{|\vec{x} - \vec{x}_i|^n} \ , \qquad n = d - p - 3 \ , \qquad \sum_{i=1}^{n} C(\vec{x}_i) = N \ , \tag{1}$$

where a is a constant that may depend, according to the type of brane we discuss, only on the Planck length $l_{\rm P}$, the string length l_s and the dimensionless string coupling constant g_s. For a generic choice of vectors $\vec{x}_i$ and weight-function $C(\vec{x}_i)$, the $SO(n+2)$ symmetry of the transverse space is broken. Here we will make the simple choice that all the centers lie in a ring of radius r_0 in the plane defined by x_{n+1} and x_{n+2} and also that $\vec{x}_i = (0, 0, \cdots, r_0 \cos\phi_i, r_0 \sin\phi_i)$, with $\phi_i = 2\pi i/N$. Moreover we choose $C(\vec{x}_i) = 1$. Hence the $SO(n + 2)$ symmetry of the transverse space is broken to $SO(n) \times Z_N$. Since the $\vec{x}_i$'s correspond to non-zero vacuum expectation values (vev's) for the scalars, the corresponding super Yang–Mills theory is broken from $SU(kN)$ to $U(k)^N$, with the vacuum having a Z_N symmetry. Then (1) can be written as

$$
\begin{aligned}
H_n &= 1 + ak \sum_{i=0}^{N-1} (r^2 + r_0^2 - 2r_0\rho\cos(2\pi i/N - \psi))^{-n/2} \ , \\
r^2 &= \vec{x}^2 \ , \qquad x_{n+1} = \rho\cos\psi, \quad x_{n+2} = \rho\sin\psi \ . \qquad (2)
\end{aligned}
$$

The finite sum in (2) can be computed for even (odd) n from that for $n = 2$ ($n = 1$). In this way we find the harmonic corresponding to N D5 (NS5) branes on a circle of radius r_0 to be [3]

$$
H_2 = 1 + \frac{kNl_s^2 g_s^\delta}{((r^2 + r_0^2)^2 - 4r_0^2\rho^2)^{1/2}} \ \Lambda_N(x, \psi) \ , \qquad (3)
$$

where

$$
\begin{aligned}
\Lambda_N &\equiv \frac{\sinh(Nx)}{\cosh(Nx) - \cos(N\psi)} = \frac{1}{2}\left(\coth(N(x + i\psi)) + \coth(N(x - i\psi))\right) \\
&= 1 + \sum_{m\neq 0} e^{-N(|m|x - im\psi)} \ , \qquad (4)
\end{aligned}
$$

and $\delta = 1$ (0) for D5 (NS5) branes. Similarly, the harmonic corresponding to N D3 branes on a circle of radius r_0 is [3]

$$
H_4 = 1 + \frac{4\pi N k g_s l_s^4 (r^2 + r_0^2)}{((r^2 + r_0^2)^2 - 4r_0^2\rho^2)^{3/2}} \ \Sigma_N(x, \psi) \ , \qquad (5)
$$

where

$$
\begin{aligned}
\Sigma_N &\equiv \Lambda_N + N\frac{((r^2 + r_0^2)^2 - 4r_0^2\rho^2)^{1/2}}{r^2 + r_0^2} \, \frac{\cosh Nx \cos N\psi - 1}{(\cosh Nx - \cos N\psi)^2} \\
&= 1 + \sum_{m\neq 0} \left(1 + \frac{((r^2 + r_0^2)^2 - 4r_0^2\rho^2)^{1/2}}{r^2 + r_0^2} \ N|m|\right) e^{-N(|m|x - im\psi)} \ . \qquad (6)
\end{aligned}
$$

The variable x appearing in (4), (6) is defined as

$$
e^x \equiv \frac{r^2 + r_0^2}{2r_0\rho} + \sqrt{\left(\frac{r^2 + r_0^2}{2r_0\rho}\right)^2 - 1} \ . \qquad (7)
$$

Note that in both (3) and (5) there is an explicit Z_N invariance under shifts of $\psi \to \psi + \frac{2\pi}{N}$. In the $\frac{1}{N}$-expansion, Λ_N and Σ_N have only a "tree-level" contribution, whereas the rest of the terms in the infinite sum are non-perturbative. In particular, the exponential factors $N(|m|x - m\psi)$ are likely to originate from configurations of the spontaneously broken gauge theory that interpolate between the N different degenerate vacua. It would be interesting, in the case of D3 branes, to find an interpretation in terms of configurations in the $\mathcal{N} = 4$ spontaneously broken SYM theory. For odd n's we have not been able to perform the corresponding finite sums exactly. However, these can be computed in the large N limit. Then we may replace the sum in (2) by an integral and give the result, for any n, in terms of a hypergeometric function

$$\begin{aligned} H_n &\approx 1 + akN \int_0^{2\pi} \frac{d\phi}{2\pi} (r^2 + r_0^2 - 2r_0\rho\cos\phi)^{-n/2} \\ &= 1 + akN(r^2 + r_0^2 + 2r_0\rho)^{-n/2} F\Big(\frac{1}{2}, \frac{n}{2}, 1, \frac{4r_0\rho}{r^2 + r_0^2 + 2r_0\rho}\Big) \ . \end{aligned} \tag{8}$$

The harmonic functions for (3), (5) reduce in the large N limit to those obtained using (8). The result for $n = 1$ will be further used in the appendix in connection with multi-instanton solutions of 4-dim gravity. The general harmonic function (1) as well as its derivatives, behave as $\frac{akN}{r^n}$ for large r's as expected. What is less obvious is that very close to the ring (8) behaves as a single-center harmonic smeared out completely along a transverse direction [3]. In other words our multicenter harmonic in E^{n+2} reduces to a single-center harmonic in E^{n+1}.

3 Branes on a disc

Consider N static D3-branes distributed, uniformly in the angular direction, inside a disc of radius l in the x_5-x_6 plane. Their centers are chosen at [3]

$$\begin{aligned} &\vec{x}_{ij} = (0,0,0,0, r_{0j}\cos\phi_i, r_{0j}\sin\phi_i) \ , \\ &\phi_i = \frac{2\pi i}{N} \ , \quad r_{0j} = l\left(j/\sqrt{N}\right)^{1/2} \ , \quad i,j = 0,1,\cdots,\sqrt{N}-1 \ . \end{aligned} \tag{9}$$

Since we are mainly interested in the large-N limit we may take $\sqrt{N}$ = integer without loss of generality. Then, the corresponding harmonic function becomes

$$\begin{aligned} H_4 &= 1 + 4\pi g_s l_s^4 \sum_{i,j=0}^{\sqrt{N}-1} \frac{1}{\Big(r^2 + r_{0j}^2 - 2\rho r_{0j}\cos(\phi_i - \psi)\Big)^2} \\ &\approx 1 + \frac{8\pi g_s l_s^4 N}{\sqrt{(r^2+l^2)^2 - 4l^2\rho^2}\left(r^2 - l^2 + \sqrt{(r^2+l^2)^2 - 4l^2\rho^2}\right)} \ , \\ & \quad r^2 = x_1^2 + \cdots + x_6^2 \ , \qquad \rho^2 = x_5^2 + x_6^2 \ , \end{aligned} \tag{10}$$

where the second line is an approximation, valid for large N. The harmonic function H_4 becomes singular in the x_5-x_6 plane inside a disc of radius $r = \rho = l$. The appropriate supergravity solution corresponds to the extremal limit of a rotating D3-brane solution of type-IIB supergravity that was found in [8] (using earlier work in [9]). The radius l of the ring becomes the angular momentum parameter of the rotating solution. We emphasize that a priori it is not obvious that a non-extremal version of the BPS solution with harmonic (10) should exist at all, since non-BPS branes exert forces against one another. In our case the gravitational attraction, which is no longer balanced by just the R–R repulsion, is now balanced by introducing angular momentum. It is tempting to attribute this balance to some kind of centrifugal force due to the rotation, but one should remember that this is intrinsic and not orbital angular momentum.[1] It would be interesting to extend the discussion of this section for the D3 brane solution with three rotating parameters [10].

4 D5's and NS5's on a circle

Now we specialize to the case of kN D5-branes on a circle of radius r_0 in the decoupling limit

$$u_i = \frac{x_i}{l_s^2} = \text{fixed} \ , \qquad U^2 = u_1^2 + u_2^2 + u_3^2 + u_4^2 \ , \qquad u^2 = u_3^2 + u_4^2 \ ,$$
$$g_{\rm YM}^2 = g_s l_s^2 = \text{fixed} \ , \qquad U_0 = \frac{r_0}{l_s^2} = \text{fixed} \ , \qquad l_s \to 0 \ . \tag{11}$$

We may take $r_0 \sim l_s/g_s^{1/2}$ or $U_0 \sim 1/g_{\rm YM}$ since the coupling constant $g_{\rm YM}$ is the only scale in the classical theory. In this limit the appropriate supergravity solution is (we omit the R–R 3-form magnetic field strength)

$$\begin{aligned} \frac{1}{l_s^2} ds^2 &= \frac{V}{\sqrt{g_{\rm YM}^2 Nk}} \, ds^2(E^{1,5}) + \frac{\sqrt{g_{\rm YM}^2 Nk}}{V} \, du_i du_i \ , \\ e^{\Phi} &= \frac{g_{\rm YM} V}{\sqrt{Nk}} \ , \end{aligned} \tag{12}$$

where V is a function of U and u defined as

$$V(U,u) = \Big((U^2 + U_0^2)^2 - 4 U_0^2 u^2 \Big)^{1/4} \Lambda_N^{-1/2}(x,\psi) \ , \tag{13}$$

and e^x is given by the corresponding expression in (3) with r, ρ and r_0 replaced by U, u and U_0 respectively. The S-dual description of (12) is in terms of kN NS5-branes with a supergravity description that uses the same harmonic function. The corresponding metric, antisymmetric tensor field strength and dilaton are

$$\begin{aligned} \frac{1}{l_s^2} ds^2 &= ds^2(E^{1,5}) + Nk V^{-2} \, du_i du_i \ , \\ \frac{1}{l_s^2} H_{ijk} &= Nk \epsilon_{ijkl} \partial_l V^{-2} \ , \\ e^{\Phi} &= \frac{\sqrt{Nk}}{g_{\rm YM} V} \ , \end{aligned} \tag{14}$$

and represent an axionic instanton [5]. Note that for energy regimes $g_{\rm YM} U - 1 \geq \frac{1}{N}$, the factor Λ_N in the expression for $V(U,u)$ can be ignored and be set to 1. Using the

[1] I thank D. Youm for a discussion on this point.

above, and assuming that $N \gg 1$ and that $U_0 \sim 1/g_{\rm YM}$, we may determine the energy ranges where the supergravity description or the "perturbative" 6-dim SYM theory one, are valid. The discussion parallels the one performed in [11] for the one-center solution and can be found in [3].

The geometrical interpretation of the solution (14) is that of a semi-wormhole with a fat throat and S^3-radius $\sqrt{Nk}l_s$ [12, 3]. However, as we get closer to any one of the centers, the solution tends to represent the throat of a wormhole with S^3-radius $\sqrt{k}l_s$. Hence, we think of (14) as a superposition of "microscopic" semi-wormholes distributed around a circle. This is to be contrasted with the zero size throat of the usual $SU(2)\times U(1)$ semi-wormhole.

4.1 Exact conformal field theory description

In the case of NS5-branes, we may find an exact CFT description for the background (14) in two limiting cases. For $N = 1$, corresponding to k NS5-branes at a single point, it is known that the exact description is in terms of the $SU(2)_k \times U(1)_Q$ WZW model, where $Q = \sqrt{\frac{2}{k+2}}$ is the background charge associated with the $U(1)$ factor [4, 5]. An exact CFT description is also possible when $N \gg 1$ [3]. After a change of variables and a T-duality transformation with respect to the vector field $\partial/\partial\tau$ we obtain a solution of type IIA supergravity, with the same six flat directions as in (14), and a non-trivial transverse part given by [3]

$$
\begin{aligned}
\frac{1}{Nk} ds_\perp^2 &= \Lambda_N (d\rho^2 + \coth^2\rho d\psi^2) + \Lambda_N^{-1}(\coth^2\rho + \tan^2\theta) d\tau^2 \\
&\quad + \Lambda_N \left(1 + (1+\coth^2\rho\cot^2\theta)\frac{\sin^2(N\psi)}{\sinh^2(Nx)}\right) d\theta^2 + \ 2\coth^2\rho\ d\tau d\psi \\
&\quad + 2\cot\theta \frac{\sin(N\psi)}{\sinh(Nx)} (\Lambda_N \coth^2\rho d\psi + (\coth^2\rho + \tan^2\theta) d\tau)\ d\theta\ , \qquad (15) \\
e^{-2\Phi} &= \frac{g_{\rm YM}^2 U_0^2}{Nk}\ \cos^2\theta \sinh^2\rho\ ,
\end{aligned}
$$

and zero antisymmetric tensor. In the limit $N \gg 1$, we obtain

$$
\begin{aligned}
\frac{1}{Nk} ds_\perp^2 &= d\theta^2 + \tan^2\theta\ d\varphi^2 + d\rho^2 + \coth^2\rho\ d\omega^2\ , \\
e^{-2\Phi} &= \frac{g_{\rm YM}^2 U_0^2}{Nk}\ \cos^2\theta \sinh^2\rho\ , \qquad (16)
\end{aligned}
$$

where $\omega = \tau + \psi$ and $\varphi = \tau$. This is the background corresponding to the exact CFT $SU(2)_{kN}/U(1) \times SL(2, \mathbb{R})_{kN+4}/U(1)$. In the opposite extreme case of $N = 1$, it can be shown that (15) reduces to the background for the exact CFT $SU(2)_k/U(1) \times U(1) \times U(1)_Q$. This is no surprise, since the latter background and the one for $SU(2)_k \times U(1)_Q$ are T-duality related [13].

For the supersymmetric properties of the solutions (14)-(16) as well as for their relation to pure gravity backgrounds and non-extremal black holes arising upon toroidal compactification we refer the reader to [3].

4.1.1 CFT for non-extremal NS5 branes:

For completeness we include the discussion of the CFT related to a certain limit of the supergravity solution representing N non-extremal one-center NS5 branes [6, 14]. The corresponding metric, antisymmetric tensor field strength and dilaton are [15]

$$\begin{aligned} ds^2 &= ds^2(E^5) - f dt^2 + H\left(f^{-1}dr^2 + r^2 d\Omega_3^2\right) , \\ H_{ijk} &= \epsilon_{ijkl}\partial_l H' , \\ e^{2\Phi} &= g_s^2 H , \end{aligned} \tag{17}$$

where

$$\begin{aligned} &H = 1 + \frac{\mu^2 \sinh^2\alpha}{r^2} , \quad H' = 1 + \frac{\mu^2 \sinh\alpha\cosh\alpha}{r^2} , \quad f = 1 - \frac{\mu^2}{r^2} , \\ &\sinh^2\alpha = \left((\alpha' N/\mu^2)^2 + 1/4\right)^{1/2} - \frac{1}{2} , \end{aligned} \tag{18}$$

where μ is the non-extremality parameter. Consider first the change of variables $r = \mu\cosh\rho$ and then the limit $\mu, g_s \to 0$ in such a way that $\frac{\mu}{g_s l_s} \equiv r_0$ is held fixed. Then (17) becomes

$$\begin{aligned} ds^2 &= ds^2(E^5) - \tanh^2\rho\, dt^2 + \alpha' N d\rho^2 + \alpha' N d\Omega_3^2 , \\ H &= -2\alpha' N \epsilon_3 , \\ e^{\Phi} &= \frac{\sqrt{N}}{r_0\cosh\rho} , \end{aligned} \tag{19}$$

where ϵ_3 is the volume form of the unit 3-sphere. The background (19) corresponds to the $SL(2,\mathbb{R})/SO(1,1) \times SU(2)$ exact CFT, with the first factor representing the famous 2-dim black hole solution [7]. Note also that, choosing N and r_0 in such a way that the relation $1 \ll N \ll r_0^2$ is satisfied, we suppress perturbative as well as string loop corrections.

5 Final comments

In [3] NS5-branes of type II and heterotic string theory whose non-trivial 4-dim part is described by the non-Abelian dual of 4-dim hyper-Kähler metrics with $SO(3)$ isometry, were constructed. In the case corresponding to the non-Abelian dual of 4-dim flat space the discrete distribution of the brane centers was found. It will be interesting to extend this analysis to more general non-Abelian duals of 4-dim hyper-Kähler metrics.

It will be interesting to compute the heavy quark–antiquark potential for the broken $\mathcal{N} = 4$ SYM in the large-N limit using the AdS/CFT correspondence. The prototype computation was done for unbroken gauge group in [16], and for $SU(N)$ broken to $SU(N/2) \times SU(N/2)$ in [17]. In the latter case a "confining" behaviour, albeit unstable, was found. Further breaking the symmetry group might result in a stabilization of this behaviour. It will be interesting to repeat these computations using the different supergravity solutions we have presented in this note.

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A Comments on gravitational multi-instantons

Consider 4-dim self-dual metrics with a translational Killing vector field $\partial/\partial\tau$. The general form of the metric is [18]

$$\begin{aligned} ds^2 &= V(d\tau + \omega_i dx_i)^2 + V^{-1} dx_i dx_i \ , \\ \partial_i V^{-1} &= \epsilon_{ijk}\partial_j \omega_k \ , \qquad i = 1,2,3 \ . \end{aligned} \tag{A.1}$$

Hence V^{-1} is a harmonic function of the form

$$V^{-1} = \epsilon + \sum_{i=1}^{N} \frac{m}{|\vec{x} - \vec{x}_i|} \ . \tag{A.2}$$

The analogous anti-self-dual metrics can be obtained by the sign change $\tau \to -\tau$, so that they will not be considered any further. With the above choice the singularities at $\vec{x} = \vec{x}_i$ are removable NUT singularities provided that the variable τ has period $4\pi m$. Hence, it follows that if the constant $\epsilon \neq 0$ (in which case it can be normalized to 1) the space is asymptotically locally flat (ALF). However, if $\epsilon = 0$, then it is asymptotically locally Euclidean (ALE) with boundary at infinity S^3/R_N with R_N being a discrete subgroup of $SO(4)$. Here we are interested in a distribution of a large number N of NUT singularities on a circle of radius r_0 in a similar fashion as in section 2. Then, except for distances $r \simeq r_0 + \mathcal{O}(\frac{1}{N})$ and smaller, the harmonic function V^{-1} is essentially given by (8) for $n = 1$. Using well known formulae we write

$$\begin{aligned} V^{-1} &\approx \frac{2mN}{\pi} \frac{K(k)}{(r^2 + 2r_0 r \sin\theta + r_0^2)^{1/2}} = mN \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{2l+1}} P_{2l}(0) P_{2l}(\cos\theta) \ , \\ k &= 2\sqrt{\frac{r_0 r \sin\theta}{r^2 + 2r_0 r \sin\theta + r_0^2}} \ , \qquad P_{2l}(0) = \frac{(-1)^l (2l)!}{4^l (l!)^2} \ , \end{aligned} \tag{A.3}$$

where $K(k)$ is the complete elliptic integral of the first kind, P_l is the usual Legendre polynomial and $r_<$ ($r_>$) denotes the smaller (larger) of r, r_0. Actually, the expression above is a classic result of electrostatics as it represents the potential due to a uniformly charged ring of radius r_0 and total charge mN. In order to compute the ω_i's appearing in (A.1) it is convenient to use spherical coordinates r, θ and ϕ in which $\omega_r = \omega_\theta = 0$. The remaining non-vanishing component $\omega_\phi(r,\theta)$ is determined by solving the differential eqs.

$$\partial_\theta \omega_\phi = r^2 \sin\theta \partial_r V^{-1} , \qquad \partial_r \omega_\phi = -\sin\theta \partial_\theta V^{-1} . \tag{A.4}$$

The most convenient way we found to present the solution is as an infinite series in terms of Legendre polynomials

$$\omega_\phi = mN \sum_{l=0}^{\infty} C_l(r) P_{2l+1}(\cos\theta) , \tag{A.5}$$

where

$$\begin{aligned} C_l &= \frac{2l+1}{4l+1} P_{2l}(0) \left(\frac{r_0}{r}\right)^{2l} - \frac{2l+3}{4l+5} P_{2l+2}(0) \left(\frac{r_0}{r}\right)^{2l+2} , \qquad \text{if } r > r_0 , \\ C_l &= -\frac{2l}{4l+1} P_{2l}(0) \left(\frac{r}{r_0}\right)^{2l+1} + \frac{2l+2}{4l+5} P_{2l+2}(0) \left(\frac{r}{r_0}\right)^{2l+3} , \qquad \text{if } r < r_0 , \end{aligned} \tag{A.6}$$

Computing the infinite sum explicitly in (A.5) seems a difficult task to perform. Note that in the limit $r_0 \to 0$ only the coefficient $C_0 = 1$ survives. Then $\omega_\phi = mN \cos\theta$, which is as expected for a one-center solution of NUT charge mN.

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