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Subleading Corrections and Central Charges in the AdS/CFT Correspondence

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Abstract: We explore subleading contributions to the two basic central charges c and a of four-dimensional conformal field theories in the AdS/CFT scheme. In particular we probe subleading corrections to the difference $c-a$ from the string-theory side. In the $\mathcal{N}=4$ CFT, $c-a$ vanish identically consistently with the string-theory expectations. However, for $\mathcal{N}=1$ and $\mathcal{N}=2$ CFTs, the $U_R(1)$ anomaly, which is proportional to $c-a$, is subleading in the large N limit for theories in the AdS/CFT context and one expects string one-loop R^2 and $B\wedge R\wedge R$ terms in the low energy effective action. We identify these terms as coming from the R^4 terms. Similar considerations apply to the $U_R(1)^3$ anomaly which is, however, subleading only for $\mathcal{N}=2$ theories. As a result, a string one-loop term $B\wedge F\wedge F$ should exist in the low energy effective action of the $\mathcal{N}=4$ five-dimensional supergravity. The $U_R(1)^3$ term is leading for the $\mathcal{N}=1$ CFT and it is indeed present in the $\mathcal{N}=2$ five-dimensional supergravity.

Keywords: 1/N Expansion, Supergravity Models, M-Theory, Anomalies in Field and String Theories .

Contents

1. Introduction and Conclusions

It has recently been argued in [1] that the large N limit of certain conformal field theories (CFT) can be described in terms of Anti de-Sitter (AdS) supergravity. The CFT lives on the AdS boundary and a precise recipe for expressing correlation functions of the boundary theory in terms of the bulk theory has been given [2],[3]. In particular, the four-dimensional $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills theory is described by the type IIB string theory on $AdS_5 \times S^5$ where the radius of both the AdS_5 and S^5 are proportional to N. A field theory formulation of the proposed AdS/CFT correspondence has been given in [4],[5]. It has also been argued that in a suitable limit, the generating functional for the boundary correlators is reproduced [2],[3] by the maximal $\mathcal{N} = 8$ $d = 5$ gauged supergravity on its anti-de Sitter vacuum [6]. The symmetry of the latter is $SO(4, 2) \times SU(4)$ which is just the even subgroup of the $SU(2, 2|4)$ superalgebra. The latter is realized by $\mathcal{N} = 4$ superconformal YM theory on the four-dimensional boundary of the anti-de Sitter space.

In addition to the $\mathcal{N} = 4$ supersymmetry algebra $SU(2, 2|4)$, there also exist the superalgebras $SU(2, 2|2)$ and $SU(2, 2|1)$. Their even subgroups are $SO(4, 2) \times U(2)$ and $SO(4, 2) \times U(1)$, respectively, and they are realized by conformal field theories with less supersymmetries, namely, $\mathcal{N} = 2$ and $\mathcal{N} = 1$ superconformal Yang-Mills theories. In this case, the boundary correlators are reproduced by the $\mathcal{N} = 4$ and $\mathcal{N} = 2$ $d = 5$ gauged supergravity [7],[8]. However, this way one may explore only the leading N^2 terms since classical supergravity arises from tree-level string theory and so there exist a $1/g_s^2$ factor in front of its effective action. Recalling that $g_s \sim 1/N$, we immediately conclude that classical supergravity is of order N^2. Thus, in order to probe the subleading structure, one has to go beyond tree-level string theory and take into account string-loop effects. Here we will explore subleading contributions to the basic central charges of a four dimensional conformal theory, commonly called c and a [9]. Quantum field theoretical knowledge is used as a guideline to identify the desired terms in the string description. The leading contributions to c and a have been calculated, in the holographic context, in refs. [2],[10].

One may ask if the field-theory/string-theory correspondence can be extended so that for any given $\mathcal{N} = 0, 1, 2$ superconformal model in four dimensions there exist a super-

gravity theory on AdS_5. It seems, however, from the known examples discussed so far [11] that the correspondence works only when $c = a$ at the leading order. We may conjecture then that *all CFT with c=a in the leading order have a supergravity dual*. This is also indicated from the present work on subleading corrections. Conformal field theories with $c = a$ are a special subclass of the more general family with c and a unconstrained [12], [13]. This and other features visible from the quantum field theoretical viewpoint fit nicely with the supergravity description and our purpose here is to show that they are consistent also with the subleading corrections that have a string origin.

The first quantity to probe is the difference $c-a$. Given that $c = a$ in the supergravity limit, the presence of subleading effects can be detected as a non-vanishing, subleading, value of the difference $c - a$. This contribution appears in theories with $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry and does not appear in theories with $\mathcal{N} = 4$. In the former cases a corresponding term, that we shall discuss in detail, is present in the string-theoretical description.

$c - a$ is a multiplicative factor of a four-derivative term in field theory. Thus, we expect that these terms correspond to four-derivative interactions in five-dimensional supergravity. In addition, the latter should be of order $\mathcal{O}(1)$ compared to the leading N^2 terms. Thus, they should emerge from one-loop in string theory. Such string one-loop four-derivative interactions in five dimensions are induced by R^4 terms in ten [14],[15] or eleven dimensions [16],[17]. Since their structure depends on the number of supersymmetries as well as on the particular compactifications, we will examine these terms separately according to the number of supersymmetries.

2. $\mathcal{R}^4$ terms in string and M-theory

The massless spectrum of the ten dimensional type II string theory contains in its NS/NS sector the graviton g_{MN}, the antisymmetric two-form B_{MN} and the dilaton ϕ. The R/R sector of the IIA theory contains a one-form A_M and a three-form A_{MNP} while the R/R sector the type IIB theory consists of a second scalar χ, a two-form $B^{(2)}_{MN}$ and a four-form A^+_{MNPQ} with self-dual field strength. In particular the massless spectrum of the type IIA theory can be obtained from the dimensional reduction of the eleven-dimensional supergravity on a circle. In that case, the dilaton is related to the radius of the circle, the one-form is the KK potential, and the two-and three-forms result from the three form of eleven-dimensional supergravity.

In the large wavelength limit, the type IIA and IIB theories are described by the non-chiral and chiral $\mathcal{N} = 2$ supergravity, respectively, and the bosonic part of their low energy effective action of the NS/NS sector is

$$S_t = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left(R + 4\partial\phi^2 - \frac{1}{12}H_{KMN}H^{KMN}\right), \tag{2.1}$$

where $\kappa_{10}^2 = 2^6 \cdot \pi^7 \cdot \alpha'^4$. The first terms in the effective action that receive quantum corrections are the eight-derivative ones. Such terms are the familiar $t_8 t_8 R^4$, $\varepsilon_{10}\varepsilon_{10} R^4$ and $t_8 \varepsilon_{10} R^4$ where

$$\begin{aligned} t_8 t_8 R^4 &= t_8^{M_1 M_2 \dots M_7 M_8} t_8^{N_1 N_2 \dots N_7 N_8} R_{M_1 M_2 N_1 N_2} \dots R_{M_7 M_8 N_7 N_8} \,, \\ \varepsilon_{10}\varepsilon_{10} R^4 &= \varepsilon^{M_1 M_2 \dots M_7 M_8 AB} \varepsilon^{N_1 N_2 \dots N_7 N_8}{}_{AB} R_{M_1 M_2 N_1 N_2} \dots R_{M_7 M_8 N_7 N_8} \,, \\ t_8 \varepsilon_{10} R^4 &= \varepsilon^{AB M_1 M_2 \dots M_7 M_8} t_8^{N_1 N_2 \dots N_7 N_8} R_{M_1 M_2 N_1 N_2} \dots R_{M_7 M_8 N_7 N_8} \,. \end{aligned} \tag{2.2}$$

The eight-tensor t_8 appears in string amplitude calculations [18] and ε_{10} is the ten-dimensional totally antisymmetric symbol. In particular, using the explicit form of t_8 we find [19],[20]

$$\begin{aligned} t_8 t_8 R^4 &= 6\, t_8 \left(4R^4 - ({}^2TrR^2) \right) \\ &= 12 \left(R_{MNPQ} R^{MNPQ} \right) - 192 R_{MNPQ} R^{MNPK} R_{ABCQ} R^{ABCQ} \dots \,, \\ \varepsilon_{10}\varepsilon_{10} R^4 &= -96 \left(R_{MNPQ} R^{MNPQ} \right) \dots \,, \end{aligned}$$

The next-to-leading order corrections to the tree effective action can be computed either by string amplitude calculations or in sigma-model perturbation theory. Both ways lead to the result that the eight-derivative term in the effective action is of the form $t_8 t_8 R^4$. However, in the case of ten-dimensional $\mathcal{N} = 1$ supergravity, it has been shown that supersymmetry relates the $t_8 t_8 R^4$ term to $\varepsilon_{10}\varepsilon_{10} R^4$ [21]. In fact what appears in the effective action are the two super-invariants with bosonic parts

$$J_0 = t_8 t_8 R^4 + \frac{1}{8} \varepsilon_{10}\varepsilon_{10} R^4 \,, \quad J_1 = t_8 X_8 - \frac{1}{4} \varepsilon_{10} B X_8 \,, \tag{2.3}$$

where

$$X_8 = \frac{1}{(2\pi)^4} \left(-\frac{1}{768} (TrR^2)^2 + \frac{1}{196} TrR^4 \right) \,, \tag{2.4}$$

is the eight-form anomaly polynomial. We will assume that that this is also the case for the $\mathcal{N} = 2$ supersymmetry and then the eight-derivative tree-level effective action turns out to be

$$S^{(0)}_{R^4} = \frac{1}{3 \cdot 2^{12} \cdot \kappa_{10}^2} \zeta(3) \int d^{10}x \sqrt{-g} e^{-2\phi} \left(t_8 t_8 + \frac{1}{8} \varepsilon_{10}\varepsilon_{10} \right) R^4 \,. \tag{2.5}$$

We see that J_1 contains a CP-odd coupling and thus it is expected to be protected from perturbative corrections and all possible corrections are non-perturbative ones. On the other hand, J_0 is believed to have only one-loop corrections. In particular, J_0 also appears in type IIB theory. There, its perturbative and non-perturbative corrections can be extracted from symmetry considerations [22], namely, the $SL(2, \mathbf{Z})$ invariance.

The latter specifies the form of the corrections not only to the R^4 term but of all eight-derivative terms [23]. The result is that there exist only one-loop corrections to these terms and the non-perturbative ones are due to the type IIB D-instantons. In fact, $SL(2,\mathbf{Z})$ symmetry is even stronger. One may prove that higher-derivative gravitational interactions [24] are the form R^{6L+4} ($L = 0, 1, ...$) and they appear at L and $2L+1$ loops [25]. For $L = 0$ this is just the statement that the tree level R^4 term has only one-loop counterpart and all other corrections are non-perturbative.

The one-loop effective action of the type IIA theory turns out to be

$$S^{(1)}_{R^4} = \frac{2\pi^2}{3}\frac{1}{3\cdot 2^{13}\cdot\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\left(t_8t_8 - \frac{1}{8}\varepsilon_{10}\varepsilon_{10}\right)R^4 - B\wedge\left((TrR^2)^2 - 4TrR^4\right)\right) , \quad (2.6)$$

which contains a CP-odd and a CP-even part. Let us note at this point that the CP-odd term is absent in the type IIB theory since the transformation $-I_{2\times 2} \in SL(2,\mathbf{Z})$ changes the sign of the two-forms (i.e., $B \to -B$) and thus the last two-terms in eq.(2.6) is absent. Moreover, the CP-even term is different also in type IIB and is proportional to $\left(t_8t_8 + \frac{1}{8}\varepsilon_{10}\varepsilon_{10}\right)R^4$.

By relating the dilaton with the radius R_{11} of an S^1 compactification of M-theory as $e^{\phi} = R_{11}^{3/2}$, one may lift the tree and one-loop effective actions eqs.(2.5,2.6) to eleven-dimensions. One then finds that in the decompactification limit $R_{11} \to \infty$ only the one-loop effective action in eq.(2.6) survives and the result is

$$S^{11}_{R^4} = \frac{2\pi^2}{3}\frac{1}{3\cdot 2^{13}\cdot\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(\left(t_8t_8 - \frac{1}{24}\varepsilon_{11}\varepsilon_{11}\right)R^4 - C\wedge\left((TrR^2)^2 - 4TrR^4\right)\right) , \quad (2.7)$$

where C is the three-form of eleven-dimensional supergravity. In fact, the last term in eq.(2.7) is needed to cancel the anomaly on the fivebrane world-volume by a bulk contribution [26].

3. $\mathcal{R}^2$ terms in $d = 5$ supergravity

There exist four supergravity theories in five dimensions, the $\mathcal{N} = 8$, $\mathcal{N} = 6$, $\mathcal{N} = 4$ and $\mathcal{N} = 2$ supergravities [27]. Since according to the AdS/CFT scheme these theories will correspond to supersymmetric $\mathcal{N}/2$ YM theories, we will only consider the $\mathcal{N} = 8$, $\mathcal{N} = 4$ and $\mathcal{N} = 2$ $d = 5$ theories. They can be obtained by compactifications of M-theory on T^6, $K3\times T^2$ and Calabi-Yau (CY), respectively. The presence of R^4 terms in eleven dimensions yield R^4 as well as R^2 terms in five dimensions after compactification. In particular the presence of the latter depends on the number of supersymmetries. Namely, R^2 terms in five dimensions, which are one-loop and thus subleading with respect to the two-derivative terms, exist, as we will see, only for the $\mathcal{N} = 4$ and $\mathcal{N} = 2$ case and not for $\mathcal{N} = 8$. We should stress here that the R^2 terms we are discussing appear in the

ungauged theory. The latter have a $USp(\mathcal{N})$ group of local symmetries and one may gauge an appropriate subgroup of it. In this case, the R^2 terms as well as the R^4 terms should also exist in the gauged theory since in the limit of vanishing gauge coupling one should recover these terms. This is also supported from the fact that these terms are needed for the consistency of the AdS/CFT correspondence. Namely, the one-loop R^2 terms which are subleading with respect to the two-derivative terms in the supergravity side provide the necessary and correct structure to produce in the CFT side the R-current anomalies which are proportional to $c-a$. Note that $c-a$ is exactly zero for the $\mathcal{N}=4$ SCFT while it is subleading in the $\mathcal{N}=1,2$ case. Thus, we expect R^2 terms in the supergravity side only for the $\mathcal{N}=2,4$ and not for the $\mathcal{N}=8$ $d=5$ supergravity. This is indeed what we find and supports the fact that the R^2 in the ungauged theory also exist in the gauged one.

3.1. The maximal $\mathcal{N}=8$ $d=5$ supergravity

The maximal $\mathcal{N}=8$ five-dimensional ungauged supergravity theory has been constructed in [27]. It can be obtained by toroidal compactification of M-theory which has the eleven-dimensional supergravity as its low-energy limit. It has a graviton, eight symplectic Majorana gravitini, 27 vectors, 48 symplectic Majorana spinors and 42 scalars. It has an $E_{6(6)}$ global and a local $USp(8)$ symmetry. The scalar fields parametrize the coset space $E_{6(6)}/USp(8)$. An $SO(p,6-p)$ $(3\leq p\leq 6)$ subgroup of $E_{6(6)}$ can be gauged resulting in the maximal gauged supergravity in five dimensions [6]. In particular, for $SO(6)=SU(4)$ gauging, the supergravity admits an AdS_5 vacuum which exhibits the $SU(2,2|4)$ superalgebra and according to the AdS/CFT correspondence, describes large N $SU(N)$ $\mathcal{N}=4$ YM theory at the boundary of AdS_5. Note that the maximal gauged supergravity may have vacua with less supersymmetries. However, there is no complete classification of the critical points of the potential of the $\mathcal{N}=8$ gauged supergravity [28],[29], [30].

Since the ungauged theory can be obtained by toroidal compactification of M-theory we do not expect four-derivative interactions. The first non-zero higher-derivative terms are eight-derivative ones which in the AdS/CFT context has been discussed in [31].

3.2. The $\mathcal{N}=4$ $d=5$ supergravity

The five-dimensional $\mathcal{N}=4$ supergravity has been constructed in [27] by truncation of the $\mathcal{N}=8$ theory and its action has explicitly be written in [34] where its coupling to n vector multiplet has also be considered. The $\mathcal{N}=4$ $d=5$ supersymmetry algebra has $USp(4)$ as its automorphism group and the graviton multiplet contains six vectors in the $5+1$ rep. of $USp(4)$. Since the bosonic subgroup of the $\mathcal{N}=4$ anti-de Sitter

supergroup $SU(2,2|2)$ is $SU(2,2) \times SU(2) \times U(1)$, an $SU(2) \times U(1)$ subgroup of $USp(4)$ can be gauged [7].

The $\mathcal{N} = 4$ $d = 5$ supergravity can be obtained by compactification of M-theory on $K3 \times T^2$, or equivalently of type IIA and IIB on $K3 \times S^1$. The R^4 terms in string or M-theory can potentially give rise to R^2 terms as well. To find the explicit form of these contributions to the effective action we will consider first compactification of the ten-dimensional type IIA theory theory on $K3$ and a further compactification on S^1. For the $K3$ compactification there exist two R^2 type of terms. The ones coming from the tree level eq.(2.5) effective action $S^{(0)}_{R^4}$ and those coming from the one-loop action $S^{(1)}_{R^4}$ in eq.(2.6). One may easily verify that the former are zero while the latter are non-zero and they are given explicitly by

$$S^{(1)}_{R^2} = \frac{\pi^2}{3 \cdot 2^7 \cdot \kappa_{10}^2} \int d^6x \sqrt{-g} \left(R_{\bar{m}\bar{n}\bar{p}\bar{q}} R^{\bar{m}\bar{n}\bar{p}\bar{q}} - \frac{1}{4} \varepsilon^{\bar{m}\bar{n}\bar{p}\bar{q}\bar{r}\bar{s}} B_{\bar{m}\bar{n}} R_{\bar{a}\bar{b}\bar{p}\bar{q}} R_{\bar{a}\bar{b}\bar{r}\bar{s}} \right) . \tag{3.1}$$

where $\bar{m}, \bar{n} \ldots = 0, ..., 5$. As a result, the only R^2 terms existing in $\mathcal{N} = 2$ six-dimensional theory are at one-loop level as has also been found by a direct string one-loop calculation [32]. This can also be infield from the heterotic/type IIA duality according to which heterotic string theory on T^4 is dual to type IIA theory on $K3$. The tree-level effective action of the ten-dimensional heterotic string has R^2 terms which upon reduction on T^4 will give similar terms in six-dimensions. By using the heterotic-type IIA mapping these terms can be written in the dual IIA theory and will become one-loop terms. These are the terms we found by direct compactification in the type IIA side on $K3$. Let us recall here, that, the compactification of the IIA string on $K3$ will give rise to a six-dimensional theory with massless sector consisting of the graviton and vector multiplets of the non-chiral $(1,1)$ supersymmetry. In particular, the ten-dimensional graviton will give rise to the six-dimensional graviton together with 58 scalars. The two-form B_{MN} will give rise to a two-form $B_{\bar{m}\bar{n}}$ and 22 scalars and together with the dilaton we get 81 scalars from the NS/NS sector. On the other hand, the R/R sector provides 24 vectors. Thus, finally, we end up with the $(1,1)$ six-dimensional graviton multiplet which contains the graviton, 1 antisymmetric two-form, 4 vectors, 1 real scalar, 4 Weyl spinors and 2 gravitini together with 20 vector multiplets each one containing 1 vector, 2 Weyl spinors and 4 scalars. Note that the scalars parametrize the space $R^+ \times SO(4,20)/(SO(4) \times SO(20))$. The six-dimensional effective action for the non-chiral six-dimensional supergravity with no vectors has been given in [33]. Then, together with the one-loop R^2 terms of eq.(3.1) we have

$$\begin{aligned} S_6 = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \Bigg(& e^{-2\phi} \left(R + 4\partial\phi^2 - \frac{1}{12} H_{\bar{k}\bar{m}\bar{n}} H^{\bar{k}\bar{m}\bar{n}} \right) \\ & - \frac{1}{2} F^I_{\bar{m}\bar{n}} F_I^{\bar{m}\bar{n}} - \frac{1}{8} \varepsilon^{\bar{m}\bar{n}\bar{p}\bar{q}\bar{r}\bar{s}} B_{\bar{m}\bar{n}} F^I_{\bar{p}\bar{q}} F_{I\bar{r}\bar{s}} \end{aligned} \tag{3.2}$$

$$+\frac{1}{8}\alpha'\left(R_{\bar{m}\bar{n}\bar{p}\bar{q}}R^{\bar{m}\bar{n}\bar{p}\bar{q}}-\frac{1}{4}\varepsilon^{\bar{m}\bar{n}\bar{p}\bar{q}\bar{r}\bar{s}}B_{\bar{m}\bar{n}}R_{\bar{a}\bar{b}\bar{p}\bar{q}}R_{\bar{a}\bar{b}\bar{r}\bar{s}}\right)\Bigg)\ ,$$

where $F^I = dA^I$, $I = 1, ..., 4$ are the field strengths of the four vectors A^I of the graviton multiplet and κ_6 is the six-dimensional gravitational coupling constant.

By a further compactification on S^1, we get the $\mathcal{N} = 4$ five-dimensional theory. In this case, the six-dimensional graviton multiplet yields the five dimensional graviton multiplet and a vector multiplet. In particular, the six-dimensional graviton will give rise to the graviton, one vector and one scalar while $B_{\bar{m}\bar{n}}$ will give rise to a vector $B_m = B_{m5}$ $(m = 0, ..., 4)$ and an antisymmetric two-form which can be dualized to a vector as well. The four vectors will result into four vectors and four scalars and in addition we will have one more scalar ϕ. The graviton together with $5 + 1$ vectors fills up the five-dimensional graviton multiplet and the rest form a vector multiplet which contains a vector and five scalars. Thus, in this case we get a $\mathcal{N} = 4$ supergravity first discussed in [27] coupled to a vector multiplet where the scalars parametrize $SO(1, 1) \times SO(5, 1)/SO(5)$ [34].

The vectors of the five-dimensional graviton multiplet transform in the 5+1 rep. of the $USp(4)$ automorphism group and it is not difficult to see by comparing the Cern-Simons term of eq.(3.2) with the corresponding term of the $\mathcal{N} = 4$ $d = 5$ supergravity [34] that the $USp(4)$ singlet is B_m. Thus, in the bosonic part of the $\mathcal{N} = 4$ $d = 5$ supergravity theory for the graviton multiplet we must also include the four-derivative interaction terms

$$S_{R^2}^{\mathcal{N}=4} \sim \int d^5x\sqrt{-g}\left(R_{mnrs}R^{mnrs}-\frac{1}{2}\varepsilon^{mnpqr}B_m R_{abnp}R_{abqr}\right)\ ,$$

Thus, unlike the maximal $\mathcal{N} = 8$ theory, the $\mathcal{N} = 4$ $d = 5$ theory has four-derivative R^2 terms. These are one-loop terms and thus, subleading with respect to the dominant two-derivative ones. The four-derivative interactions exist in the ungauged theory and we expect to exist in the gauge one as well. This is supported also from the CFT side as we will see later.

3.3. The $\mathcal{N} = 2$ $d = 5$ supergravity

This theory has also been constructed in [27] and it is very similar to eleven-dimensional supergravity. The five-dimensional action can be found by compactification of M-theory on a Calabi-Yau manifold. Then, the reduced five-dimensional theory is described by the $\mathcal{N} = 2$ supergravity coupled to $h_{(1,1)} - 1$ vectors and $h_{(2,1)} + 1$ hypermultiplets [35],[36]. Since there are higher-derivative terms in M-theory given in eq.(2.7), one expects that there should be similar terms in the $\mathcal{N} = 2$ $d = 5$ theory as well. For a Calabi-Yau compactification, we may express the three-form of eleven-dimensional supergravity C as $C = \sum_\Lambda A_1^\Lambda \wedge \omega_\Lambda$ where $\Lambda = 1, ..., h_{(1,1)}$ and ω_Λ are the corresponding $(1, 1)$ harmonic

forms on the CY_3. In this case, an interaction of the form

$$S_5^L \sim \int \alpha_\Lambda A_1^\Lambda \wedge TrR^2\,, \tag{3.3}$$

where

$$\alpha_\Lambda = \int_{CY_3} \omega_\Lambda \wedge TrR^2\,, \tag{3.4}$$

is generated in five dimensions. This term is actually the bulk term needed to cancel the anomalies due to wrapped fivebranes around the CY four-cycles [37]. On the other hand, four-derivative terms can also be emerge, as has been shown in [38] by integrating the $\varepsilon_{11}\varepsilon_{11}R^4$ term in eq.(2.7). Indeed, the integration over the CY_3 produces the effective term

$$S_5' \sim \int \varepsilon^{mnpq}{}_r R_{mn} \wedge R_{pq} \wedge dx^r = \frac{1}{2}\int d^5x\sqrt{-g}\,(R_{mnpq}R^{mnpq} + \ldots)\ . \tag{3.5}$$

As a result, the ungauged $\mathcal{N}=2$ five-dimensional supergravity contains the four-derivative interactions

$$S_{R^2}^{\mathcal{N}=2} \sim \int d^5x\sqrt{-g}\left(\beta R_{mnpq}R^{mnpq} + \alpha_\Lambda A_1^\Lambda \wedge TrR^2\right)\,, \tag{3.6}$$

where β is proportional to $c_2 \cdot k$ [38] with c_2, k the second Chern class and Kähler class of the CY_3, respectively. In the gauged theory we expect that eq.(3.6) survives consistently with the field theory expectations as will see below. The gauged U(1) theory have a one-form potential B_1 which is a combination of the $h_{1,1}$ vectors of the theory [8] and the relative coefficients of the two terms in eq.(3.6) are related by supersymmetry in the same way that the R-current anomaly is related to the trace anomaly in the field theory side.

4. Field Theory Results

We recall the expression for the trace anomaly of a four-dimensional conformal field theory in external gravitational field (see [9] for the notation),

$$\Theta = \frac{1}{16\pi^2}\left[c(W_{\mu\nu\rho\sigma})^2 - a(\tilde{R}_{\mu\nu\rho\sigma})^2\right] + \frac{c}{6\pi^2}(F_{\mu\nu})^2, \tag{4.1}$$

where $W_{\mu\nu\rho\sigma}$ and $R_{\mu\nu\rho\sigma}$ are the Weyl and Riemann tensors, respectively, while $F_{\mu\nu}$ is the field strength of the $U(1)$ field B_μ coupled to the R-current. We recall that $\mu,\nu\ldots = 0,...,3$. In the free-field limit we have $c = \frac{1}{24}(3N_v + N_\chi)$ and $a = \frac{1}{48}(9N_v + N_\chi)$, where N_v and N_χ are the numbers of vector multiplets and chiral multiplets, respectively. c and a are marginally uncorrected [9], so their values are independent of the coupling constant in the theories that we are considering. We can rewrite (4.1) as

$$\Theta = \frac{1}{16\pi^2}\left[2(2a-c)R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}(c-3a)R^2 + (c-a)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] + \frac{c}{6\pi^2}(F_{\mu\nu})^2. \tag{4.2}$$

The factor $c-a$ multiplies the term containing the Riemann tensor and this is one way to detect the subleading corrections.

Supersymmetry relates the trace anomaly to the R-current anomaly. Now, in $\mathcal{N}$=1 supersymmetric theories, there is only one such current, in general. It reads [9]

$$R_\mu^{(\mathcal{N}=1)} = \frac{1}{2}\bar{\lambda}\gamma_\mu\gamma_5\lambda - \frac{1}{6}(\bar{\psi}\gamma_\mu\gamma_5\psi + \bar{\tilde{\psi}}\gamma_\mu\gamma_5\tilde{\psi}) + \text{scalars},$$

where λ is the gaugino and $\psi, \tilde{\psi}$ are the matter fermions. The anomaly formula is [9]

$$\partial_\mu(\sqrt{g}R^\mu)_{(\mathcal{N}=1)} = \frac{1}{24\pi^2}(c-a)\,\varepsilon_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\beta\gamma}R^{\rho\sigma\beta\gamma} + \frac{1}{9\pi^2}(5a-3c)F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{4.3}$$

On the other hand, in $\mathcal{N}$=2 theories one has an $SU(2)\otimes U(1)$-group of R-currents and the $U(1)$ R-current reads, in the notation of [12],

$$R_\mu^{(\mathcal{N}=2)} = \frac{1}{2}\bar{\lambda}_i\gamma_\mu\gamma_5\lambda_i - \frac{1}{2}(\bar{\psi}\gamma_\mu\gamma_5\psi + \bar{\tilde{\psi}}\gamma_\mu\gamma_5\tilde{\psi}) + \text{scalars},$$

where λ_i, $i=1,2$, are the two gauginos. It satisfies [12]

$$\partial_\mu(\sqrt{g}R^\mu)_{(\mathcal{N}=2)} = -\frac{1}{8\pi^2}(c-a)\,\varepsilon_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\beta\gamma}R^{\rho\sigma\beta\gamma} + \frac{3}{\pi^2}(c-a)F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{4.4}$$

The above relationships provide alternative ways to detect the subleading corrections to $c-a$, and exhibit some difference between the $\mathcal{N}$=1 and $\mathcal{N}$=2 cases.

The divergence of the R-current couples to the longitudinal component of the $U(1)$-field B_μ. Indeed, taking B_μ to be pure gauge, $B_\mu = \partial_\mu\Lambda$, we have

$$\int \mathrm{d}^4x\,\sqrt{g}R^\mu B_\mu \to -\int_M \mathrm{d}^4x\,\Lambda\partial_\mu(\sqrt{g}R^\mu).$$

The external sources are viewed as boundary limits of fields in five-dimensional supergravity.

The string-one-loop subleading correction derived from ten dimensions reads in five dimensions

$$\int_{\mathcal{M}} \mathrm{d}^5x\,\varepsilon^{mnpqr}B_m R^a{}_{bnp}R^b{}_{aqr}. \tag{4.5}$$

After replacing A_m with $\partial_m\Lambda$, the boundary limit is straightforward, in the sense that we do not need to use explicit Green functions. This is true in general for anomalies, since it is sufficient to look at the local part of the triangle diagram to reconstruct the full correlator. In particular, in a conformal field theory the three-point function $\langle RTT\rangle$ is unique up to a factor and therefore uniquely determined by the anomaly we are considering. We have

$$-\int_{\mathcal{M}} \mathrm{d}^5x\,\varepsilon^{mnpqr}\partial_m\left(\Lambda R^a{}_{bnp}R^b{}_{aqr}\right) \to -\int_M \mathrm{d}^4x\,\varepsilon^{\mu\nu\rho\sigma}\Lambda R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\sigma}$$

where $M = \partial\mathcal{M}$. The anomaly correlator $\langle \partial R(x)\, T_{\mu\nu}(y)\, T_{\rho\sigma}(z)\rangle$ is derived by taking one functional derivative with respect to Λ and two functional derivatives with respect to the metric tensor. The result can be written in the form of an operator equation

$$\partial_\mu(\sqrt{g}R^\mu) = f\,\varepsilon_{\mu\nu\rho\sigma}R^{\mu\nu}{}_{\beta\gamma}R^{\rho\sigma\beta\gamma}.$$

for some factor f. Quantum field theory, formula (4.4), says that $f = \frac{1}{24\pi^2}(c-a)$ for $\mathcal{N}$=1 and $f = -\frac{1}{8\pi^2}(c-a)$ for $\mathcal{N}$=2. String theory gives the geometrical interpretation of this number via formula (3.4).

Similar remarks can be repeated for the contribution $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ in the trace anomaly (4.2). Instead, the coefficient of the term $F_{\mu\nu}\tilde{F}^{\mu\nu}$ in the R-current anomalies presents two different behaviours: it is leading for $\mathcal{N}$=1 and subleading for $\mathcal{N}$=2. The string/supergravity description is in agreement with this fact (see below), which is a nontrivial cross-check of the consistency of our picture and, as a bonus, provides a precise prediction for some string-loop corrections.

4.1. $\mathcal{N} = 4$

The coefficient f is subleading in the large N limit and identically zero in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. With $G = SU(N)$, we have $c = a = \frac{1}{4}(N^2-1)$ so that $f = 0$. Since the supergravity dual of this theory is the $\mathcal{N} = 8$ $d = 5$ gauge supergravity, we do not have four-derivative interactions. Indeed, as we have discussed in section 2, there are no such interactions as a result of the toroidal compactification, consistently with the field-theory expectations.

4.2. $\mathcal{N} = 2$

In general in this case c and a is not exactly equal. Consider for example the $\mathcal{N} = 2$ finite theory with $G = SU(N) \otimes SU(N)$ and two copies of hypermultiplets in the $R = (N, \bar{N})$ representation. We have

$$c = \frac{1}{2}N^2 - \frac{1}{3}, \qquad a = \frac{1}{2}N^2 - \frac{5}{12}, \qquad c - a = \frac{1}{12}.$$

The supergravity dual of this theory has been found to be type IIB theory on $AdS_5 \times S^5/Z_2$ [39],[40] and thus it is $\mathcal{N} = 4$ $d = 5$ gauged supergravity. The latter has, as we have seen in section 3, R^2 interactions of precisely the correct form to account for R-current anomaly eq.(4.4) and the $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$-term in (4.2).

The $U(1)^3_R$ term $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is subleading, as we see in (4.4). Thus we expect that a term of the form $B \wedge F \wedge F$ should exist in the low-energy string effective action. The absence of this term in the five-dimensional $\mathcal{N} = 4$ leading supergravity action has been noticed

in [5][1]. However, as we see here, this term is actually subleading and should come from the string one-loop computation.

4.3. $\mathcal{N}=1$

As in the previous case, here we have also, in general, non-vanishing $c-a$. One can take for example the $\mathcal{N}=1$ theory with $G=SU(N)\otimes SU(N)\otimes SU(N)$ and three copies of $R=(N,\bar{N},1)\oplus$ cyclic perm.s [39]. We have

$$c=\frac{3}{4}N^2-\frac{3}{8},\qquad a=\frac{3}{4}N^2-\frac{9}{16},\qquad c-a=\frac{3}{16}.$$

The supergravity dual is the $\mathcal{N}=2$ $d=5$ gauged supergravity theory. As above, this theory has the correct R^2 terms eq.(3.6) to produce the R-current anomaly.

The $U(1)_R^3$ term $F_{\mu\nu}\tilde{F}^{\mu\nu}$ is leading, by formula (4.3), and the corresponding bulk term $B\wedge F\wedge F$ is indeed present in the supergravity Lagrangian [8],[5].

4.4. Interpretation at the level of quantum conformal algebras.

The quantum field theoretical origin of the subleading corrections to $c-a$ was explained in [12]. One can study the OPE of conserved currents, or in general finite operators, which generate a hierarchy of higher spin tensor currents organized into supersymmetric multiplets. In the $\mathcal{N}$=2 case there is one pair of current multiplets for each even spin and one current multiplet for each odd-spin. All multiplets have length 2 in spin units. In $\mathcal{N}=4$ [13], instead, there is one 4-spin-long multiplet for each even spin, plus the stress-tensor. Some powerful theorems in quantum field theory imply that a relevant part of the hierarchical structure is preserved to all orders in the coupling constant (see [13, 12] for details) and therefore should be visible around the strongly coupled limit.

In particular, the $\mathcal{N}=2$ algebra contains a current multiplet $\mathcal{T}^*$ that mixes with the multiplet $\mathcal{T}$ of the stress-tensor. The mixing is responsible of the desired effect. We recall here the argument.

The central charges c and a are encoded, as we see from (4.1), into the three-point function $\langle T(x)T(y)T(z)\rangle$, which we can study by taking the $x\to y$ limit and using the operator product expansion of [12], written schematically as $T(x)T(y)=\sum_n c_n(x-y)\mathcal{O}_n\left(\frac{x+y}{2}\right)$. The operators $\mathcal{O}_n$ that mix with the stress-tensor contribute via the two-point functions $\langle\mathcal{O}_n\,T\rangle$. In the $\mathcal{N}=2$ quantum conformal algebra these are the stress-tensor $T_{\mu\nu}$ itself and a second operator $T^*_{\mu\nu}$, which in the free-field limit is proportional to T_v-2T_m, T_v and T_m being the vector-multiplet and hypermultiplet contributions to the stress-tensor. $\mathcal{O}_n=T$ produces a contribution $\langle TT\rangle$, which is leading and actually equal to c. The contribution from $\mathcal{O}_n=\mathcal{T}^*$, $\langle\mathcal{T}^*\,\mathcal{T}\rangle$, is non-vanishing and subleading, precisely $\mathcal{O}(1)$.

[1]We thank S. Ferrara for clarifying discussions on this point

Apart from this remnant, $\mathcal{T}^*$, as well as the other non-conserved operators $\mathcal{O}_n$, decouple from the theory in the strongly coupled large-N limit.

The identification between string corrections and $\mathcal{T}^*$ is a step towards the reconstruction of the spin hierarchies of [13, 12] as string excitations around the supergravity limit. We expect that the full hierarchies of [13, 12] can be found in the string description. Roughly, the vocabulary should be as follows. i) Higher-spin current multiplets that are orthogonal to the stress-tensor correspond to α'-corrections. ii) The renormalization mixing between pairs of current multiplets is mapped onto $1/N$-corrections.

There is however an effect that is not included in this classification. c and a receive, separately, a $1/N$-correction that is not detectable via the difference $c-a$ and is present also in $\mathcal{N}=4$ (where no renormalization mixing takes place [13]). The identification of the string origin of this kind of subleading corrections is still missing.

We recall, finally, that there are theories in which $c-a$ is leading. They have a quite different quantum conformal algebra [12] and the difference persists in the closed (large N, large g^2N) limit. At the moment, there is no string interpretation to this more general class of conformal field theories in four dimensions.

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