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# SHAPE COEXISTENCE IN ATOMIC NUCLEI AND ITS SPECTROSCOPIC FINGERPRINTS

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## Abstract

In the present discussion we concentrate on shape coexistence as obtained within a deformed single-particle field as well as starting from the spherical shell-model, incorporating deformation effects via the residual proton-neutron quadrupole interaction. We discuss in particular the appearance of shape coexisting phenomena in the Pb region. In a second part then, we present a number of experimental fingerprints that allow to recognize the appearance of shape coexisting phenomena or of shape mixing through the use of selective experiments (e.g. band structure, spectroscopic factors, static moments, E0 properties and alpha-decay).

**(ISOLDE GENERAL)**

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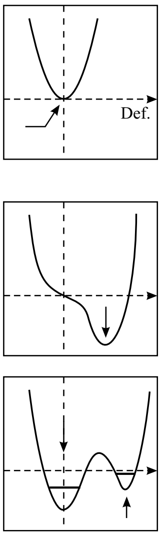

Figure 1: Schematic potential energy surfaces for spherical, quadrupole deformed and transitional (shape coexisting) nuclei, respectively.

# 1 Introduction

Even though the study of the nuclear many-body problem, starting from a given two-body nucleon-nucleon interaction in order to derive a mean-field and study detailed nuclear properties within a shell-model framework, can produce a large number of observable quantities (energies, transition rates, ...), the introduction of model concepts to describe the atomic nucleus still plays an important role in capturing the essential physics. Nuclear shape variables and the study of nuclear dynamics using collective shape variables have proven to provide a very interesting framework for describing the atomic nucleus and its internal excitations [1].

When studying the properties of nuclei in various parts of the nuclear mass table, the total energy associated with various shapes and shape changes, can give interesting insight in the nuclear dynamics. Conversely, the nuclear shell structure, using a liquid-drop model plus Strutinsky corrections [2] (shell and pairing corrections), leads to specific variations in the energy surface defined in the space of quadrupole deformation variables. So, at closed shells, a rather stiff spherical shape results whereas for regions with many valence protons and neutrons outside of closed shells, a deformed quadrupole equilibrium shape results. In certain mass regions, often called transitional mass regions, various shapes can result and often may even coexist. In other cases (see eg. the Sr,Zr region), mixing between the various possible shapes can show up (see fig. 1 for an illustration of the three major classes).

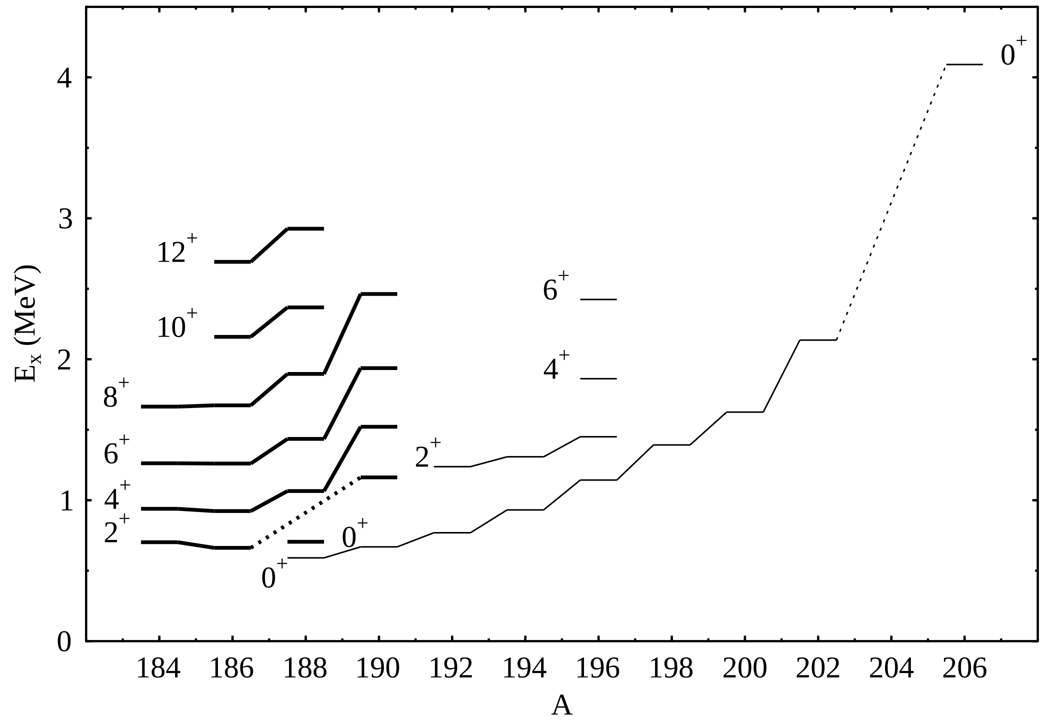

Figure 2: Systematics of low-lying intruder bands in the even-even Pb nuclei. The heavy (thin) lines denote the prolate 4p-4h and oblate 2p-2h configurations ([6] and refs. therein.

## 2 Nuclear Shape Coexistence

The deformed single-particle model (Nilsson model, deformed Woods-Saxon, mean-field Hartree-Fock approach, ...) is a most interesting avenue in order to study the appearance of various shapes in a given nucleus, albeit associated with a different excitation energy region [1]. The strong up- and downsloping orbitals just below and above the closed shells are the initial elements that, even in an intuitive way, show the natural tendency for nuclei to acquire a deformed shape next to the spherical shape corresponding to the closed shell itself. A large number of calculations have been carried out in that respect but here we mainly concentrate on the Pb region. Calculations of Bengtsson and Nazarewicz [3] and Nazarewicz [4] have clearly indicated the presence of slightly deformed oblate and more strongly deformed prolate bands to show up and mainly coexist in the neutron deficient Pb region (see also J.Wood et al. for a review on this subject and many references to the literature [5]). In the Hg nuclei, the ground-band structure relates to the oblate shape while the intruder configuration connects to the prolate shape.

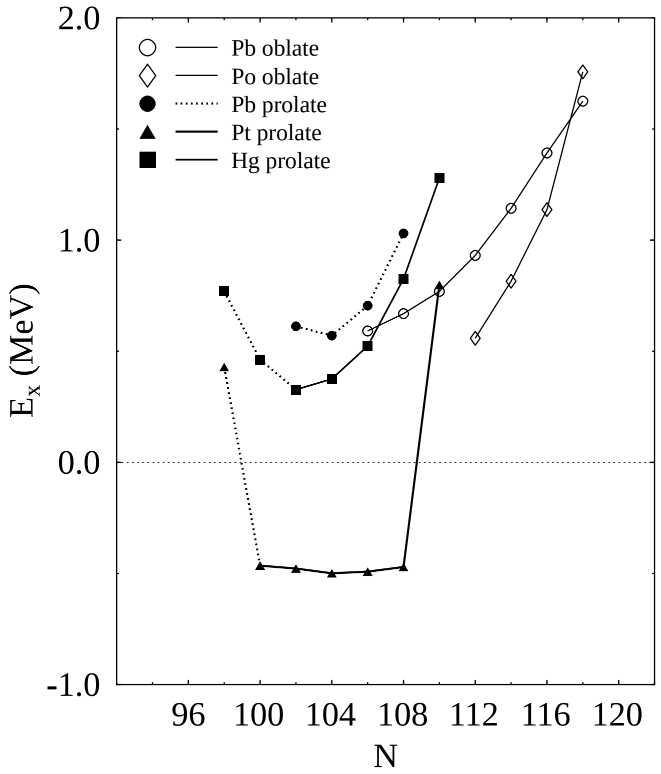

Figure 3: Energies of the $0^+$ band-heads in the even-even Po, Pb, Hg and Pt nuclei. The change of scale for the Pt nuclei assumes that the change between oblate and prolate ground-state happens between mass number A=188 and A=186.

In the Pt nuclei, the prolate configuration even becomes the lowest-lying configuration in the mass region $178 \leq A \leq 186$. In the Pb nuclei, one observes both deformed shapes but here, the spherical configuration remains the determining factor for the lowest-lying configurations that have been observed. A collection of the most recent data on the Pb nuclei is shown in fig. 2 [6]. In this figure, one clearly distinguishes the rapid lowering of the first excited $0^+$ state (associated to the oblate shape) while from mass number A=190 onwards, a well-deformed band structure shows up that can be connected to the more strongly deformed prolate band. This very neutron deficient Pb region, encompassing both even-even and odd-mass nuclei [7] in the Pt, Au, Hg, Tl, Pb, Bi, Po isotopes forms a most active region for research with experiments carried out at a number of laboratories.

In fig. 3, we present the excitation energy for the lowest-lying excited $0^+$ states in these nuclei and an interesting feature shows up. Irrespective of the proton number, the energy of these states drops towards the neutron mid-

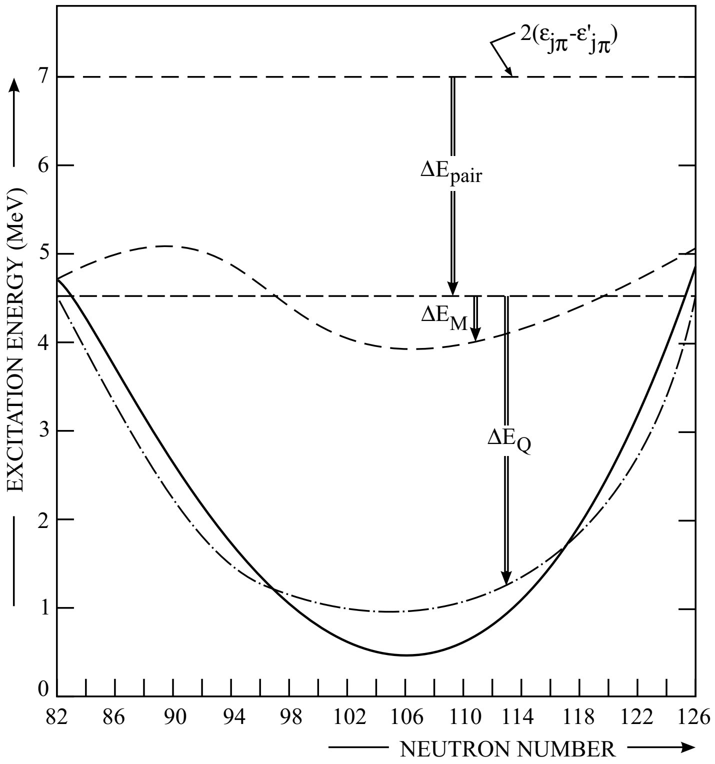

Figure 4: The various energy terms contributing to the energy of the $0^+$ lowest intruder 2p-2h configuration. The separate unperturbed, pairing, monopole and quadrupole energy contributions are given [8].

shell configuration at N=104. This experimental fact, related to the clear-cut observation of particle-hole excitations at and near to the closed shell refer to a possible alternative but equivalent formulation for the description of shape coexisting phases near closed shells.

A spherical shell-model approach has been formulated in which the underlying microscopic structure of low-lying intruder $0^+$ states is largely connected to the presence of 2p-2h excitations across the proton closed shell at Z=82 (for the Pb nuclei) [8]. Similar and compelling evidence has been obtained at the Z=50 shell closure too [5]. It was suggested that this may well form a rather general basis for the formation of low-lying intruding configurations. In fig. 4, the essential various energy terms are shown that contribute to the lowering of the rather high-lying unperturbed 2p-2h excitations in a shell-model framework. Besides the pairing energy gain and smaller variations due to the nuclear monopole corrections originating in the proton-neutron interaction, the most dramatic lowering of the energy origi-

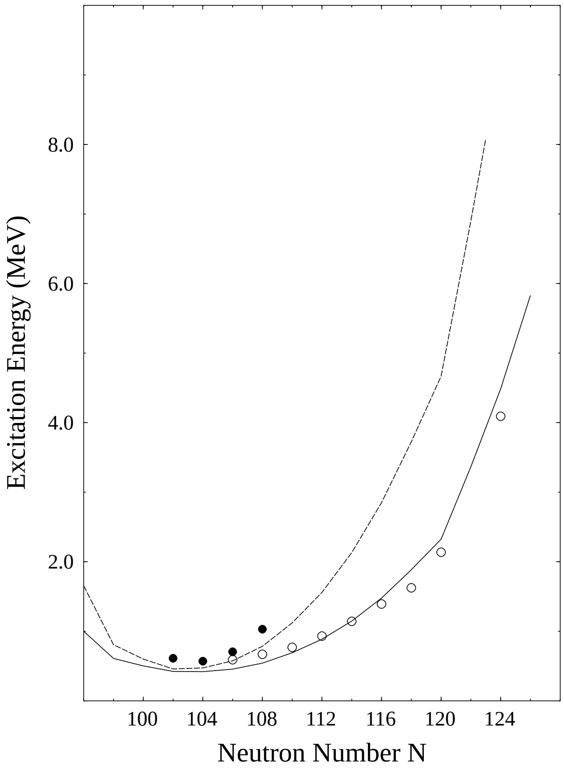

Figure 5: The energy dependence of the lowest 2p-2h and 4p-4h intruder $0^+$ excitations in the even-even Pb nuclei, according to the use of eqs. (1) and (2) [8].

nates from the attractive quadrupole proton-neutron force. In the simplified case of a separable quadrupole-quadrupole force, the energy difference between the regular and intruder configuration can be depicted (see also fig. 4) as

$$E_{intr.}(0^+) = \langle 0_I^+|\hat{H}|0_I^+\rangle - \langle 0_{GS}^+|\hat{H}|0_{GS}^+\rangle \quad , \tag{1}$$

$$\Delta E_Q \simeq 2\kappa\,(\Delta N_\pi)\,N_\nu \quad , \tag{2}$$

where the latter expression is a very good approximation to the more realistic evaluation of the energy difference. Here, $\kappa$ denotes the quadrupole force strength, $N_\nu$ the number of neutron pairs and $\Delta N_\pi$, the number of extra pairs formed in creating the intruder state. The use of the above prescription, discussed in detail in ref. [8] for the Pb nuclei, is illustrated, in fig. 5, for the 2p-2h (oblate) states and the 4p-4h (prolate) states. In the reproduction of the data, only a single parameter i.e. the quadrupole force strength $\kappa$, shows up.

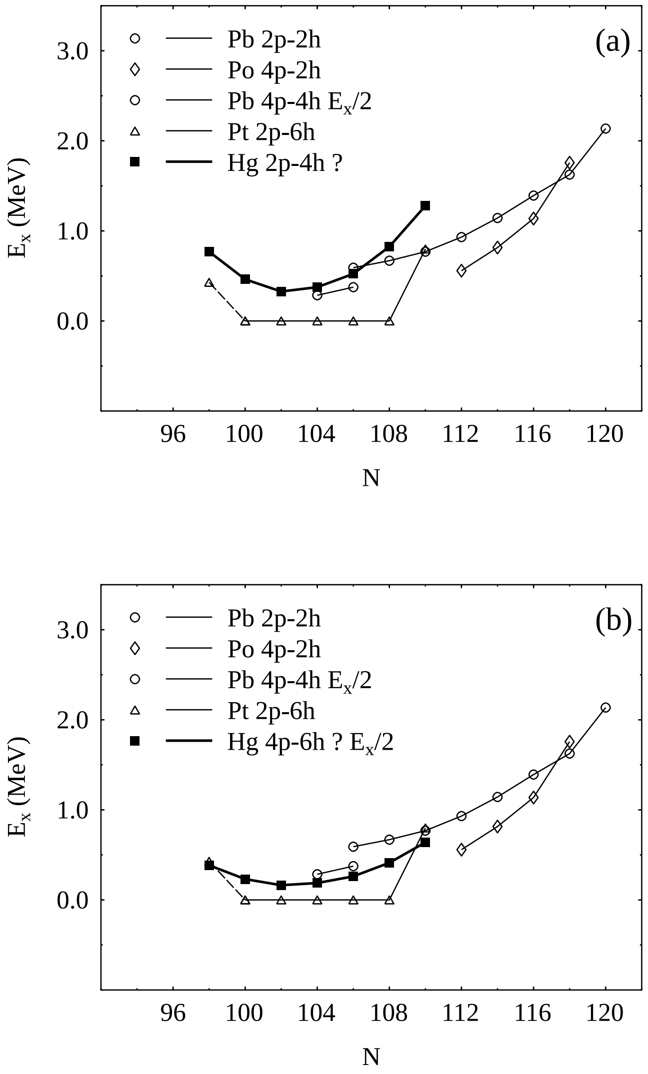

Figure 6: Illustration of the quadrupole energy scaling law (eq. (2)) as applied to the Pb region. In part (a), we assume a 2p-4h structure for the $0^+$ intruder band in Hg nuclei whereas in (b) the assumption of a 4p-6h structure is made in evaluating the scaled energy.

We can now make use of the result obtained within the spherical shell-model approach in order to suggest a possible explanation for the conspicuous behaviour of the $0^+$ excitation energy in the Pb region as a function of neutron number as was illustrated in Figure 3. If indeed, the quadrupole energy gain of eq. (2) is the determining factor, then a scaling law should describe the energy of the various mp-nh intruder configurations relative to the reference state for a given nucleus. This scaling factor is exactly the number of extra p-h pairs that are formed in creating the excited state (see eq. (2)).

So, by dividing the excitation energy of the 4p-4h excitations by a factor 2, all excitation energies should ideally fall on a single curve. We present resuls in fig. 6a,b for two different assumptions concerning the excited $0^+$ band in the Hg nuclei since the microscopic origin is not unambigously defined. The figure 6b illustrates that the origin as a 4p-4h excitation on top of the existing 2 holes in the Hg nuclei gives an even better illustration of the apparent scaling law.

We finish this section by pointing out that the approach starting from a deformed single-particle mean field, incorporating residual pairing correlations or starting from a spherical shell-model approach but including the important effects caused by the strong and attractive proton-neutron force give very equivalent results. Also the application to the Pb region, at present forming the most documented region in the nuclear mass table, is not unique. Applications to the N=50, the N=82 mass regions, the Sn region but also to much lighter nuclei such as the N=20 nuclei have been carried out [5].

## 3 Experimental Fingerprints

A number of very specific experimental fingerprints provide the necessary elements to classify an observed excited band as a shape coexisting band (although quite often the presence of mixing between the normal and the intruding bands shows up through specific perturbations in standard energy and decay patterns).

First, on the level of the excitation energy and the decay properties, it is clear that the band, corresponding to a much more quadrupole deformed shape, can show rotational features. Even more important is the presence of very strong intraband B(E2) values and hindered or much reduced interband decay [5]. A recent study of the intruder band in $^{114,116}$Sn is a very nice example and we illustrate part of the band, incorporating also decay properties, in fig. 7 (taken from reference [9]). This situation as well as the one in the other nearby even-even Sn nuclei shows overwhelming data as to the observation of a shape coexisting band exactly at the Z=50 proton closed shell (in $^{116}$Sn, the band extension all the way up to spin $20^+$ has been

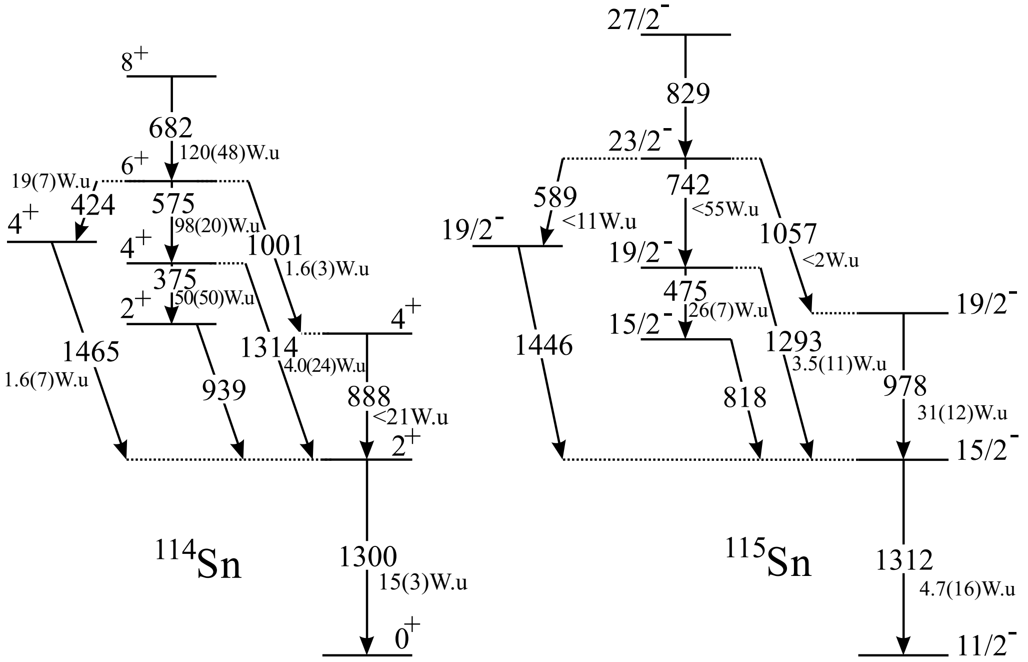

Figure 7: Comparison of the low-lying intruder bands in $^{114}$Sn and $^{115}$Sn. The B(E2) values are also given in Weisskopf units (W.u.) (taken from [9]).

observed [9]). The Hg and Pt nuclei as well as the more recently studied Pb nuclei are also very nice examples on this fingerprint of band structure.

A second and equally important fingerprint is brought about by one-, two- and even cluster-transfer reactions. Those reactions can very selectively populate final states with a given particle-hole structure by appropriate selection of the initial nucleus and the transferred particle(s).In this context, it was shown clearly that e.g. in the Sn nuclei, one-nucleon transfer reactions starting with In or Sb as target nuclei, preferentially populate states above 4 MeV [5] (the energy gap at Z=50) and do not feed the intruder band structure whereas two-proton transfer ($^3$He,n) reactions [10] starting from the even-even Cd nuclei, rather strongly populate the $0^+$ band-head of these intruder bands. We like to stress the importance of getting more systematic data on selective transfer into the proposed intruder bands since those experiments can give specific answers as to the underlying microscopic structure for many of the observed bands. Unfortunately, in many cases, the particular nuclei of interest are far from the region of stable nuclei and those experiments cannot be carried out.

A specific fingerprint is found in the study of static moments such as nuclear radii, electric quadrupole moments (indicating direct evidence for quadrupole deformation) and also magnetic dipole moments. The latter can play a selective role as they are ideal indicators for the single-particle structure of a given state. In particular, the important work on systematic studies of radii over many long mass chains has given unvaluable insight in the deformation behaviour and the effect that shape changes may cause to the atomic nucleus, even in its ground-state [11]. The experiments in the Z=40 region (Sr, Zr, ...) and also in the Pb region (Pb, Tl, Hg, Au and Pt) have brought dramatic examples for shape coexistence and also for very abrupt shape changes (see e.g. at N=60 in the mass A=100 region and near the neutron mid-shell N=104 in the Pb region in particular the Au odd-mass nuclei [11]). Quadrupole moments are known to a much lesser extent but out of B(E2) rates in the intruder band, transition quadrupole moments can be derived (see e.g. the odd-mass In and Sb nuclei, the even-even Hg and Pt nuclei [5]) and in all cases, the derived moment indicates quadrupole deformations that are typical for strongly deformed nuclei and not at all what one would expect for nuclei at or near to closed-shell configurations.

A most interesting fingerprint follows from studying E0 transitions and the reduced $\rho^2$ values (similar to reduced matrix elements for the other electromagnetic transitions). It has been shown that very large $\rho^2$(E0) values are an optimal indication for strong mixing between states for which the wavefunctions are localized at largely different values of the deformation shape coordinate [12, 13]. By studying a particular two-level model the salient features of such mixing already show up. More precise calculations, using either the nuclear shell-model or some other collective model approach, support this idea. We refer to a forthcoming paper of Wood et al. for an extensive discussion on E0 monopole properties in nuclei [14]. The essential expression that relates the $\rho^2$ value to the difference in mean-square deformation reads

$$\rho^2_{1,2} = a^2\left(1-a^2\right)\left(\frac{3}{4\pi}\right)^2 Z^2\left(\beta_2^2-\beta_1^2\right)^2 \quad , \tag{3}$$

where $a$ indicates the mixing between the two unperturbed levels and we use the definition for $\beta_i^2$ as

$$\beta_i^2 \equiv \langle 0_i^+ | \sum_\mu |\alpha_{2\mu}|^2 | 0_i^+ \rangle \quad . \tag{4}$$

In ref. [14] besides a number of interesting examples of how the observed E0 rates may well correlate with this simple expression, it is shown that one can relate the $\rho^2$ value with the isotopic shifts. Here, we restrict ourselves to illustrate E0 transitions in the mass A=100 region. Fig. 8 very clearly illustrates the observation of the largest $\rho^2$ values observed in atomic nuclei, a result that most probably serves as a good fingerprint for strong shape mixing near N=58,60.

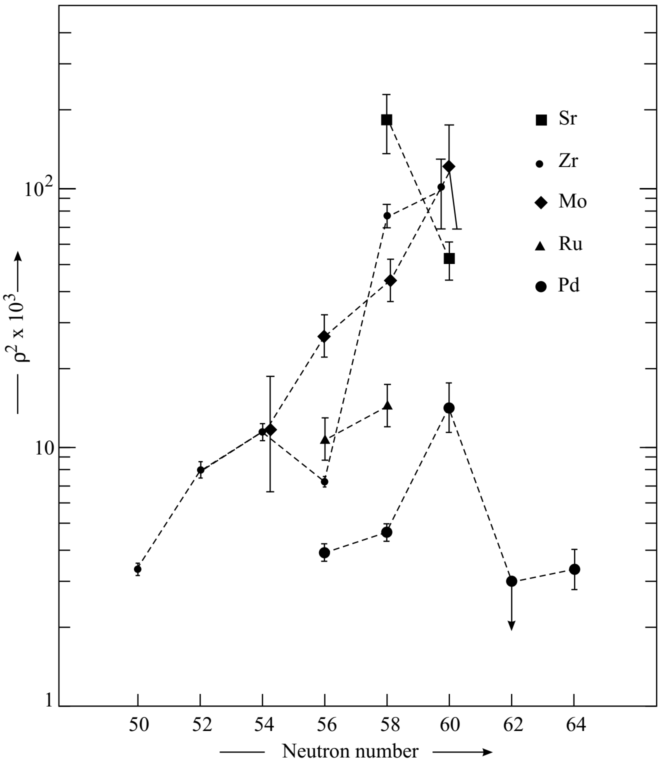

Figure 8: Experimental values for $\rho^2$(E0) in the mass A=100 region. References for the data can be found in [14].

Finally, $\alpha$-decay, much like the $\alpha$-particle transfer reactions, is able to probe shape coexistence or shape mixing. The study of the very neutron-deficient nuclei in the Pb region has been very intensively pursued over the last years, in particular as the neutron-deficient highly unstable nuclei have $\alpha$-decay as a major decay mode. For a compilation of the most recent results, we refer to the Ph.D. thesis of N.Bijnens where the intruding configurations and more important, their mixing, can be studied over a large mass interval [15].

## 4 Conclusion

In the present contribution, we have shown ample evidence for the presence of shape coexisting configurations, in particular near closed shells in many regions of the nuclear mass table. We have indicated that an approach which starts from a deformed single-particle field, in order to evaluate the total energy of the nucleus as a function of deformation, as well as an approach starting from a spherical shell model accentuating the proton-neutron quadrupole force can give a good account of those intruder structures. Moreover, both methods are equivalent. From the spherical shell-model approach, moreover, an interesting scaling law follows that is qualitatively able to describe the mass behaviour of the excitation energy for the $0^+$ band-head in many of these intruding configurations.

In a second part, we have presented and illustrated a number of interesting spectroscopic fingerprints in order to obtain unambiguous and also selective evidence for the fact that a number of observed low-lying bands indeed can be described as mainly shape coexisting configurations. We also show the need to combine the outcome of a large number of different probes (band structure, E2 transitions, spectroscopic factors in transfer reactions or $\alpha$-decay, the study of moments and E0 transitions if possible, ...) in order to come to reliable conclusions about the precise nature of low-lying bands in the vicinity of closed shells.

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