A SIMPLE MODEL FOR TRANSVERSE ENERGY DISTRIBUTION IN HEAVY ION COLLISIONS

Dedicated to Ján Pišút on the occasion of his 60th birthday

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A simple geometrical model (often quoted in literature as the Glauber model) of heavy ion collisions is recapitulated. It is shown that the transverse energy distribution of heavy ion collisions follow the geometry of the collision. An extension of the model to include rapidity and transverse mass particle spectra is discussed.

1 Introduction

Detailed studies of nucleus-nucleus interactions require the use of detailed event generators such as QGSM, RQMD or VENUS. These models incorporate many features, and it can be difficult to understand the origin of relations between variables. In the following, we describe a simple geometrical model relating the number of collisions to the cross section.

We will assume a Glauber model. This model is widely used in heavy ion community [1]-[10]. Present estimates of the collision centrality by the WA97 and NA50 collaborations are based on this kind of model. The physical ideas behind the model are found in [11].

2 Model

A good introduction to the model is given in [12]. Here we recapitulate the basic assumptions of the model. The nucleus-nucleus collision is described in terms of nucleon-nucleon collisions. The nucleons of the nucleus with mass number A are distributed in nucleus with probability density $\rho(\vec{b}, z)$ normalised to unity. It is convenient to introduce the thickness function for a nucleus A as

$$T_A(\vec{b}) = \int \rho(\vec{b}, z)dz$$

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If the nucleus A collide with a nucleus B at impact parameter b the probability of having at least one inelastic nucleon-nucleon collision is

$$p = \sigma_{in} \int T_A(\vec{s}) T_B(\vec{b} - \vec{s}) d^2\vec{s}$$

The probability for the occurrence of n inelastic baryon-baryon collisions in the A-B collision is then

$$P(n, \vec{b}) = \left(\begin{array}{c} AB \\ n \end{array} \right) p^n (1-p)^{AB-n}$$

It is easy to represent above formulas by a Monte-Carlo program:

- the nucleons in nucleus A are generated according the probability density $\rho_A(\vec{b}, z)$

- the nucleons in nucleus B are generated according the probability density $\rho_B(\vec{b}, z)$

- nucleons from nucleus A are "collided" with nucleons from nucleus B. The collision of nucleon i in nucleus A at coordinates $(\vec{b}_A^i, z_A^i)$ with nucleon of nucleus B at coordinates $(\vec{b}_B^j, z_B^j)$ takes place if $|\vec{b}_A^i - \vec{b}_B^j| < \sqrt{(\sigma_{in}/\pi)}$

The number of participants N_{part} (sometimes they are called "wounded nucleons") is defined as the number of nucleons in the target and projectile which have at least one inelastic collision. The number of participants at impact parameter $\vec{b}$ is

$$N_{part} = A \int d^2\vec{s} T_A(\vec{s}) \left(1 - (1 - \sigma_{in} T_B(\vec{s} - \vec{b}))^B \right) +$$

$$B \int d^2\vec{s} T_B(\vec{s} - \vec{b}) \left(1 - (1 - \sigma_{in} T_A(\vec{s}))^A \right).$$

The number of collisions is the number of binary nucleon-nucleon collisions. The number of collisions at impact parameter $\vec{b}$ is

$$N_c = AB\sigma_{in} \int T_A(\vec{s}) T_B(\vec{b} - \vec{s}) d^2\vec{s}$$

Both the number of participants and the number of collisions are easily calculated using a Monte-Carlo code.

3 Relation with Transverse Energy

It is not possible to relate the geometrical model described above to physically observable quantities without further assumptions. To proceed we shall make two assumptions both based on empirical observations:

> The number of produced particles is proportional to the number of participants

> The number of produced particles is proportional to the transverse energy in a given phase space window.

The NA35 collaboration has studied multiplicity distributions in SS collisions using detailed simulations in terms of the FRITIOF and VENUS models [13]. In both models they find that the average charged multiplicity per participant is approximately constant as a function of the number of participants. The simple proportionality of $< n_- >$ and $< N_p >$ was also assumed by Białas in the wounded nucleon model [1] and a similar result is obtained in the Dual Parton Model [14]. Comparison of several detailed models (FRITIOF, DTNUC 1.02, VENUS 4.12) with data is done in [15]. The proportionality between the transverse energy and number of produced particles was discussed in [16].

As an example of measured transverse energy distribution, fig.1 shows the differential cross section of the transverse energy produced in Pb-Pb and S-Au collisions at central rapidity as measured by NA49 collaboration [17].

The differential cross section of the number of participants for the Pb-Pb and S-Au collisions calculated in our model is shown in fig.2. The density of the nucleons in nucleus was taken to be uniform in a sphere of radius $R = 1.2A^{1/3}$. The inelastic cross section is $\sigma_{in} = 30$ mbarn. Comparing fig.1 and fig.2 it is seen that energy per participant is about 1.5 GeV. This number fixes the y-scale absolutely. Knowing that the number of generated events in fig.2 was 10^6 and that they were generated in a circle with the area 6.35 barn we see that the agreement of the cross section (measured in barn/GeV) is good.

We also present a variation of this model based on [7, 10]. We assume that particles are produced in each inelastic nucleon-nucleon collision. When traversing the nucleus, a nucleon loses its energy. We assume a simple law according to which the momentum of nucleon in the CMS nucleon-nucleon system is reduced by by a factor 1.7 ($p_{old}/p_{new} = 1.7$) in each collision. This corresponds to the lost of about 0.5 units of rapidity per collision. The number of charged particles (normalized to 1. at $\sqrt{(s)} = 20$.) emitted in each collision is then calculated according to the empirical rule:

$$n(\sqrt{(s)}) = \ln(\sqrt{(s)}/2.3)/\ln(8.7)$$

where $n(2.3) = 0$. and $n(20) = 1$. , note that $\sqrt{(s)} = 20 GeV$ is the SPS heavy ion energy for sulphur-sulphur collision and $\sqrt{(s)} = 17 GeV$ is SPS energy for lead-lead collision.
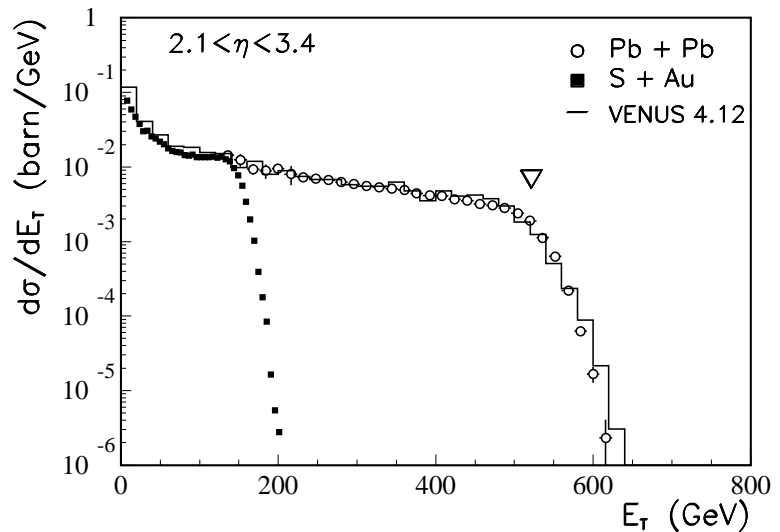
Fig1 1. Differential cross section of the transverse energy produced in Pb-Pb and S-Au at central rapidity as measured by NA49 coll. [17]

In fig. 3 the differential cross section of the quantity

$$w = \sum_{collisions} n(s) \tag{1}$$

is plotted. The choice of this quantity is motivated by the fact that if there is no degradation of the energy the quantity w is equal to the number of collisions. Comparison of fig.3 and fig.2 shows that the modified model is also able to reproduce the data.

4   Conclusions

We show that the transverse energy and charged particle distributions of heavy ion collisions are determined by the geometry of nuclei collisions. Assuming that the number of produced particles and the transverse energy are proportional to the number of participants the model is able to describe the data for different heavy ion systems.

This model allows us to estimate the number of participants and impact parameters corresponding to different centrality triggers [18]. Models based on the same principle were used to quantify the centrality of the collision in the WA97 [19] and NA50 [20] collaborations. (The NA49 collaboration uses a
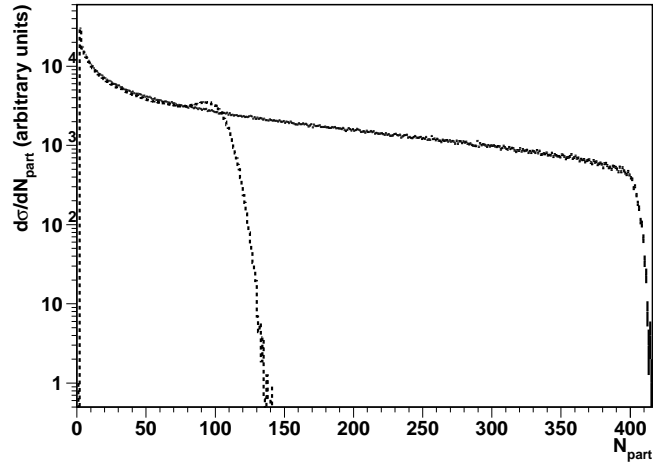
Fig 2. Differential cross section of the number of participants in Pb-Pb collisions (crosses) and S-Au collisions (histogram)

different approach as they are able to measure the number of participants using either a zero degree calorimeter or the number of measured protons [21].

The phenomenological observation that the number of produced particles is proportional to the number of participants can be obtained in a modified model where

> nucleons loose their energy in each collision when traversing the nucleus

> particles are produced in each nucleon-nucleon collision in amount dependent on the CMS energy of the nucleon-nucleon collision.

The extension of the modified model to include the rapidity and transverse momentum spectra of produced particles is straightforward.
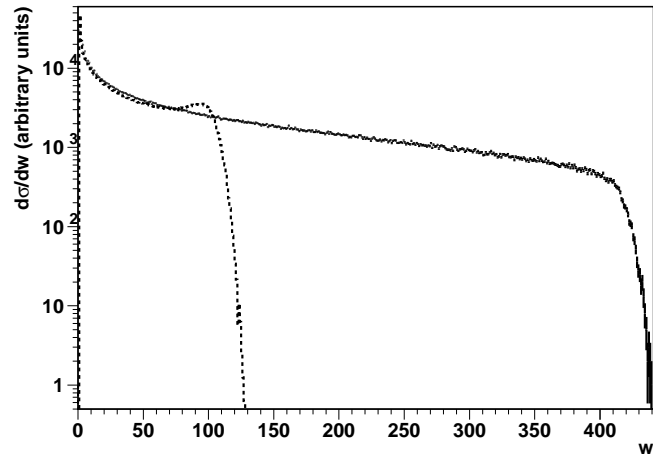
5   Acknowledgement

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Fig1 3. Differential cross section of the transverse energy in modified model, for explanation see text.

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