

Coulomb Final State Interactions for Gaussian Wave Packets

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Two-particle like-sign and unlike-sign correlations including Coulomb final state interactions are calculated for Gaussian wave packets emitted from a Gaussian source. We show that the width of the wave packets can be fully absorbed into the spatial and momentum space widths of an effective emission function for plane wave states, and that Coulomb final state interaction effects are sensitive only to the latter, but not to the wave packet width itself. Results from analytical and numerical calculations are compared with recently published work by other authors.

1. Introduction. – To analyze the geometry and dynamics of the collision region, two-particle correlations $C(\mathbf{q}, \mathbf{K})$ of like-sign and unlike-sign hadrons have been studied extensively in relativistic heavy ion collisions at AGS¹ and CERN SPS^{2,3} energies. They show \mathbf{q} -dependent structures at relative pair momenta $|\mathbf{q}| < 100$ MeV. These originate mainly⁴ from (i) final state interactions (which for pions at $|\mathbf{q}| < 100$ MeV are dominated completely by the Coulomb force) and (ii) the quantum statistics of identical particles.

Practical attempts to reconstruct space-time information from two-pion correlation data in momentum space so far exploit mostly the quantum statistical Hanbury Brown – Twiss (HBT) effect between identical bosons^{3,5,6}. This requires a prior subtraction of final state Coulomb interaction effects from the measured correlation functions, with proper account for the finite size of the emission region^{4,7–11}. Up to now this is done directly in the experimental analysis, either by taking experimental unlike-sign pion correlations to correct the like-sign ones³, or by a model calculation for the Coulomb effect expected for a finite size emission region¹.

This approach was recently questioned by Merlitz and Pelte^{12–14}. From a numerical analysis based on Gaussian wave packets emitted from a Gaussian source, they concluded that¹² “the expected Coulomb distortion in the momentum correlation . . . becomes unobservable” and that therefore “experimental data, which are published after Coulomb correction, are wrong for small momentum differences”. If correct, this conclusion would invalidate a substantial part of the existing work on the analysis of two-particle correlation

data since it implies that either (i) any attempt to base a space-time interpretation of identical two-particle correlations on Coulomb corrected data is ill-founded or that (ii) any attempt to describe the particle emitting source in heavy ion collisions by a set of Gaussian wave packets is inconsistent.

This dramatic perspective has led us to reconsider the calculation of two-particle correlations for Gaussian wave packets. Following Merlitz and Pelte we describe the particle emitting source in terms of a distribution of wave packet centers and a characteristic wave packet width σ of the emitted particles. The emitted Gaussian wave packets are propagated into the detector under the influence of mutual Coulomb final state interactions. We derive analytical expressions which show that the wave packet size σ can always be absorbed by a redefinition of the model parameters characterizing the source size. For Gaussian source models, two-particle correlation measurements cannot differentiate between “source size” and “wave packet width”. In a limiting case we further prove analytically the equivalence of the Gaussian wave packet formalism and the usually adopted plane wave calculations *irrespective of the size of the wave packet width*. These analytical calculations show quite generally that the problem pointed out by Merlitz and Pelte does not exist. Their results disagree with our numerical calculations as well as with analytical formulae which we derive without approximations from the same starting point as the calculation presented in Ref. ¹².

2. *Unlike-sign pion correlations.* – Our starting point is a set of Gaussian one-particle wave packets ^{16–19}

$$f_i(\mathbf{x}, t_0) = \frac{1}{(\pi\sigma^2)^{3/4}} e^{-(\mathbf{x}-\check{\mathbf{r}}_i)^2/(2\sigma^2)+i\check{\mathbf{p}}_i\cdot\mathbf{x}}, \quad (1)$$

which are centered at initial time $t = t_0$ at phase-space points $(\check{\mathbf{r}}_i, \check{\mathbf{p}}_i)$. We expand the time evolution of the corresponding two-particle state $\Psi_{ij}(\mathbf{x}_1, \mathbf{x}_2, t_0) = f_i(\mathbf{x}_1, t_0) f_j(\mathbf{x}_2, t_0)$ in terms of plane waves $\phi_{\mathbf{p}_1, \mathbf{p}_2}$,

$$\Psi_{ij}(\mathbf{x}_1, \mathbf{x}_2, t) = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \mathcal{A}_{ij}(\mathbf{p}_1, \mathbf{p}_2, t) \phi_{\mathbf{p}_1, \mathbf{p}_2}(\mathbf{x}_1, \mathbf{x}_2, t), \quad (2)$$

$$\phi_{\mathbf{p}_1, \mathbf{p}_2}(\mathbf{x}_1, \mathbf{x}_2, t) = e^{-iEt} \phi_{2\mathbf{K}}(\mathbf{X}) \phi_{\mathbf{q}/2}(\mathbf{r}) \equiv e^{-i(E_1+E_2)t} e^{2i\mathbf{K}\cdot\mathbf{X}} e^{i\mathbf{q}\cdot\mathbf{r}}, \quad (3)$$

which we write in terms of center of mass coordinates $\mathbf{X} = \frac{1}{2}(\mathbf{x}_1 + \mathbf{x}_2)$, $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2)$, and relative coordinates $\mathbf{r} = (\mathbf{x}_1 - \mathbf{x}_2)$, $\mathbf{q} = (\mathbf{p}_1 - \mathbf{p}_2)$. The probability \mathcal{P}_{ij} for detecting at time $t \rightarrow \infty$ the two particles prepared in the state Ψ_{ij} with momenta \mathbf{p}_1 and \mathbf{p}_2 is given by ¹¹

$$\mathcal{P}_{ij}(\mathbf{p}_1, \mathbf{p}_2) = \mathcal{P}_{ij}(\mathbf{q}, \mathbf{K}) = \lim_{t \rightarrow \infty} |\mathcal{A}_{ij}^*(\mathbf{p}_1, \mathbf{p}_2, t)|^2, \quad (4)$$

$$\begin{aligned}
\lim_{t \rightarrow \infty} \mathcal{A}_{ij}(\mathbf{p}_1, \mathbf{p}_2, t) &= \lim_{t \rightarrow \infty} \langle e^{-i\hat{H}_0(t-t_0)} \phi_{\mathbf{p}_1, \mathbf{p}_2}(t_0) | e^{-i\hat{H}(t-t_0)} \Psi_{ij}(t_0) \rangle \\
&= \langle \phi_{2\mathbf{K}} | \Psi_{ij}^{\text{pair}} \rangle \langle \Omega_+ \phi_{\mathbf{q}/2} | \Psi_{ij}^{\text{rel}} \rangle. \tag{5}
\end{aligned}$$

Here we separated the state $\Psi_{ij}(\mathbf{x}_1, \mathbf{x}_2, t_0)$ into relative and center of mass wave functions,

$$\Psi_{ij}^{\text{pair}}(\mathbf{X}) = \frac{1}{(\pi\sigma^2)^{3/4}} e^{-(\mathbf{X} - \check{\mathbf{X}}_{ij})^2/\sigma^2 + 2i\check{\mathbf{K}}_{ij} \cdot \mathbf{X}}, \tag{6}$$

$$\Psi_{ij}^{\text{rel}}(\mathbf{r}) = \frac{1}{(\pi\sigma^2)^{3/4}} e^{-(\mathbf{r} - \check{\mathbf{r}}_{ij})^2/(4\sigma^2) + \frac{i}{2}\check{\mathbf{q}}_{ij} \cdot \mathbf{r}}, \tag{7}$$

where $\check{\mathbf{X}}_{ij}, \check{\mathbf{K}}_{ij}, \check{\mathbf{r}}_{ij}, \check{\mathbf{q}}_{ij}$ are the corresponding center-of-mass and relative coordinates constructed from the wave packet centers. In (5) we have also introduced the Møller scattering operator $\Omega_+ = \lim_{t \rightarrow \infty} e^{i\hat{H}(t-t_0)} e^{-i\hat{H}_0(t-t_0)}$ for the final state interaction hamiltonian $\hat{H} = \hat{H}_0 + V(\mathbf{r})$, $\hat{H}_0 = -\frac{1}{4m}\Delta_{\mathbf{X}} - \frac{1}{m}\Delta_{\mathbf{r}}$. This Møller operator maps the plane wave $\phi_{\mathbf{q}/2}$ onto the solution of the Lippmann-Schwinger equation for the corresponding stationary scattering problem. For two-particle Coulomb interactions this is the Coulomb scattering wave

$$(\Omega_+ \phi_{\mathbf{q}/2})(\mathbf{r}) = \Phi_{\mathbf{q}/2}^{\text{coul}}(\mathbf{r}) = \Gamma(1 - i\eta) e^{\frac{1}{2}\pi\eta} e^{\frac{i}{2}\mathbf{q} \cdot \mathbf{r}} F(i\eta; 1; iz_-), \tag{8}$$

$$z_{\pm} = \frac{1}{2}(qr \pm \mathbf{q} \cdot \mathbf{r}), \quad \eta = \frac{me^2}{4\pi q}, \tag{9}$$

where $e^2/4\pi = \alpha = 1/137$, $r = |\mathbf{r}|$, $q = |\mathbf{q}|$, $F(i\eta; 1; iz_-)$ is the confluent hypergeometric function, and η is the Sommerfeld parameter. Eq. (8) applies for pairs with opposite charges; for like-sign pairs one replaces $\eta \mapsto -\eta$. With the help of Eq. (8), the calculation of the amplitudes (5) is reduced to a six-dimensional integral.

2.1. Gaussian source model. – We consider a toy model of simultaneous particle emission at time $t = t_0$ for which initially the wave packet centers are distributed with Gaussians of widths R and Δ in coordinate and momentum space, respectively. It can be specified by the following normalized distributions of relative distances and pair coordinates:

$$S_{\text{rel}}(\check{\mathbf{r}}, \check{\mathbf{q}}) = \frac{1}{(4\pi R\Delta)^3} e^{-\frac{\check{\mathbf{r}}^2}{4R^2} - \frac{\check{\mathbf{q}}^2}{4\Delta^2}}, \quad S_{\text{pair}}(\check{\mathbf{X}}, \check{\mathbf{K}}) = \frac{1}{(\pi R\Delta)^3} e^{-\frac{\check{\mathbf{X}}^2}{R^2} - \frac{\check{\mathbf{K}}^2}{\Delta^2}}. \tag{10}$$

With the choice $R^2 = R_s^2/2$, $\Delta^2 = mT$, and $\sigma^2 = 2\sigma_0^2$, this model coincides with the one considered by Merlitz and Pelte¹². We calculate the unlike-sign two-particle correlator via the two-particle spectrum $\mathcal{P}_{ij}(\mathbf{p}_1, \mathbf{p}_2)$ of Eq. (4),

averaged over the distributions (10) and normalized to the corresponding spectrum for pairs of non-interacting particles:

$$C^{+-}(\mathbf{q}, \mathbf{K}) = \frac{I^{\text{int}}(\mathbf{q}, \mathbf{K})}{I^{\text{nonint}}(\mathbf{q}, \mathbf{K})}, \quad (11)$$

$$I(\mathbf{q}, \mathbf{K}) = \int d^3\check{r}_{ij} d^3\check{q}_{ij} d^3\check{X}_{ij} d^3\check{K}_{ij} S_{\text{rel}}(\check{\mathbf{r}}_{ij}, \check{\mathbf{p}}_{ij}) S_{\text{pair}}(\check{\mathbf{X}}_{ij}, \check{\mathbf{K}}_{ij}) \mathcal{P}_{ij}(\mathbf{q}, \mathbf{K}). \quad (12)$$

For the non-interacting case Eq. (12) must be evaluated with $\mathcal{P}_{ij}^{\text{nonint}}$ which is obtained by replacing in the amplitude (5) the relative Coulomb wave $\Omega_+ \phi_{\mathbf{q}/2}$ by the plane wave $\phi_{\mathbf{q}/2}$. Since the center of mass coordinate is not affected by the two-particle final state interaction, the integrations over $S_{\text{pair}}(\check{\mathbf{X}}_{ij}, \check{\mathbf{K}}_{ij})$ drop out in the ratio (11), and the correlator $C^{+-}(\mathbf{q}, \mathbf{K})$ becomes independent of the pair momentum \mathbf{K} . One finds

$$C^{+-}(\mathbf{q}) = G(-\eta) \left(\frac{\bar{\Delta}}{4\pi\bar{R}} \right)^3 e^{\frac{q^2}{4\bar{\Delta}^2}} \int d^3r e^{-\frac{r^2}{8\bar{R}^2} + \frac{i}{2}\mathbf{r}\cdot\mathbf{q}} F(i\eta; 1; iz_-) \\ \times \int d^3r' e^{-\frac{r'^2}{8\bar{R}^2} - \frac{i}{2}\mathbf{r}'\cdot\mathbf{q}} [F(i\eta; 1; iz'_-)]^* e^{-\left(\frac{\bar{\Delta}^2}{4} - \frac{1}{16\bar{R}^2}\right)(\mathbf{r}-\mathbf{r}')^2}, \quad (13)$$

where $G(\eta) = |\Gamma(1+i\eta)e^{-\frac{1}{2}\pi\eta}|^2 = 2\pi\eta/(e^{2\pi\eta}-1)$ is the Gamow factor. It is important that this correlator depends only on the parameter combinations

$$\bar{R}^2 = R^2 + \frac{\sigma^2}{2}, \quad \bar{\Delta}^2 = \Delta^2 + \frac{1}{2\sigma^2}. \quad (14)$$

This shows that for Gaussian models of particle emission, the wave packet width σ can be absorbed in a redefinition of the model parameters. There is no measurement which allows to determine σ independent of R and Δ . Of course, the specific σ -dependence in (14) still constrains the values which \bar{R} and $\bar{\Delta}$ can take. In particular, $\bar{R}, \bar{\Delta}$ always satisfy the uncertainty relation $\bar{R}\bar{\Delta} \geq \hbar/2$, rendering the exponent of the last term in (13) always negative.

In the absence of final state interactions, the dependence of the correlator on the parameter combinations (14) has been noted repeatedly^{16,18,20}. The momentum spectra and correlations are entirely determined by the ‘‘effective’’ emission function^{18,20}

$$S_{\text{eff}}(\mathbf{r}, \mathbf{p}) = \int d^3\check{r} d^3\check{p} S(\check{\mathbf{r}}, \check{\mathbf{p}}) S_{\text{w.p.}}(\mathbf{r} - \check{\mathbf{r}}, \mathbf{p} - \check{\mathbf{p}}), \quad (15)$$

which is a folding integral between the distribution of wavepacket centers $S(\check{\mathbf{r}}, \check{\mathbf{p}})$ and the Wigner density $S_{\text{w.p.}}$ of a single particle wave packet. For

Gaussian source models, S_{eff} depends on \bar{R} and $\bar{\Delta}$ only²⁰. The same holds true in the presence of final state interactions where the correlator can be written for arbitrary model distributions $S(\bar{\mathbf{r}}, \bar{\mathbf{p}})$ as a quite involved expression depending on S_{eff} only (see Eq. (60) in Ref.¹¹). Our result (13) is an explicit representation of this general relation, obtained for the Gaussian source models (10). It allows further analytical and numerical studies:

2.2. Limiting cases. – In two interesting limits the correlator $C^{+-}(\mathbf{q})$ can be further simplified analytically. Using $\lim_{\bar{\Delta} \rightarrow \infty} (\bar{\Delta}^2/4\pi)^{3/2} \exp\left[-\frac{\bar{\Delta}^2}{4}(\mathbf{r} - \mathbf{r}')^2\right] = \delta^{(3)}(\mathbf{r} - \mathbf{r}')$ we find

$$\lim_{\bar{\Delta} \rightarrow \infty} C^{+-}(\mathbf{q}) = \frac{1}{(4\pi\bar{R}^2)^{3/2}} \int d^3r e^{-\frac{\pi^2}{4\bar{R}^2}} |\Phi_{\mathbf{q}/2}^{\text{Coul}}(\mathbf{r})|^2. \quad (16)$$

This expression, first written down by Koonin²³, is the usually adopted starting point for plane wave calculations^{1–3,7,8,10,21–23}; it was shown by Baym and Braun-Munzinger¹⁰ to be well approximated by a semi-classical approach. The limit $\bar{\Delta} \rightarrow \infty$ can be taken for arbitrary values of the wave packet width σ and is equivalent to the limit $\Delta \rightarrow \infty$ which describes an emission function S without momentum dependence. In this limit, the correlator $C^{+-}(\mathbf{q})$ for simultaneously emitted Gaussian wave packets coincides exactly with the starting point of conventional plane wave calculations^{10,11}, *irrespective of the size σ of the wave packet*. Rescaling σ then simply amounts to a change of the effective spatial size \bar{R} of the source.

What happens if the source size becomes large? Changing in Eq. (13) the integration variables $\mathbf{r} \rightarrow \sqrt{\bar{R}}\mathbf{r}$, $\mathbf{r}' \rightarrow \sqrt{\bar{R}}\mathbf{r}'$, and replacing the Coulomb wave function by its leading contribution, $\Phi_{\mathbf{q}/2}^{\text{Coul}}(\sqrt{\bar{R}}\mathbf{r}) \rightarrow \exp\left(\frac{i}{2}\sqrt{\bar{R}}\mathbf{r} \cdot \mathbf{q} + i\eta \ln\left(\frac{\sqrt{\bar{R}}}{2}(qr - \mathbf{q} \cdot \mathbf{r})\right)\right) + O\left(\frac{1}{\sqrt{\bar{R}}}\right)$, we find

$$\lim_{\bar{R} \rightarrow \infty} C^{+-}(\mathbf{q}) = 1. \quad (17)$$

This is expected: as the source becomes larger, the average spatial separation between particles increases and their Coulomb attraction decreases, leading to a flat correlator in the limit of infinite source size.

2.3. Numerical results. – One may wonder whether a large but realistic effective source size \bar{R} can come sufficiently close to the limiting case $\bar{R} \rightarrow \infty$ of (17) to support the claim of Merlitz and Pelte^{12,13} that the Coulomb repulsion becomes effectively unobservable. To study this question we have calculated the correlator (13) numerically, after doing the azimuthal integrations ($q = |\mathbf{q}|$):

$$C^{+-}(q) = 4\pi^2 G(-\eta) \left(\frac{\bar{\Delta}}{4\pi\bar{R}}\right)^3 e^{\frac{q^2}{4\bar{\Delta}^2}}$$

$$\begin{aligned}
& \times \int_0^\infty r^2 dr e^{-\frac{r^2}{8\bar{R}^2}} \int_{-1}^1 dx e^{\frac{i}{2}qr x} F(i\eta; 1; \frac{i}{2}qr(1-x)) \\
& \times \int_0^\infty r'^2 dr' e^{-\frac{r'^2}{8\bar{R}^2}} \int_{-1}^1 dy e^{-\frac{i}{2}qr' y} [F(i\eta; 1; \frac{i}{2}qr'(1-y))]^* \\
& \times I_0 \left(2B^2 rr' \sqrt{1-x^2} \sqrt{1-y^2} \right) e^{-B^2(r^2 - 2rr'xy + r'^2)}. \quad (18)
\end{aligned}$$

Here I_0 is the modified Bessel function and $B^2 = \bar{\Delta}^2/4 + 1/(16\bar{R}^2)$. The numerical results for $C^{+-}(\mathbf{q})$ are shown in Fig. 1 for the model parameters $R = 3.5$ fm, $\Delta = 84$ MeV and different values of the wave packet width σ . For $\sigma = 2.5$ fm, these values correspond to the model parameters chosen in Ref. ¹².

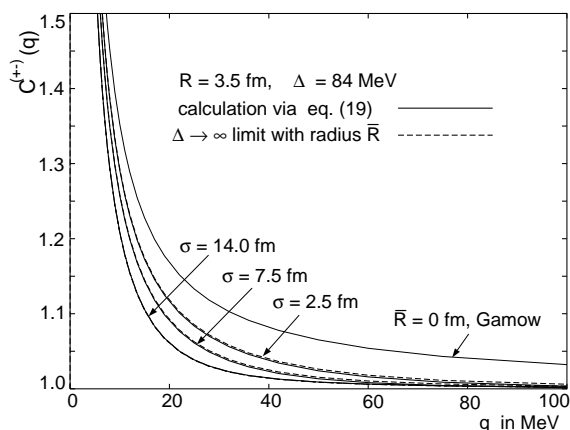


Figure 1: Two-particle correlator of unlike-sign pion pairs for the Gaussian model (10). The correlation depends only on the model parameter combinations $\bar{R}^2 = R^2 + \sigma^2/2$ and $\bar{\Delta}^2 = \Delta^2 + 1/(2\sigma^2)$. The results for $\Delta = 84$ MeV agrees well with Koonin's expression (16) obtained in the limit $\Delta \rightarrow \infty$.

One sees that, with increasing source size $\bar{R}^2 = R^2 + \sigma^2/2$, the resulting Coulomb correlation indeed becomes smaller than the Gamow factor, but it remains clearly observable even for a very large wave packet size $\sigma = 14$ fm. We also compare in Fig. 1 the full correlator $C^{+-}(q)$ of Eqs. (13/18) to the limit (16), calculated for the same value of \bar{R} . At least for the model parameters studied here, both expressions agree almost exactly. There seems to be no possibility to distinguish on the basis of final state correlations between the Gaussian wave packet formalism and the plane wave calculation based on the approximate expression (16) which was used in other studies ^{1,3,10}. As long as the Gaussian wave packet width σ is included consistently in the definition

of the source size $\bar{R} = \sqrt{R^2 + \sigma^2/2}$, both formalisms lead qualitatively and quantitatively to the same result. We conclude that the differing results derived in Ref. ¹² with the help of a numerical simulation of the time evolution of wave packets are incorrect.

3. *Like-sign pion correlations.* – For pairs of pions of identical charge the Coulomb final state effects are superimposed on the quantum statistical effects resulting from the symmetrization of the two-particle wave function. Paralleling the calculation of section 2. with Bose-Einstein symmetrized Gaussian wavepackets Ψ_{ij} and plane waves $\phi_{\mathbf{p}_1, \mathbf{p}_2}$, one arrives at the symmetrized asymptotic two-particle amplitude

$$\lim_{t \rightarrow \infty} \mathcal{A}_{ij}^{\text{BE}}(\mathbf{p}_1, \mathbf{p}_2, t) = \langle \phi_{\mathbf{K}} | \Psi_{ij}^{\text{pair}} \rangle [\langle \Omega_+ \phi_{\mathbf{q}/2} | \Psi_{ij}^{\text{rel}} \rangle + \langle \Omega_+ \phi_{-\mathbf{q}/2} | \Psi_{ij}^{\text{rel}} \rangle], \quad (19)$$

from which the two-particle momentum-space probability $\mathcal{P}_{ij}^{\text{BE}}(\mathbf{p}_1, \mathbf{p}_2)$ is again calculated according to Eq. (4). Averaging $\mathcal{P}_{ij}^{\text{BE}}$ according to (12) over the model distribution gives the numerator of the two-particle correlator which we call $I^{\text{BE,int}}(\mathbf{q}, \mathbf{K})$. We normalize it by the method of “mixed pairs”: an uncorrelated (mixed) pair is described by an unsymmetrized product state without Coulomb interaction and leads, after averaging over the model distribution, to $I^{\text{nonint}}(\mathbf{q}, \mathbf{K})$ (see Eqs. (11/12)). Taking both distinguishable states $f_i(\mathbf{x}_1, t_0) f_j(\mathbf{x}_2, t_0)$ and $f_i(\mathbf{x}_2, t_0) f_j(\mathbf{x}_1, t_0)$ into account we have

$$C^{++}(\mathbf{q}, \mathbf{K}) = \frac{I^{\text{BE,int}}(\mathbf{q}, \mathbf{K})}{I^{\text{nonint}}(\mathbf{q}, \mathbf{K}) + I^{\text{nonint}}(-\mathbf{q}, \mathbf{K})} = C^{\text{dir}}(\mathbf{q}) + C^{\text{ex}}(\mathbf{q}). \quad (20)$$

For the Gaussian model (10), the center of mass coordinate is affected neither by two-particle final state interactions nor by two-particle Bose-Einstein symmetrization. The correlator (20) hence does not depend on the pair momentum \mathbf{K} . It splits into two contributions. The “direct term” can be obtained from $C^{+-}(\mathbf{q})$ in Eq. (13) by changing the sign of the Sommerfeld parameter, $-\eta \rightarrow \eta$. The “exchange term” is given by

$$C^{\text{ex}}(\mathbf{q}) = \text{Re} \left\{ G(\eta) \left(\frac{\bar{\Delta}}{4\pi\bar{R}} \right)^3 e^{\frac{\mathbf{q}^2}{4\bar{\Delta}^2}} \int d^3r e^{-\frac{\mathbf{r}^2}{8\bar{R}^2} + \frac{i}{2}\mathbf{r}\cdot\mathbf{q}} F(-i\eta; 1; iz_-) \right. \\ \left. \times \int d^3r' e^{-\frac{\mathbf{r}'^2}{8\bar{R}^2} + \frac{i}{2}\mathbf{r}'\cdot\mathbf{q}} [F(-i\eta; 1; iz'_+)]^* e^{-\left(\frac{\bar{\Delta}^2}{4} - \frac{1}{16\bar{R}^2}\right)(\mathbf{r}-\mathbf{r}')^2} \right\}. \quad (21)$$

This integral can be simplified to a 4-dimensional expression similar to (18). The limits $\bar{\Delta} \rightarrow \infty$ and $\bar{R} \rightarrow \infty$ of the first term $C^{\text{dir}}(\mathbf{q})$ are obtained from (16/17) by replacing $-\eta \rightarrow \eta$. The corresponding limits for the exchange term

are given by

$$\lim_{\bar{\Delta} \rightarrow \infty} C^{\text{ex}}(\mathbf{q}) = \frac{1}{(4\pi\bar{R}^2)^{3/2}} \int d^3r e^{-\frac{r^2}{4\bar{R}^2}} \cos(\mathbf{r} \cdot \mathbf{q}) |\Phi_{\mathbf{q}/2}^{\text{coul}}(\mathbf{r})|^2, \quad (22)$$

$$\lim_{\bar{R} \rightarrow \infty} C^{\text{ex}}(\mathbf{q}) = 0. \quad (23)$$

As in the case of unlike-sign correlations, the correlator $C^{++}(\mathbf{q})$ depends only on the parameter combinations \bar{R} and $\bar{\Delta}$, but not explicitly on the wave packet width σ . In the limit $\bar{\Delta} \rightarrow \infty$, the Gaussian wave packet formalism again coincides with the Koonin formula²³ (now with the additional symmetrization factor $1 + \cos(\mathbf{q} \cdot \mathbf{r})$ under the integral) which is the starting point of most conventional plane wave calculations.

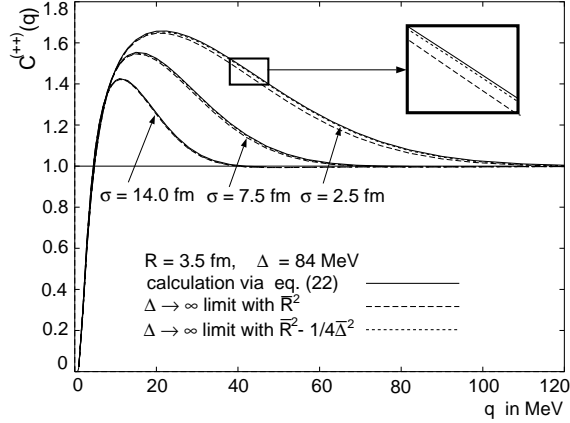


Figure 2: Two-particle correlator of like-sign pion pairs for the Gaussian model (10). The correlation depends only on the model parameter combinations $\bar{R}^2 = R^2 + \sigma^2/2$ and $\bar{\Delta}^2 = \Delta^2 + 1/(2\sigma^2)$. The full correlator is well approximated by the standard Koonin expression for a static momentum-independent source of radius squared $\bar{R}^2 - 1/(4\bar{\Delta}^2)$.

In Fig. 2 we compare numerically the full correlator $C^{++}(q)$ from Eq. (20), for $\Delta = 84$ MeV, with the limit $\Delta \rightarrow \infty$, both calculated for the same value of $\bar{R}^2 = R^2 + \sigma^2/2$. (Due to spherical symmetry of the source the correlator depends only on $q = |\mathbf{q}|$.) For small values of σ one observes a small, but significant difference. The reason is that even in the absence of Coulomb final state interactions, the HBT radius parameter (which gives the q -width of the correlator) is not exactly given by the source size \bar{R}^2 but rather by¹⁸

$$R_{\text{HBT}}^2 = \bar{R}^2 - \frac{1}{4\bar{\Delta}^2}. \quad (24)$$

For large values of R or σ , the term \bar{R}^2 dominates this expression, and the difference between R_{HBT}^2 and \bar{R}^2 disappears (see Fig. 2). In fact, when computing the limit $\Delta \rightarrow \infty$ of $C^{++}(q)$ using R_{HBT}^2 instead of \bar{R}^2 , the agreement with the full correlator $C^{++}(q)$ becomes almost exact even for small values of σ (see inset in Fig. 2). We note that even for the smallest value studied here, $\sigma = 2.5$ fm, the term $1/(4\bar{\Delta}^2)$ contributes only $\approx 5\%$ to R_{HBT}^2 . This small difference is clearly visible in the exchange term (21) of $C^{++}(q)$, whereas its influence on the direct term C^{dir} (and hence on the correlator C^{+-}) is found numerically to be an order of magnitude smaller. This illustrates that the two-particle correlator C^{++} of identical pions is more sensitive than C^{+-} to a small change in the Gaussian width of the phase space density.

The modification (24) of the radius parameter to be used as input in the plane wave calculation can also be obtained from the Koonin expression²³ or, most explicitly, from Eq. (65) of Ref.¹¹. The consistency of Koonin's expression with the full correlator in the present model calculation is a non-trivial check of the so-called smoothness approximation used in Ref.¹¹ to derive Koonin's expression from a general treatment of two-body final state interactions.

To sum up: as long as the Gaussian wave packet width σ is included consistently in the definition of the source size, both the plane wave calculations^{1-3,7,8,10,21-23} and the Gaussian wave packet formalism¹⁶⁻¹⁹ lead to qualitatively and quantitatively equivalent results. While the present study proved this only for Gaussian source models, we expect it to be true quite generally since we know that¹¹ the relation (15) between the effective emission function and the Wigner density of single particle wave packets holds for arbitrary model distributions and that¹⁵ two-particle momentum correlations are mostly sensitive to the Gaussian characteristics of the source in space-time.

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