Five-Dimensional Gauged Supergravity and Supersymmetry Breaking in M Theory

John Ellis[a], Zygmunt Lalak[b,c] and Witold Pokorski[d]

ABSTRACT

We extend the formulation of gauged supergravity in five dimensions, as obtained by compactification of M theory on a deformed Calabi-Yau manifold, to include non-universal matter hypermultiplets. Even in the presence of this gauging, only the graviton supermultiplets and matter hypermultiplets can couple to supersymmetry breaking sources on the walls, though these mix with vector supermultiplets in the bulk. Whatever the source of supersymmetry breaking on the hidden wall, that on the observable wall is in general a combination of dilaton- and moduli-dominated scenarios.

[a] Theory Division, CERN, Geneva, Switzerland
[b] Physikalisches Institut, Universität Bonn, Germany
[c] Institute of Theoretical Physics, Warsaw University, Poland
[d] Department of Theoretical Physics, University of Oxford, United Kingdom

1 Introduction

One of the most promising recent developments for attempts to construct satisfactory unified models in the context of string theory has been the realization that the strong-coupling limit can be treated using an eleven-dimensional approach [1, 2, 3, 4]. In particular, this offers the possibility of reconciling the GUT scale M_{GUT}, estimated on the basis of low-energy data from LEP and elsewhere, with the string unification scale calculated in terms of the four-dimensional Planck scale [4, 5, 6, 7]. This reconciliation is possible in the strong-coupling limit with a fifth dimension L_5 that is considerably larger than M_{GUT}^{-1}. According to this scenario, six of the original eleven dimensions are compactified at a length scale comparable to M_{GUT}^{-1}, beyond which physics is described by an effective five-dimensional supergravity, that is reduced further to an effective four-dimensional theory at length scales larger than L_5 [2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

Physics on the boundaries of the fifth dimension are also described by effective four-dimensional supersymmetric theories. The effective five-dimensional supergravity theory in the bulk space between these boundary walls serves to communicate between them, and provides, e.g., the essential framework for describing the mediation of supersymmetry breaking by gravitational interactions through the bulk, between a suspected source on the hidden wall and physics in the observable sector [19, 20, 6, 8, 13, 14, 18, 21]. The general structure of five-dimensional supergravity theories has been studied, as have specific features of the effective theory obtained from the original eleven-dimensional theory by compactification on a six-dimensional Calabi-Yau manifold [20, 22, 23, 24]. In this case, the multiplicities of vector supermultiplets and matter hypermultiplets are related to the topological data $h_{(1,1)}$ and $h_{(2,1)}$ of the Calabi-Yau manifold, and the structure of the Chern-Simons terms and the geometries of the scalar-field manifolds are also related to properties of the Calabi-Yau manifold. Furthermore, consistent compactification requires a deformation [2] of the Calabi-Yau manifold along the fifth dimension that induces a gauging [22, 24] of the effective five-dimensional supergravity theory.

In a previous paper [23], we discussed the issue of mediation of supersymmetry breaking through the five-dimensional bulk, stressing in particular that only the gravity supermultiplet and the universal and non-universal matter hypermultiplets can couple to supersymmetry breaking on the walls. The vector supermultiplets lack such a coupling because a parity symmetry forbids their supersymmetry variations from having expectation values on either boundary. This previous discussion was not formulated explicitly in the gauged form of the five-dimensional supergravity theory.

In this paper, we supplement this previous discussion, first by extending the construction of the gauged supergravity [22, 24] to include non-universal hypermultiplets, and then by discussing the ensuing coupled dynamics of the gravity and vector supermultiplets and the matter hypermultiplets in the bulk, including terms related to the Calabi-Yau deformation [2]. We find that there is non-trivial dynamical mixing in the bulk, but confirm that the vector hypermultiplets cannot couple directly to the breaking of supersymmetry to the walls. The possible types of supersymmetry breaking correspond to the dilaton- and moduli-dominated scenarios

for supersymmetry breaking discussed originally in the context of weakly-coupled heterotic string theory. However, even if just one of these is dominant on the hidden wall, the dynamical mixing in the bulk may cause both of them to be present on the observable wall.

In particular, we are interested in the specific source of supersymmetry breaking provided by a condensate of strongly-interacting gauge fermions on the hidden wall. We demonstrate that, in the standard-embedding version of the Horava-Witten model [1, 2, 3], the reduction from eleven dimensions down to five dimensions of the coupling between the bulk moduli and the gaugino condensate living on the wall is the same in both gauged and non-gauged versions of the effective five-dimensional supergravity. We stress also the fact, already demonstrated in our previous paper, that the effective five-dimensional coupling of the condensate to moduli includes a direct coupling of the condensate, not only to the universal hypermultiplet scalars and to scalars from the gravity and vector multiplets, but also to Z_2-even and Z_2-uneven scalars from the non-universal hypermultiplets, including the type-$(2, 1)$ moduli.

2 Primer of Five-dimensional Supergravity

We first recall some essential features of the five-dimensional supergravity theory that describes M-theory dynamics in the bulk after compactification on a Calabi-Yau manifold. It contains $h_{(1,1)}$ vector fields $\mathcal{A}^i_\mu$, of which one is the graviphoton and the remaining $h_{(1,1)} - 1$ belong to vector supermultiplets. [1]

These are accompanied by $h_{(1,1)}$ scalars X^I, a complex scalar C and the three-form $C_{\alpha\beta\gamma}$, which is dual to a scalar D. The five-dimensional supergravity theory contains then a universal hyperplet whose bosonic components are $(V, D; C, \bar{C})$, where $V \equiv \frac{1}{6}d_{IJK}X^I X^J X^K$ represents the Calabi-Yau volume. The shape moduli

$$t^A \equiv \frac{1}{V^{1/3}}X^A : \quad d_{ABC}t^A t^B t^C = 6 \tag{1}$$

represent the $h_{(1,1)} - 1$ independent scalar components of the vector supermultiplets, and the graviphoton is the model-dependent combination

$$\mathcal{B}_\mu \equiv t_A \mathcal{A}^A_\mu \tag{2}$$

which is orthogonal to the hypersurface (1), with respect to the metric

$$G_{AB} = \frac{1}{2V}\int_X V_A \wedge \star V_B \tag{3}$$

where the V_A form a basis for the $(1, 1)$ forms and $A = 1, ..., h_{(1,1)}$. The combination (2) is, however, not the same as the combination of vector fields that participates in the gauging induced by the deformation of the Calabi-Yau manifold, as we now discuss.

---

[1] All the notation we use in this paper is compatible with that in [23] and [25].

The linearized solution for the eleven-dimensional Bianchi identities in the standard-embedding case is

$$G_{ABCD} = -\frac{3}{4\sqrt{2}\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3} tr(F^{(1)}_{[AB}F^{(1)}_{CD]}), \quad G_{ABC11} = 0, \tag{4}$$

where now the index A corresponds to the ten-dimensional space-time (since it is not ambiguous in any case, we use here the same symbol as in the equation (3)). This is the expression appropriate on the half-circle $x^{11} \in (0, \pi\rho_0)$. To continue to the other half-circle, we have to remember that G_{MNPQ} is Z_2-odd, and hence has to change sign when it crosses any of the fixed planes. It is important to note that the G_{ABCD} vacuum does not depend explicitly on the coordinate x^{11}. On a Calabi-Yau manifold, the vacuum configuration for G must be a $(2,2)$ form. Since $h_{(2,2)} = h_{(1,1)}$ on a three-fold, it is convenient to choose as a basis of $H^{(2,2)}$ the forms Y^B related to duals of V_A: $Y^B = 1/(2V)G^{BA} \star V_A$. In this way, one has $\int_X Y^B \wedge V_A = \delta^B_A$, and

$$G_{MNPQ} = \frac{1}{4V}\alpha_B G^{BC}\frac{1}{24}\epsilon_{MNPQ}{}^{EF}V_{(C)\,EF} \tag{5}$$

The constants α_B are given a geometric interpretation through the representation

$$\alpha_B = -\sqrt{2}\pi\left(\frac{\kappa}{4\pi}\right)^{2/3}\left(\frac{1}{8\pi^2}\int_{\mathcal{C}_B} tr F^{(1)} \wedge F^{(1)}\right)\epsilon(x^{11}) \tag{6}$$

where $\mathcal{C}_B$ is the four-cycle dual to the form Y^B, and we have included explicitly the antisymmetric step function $\epsilon(x^{11})$ in the formula (6), in order to recall that α_B is also Z_2-odd, like the background G_{MNPQ} itself.

We now recall briefly the way the gauging arises [22, 24] in connection with the non-trivial vacuum solution for the components of the antisymmetric-tensor field and its strength, which is linear in x^{11} to lowest non-trivial order in $\kappa^{2/3}$ (4). As we discuss in more detail later, in order to construct the effective five-dimensional theory, one expands the Lagrangian around this non-trivial eleven-dimensional background, and treats five-dimensional zero modes as fluctuations in that non-vanishing background [22, 24]. Substituting such an expansion into the topological $C \wedge G \wedge G$ term in the eleven-dimensional supergravity Lagrangian, one finds, among other terms, a new coupling between zero modes of the form $\partial_\mu D\, \mathcal{C}^\mu$, where D is in our language the imaginary part of the complex even scalar S from the universal hypermultiplet, and in the language of the effective four-dimensional theory on a wall is simply the universal axion, and $\mathcal{C}_\mu$ is the combination

$$\mathcal{C}_\mu \equiv \alpha_B \mathcal{A}^B_\mu \tag{7}$$

of the $h_{(1,1)}$ $U(1)$ gauge fields in the bulk, where the coefficients α_B are given by (6). Thus, its composition depends on the orientation of the gauge and gravitational instantons with respect to the cohomology basis used to define the zero modes.

This mixing between a vector boson and a derivative of a pseudoscalar, which is dual to the component of $G_{\alpha\beta\gamma\delta}$ with all indices five-dimensional, is reminiscent of a Higgs mechanism. The only way to accommodate it in an explicitly supersymmetric theory is as part of a squared covariant derivative in the gauged five-dimensional supergravity, where the gauging is of translations along the imaginary direction of the complex Z_2-even scalar $S = V + iD$. There are

3

other terms in the Lagrangian which arise from the gauging, for instance the scalar potential coming from the Killing prepotentials, and these terms can also be found via the reduction on the nontrivial background given above [24]. In this paper, we extend the analysis of [24] to include fields coming from the non-universal hypermultiplets. Since some important expressions in the effective Lagrangian become notably more complex, we discuss some key points of the reduction in more detail in the following sections of this paper.

Since the coupling of the scalar D to the gauge boson $\mathcal{C}$ is of order $\mathcal{O}(\kappa^{2/3})$, it is of higher order than the kinetic couplings in the bulk which we considered in [23]. Likewise, the necessary supersymmetrization involves higher-order bulk couplings. These can be obtained from formulae given in [26], as well as the corresponding modifications to the supersymmetry transformation laws [24] discussed in Section 6. In particular, we note that the potential term related by supersymmetry to the $\mathcal{O}(\kappa^{2/3})$ mixing, analogous to D terms in four-dimensional supersymmetry, is of order $\mathcal{O}(\kappa^{4/3})$: see Section 5. This exemplifies the fact that the new couplings in the gauged theory are of higher order in $\kappa^{2/3}$ than the σ-model couplings we considered in [23].

3   Couplings to Non-Universal Hypermultiplets

We start with key steps in the dimensional reduction in the presence of non-universal $(2,1)$-moduli. The scheme for the reduction follows [25] closely, as in [23]. The basic modifications compared to the compactifications with just a single universal hypermultiplet are already visible in the reduction of the three-form field. We consider the expansion of the three-form field C_{IJK} into harmonics, distinguishing between three different configurations of the indices I, J, K. To give non-vanishing zero modes, the indices have to be either all non-compact, one non-compact and two compact, or all compact. This is because, on a Calabi-Yau manifold, only the (3,0), (2,1), (1,1), (0,0) harmonic forms and their Hodge duals are non-vanishing. Taking this into account, we may write the following decomposition

$$
\begin{aligned}
C_{IJK}(x^M)\, dx^I \wedge dx^J \wedge dx^K \;=\;& C_{\mu\nu\rho}(x^\sigma)\, dx^\mu \wedge dx^\nu \wedge dx^\rho + C_{\mu ab}(x^M)\, dx^\mu \wedge dx^a \wedge dx^b \\
+\;& C_{abc}(x^M)\, dx^a \wedge dx^b \wedge dx^c.
\end{aligned}
\tag{8}
$$

Using the basis $V^A : A = 1, ..., h_{(1,1)}$ of harmonic (1,1) forms, we can write the above expansion as

$$
C_{IJK}\, dx^I \wedge dx^J \wedge dx^K = C_{\mu\nu\rho}\, dx^\mu \wedge dx^\nu \wedge dx^\rho + C_\mu^A\, V^A + C_{abc}\, dx^a \wedge dx^b \wedge dx^c.
\tag{9}
$$

The non-trivial part of (9) is the term with three compact indices. We concentrate on its expansion in terms of non-vanishing harmonic (2,1) forms in the Dolbeault cohomology basis in $H^{2,1}$:

$$
\Phi_I = \frac{1}{2!}\Phi_{Iij\bar{k}}\, dz^i \wedge dz^j \wedge d\bar{z}^k \qquad I = 0, ..., h_{(2,1)}
\tag{10}
$$

and the (3,0) form

$$
\Omega = \frac{1}{3!}\Omega_{ijk} dz^i \wedge dz^j \wedge dz^k,
\tag{11}
$$

4

which constitutes the Dolbeault cohomology basis for $H^{(3,0)}$. In is important to notice that (10) includes $h_{(2,1)} + 1$ forms, which are not all linearly independent, since by definition we only have $h_{(2,1)}$ non-vanishing harmonic (2,1) forms. The convenience of the choice (10) is due to an obvious analogy with homogeneous coordinates, which we discuss later below.

In order to discuss the H^3 cohomology sector, we introduce a *real* deRham cohomology basis (α_I, β^I), where $I = 0, ..., h_{(2,1)}$, one of our aims being to impose the invariance of C under simplectic transformations $Sp(2h_{(2,1)} + 2)$ [25]. This basis is dual to a canonical homology basis for $H_3(\mathcal{M}, Z)$ which we denote by (A^I, B_I). The two bases are defined in such a way that

$$\int_{A^J} \alpha_I = \int \alpha_I \wedge \beta^J = \delta_I^J \tag{12}$$

and

$$\int_{B_I} \beta^J = \int \beta^J \wedge \alpha_I = -\delta_I^J. \tag{13}$$

We introduce the periods $\tilde{Z}^I$ and F_I of the holomorphic (3,0) form Ω (11) via

$$\tilde{Z}^I \equiv \int_{A_I} \Omega \tag{14}$$

and

$$-iF_I \equiv \int_{B_I} \Omega. \tag{15}$$

Following [27], one can show that the complex structure of the manifold $\mathcal{M}$ is entirely determined by the $\tilde{Z}^I$, implying that $F_I = F_I(\tilde{Z}^I)$. It is clear from the definition (14) that rescaling $\tilde{Z}^I \to \lambda \tilde{Z}^I$, where λ is a non-zero number, corresponds to a rescaling of Ω. Therefore the $\tilde{Z}^I$ can be regarded as projective coordinates for the complex structure: $\tilde{Z}^I \in P_{H_{(2,1)}}$, with Ω being homogeneous of degree one in these coordinates:

$$\Omega(\lambda \tilde{Z}) = \lambda \Omega(\tilde{Z}). \tag{16}$$

As already mentioned, these homogeneous coordinates, although convenient in our case, cannot be a good coordinate system, since the space $P_{H_{(2,1)}}$ is a $h_{(2,1)}$-dimensional quaternionic manifold, whilst there are $h_{(2,1)} + 1$ coordinates $\tilde{Z}^I$. We can define inhomogeneous coordinates by

$$Z^a = \frac{\tilde{Z}^{I=a}}{\tilde{Z}^0} \quad a = 1, ..., h_{(2,1)}. \tag{17}$$

for example.

We use now the real cohomology basis (α_I, β^I) to expand the holomorphic (3,0) form (11). Since Ω is a complex form, we can perform the expansion only if we complexify the real basis. We therefore write it as

$$\Omega(\tilde{Z}) = \tilde{Z}^I \alpha_I + iF_I(\tilde{Z})\beta^I. \tag{18}$$

Kodaira has derived [28] the following decomposition:

$$\Omega_I = \frac{\partial \Omega}{\partial Z^I} = K_I \Omega + \Phi_I, \tag{19}$$

where the K_I are coefficients that depend on the $\tilde{Z}^I$, but not on the coordinates of the Calabi-Yau space, and the Φ_I are (2,1) forms. As mentioned before, the forms Φ_I are not linearly independent. It is, however, convenient to use the above set of $h_{(2,1)} + 1$ forms, remembering that they satisfy the condition

$$\tilde{Z}^I \Phi_I = 0, \tag{20}$$

which leaves the right number of linearly independent degrees of freedom. We will show in the following paragraphs that the constraint (20) is consistent with previous definitions.

Using (19), and recalling that

$$(\Phi_I, \Omega) = 0, \tag{21}$$

and

$$(\Omega, \Omega) = 0. \tag{22}$$

one can easily show that

$$\left(\Omega, \frac{\partial \Omega}{\partial \tilde{Z}^I}\right) = \int \Omega \wedge \frac{\partial \Omega}{\partial \tilde{Z}^I} = 0. \tag{23}$$

Using the expansion (18), we conclude from (23) that the functions $F_I(\tilde{Z})$ have the following property

$$2F_I = \frac{\partial}{\partial \tilde{Z}^I}(\tilde{Z}^J F_J). \tag{24}$$

It follows from (24) that F_I is the gradient of a homogeneous function of degree two, i.e.,

$$F_I = \frac{\partial F}{\partial \tilde{Z}^I} \quad : \quad F(\lambda \tilde{Z}) = \lambda^2 F(\tilde{Z}). \tag{25}$$

It is also useful to notice that we can write (25) in the following form

$$F_I = \tilde{Z}^J F_{IJ}. \tag{26}$$

As stated previously, we use the following Dolbeault cohomology basis in $H^{(3,0)}$ and $H^{(2,1)}$:

$$\Omega(\tilde{Z}), \tag{27}$$

and

$$\Phi_I(\tilde{Z}) = \Omega_I - \frac{(\Omega_I, \overline{\Omega})}{(\Omega, \overline{\Omega})}\Omega, \tag{28}$$

with the additional condition (20). In writing (28), we used the the fact that $(\Phi_I, \overline{\Omega}) = 0$ and expressed K_I in terms of Ω, by taking the inner products of both sides of (28) with $\overline{\Omega}$. The condition (20) follows from equations (28, 18, 26). It is easy to see that $\tilde{Z}^I \Omega_I = \Omega$ which immediately gives (20).

We can now use the integrals over the real cohomology basis (α, β) to express everything in terms of moduli $\tilde{Z}^I$ and the holomorphic function $F(\tilde{Z})$. In the rest of this section, we drop the tilde from $\tilde{Z}$ in order to make the equations more readable, not forgetting that at the end we must pass to inhomogeneous coordinates given by (17). One easily finds the following relations

$$(\Omega, \overline{\Omega}) = -4i(\overline{Z}NZ), \tag{29}$$

6

$$(\Phi_I, \overline{\Phi}_I) = -4i \left(N_{IJ} - \frac{(N\overline{Z})_I (NZ)_J}{(\overline{Z}NZ)} \right), \tag{30}$$

$$(\Omega_I, \overline{\Omega}) = -4i(N\overline{Z})_I, \tag{31}$$

$$K_I = \frac{(N\overline{Z})_I}{(\overline{Z}NZ)}. \tag{32}$$

which will be useful in the following.

The real cohomology basis which we have introduced in this section proves to be very useful [25] in performing the expansion of the three-form field C_{abc} in terms of harmonic forms. As was argued in [25], it enables us to fix arbitrary coefficients appearing in the expansion. We do not repeat here all the technical discussion, and present here only the result. We use the notation $\hat{C}$ for the three-form field and C for the five-dimensional scalar field. The expansion derived in [25] then reads

$$\hat{C} = (Re\,C)_I(2a^{IJ})\alpha_J + ((Re\,C)_I(b_J^I + \overline{b}_J^I) + i(Im\,C)_I(b_J^I - \overline{b}_J^I))\beta^J, \tag{33}$$

where the $C_I : I = 0, ..., h_{(2,1)}$ are the complex five-dimensional scalar fields in the bulk hypermultiplets, and a^{IJ}, b_J^I are coefficients which, as was argued in [27], depend explicitly on the moduli Z but not on the coordinates of the Calabi-Yau space.

Using

$$\begin{aligned} K_I\Omega + \Phi_I &= \alpha_I + iF_{IJ}\beta^J \\ \overline{K}_I\overline{\Omega} + \overline{\Phi}_I &= \alpha_I - i\overline{F}_{IJ}\beta^J \end{aligned} \tag{34}$$

we can express the basis (α, β) in terms of (2,1) and (3,0) forms:

$$\begin{aligned} \beta &= -iN^{-1}[K\Omega + \Phi - \overline{K\Omega} - \overline{\Phi}] \\ \alpha &= N^{-1}[\overline{F}(K\Omega + \Phi) + F(\overline{K\Omega} + \overline{\Phi})], \end{aligned} \tag{35}$$

where $N_{IJ} = \frac{1}{4}(F_{IJ} + \overline{F}_{IJ})$, and we have omitted all the indices.

Using the above expressions, we can write

$$\hat{C} = \frac{1}{4}C((aF'' - ib)N^{-1}(\Phi + K\Omega) + (aF'' + ib)N^{-1}(\overline{\Phi} + \overline{K\Omega})) + h.c. \tag{36}$$

where $F'' = F_{IJ}$. Using arguments given in [25], one can show finally that

$$\hat{C} = \gamma\,C N^{-1}(-\Phi + \overline{K\Omega}) + \gamma\,\overline{C}N^{-1}(-\overline{\Phi} + K\Omega), \tag{37}$$

where γ is some numerical factor. We can also write the above expression in terms of the real basis (α, β)

$$\hat{C} = \gamma\,(Re\,C)R^{-1}\alpha + \gamma\left(i\,(Re\,C)[(1 - 2KZ)F'' - (1 - 2\overline{KZ})F''] - 4\,(Im\,C)\right)\beta, \tag{38}$$

where $(R^{-1})^{IJ} = 2(N^{-1}(1 - \overline{KZ} - KZ))^{IJ}$.

4 Coupling to the Gaugino Condensate

In addition to extending the study of this gauged supergravity to include the non-universal hypermultiplets, and to calculate explicitly the potential for the scalar fields associated with the vector multiplets and the hypermultiplets, we also include into the gauged supergravity picture the coupling of the bulk moduli to the gaugino condensate living on the hidden wall. This is of particular phenomenological importance, as hidden-sector gaugino condensation remains a primary candidate source of supersymmetry breaking in M-theory models. We shall treat the reduction of the wall-bulk coupling rather completely, in order to make explicit the additional couplings of the condensate to non-universal hypermultiplets, which have, so far, only been studied in our previous paper [23].

We use (37) to write down the following expression for the field strength of the field C_{IJK}:

$$
\begin{aligned}
G_M &= \gamma \left[\partial_M C N^{-1} - (C + \overline{C}) N^{-1} (K \partial_M Z + \frac{1}{4} F_3 \partial_M \overline{Z} N^{-1}) \right] \Phi \\
&+ \gamma \left[\partial_M C (N^{-1} \overline{K}) - (C + \overline{C})(N^{-1}) \frac{\partial \overline{K}}{\partial Z^L} \partial_M Z^L \right] \overline{\Omega} + h.c.,
\end{aligned}
\tag{39}
$$

where $F_3 \equiv F_{IJK}$, and we have not written explicitly the three internal indices I, J, K. We see in (39) that the term which is propotional to the holomorphic three-form, which will couple to the gaugino condensate reads

$$
\partial_M C (N^{-1} \overline{K}) - (C + \overline{C})(N^{-1}) \frac{\partial \overline{K}}{\partial Z^L} \partial_M Z^L.
\tag{40}
$$

The fields C, being odd, have to vanish or to have a discontinuity on the walls, as do the derivatives with respect to x^5 (x^{11}) of the even moduli Z and S. However, each part of the above equation contains an even number of Z_2-odd objects, so each can have a well-defined nonvanishing limit on the wall, and couple there to any gaugino condensate.

The above calculation shows that the coupling to the gaugino condensate involves not only the universal hypermultiplet, but also scalar fields from non-universal hypermultiplets. To be more explicit, we consider the function $F(\tilde{Z})$ that characterizes the simple model discussed in [23], namely

$$
F(\tilde{Z}) = (\tilde{Z}^0)^2 - (\tilde{Z}^a)^2, \quad a = 1, ..., h_{(2,1)}.
\tag{41}
$$

This gives a Calabi-Yau space with a nontrivial moduli sector, sufficient to study the questions we want to ask. although the corresponding Yukawa couplings vanish, since these are given by the third derivatives of F. The choice (41) of F leads to the following form of the $(h_{(2,1)} + 1)$-dimensional matrix N and its inverse:

$$
N = \frac{1}{2} \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} = \frac{1}{4} N^{-1}
\tag{42}
$$

The combination of moduli and their derivatives which couples directly to the condensate is

$$
\frac{2}{1 - |Z^a|^2} (\partial_{11} C_0 + \partial_{11} C_a Z^a) + \frac{4}{(1 - |Z^a|^2)^2} (C_{0r} + C_{ar} Z^a) \bar{Z}^b \partial_{11} Z^b
\tag{43}
$$

8

in terms of physical, untilded, quantities, which should also multiplied by a factor V_{CY}, since above we have been working in the metric which is canonically normalized in eleven dimensions. We note that the above expression contains different powers of moduli fields and their derivatives. The lowest-order part is simply $2\,\partial_{11}C_0$, i.e., the derivative of the Z_2-odd component of the universal hypermultiplet with respect to x^{11}. The result (43) is nothing other than the component form of the five-dimensional σ-model expression given in [23]:

$$L_{coupling} = -\frac{1}{2}g_{xy}g^{55}(\partial_5\sigma^x - \mathcal{L}\delta(x^5 - \pi\rho)\delta^{xx_0})(\partial_5\sigma^y - \mathcal{L}\delta(x^5 - \pi\rho)\delta^{yx_0}) \tag{44}$$

where we assume, as mentioned above, the conventional wisdom that the four-dimensional gaugino condensate must be proportional to the Calabi-Yau $(3,0)$ form Ω_{ijk}. We note that the coupling (44) includes also the possibility of switching on the part of the background for the Chern-Simons forms which is proportional to Ω_{ijk}, $\mathcal{L} \rightarrow \mathcal{L} + < \Omega_{CS} >$, as discussed in [14] and in [18]. If one considers switching on a part of the background for the Chern-Simons form that is proportional to the $(2,1)$ forms Φ_I, such a background would couple to the following combinations of the massless modes

$$\left(\partial_M C N^{-1} - (C + \overline{C})N^{-1}(K\partial_M Z + \frac{1}{4}F_3\partial_M\overline{Z}N^{-1})\right)^I \tag{45}$$

The components of the background proportional to heavy modes of the Laplacian on the Calabi-Yau space decouple from the massless modes.

The calculation given in some detail above constitutes the derivation of the effective five-dimensional coupling (44) from the eleven-dimensional Lagrangian given in [1, 19]. The result of this procedure is not sensitive to the gauging of the five-dimensional supergravity, as the background value of G_{ABC11} which solves the consistency equations to order $\kappa^{2/3}$ in eleven dimensions vanishes for the standard embedding. Since in eleven dimensions only G_{ABC11} couples to the condensates, the non-trivial backgrounds for the other components of the antisymmetric tensor field strength G do not affect the coupling.

We return at this point to the reduction of the $C \wedge G \wedge G$ term from the original eleven-dimensional action, to see in more detail how the coupling to the gauge boson arises. This coupling must be proportional to the background value of the field strength G, and the only components of G that have vacuum expectation values are these with all indices tangent to the Calabi-Yau space. Hence, from the decomposition of the three-form field into zero modes, we see that the terms affected by the background are of the form

$$\epsilon^{\mu\alpha\beta\gamma\delta ABMNPQ}\,C_{\mu AB}G_{\alpha\beta\gamma\delta}G_{MNPQ} \tag{46}$$

Using the decompositions (9) of $C_{\mu AB}$ and (39) of G_{MNPQ} and integrating over the Calabi-Yau space, we immediately find the five-dimensional coupling

$$\epsilon^{\mu\alpha\beta\gamma\delta}G_{\alpha\beta\gamma\delta}(\alpha_B\mathcal{A}_\mu^B) \tag{47}$$

Remembering that, upon using the equations of motion, the four-form $G_{\alpha\beta\gamma\delta}$ is seen in five dimensions to be dual to a closed one-form, which may be represented locally by the derivative

9

of a scalar, we see that we have found the mixed bilinear term. Taking into account the possible index structures of the four-form G, we see that this scalar, which we shall call D, is the only one which can couple directly to any vector field. To describe the correspondence between D and $G_{\alpha\beta\gamma\delta}$ more precisely, we note that to perform correctly the duality transformation we have to take into account two other terms. The first and obvious one is the square $G_{\alpha\beta\gamma\delta}G^{\alpha\beta\gamma\delta}$ from the kinetic term, and the second one is

$$\epsilon^{\mu\alpha\beta\gamma\delta ABMNPQ}\, C_{ABM}G_{\alpha\beta\gamma\delta}G_{\mu NPQ} \tag{48}$$

from the topological $C \wedge G \wedge G$ term. Using again the decompositions of C and G which we have given earlier, we obtain

$$
\begin{aligned}
G_{\alpha\beta\gamma\delta} &= \frac{1}{\sqrt{2}V}\epsilon_{\alpha\beta\gamma\delta}{}^{\mu}\left(\partial_\mu D - 2\alpha_B \mathcal{A}_\mu^B \right. \\
&\quad - i(CN^{-1})_I(\partial_\mu \bar{C}N^{-1})_{\bar{J}}G^{I\bar{J}} + i(CN^{-1})_I(C+\bar{C})N^{-1}(\bar{K}\partial_\mu \bar{Z})_{\bar{J}}G^{I\bar{J}} \\
&\quad - \left. \bar{C}N^{-1}K[\partial_\mu C(N^{-1}\bar{K}) - (C+\bar{C})N^{-1}\frac{\partial \bar{K}}{\partial Z^L}\partial_\mu Z^L] + \text{ h.c. }\right)
\end{aligned}
\tag{49}
$$

and the only non-trivial gauge-covariant derivative is

$$D_\mu D = \partial_\mu D - 2\alpha_B \mathcal{A}_\mu^B \tag{50}$$

It is straightforward to see that, in the case where $h_{(2,1)} = 0$, the complicated expression in the bracket in (49) reduces to $-4i(C_0\partial_\mu \bar{C}_0 - \bar{C}_0\partial_\mu C_0)$, which is the limit considered in [24].

This completes the construction of the coupling of the gauged five-dimensional supergravity to a gaugino condensate living on a four-dimensional boundary. This coupling is the only part of the construction where the enhancement of the hypermultiplet sector plays an important role. However, it is precisely this part that turns out to be insensitive to the gauging. The nature of the gauging does not change either, as it is still the gauging of the translation of the scalar dual to $G_{\alpha\beta\gamma\delta}$, which is the imaginary part of the complex modulus S [23].

We conclude this section with the observation that the symmetry of the quaternionic manifold which is gauged, the translation of $Im(S)$, is broken down to a discrete subgroup on the boundaries by the instantons of the gauge bundles living there.

5   Scalar Potential in Gauged Supergravity

We now construct the scalar potential in the bulk which appears due to the gauging, including the non-universal hypermultiplets. This will provide the final ingredient needed for a discussion of the modifications to the analysis of supersymmetry breaking and its transmission given in [23].

First we recapitulate the basics of the gauged supergravity structure given in [26]. When one compactifies supergravity from eleven dimensions down to five dimensions, one finds vectors, moduli scalars and associated fermions in the five-dimensional gravity supermultiplet, $h_{(1,1)} -$

1 vector multiplets which also contain associated scalars, and the $h_{(2,1)} + 1$ hypermultiplets discussed above. The complex scalars (zero-forms) $z^i : i = 1, ..., n$ where $n \equiv h_{(1,1)} - 1$ that come from the n vector multiplets span a special Kahler manifold $\mathcal{SM}$. The real scalar fields q^u ($u = 1, ..., 4m$) coming from the $m = h_{(2,1)} + 1$ hypermultiplets can be regarded as coordinates of a quaternionic manifold HM.

As shown in [26], the gauge potential can be expressed in the following form, using purely geometrical objects:

$$V(z, \overline{z}, q) = g^2 \left[(g_{ij^*} k_\Lambda^i k_\Sigma^{j^*} + 4 h_{uv} k_\Lambda^u k_\Sigma^v) \overline{L}^\Lambda L^\Sigma + g^{ij^*} f_i^\Lambda f_{j^*}^\Sigma \mathcal{P}_\Lambda^x \mathcal{P}_\Sigma^x - 3 \overline{L}^\Lambda L^\Sigma \mathcal{P}_\Lambda^x \mathcal{P}_\Sigma^x \right]. \tag{51}$$

Here, the indices Λ and Σ run from 0 to n (they correspond to the vector fields including the graviphoton), the indices i and j take values 1 to n, and the indices u and v take values 1 to $4m$, corresponding to the hypermultiplets. Additionally, we note that g_{ij^*} in (51) is the special Kähler metric for the scalars z^i coming from the vector multiplets, h_{uv} is the metric on the quaternionic manifold, and k_Λ^i and k_Λ^u are the Killing vectors for the special Kahler and quaternionic manifold, repectively. The vectors L^Λ are (parts of) covariantly holomorphic sections: $(\partial_{i^*} - \frac{1}{2}\partial_{i^*}\mathcal{K})L^\Lambda = 0$ where $\mathcal{K}$ is the Kahler potential (satisfying condition (4.27) from [26]), of the $2n+2$-dimensional symplectic vector bundle with the structure group $Sp(2n+2, \mathbf{R})$ over the special Kahler manifold SM, and the f_i^Λ are covariant derivatives of L^Λ: $f_i^\Lambda = (\partial_i - \frac{1}{2}\partial_i\mathcal{K})L^\Lambda$. Finally, the $\mathcal{P}_\Lambda^x$ are triplets ($x = 1, 2, 3$) of prepotentials associated with each Killing vector on the quaternionic manifold $\mathcal{QM}$.

The *non-holomorphic* sections L^Λ can be related to *holomorphic* sections X^Λ in the following way:

$$L^\Lambda = e^{\frac{\mathcal{K}}{2}} X^\Lambda, \tag{52}$$

where $\mathcal{K}$ is the Kahler potential. In the case of most interest here, we can regard X^Λ as a set of homogeneous coordinates on $\mathcal{SM}$. This means [26, 29] that we can write

$$L^\Lambda = e^{\frac{\mathcal{K}}{2}} z^\Lambda \tag{53}$$

with $z^0 = 1$. Using the holomorphic function $F(X)$, we can determine the rest of the geometric structure of $\mathcal{SM}$, and in particular the functions f_i^Λ. The object we need to determine the scalar potential is $\mathcal{P}_\Lambda^x$. Following [26], the triplet of zero-form prepotentials $\mathcal{P}_\Lambda^x$ associated to each Killing vector is given by

$$k_\Lambda^u \Omega_{uv}^x = -(\partial_v \mathcal{P}_\Lambda^x + \epsilon^{xyz} \omega_v^y \mathcal{P}_\Lambda^z). \tag{54}$$

where $\omega^y = \omega_v^y dq^v$ is the $Sp(2)$ connection, and $\Omega^x = \Omega_{uv}^x dq^u \wedge dq^v$ is the coresponding curvature.

The quaternionic manifold admits three complex structures $J^x : x = 1, 2, 3$, that satisfy the quaternionic algebra

$$J^x J^y = -\delta^{xy} \mathbf{1} + \epsilon^{xyz} J_z. \tag{55}$$

As a generalization of the Kähler form on a complex manifold, one can define a triplet of two-forms, called the Hyper-Kähler form:

$$K^x = K_{uv}^x dq^u \wedge dq^v \; ; \; K_{uv}^x = h_{uw}(J^x)_v^w. \tag{56}$$

11

Part of the definition of a quaternionic manifold is the requirement that the curvature of the principal $SU(2)$ bundle be proportional to the Hyper-Kähler two-form:

$$\Omega^x = \lambda K^x. \tag{57}$$

It is useful to define the vielbein one-form

$$\mathcal{U}^{A\alpha} = \mathcal{U}_u^{A\alpha}(q) dq^u \tag{58}$$

which satisfies

$$h_{uv} = \mathcal{U}_u^{A\alpha} \mathcal{U}_v^{B\beta} C_{\alpha\beta} \epsilon AB \tag{59}$$

where $C_{\alpha\beta}$ is the flat $Sp(2m)$-invariant metric, and ϵ_{AB} is the flat $Sp(2) \approx SU(2)$-invariant metric. We can express the curvature Ω^x in terms of the vielbein

$$\frac{i}{2}\Omega^x (\sigma_x)_A^B = \lambda \mathcal{U}_{A\alpha} \wedge \mathcal{U}^{B\alpha}. \tag{60}$$

On the other hand, we can easily find the connection ω^y by requiring the vielbein to be covariantly closed with respect to both the $SU(2)$ connection ω^z and some $Sp(2m, \mathbf{R})$-Lie algebra valued connection $\Delta^{\alpha\beta}$:

$$\nabla\mathcal{U}_{A\alpha} = d\mathcal{U}_{A\alpha} + \frac{i}{2}\omega^x(\epsilon\sigma_x\epsilon^{-1})_B^A \wedge \mathcal{U}^{B\alpha} + \Delta^{\alpha\beta} \wedge \mathcal{U}^{A\gamma}\mathbf{C}_{\beta\gamma}. \tag{61}$$

This connection has been calculated explicitly in terms of scalar fields in [29], and approximete explicit expressions can be found in [23].

Having calculated both Ω^x and ω^x, we are now able to determine the prepotential $\mathcal{P}_\Lambda^x$ using (54). To keep the discussion simple, we concentrate hereafter on the specific example with just one non-universal hypermultiplet. Our scalar fields q^u we treat as real and imaginary parts of complex fields: (S, C_0) from the universal hypermultiplet and (Z_1, C_1) from the non-universal one. The complex field S is a combination of the real scalars V and D introduced previously, defined by $S = V + iD$. The vielbein $\mathcal{U}^{A\alpha}$ has the following form [29]

$$\mathcal{U} = \begin{pmatrix} u & e & \overline{v} & \overline{E} \\ v & E & -\overline{u} & -\overline{e} \end{pmatrix}, \tag{62}$$

where we have introduced four one-forms defined as follows

$$u = 2e^{(\tilde{K}+K)/2}Z\left(dC - \frac{1}{2}d\mathcal{N}R^{-1}(C + \overline{C})\right), \tag{63}$$

$$v = e^{\tilde{K}}\left(dS + (C + \overline{C})R^{-1}dC - \frac{1}{4}(C + \overline{C})R^{-1}d\mathcal{N}R^{-1}(C + \overline{C})\right), \tag{64}$$

$$e = PdZ, \tag{65}$$

$$E = e^{(\tilde{K}-K)/2}PN^{-1}\left(dC - \frac{1}{2}d\mathcal{N}R^{-1}(C + \overline{C})\right), \tag{66}$$

where

$$K = -\ln 2\overline{Z}NZ, \tag{67}$$

$$\tilde{K} = -\ln(S + \overline{S} + \frac{1}{2}(C + \overline{C})R^{-1}(C + \overline{C})), \tag{68}$$

$$P = \begin{pmatrix} -\frac{Z_1}{1-Z_1\overline{Z}_1} \\ \frac{1}{1-Z_1\overline{Z}_1} \end{pmatrix}, \tag{69}$$

$$N = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{70}$$

$$R^{-1} = -\frac{2}{1-Z_1\overline{Z}_1}\begin{pmatrix} 1 + Z_1\overline{Z}_1 & Z_1 + \overline{Z}_1 \\ Z_1 + \overline{Z}_1 & 1 + Z_1\overline{Z}_1 \end{pmatrix}, \tag{71}$$

$$Z = \begin{pmatrix} 1 \\ Z_1 \end{pmatrix}, \tag{72}$$

$$C = \begin{pmatrix} C_0 \\ C_1 \end{pmatrix}, \tag{73}$$

and

$$\mathcal{N} = \frac{1}{2}\frac{1}{1-Z_1^2}\begin{pmatrix} -1 - Z_1^2 & 2Z_1 \\ 2Z_1 & -1 - Z_1^2 \end{pmatrix}. \tag{74}$$

In the vielbein (62), the index A takes values 1 and 2 corresponding to the fundamental representation of $Sp(2) \approx SU(2)$, and the index a corresponds to the fundamental representation of $Sp(4, \mathbf{R})$ and takes values from 1 to 4. Using (60), we are now able to calculate the curvature entering into (54). We have

$$\mathcal{U}_{A\alpha} = \epsilon_{AB}C_{\alpha\beta}\mathcal{U}^{B\beta} \tag{75}$$

where the $Sp(4)$ and $Sp(2)$ metrics have the following forms

$$\epsilon_{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{76}$$

$$C_{\alpha\beta} = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}, \tag{77}$$

and using (60) we find the following result

$$\frac{i}{2}\Omega = \lambda \begin{pmatrix} \frac{1}{2}(u \wedge \overline{u} + e \wedge \overline{e} - v \wedge \overline{v} - E \wedge \overline{E}) & u \wedge \overline{u} + e \wedge \overline{E} \\ v \wedge \overline{u} + E \wedge \overline{e} & -\frac{1}{2}(u \wedge \overline{u} + e \wedge \overline{e} - E \wedge \overline{E}) \end{pmatrix}, \tag{78}$$

where $\Omega = \Omega^x \sigma_x$. The $Sp(2)$ connection is given by [29]

$$\omega = \begin{pmatrix} \frac{1}{4}(v - \overline{v}) - \frac{1}{4}\frac{\overline{Z}NdZ - ZNd\overline{Z}}{\overline{Z}NZ} & -u \\ \overline{u} & -\frac{1}{4}(v - \overline{v}) + \frac{1}{4}\frac{\overline{Z}NdZ - ZNd\overline{Z}}{\overline{Z}NZ} \end{pmatrix}, \tag{79}$$

where $\omega = \omega^x \sigma_x$.

13

We can read the form of the Killing vector generating isometries. off from the covariant derivative appearing in the reduction to five dimensions:

$$k_\Lambda = -\frac{2}{g}\alpha_\Lambda \partial_D, \tag{80}$$

where $D = \frac{1}{2i}(S - \overline{S})$. Changing variables, we find

$$k_\Lambda = -\frac{2i}{g}\alpha_\Lambda (\partial_S - \partial_{\overline{S}}), \tag{81}$$

which gives

$$k_\Lambda^S = -\frac{2i}{g}\alpha_\Lambda, \ k_\Lambda^{\overline{S}} = \frac{2i}{g}, \ k_\Lambda^{u \neq S, \overline{S}} = 0. \tag{82}$$

Anticipating the final result, we assume that only the $x = 3$ component of $\mathcal{P}_\Lambda^x$ is non-zero, i.e., $\mathcal{P}_\Lambda \propto \sigma_3$. This is an Ansatz, but it is straightforward to see that in this way we obtain an exact solution. In this case, (54) reduces to

$$k_\Lambda^u \Omega_{uv}^{1,2} = \mp \omega_v^{2,1} \mathcal{P}_\Lambda^x, \tag{83}$$

$$k_\Lambda^u \Omega_{uv}^3 = -\partial_v \mathcal{P}_\Lambda^3. \tag{84}$$

The components Ω_{uv}^x and ω_v^x are easily read off (78) and (79):

$$\frac{i}{2}\Omega^1 = \frac{1}{2}(u \wedge \overline{v} + v \wedge \overline{u} + e \wedge \overline{E} + E \wedge \overline{e}), \tag{85}$$

$$\frac{i}{2}\Omega^2 = -\frac{i}{2}(u \wedge \overline{v} - v \wedge \overline{u} + e \wedge \overline{E} - E \wedge \overline{e}), \tag{86}$$

$$\frac{i}{2}\Omega^3 = \frac{1}{2}(u \wedge \overline{u} + e \wedge \overline{e} - v \wedge \overline{v} - E \wedge \overline{E}) \tag{87}$$

and

$$\omega^1 = \frac{1}{2}(\overline{u} - u), \tag{88}$$

$$\omega^2 = \frac{i}{2}(\overline{u} + u), \tag{89}$$

$$\omega^1 = \frac{1}{4}(v - \overline{v}) - \frac{1}{4}\frac{\overline{Z}NdZ - ZNdZ}{\overline{Z}NZ}. \tag{90}$$

It is useful to note that, because of (82), the only relevant parts of Ω^3 are those proportional to dS and $d\overline{S}$. On the other hand, it is only the one-form v which contains dS. This simplifies greatly the whole analysis, and, for example, (84) can be written now as

$$ik_\Lambda^u (v \wedge \overline{v})_{uv} = \partial_v \mathcal{P}_\Lambda^3. \tag{91}$$

Finally, we obtain the following simple and exact solution

$$\mathcal{P}_\Lambda^3 = \frac{2i}{g}\alpha_\Lambda \left(S + \overline{S} + \frac{1}{2}(C + \overline{C})R^{-1}(C + \overline{C})\right). \tag{92}$$

14

This result and (51) lead immediately to the following form for the scalar potential:

$$g^2 V = \frac{1}{S + \overline{S} + \frac{1}{2}(C + \overline{C})R^{-1}(C + \overline{C})}[(Im\mathcal{F})^{-1}]^{\Lambda\Sigma}\alpha_\Lambda\alpha_\Sigma, \tag{93}$$

where $\mathcal{F}$ is the so-called kinetic matrix, which is related to the objects from the potential (51) by

$$f_i^\Lambda f_{j^*}^\Sigma g^{ij^*} = -\frac{1}{2}(Im\mathcal{F})^{-1\,\Lambda\Sigma} - \overline{L}^\Lambda L^\Sigma. \tag{94}$$

The above potential can be written less formally in terms of the objects defined at the begining of this paper. In particular, in the case where we have just one non-universal hypermultiplet, we get

$$R^{-1} = -\frac{2}{1 - |Z|^2}\left(\begin{array}{cc} 1 + |Z|^2 & Z + \overline{Z} \\ Z + \overline{Z} & 1 + |Z|^2 \end{array}, \right) \tag{95}$$

$$C = \left(\begin{array}{c} C_0 \\ C_1 \end{array}\right), \tag{96}$$

and the potential reads

$$g^2 V = \frac{1}{S + \overline{S} + \frac{1}{2}(C + \overline{C})R^{-1}(C + \overline{C})}G^{AB}\alpha_A\alpha_B, \tag{97}$$

where G^{AB} is the metric of the Kähler manifold spanned by the scalars in the vector multiplets, defined in (3), and α_A has the geometrical interpretation given in (6)[2].

6 Supersymmetry Breaking Transmission between Walls

We now examine the modifications to the supersymmetry transformation laws for fermionic fields that are induced by the gauging. First we simply list the relevant new parts of the respective transformation laws

$$\delta_{(g)}\psi_{i\mu} = \mathrm{i}\,g\,S_{ij}\eta_{\mu\nu}\gamma^\nu\epsilon^j \tag{98}$$
$$\delta_{(g)}\lambda^{ai} = gW^{aij}\epsilon_j \tag{99}$$
$$\delta_{(g)}\lambda_b = g\,N_b^i\,\epsilon_i \tag{100}$$

where

$$S_{ij} = \frac{\mathrm{i}}{2}(\sigma_x)_i{}^k\epsilon_{jk}\mathcal{P}_\Lambda^x L^\Lambda$$
$$W^{aij} = \mathrm{i}(\sigma_x)_k{}^j\epsilon^{ki}\mathcal{P}_\Lambda^x g^{ij^\star}\bar{f}_{j^\star}^\Lambda$$
$$N_b^i = 2\mathcal{U}_{bu}^i k_\Lambda^u \bar{L}^\Lambda \tag{101}$$

---

[2]This form of the scalar potential is given in the Kählerian frame, where all the kinetic terms can be obtained from the Kähler potential.

and g is, as in [26] and as in earlier parts of this paper, a formal gauge coupling which counts gauging-induced terms. Using (6),(80),(92), we see immediately that all the above new contributions are Z_2-odd, and hence discontinuous across, or vanishing on, the fixed planes. This means that they provide no new channels for communication of the supersymmetry breaking from the visible wall to the five-dimensional bulk, or from the bulk to the observable wall, beyond the channels already identified in [23] in the context of ungauged supergravity. Similarly, the covariant derivatives do not introduce any new complication in the analysis, because the fifth component of the new gauge-field term $g\mathcal{A}^\Lambda k_\Lambda$ is Z_2-odd, so that $\mathcal{D}_\mu\Phi$ has the same Z_2 properties as $\partial_\mu\Phi$, for any field Φ, and hence behaves in exactly the same way.

As a result, the only effects of the gauging on the transmission of supersymmetry breaking are indirect, through the mixing of moduli scalars in the newly-created scalar potential, and possibly via other higher-order interactions in the bulk. We recall that the gauge potential itself is of order $\kappa^{4/3}$ relative to the σ-model interactions which were take into account in the earlier analysis [23]. Therefore, it seems that the gauging introduces higher-order effects that do not affect qualitatively the previous analysis.

However, one should also bear in mind the qualitatively new possibility that supersymmetry breaks down in the bulk, and that this gets communicated to the walls via the channels discussed previously. Unfortunately, it is not obvious how to generate in this way the hierarchy of supersymmetry breaking required in the observable sector. The only possibility appears to be the introduction of new parameters having the form of generalized Fayet-Iliopoulos terms, through $\mathcal{P}_\Lambda^x \to \mathcal{P}_\Lambda^x + \xi_\Lambda^x$, see [26] for technical details. However, the analysis of this possibility involves a more complex study of the dynamics of the bulk σ model, that lies beyond the scope of this paper.

To visualize the relevance of the lowest-order solution to the equations of motion in the bulk, even in the presence of sources and nonlinearities, let us consider the equation of motion for the volume modulus of the Calabi-Yau space, $S_r = V(x^{11})$, in the model obtained explicitly in [22, 24], freezing all the other variables at some specific expectation values. The sources given there are due to F^2 and R^2 terms on the walls, and are of order $\kappa^{2/3}$. The simplified Lagrangian is

$$L = \frac{1}{2}\left(\frac{\partial_{11}S_r}{S_r}\right)^2 - \frac{\tilde{\alpha}}{S_r}\delta(x^{11}) + \frac{\tilde{\alpha}}{S_r}\delta(x^{11} - \pi\rho_0) \tag{102}$$

which gives the equation of motion

$$\partial_{11}^2 S_r - \frac{1}{S_r}(\partial_{11}S_r)^2 = \tilde{\alpha}(\delta(x^{11} - \pi\rho_0) - \delta(x^{11})) \tag{103}$$

Since the first derivative of S_r is already of order $\kappa^{2/3}$, the middle term in (103), being quadratic, is of order $\kappa^{4/3}$, and hence subdominant compared with the other two, at least formally. Thus, at the lowest non-trivial order $\kappa^{2/3}$, (103) is exactly of the form which has been studied in [21, 23], and is given by a linear combination $a\,|x^{11}| + b\,|x^{11} - \pi\rho_0| + c$. By choosing properly the coefficients of this linear combination one recovers exactly the linear part of the full solution announced in [22, 24], which corresponds to Witten's solution. It is likely that adding any additional nonlinear terms in the bulk at order $\kappa^{4/3}$ is not going to affect the leading linear behaviour of the background.

16

We now complete this analysis by giving the complete equations of motion between walls for the moduli which are relevant for the supersymmetry breaking transmission, S and C_i, $i = 0, 1, ..., h_{(1,1)}$. We consider for simplicity the case when the expectation values of the complex structure moduli Z_i are set to zero. The interesting observation is that after diagonalization of the matrix of second derivatives the contribution to the equations coming from the scalar potential survives only in the equation of motion for the volume modulus S. It drops out from the equations of motion for the moduli C_i. Hence, the only equation which gets modified with respect to the equations considered in [23] is the one which contains S''. The modified equation with the sources is

$$
\begin{aligned}
S''(x^5) &- \alpha^2 + \delta(x^5 - \pi\rho)^2 \left(\frac{4\,\vartheta^2 \sum_i C_i(x^5)^2}{2\sum_i C_i(x^5)^2 - S(x^5)} - \frac{2\,\vartheta^2 S(x^5)}{2\sum_i C_i(x^5)^2 - S(x^5)} \right) \\
&- \sum_{i,j} \frac{4\,C_i(x^5)^2 C_j'(x^5)^2}{2\sum_i C_i(x^5)^2 - S(x^5)} + \sum_i \frac{2\,S(x^5) C_i'(x^5)^2}{2\sum_i C_i(x^5)^2 - S(x^5)} \\
&- \sum_i \frac{4\,C_i(x^5) C_i'(x^5) S'(x^5)}{2\sum_i C_i(x^5)^2 - S(x^5)} + \frac{S'(x^5)^2}{2\sum_i C_i(x^5)^2 - S(x^5)} \\
&= -\varrho_v \delta(x^5) + \varrho_h \delta(x^5 - \pi\rho),
\end{aligned}
\tag{104}
$$

where $i, j = 0, 1, ..., h_{(1,1)}$, and the corresponding boundary conditions on the half-circle are

$$
\lim_{x^5 \to 0} S' = -\frac{\varrho_v}{2}, \quad \lim_{x^5 \to \pi\rho} S' = -\frac{\varrho_h}{2}
\tag{105}
$$

with $\varrho_{v,h}$ determined by the source terms on the walls. Again, exactly as found out in [23], one can check that the singularities cancel between themselves. The new term is $\alpha^2 \equiv G^{AB}\alpha_A\alpha_B$ where G^{AB} is the metric of the Kähler manifold spanned by the scalars in from the vector multiplets, defined in (3, and α_A can be given a geometrical interpretation as in (6). This constitutes an analog of a bulk 'charge' density, and in general depends on the vacuum configuration of the shape moduli t^A. We note that, although superficially the scalar potential in the Kählerian frame looks somewhat pathological, with hypersurfaces of singularities and a potential run-away along the direction of S, its contribution to the equations of motion can be, at least in the cases we consider here, quiet regular. Using sources on the walls, one obtains, upon integration of the modified equations, configurations which are qualitatively very similar to those discussed in [23]. Hence we expect the main features of the physics of the transmission of supersymmetry breaking to remain unchanged.

When one sets the expectation values of the Z_2-odd fields $C_{0,i}$ and of the moduli Z_i to zero, one recovers the BPS solution found in the papers [22, 24]. However, as in the ungauged case of [23], when a condensate on any wall is switched on, it becomes impossible to set all these expectation values to zero. This means that the actual solution in that case has to depart from the BPS solution found in [22, 24]. Unfortunatly, it is difficult to find analytically the solution corresponding to non-zero condensates: straightforward numerical integration gives backgrounds which again break all supersymmetries in the bulk. This point merits further study.

17

We would like to emphasize an important point where the gauged supergravity constitutes an essential advance over the leading linear solution. If one substitutes the simple linear backgrounds for the S and t^A fields, as dictated by the leading solutions to the equation of motion, into the *ungauged* supersymmetry transformations in five dimensions, one finds that supersymmetry is apparently completely broken in the bulk. This does not agree with the original result of Witten [2]. The point is that, when one works directly in eleven dimensions [2], one can cancel the harmful contributions to supersymmetry transformations by rotating the internal spinor η lying in the space tangent to the Calabi-Yau three-fold. On the other hand, when one works directly in five dimensions, there is no spinor η which could be rotated. Suitable counterterms which would restore supersymmetry must then be added by hand to the transformation rules. This is easier said than done, since one has to worry about the closure of the supersymmetry algebra if one modifies the rules. However, the gauged supergravity comes to the rescue. Supersymmetrizing the gauging introduces corrections to the transformations laws which act precisely in the way one needs. They restore part of the supersymmetry in the bulk, in the spirit of the original eleven-dimensional results of [2].

7 Conclusions

To summarize the outcome of this analysis, we first recall that the coupling to bulk fields of a source on the wall, such as a gaugino condensate, is already suppressed by a power of $\kappa^{2/3}$. The effects of the new gauge-related terms in the bulk on the transmission of supersymmetry breaking are formally of higher order, and hence unlikely to change qualitatively the conclusions we reached in [23], working with only the leading-order Lagrangian. They do contribute additional mixing of the scalars and vectors living in the bulk, and one should check further the supersymmetry transformations laws, which are modified. However, as as has been already noticed in [22], the corrections to these transformations, which are linear in the non-trivial background, are not only formally of higher order but also discontinous across the walls, since the background to which they are proportional is itself discontinuous. This means that these corrections do not appear on the walls, and hence do not open up any new channels of communication of supersymmetry breaking from the hidden wall to the bulk, or from the bulk to the fields living on the visible wall, beyond those already identified in our earlier paper [23].

We observe that the origins of the non-trivial backgrounds of certain five-dimensional zero modes, such as the real part of S which represents the Calabi-Yau volume, are traceable to non-trivial sources living on the walls. These are coupled to zero modes that change quasi-linearly across the bulk. The roles of such sources, which we studied in our previous paper in the leading-order Lagrangian, continue to hold to leading order also in the presence of the terms associated with the gauging, as do our conclusions.

In connection with the analysis of the transmission of supersymmetry breaking in the presence of gauging, we have found the extension of the gauged supergravity model of [22, 24] which includes a minimal sector of zero modes associated with $(2, 1)$ forms on the Calabi-Yau space, which manifest themselves as non-universal hypermultiplets in five dimensions. In particular,

we have determined the way in which the non-universal multiplets couple to gaugino condensates, which are primary candidate for hierarchical supersymmetry breaking in the framework of M theory.

The results of this paper open the way to a more phenomenological analysis of the transmission of supersymmetry breaking from the hidden wall to the observable wall through the bulk.

References

[1] P. Horava and E. Witten, *Nucl.Phys.* **B475** (1996) 94.

[2] E. Witten, *Nucl.Phys.* **B471** (1996) 135.

[3] P. Horava and E. Witten, *Nucl.Phys.* **B460** (1996) 506.

[4] T. Banks and M. Dine, *Nucl.Phys.* **B479** (1996) 173.

[5] H. P. Nilles, S. Stieberger, *Nucl.Phys.* **B499** (1997) 3.

[6] T. Li, J.L. Lopez and D.V. Nanopoulos, *Mod.Phys.Lett.* **A12** (1997) 2647; *Phys.Rev.* **D56** (1997) 2602.

[7] J. Ellis, A. E. Faraggi and D.V. Nanopoulos, *Phys.Lett.* **B419** (1998) 123.

[8] E. Dudas and C. Grojean, *Nucl.Phys.* **B507** (1997) 553.

[9] I. Antoniadis and M. Quiros, *Phys. Lett.* **B392** (1997) 61.

[10] I. Antoniadis and M. Quiros, *Nucl. Phys.* **B505** (1997) 109.

[11] I. Antoniadis and M. Quiros, *Phys.Lett.* **B416** (1998) 327.

[12] K. Choi, *Phys. Rev.* **D56** (1997) 6588.

[13] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Phys. Lett.* **B415** (1997) 24.

[14] Z. Lalak and S. Thomas, *Nucl.Phys.* **B515** (1998) 55.

[15] K. Choi, H.B. Kim, C. Munoz, *Phys. Rev.* **D57** (1998) 7521.

[16] A. Lukas, B.A. Ovrut and D. Waldram, *Nucl.Phys.* **B532** (1998) 43.

[17] A. Lukas, B. A. Ovrut and D. Waldram, *Phys. Rev.* **D57** (1997) 7529.

[18] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Nucl. Phys.* **B530** (1998) 43.

[19] P. Horava, *Phys. Rev.* **D54** (1996) 7561.

[20] E. Sharpe, *Nucl. Phys.* **B523** (1998) 211.

[21] E. A. Mirabelli and M. E. Peskin, *Phys. Rev.* **D58** (1998) 065002.

[22] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, preprint UPR-797-T, hep-th/9803235.

[23] J. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, preprint CERN-TH/98-118, hep-ph/9805377, to be published in *Nucl.Phys.* **B**.

[24] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, preprint UPR-804-T, hep-th/9806051.

[25] M. Bodner, A. C. Cadavid and S. Ferrara, *Class. Quantum Grav.* **8** (1991) 789.

[26] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fre and T. Magri, *J. Geom. Phys.* **23** (1997) 111.

[27] P. Candelas and X. de la Ossa, *Nucl. Phys.* **B355** (1991) 455.

[28] G. Tian, *Mathematical Aspects of String Theory,* ed S.T. Yau (World Scientific, Singapore, 1987), p.629.

[29] S. Ferrara and S. Sabharwal, *Nucl.Phys.* **B332** (1990) 316.