EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/98-158
06/10/98

# SU(4) SYMMETRY IN THE EXTENDED PROTON-NEUTRON INTERACTING BOSON MODEL : MULTIPLETS AND SYMMETRY BREAKING.

**B. Decroix[(1)], J. De Beule[(1)], C. De Coster[(1),*] and K. Heyde[(1,2)].**

[(1)] Institute for Theoretical Physics, Vakgroep Subatomaire en Stralingsfysica, Proeftuinstraat 86, B-9000 Gent (Belgium)
[(2)] present address: EP-ISOLDE, CERN, CH 1211 Geneva 23 (Switzerland)
[*] Postdoctoral research fellow of the Fund for Scientific Research - Flanders (Belgium)

**Abstract**

The manifestation of $SU(4)$ symmetry within an interacting boson model including particle-like and hole-like $\pi$- and $\nu$-bosons is shown for light nuclei around the Z=N=8 shell. We also present a consistent description of the particle-hole (intruder spin or $I$ spin) multiplets in the Extended Interacting Boson Model (EIBM) and of $\pi$-$\nu$ ($F$ spin) multiplets in the IBM-2 as a breaking of this $SU(4)$ symmetry.

**(ISOLDE GENERAL)**

(Accepted for publication in Physics Letters B.)

The Interacting Boson Model has been introduced to describe quadrupole collective phenomena observed in medium-heavy and heavy nuclei. The building blocks are nucleon-pairs with angular momentum $L^\pi = 0^+$ and $2^+$ which are mapped onto s- and d-bosons, respectively. In the original Interacting Boson Model (IBM-1), only one kind of s- and d-bosons are considered [1, 2].
Using the charge of the nucleons as a supplementary but essential degree of freedom, the IBM-2 was constructed enabling a classification of states with the introduction of a new quantum label $F$, associated with the $F$ spin algebra $SU_F(2)$. The symmetry of the spatial part and of the charge part of the wave function is determined by this $F$ spin label. The total wave function, being the product of an orbital wave function with the $F$ spin wavefunction, is symmetric and thus allows to consider states with possible values $F = N/2, N/2-1, ..., \mid F_z \mid$ with $F_z = \frac{N_\pi - N_\nu}{2}$. IBM-1 is incorporated in IBM-2 by the condition on the wave functions : $F = F_{max}$, hence demanding a totally symmetric orbital wave function. All the other possible states have a 'mixed $F$ spin symmetry'.
In an attempt to describe particle-hole excitations across the closed shell in the IBM-picture, an Extended Interacting Boson Model (EIBM) was proposed [3] in which a particle or hole character gets assigned to the s- and d-bosons. The symmetries in this model can be developed in analogy with the IBM-2, where now we introduce the $I$ spin (intruder spin) algebra $SU_I(2)$. By doing so we can consider states with a 'mixed $I$ spin symmetry', corresponding with a classification $I < I_{max}$ where $I_{max} = N/2$ and $N$ the total number of bosons. Applications of this model have shown to be successful in describing the coexistence of different symmetries for the groundstates and the 2p-2h intruder states in both the Z=50 and Z=82 region [4, 5]. Since the EIBM states are restricted to those members of the $I$ spin multiplet where the neutrons are considered as the reference state, i.e. the $I$ spin label is restricted to the proton configuration only and we deman! d $I^\nu = 0$, it shows to be ve ry important to construct a full Extended Proton-Neutron Interacting Boson Model (EIBM-2) by introducing both the particle-hole character and the charge of the nucleons as a degree of freedom for the bosons on equal footing. By doing so a new reduction scheme, containing an $SU(4)$ algebra, has been suggested [6].
In the present article we present an example of this $SU(4)$ symmetry in the Z=N=8 mass region. Also, the EIBM-2 allows us to describe the assumption $I^\nu = 0$ in both [4, 5] in a consistent way within the model. In the same way it is shown how the $F$ spin multiplets in IBM-2 applications are consistent with an SU(4) symmetry breaking.
By introducing both the particle-hole character and the charge character of the bosons, the EIBM-2 was constructed [6]. A first possible reduction scheme for the dynamical algebra $U(24)$ is :

$$U(24) \supset \begin{array}{c} U_L(6) \quad \supset G_\lambda \supset O_L(3) \\ \otimes \\ SU(4) \quad . \end{array} \tag{1}$$

The $G_\lambda$ stands for the IBM-1 reductions of $U(6)$ [1].

The generators for the $SU(4)$ algebra can be considered as follows :
(i) The generators of the $SU(2)$ algebra for the intruder spin ($I$ spin) for the $\pi$-bosons and for the $\nu$-bosons :

$$\hat{I}_z^{(\beta)} = 1/2(\hat{N}_p^{(\beta)} - \hat{N}_h^{(\beta)}), \tag{2}$$
$$\hat{I}_+^{(\beta)} = s^\dagger_{p,\beta} s_{h,\beta} + d^\dagger_{p,\beta} \cdot \tilde{d}_{h,\beta}, \tag{3}$$
$$\hat{I}_-^{(\beta)} = s^\dagger_{h,\beta} s_{p,\beta} + d^\dagger_{h,\beta} \cdot \tilde{d}_{p,\beta}, \tag{4}$$

with $\beta = \pi, \nu$.

(ii) The generators of the $SU(2)$ algebra for the $F$ spin for the particle bosons and for the hole bosons :

$$\hat{F}_z^{(\alpha)} = 1/2(\hat{N}_\alpha^{(\pi)} - \hat{N}_\alpha^{(\pi)}), \tag{5}$$

$$\hat{F}_{+}^{(\alpha)} = s_{\alpha,\pi}^{\dagger} s_{\alpha,\nu} + d_{\alpha,\pi}^{\dagger} \cdot \tilde{d}_{\alpha,\nu}, \tag{6}$$

$$\hat{F}_{-}^{(\alpha)} = s_{\alpha,\nu}^{\dagger} s_{\alpha,\pi} + d_{\alpha,\nu}^{\dagger} \cdot \tilde{d}_{\alpha,\pi}, \tag{7}$$

with $\alpha = p, h$.

(iii) The generators of the $SU_B(2)$ algebra :

$$\hat{B}_z = 1/2(\hat{N}^{(\pi)} - \hat{N}^{(\nu)}), \tag{8}$$

$$\hat{B}_+ = s_{p,\pi}^{\dagger} s_{h,\nu} + d_{p,\pi}^{\dagger} \cdot \tilde{d}_{h,\nu} + s_{h,\pi}^{\dagger} s_{p,\nu} + d_{h,\pi}^{\dagger} \cdot \tilde{d}_{p,\nu}, \tag{9}$$

$$\hat{B}_- = s_{p,\nu}^{\dagger} s_{h,\pi} + d_{p,\nu}^{\dagger} \cdot \tilde{d}_{h,\pi} + s_{h,\nu}^{\dagger} s_{p,\pi} + d_{h,\nu}^{\dagger} \cdot \tilde{d}_{p,\pi}. \tag{10}$$

We denote these algebras respectively by $SU_{I^{(\pi)}}(2), SU_{I^{(\nu)}}(2), SU_{F^{(p)}}(2), SU_{F^{(h)}}(2)$ and $SU_B(2)$.
Taken together with the $SU_B(2)$ generators, the algebras $SU_I(2)$ and $SU_F(2)$, generated by symmetrising the operators in ($\pi$, $\nu$) and (p,h) degrees of freedom respectively, now form an $SU(3)$ algebra. So we can consider a second reduction in EIBM-2 :

$$U(24) \supset \begin{array}{ll} U_L(6) & \supset G_\lambda \supset O_L(3) \\ \otimes & \\ SU(4) & \supset SU(3) \supset (SU_I(2) \otimes SU_F(2)). \end{array} \tag{11}$$

The above symmetries, as depicted in equations (1) and (11), can now be used to identify a number of multiplets according to the $SU(4)$ and $SU(3)$ symmetries. We denote the configuration of a nucleus with a certain number of particle-like and hole-like $\pi$- and $\nu$-bosons as $(N_p^{(\pi)}, N_h^{(\pi)}, N_p^{(\nu)}, N_h^{(\nu)})$ and can visualize this in a schematical way as presented in figure 1.

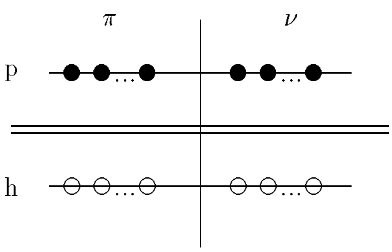


Figure 1: *Configuration of a nucleus with a given number of proton (particles and holes) and neutron (particles and holes) pairs :* $N_p^{(\pi)}, N_h^{(\pi)}, N_p^{(\nu)}, N_h^{(\nu)}$ *respectively.*

An $SU(4)$ multiplet is defined as all the eigenstates in all the nuclei for a configuration with a constant total number of bosons $N = \sum_{\alpha,\beta} N_\alpha^{(\beta)}$ which are connected by the 10 ladder operators $\hat{I}_{\pm}^{(\beta)}, \hat{F}_{\pm}^{(\alpha)}, \hat{B}_{\pm}$. If we consider the action of the operator $\hat{B}_+$ on a nucleus which has a configuration described as in figure 1, we generate the configuration $(N_p^{(\pi)}+1, N_h^{(\pi)}, N_p^{(\nu)}, N_h^{(\nu)}-1) + (N_p^{(\pi)}, N_h^{(\pi)}+1, N_p^{(\nu)}-1, N_h^{(\nu)})$ (see figure 2). It is obvious that the same configuration can be established by action of the operator $\hat{I}_+^{(\pi)}\hat{F}_+^{(h)} + \hat{I}_-^{(\pi)}\hat{F}_+^{(p)}$. To find the equivalent operators for the action of the ladder operator $\hat{B}_-$ we point out that it is the hermitian conjugate of $\hat{B}_+$. Here, an important feature has to be pointed out : when one of! the ladder operators $\hat{B}_\pm$ acts on a state of a nucleus with a certain configuration as indicated in figure 1, the result will be the sum of two eigenstates of two different nuclei, as shown in figure 2. In order to avoid any further problems we will use the convention that this 'mixing of states belonging to different nuclei' does not influence the definition of the multiplet. Of course, when considering the Hamiltonian and its symmetries, we have to consider the implications of this. Using this particular convention, it becomes clear that all members of a certain $SU(4)$ multiplet are connected by use of only 8 ladder operators associated with $\hat{I}^{(\beta)}$ and $\hat{F}^{(\alpha)}$spin.

Another implication of this convention is that the $SU(4)$ multiplet can also be identified as identical with the $SU(3)$ multiplet and also with the $SU_I(2)\otimes SU_F(2)$ multiplet. For this reason, we shall only speak of $SU(4)$ multiplets, characterized by a total number of bosons $N$.

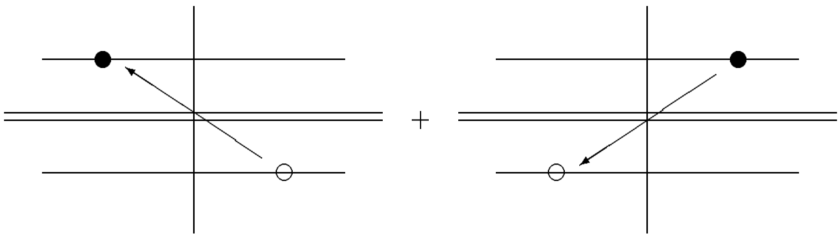

Figure 2: *The configuration generated by acting with the operator $\hat{B}_+$ on the above configuration, i.e. the resulting configuration* $(N_p^{(\pi)}+1, N_h^{(\pi)}, N_p^{(\nu)}, N_h^{(\nu)}-1)+(N_p^{(\pi)}, N_h^{(\pi)}+1, N_p^{(\nu)}-1, N_h^{(\nu)})$.

In order to describe a system with a constant number of particle-like and hole-like $\pi$- and $\nu$-bosons, we can write the following Hamiltonian :

$$\hat{H}=\sum_{\alpha,\beta}\hat{H}_{\alpha}^{(\beta)}+\sum_{\alpha',\alpha'',\beta',\beta''}\hat{V}_{\alpha',\alpha''}^{(\beta',\beta'')}, \tag{12}$$

where $\alpha,\alpha',\alpha''=p,h$ and $\beta,\beta',\beta''=\pi,\nu$, $\alpha'\neq\alpha''$ or $\beta'\neq\beta''$. By using the creation and annihilation operators for the four types of bosons and considering up to second order, one obtains :

$$\begin{aligned}\hat{H}_{\alpha}^{(\beta)} &= E_0^{\alpha,\beta}+\textstyle\sum_{lm,l'm'}\epsilon_{lm,l'm'}^{\alpha,\beta}b^{\dagger}_{\alpha\beta,lm}b_{\alpha\beta,l'm'}\\ &\quad +1/2\textstyle\sum_{l_im_i}u_{(l_i,m_i)}^{\alpha,\beta}b^{\dagger}_{\alpha\beta,l_1m_1}b^{\dagger}_{\alpha\beta,l_2m_2}b_{\alpha\beta,l_3m_3}b_{\alpha\beta,l_4m_4}, \qquad (13)\\ \hat{V}_{\alpha',\alpha''}^{(\beta',\beta'')} &= \textstyle\sum_{l_im_i}w_{\alpha',\alpha'',(l_i,m_i)}^{(\beta',\beta'')}b^{\dagger}_{\alpha'\beta',l_1m_1}b_{\alpha'\beta',l_2m_2}b^{\dagger}_{\alpha''\beta'',l_3m_3}b_{\alpha''\beta'',l_4m_4}.\end{aligned}$$

Since this Hamiltonian conserves all $N_{\alpha}^{(\beta)}$, it satisfies :

$$[\hat{H},\hat{I}_z^{(\beta)}]=[\hat{H},\hat{F}_z^{(\alpha)}]=[\hat{H},\hat{B}_z]=0. \tag{14}$$

This also implies :

$$[\hat{H},\hat{I}_z]=[\hat{H},\hat{F}_z]=0. \tag{15}$$

All these conditions express particle-number conservation which is fundamental to describe eigenstates of a nucleus with a certain configuration.
We now call $SU(4)$ a dynamical symmetry for the system described by the Hamiltonian (12) if the following conditions are satisfied :

$$[\hat{H},(\hat{I}^{(\beta)})^2]=[\hat{H},(\hat{F}^{(\alpha)})^2]=[\hat{H},\hat{B}^2]=0. \tag{16}$$

We note that, since for example $[(\hat{I}^{(\pi)})^2,\hat{B}^2]\neq 0$, we cannot have eigenstates of the Hamiltonian with both good $I^{(\pi)}$ spin and good $B$ spin.
We call $SU(4)$ a real symmetry for the system described by the Hamiltonian (12) if $SU(4)$ is not only a dynamical symmetry but if also the following conditions are fulfilled :

$$[\hat{H},\hat{I}_{\pm}^{(\beta)}]=[\hat{H},\hat{F}_{\pm}^{(\alpha)}]=[\hat{H},\hat{B}_{\pm}]=0. \tag{17}$$

Since it has been shown that

$$\hat{B}_+ = \hat{I}_+^{(\pi)}\hat{F}_+^{(h)}+\hat{I}_-^{(\pi)}\hat{F}_+^{(p)}, \tag{18}$$

$$\hat{B}_- = \hat{F}_-^{(h)}\hat{I}_-^{(\pi)}+\hat{F}_-^{(p)}\hat{I}_+^{(\pi)}. \tag{19}$$

we can conclude that $SU(4)$ is a real symmetry for the system described by the Hamiltonian (12) when $SU(4)$ is a dynamical symmetry and when the hamiltonian commutes with the 8 ladder operators $\hat{I}_{\pm}^{(\beta)}$ and $\hat{F}_{\pm}^{(\alpha)}$. These are exactly the ladder operators needed to define the $SU(4)$ multiplet.
Similarly, we call $SU(3)$ a dynamical symmetry for the system described by the Hamiltonian (12) if the following conditions are satisfied :

$$[\hat{H}, \hat{I}^2] = [\hat{H}, \hat{F}^2] = [\hat{H}, \hat{B}^2] = 0, \tag{20}$$

and $SU(3)$ is called a real symmetry for the system described by the Hamiltonian (12) if not only $SU(3)$ is a dynamical symmetry but if the following conditions are also satisfied :

$$[\hat{H}, \hat{I}_{\pm}] = [\hat{H}, \hat{F}_{\pm}] = [\hat{H}, \hat{B}_{\pm}] = 0. \tag{21}$$

We remark that $SU(4)$, as a real symmetry,immediately implies $SU(3)$ as a real symmetry. Technically, there remains a problem : Applying $\hat{I}_{\pm}, \hat{F}_{\pm}$ or $\hat{B}_{\pm}$ on an eigenstate belonging to one nucleus leads to the mixing of two eigenstates belonging to two different nuclei. This means that, though $SU(3)$ can be considered as a real symmetry, the application of the ladder operators is ill-defined in the context of symmetries. Therefore, we shall restrict ourselves to examining the $SU(4)$ as real or dynamical symmetry.
We shall now illustrate the above, more general discussion, showing (i) experimental evidence for an $SU(4)$ symmetry at the N=Z=8 shell closure, and (ii) discuss the appearance of the EIBM and IBM-2 structures as resulting from $SU(4)$ symmetry breaking.

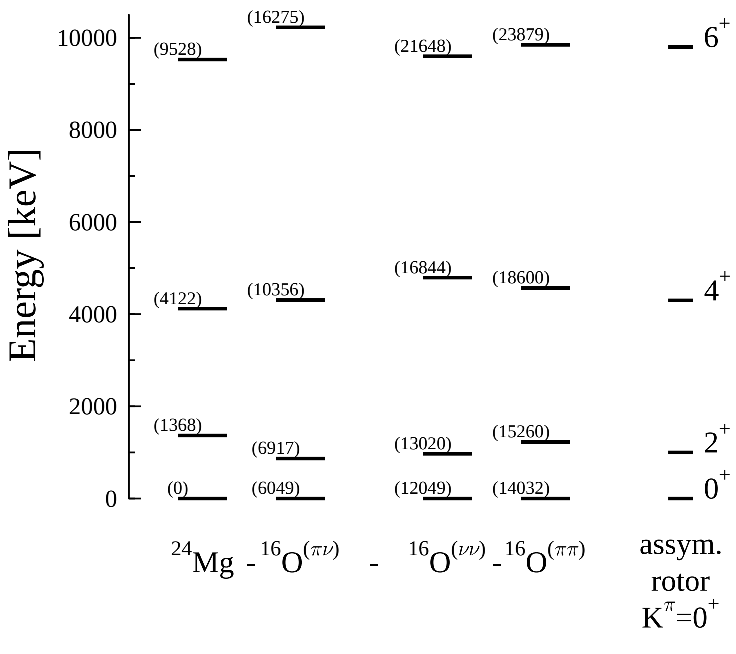


Figure 3: *Comparison of the $K^\pi = 0^+$ $N = 4$ $SU(4)$ multiplet members in $^{16}O$ and $^{24}Mg$. Thereby we can put the $\pi$(2p-2h)$\nu$(2p-2h), $\pi$(4p-4h), $\nu$(4p-4h) excitations in $^{16}O$ (denoted respectively by $^{16}O^{(\pi\nu)}$, $^{16}O^{(\pi\pi)}$, $^{16}O^{(\nu\nu)}$) and $\pi$(4p)-$\nu$(4p) ($^{24}Mg$) band members in a given multiplet.*

(i) In the nucleus $^{16}$O, there is experimental evidence for a $K^\pi = 0^+$ band and a $K^\pi = 2^+$ band associated with 4p-4h excitations, featuring the properties of an asymmetric rotor [7, 8]. The microscopic structure associated herewith is a $\pi$(2p-2h)$\nu$(2p-2h) excitation [9]. In the EIBM-2 context, this means that these levels can belong to the SU(4) multiplet with $N = 4$. If the SU(4) symmetry is a real symmetry for this multiplet, the same structure should be observed in the nucleus $^{24}_{12}$Mg$_{12}$ too. Also the levels associated with the $\pi$(4p-4h)

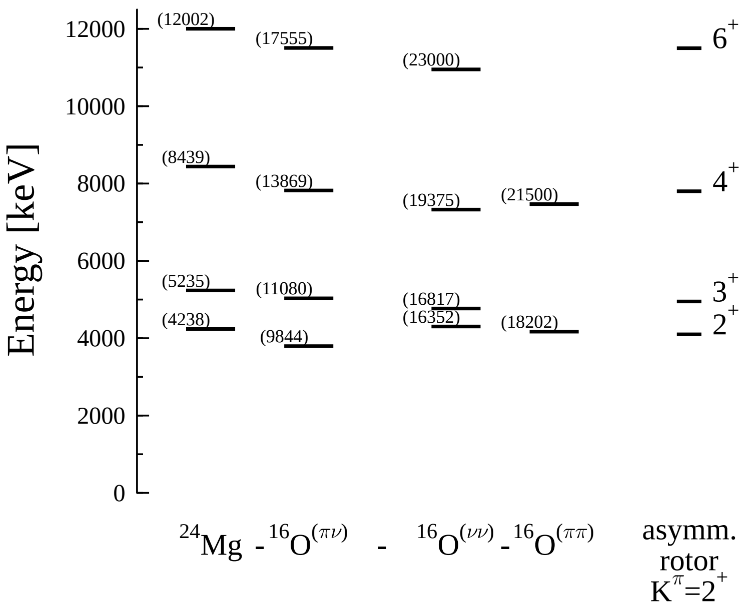


Figure 4: *Comparison of the triaxial $K^\pi = 2^+$ band for the same $SU(4)$ multiplet members as mentioned in figure 3.*

and $\nu$(4p-4h) excitation of ${}^{16}_{8}O_8$ belong to the same multiplet and are good candidates to establish SU(4) as a real symmetry. Results are presented in figure 3 when comparing the $K^\pi = 0^+$ 4p-4h multiplets in ${}^{16}$O ($\pi$(2p-2h)$\nu$(2p-2h); $\pi$(4p-4h); $\nu$(4p-4h)) with the $\pi$(4p)-$\nu$(4p) groundstate $K^\pi = 0^+$ band in ${}^{24}$Mg. Levels are! tentatively associated with the $\nu$(4p-4h) and the $\pi$(4p-4h) configurations according to the $SU(4)$ multiplet structure. Similar comparisons are carried out for the $K^\pi = 2^+$ triaxial bands as appearing in ${}^{16}$O and in ${}^{24}$Mg (figure 4). The comparison of experimental data gives good indications for the presence of a rather well established $SU(4)$ symmetry in these light nuclei.

(ii) Since $SU(4)$ can be reduced to the algebras $SU_{I^{(\pi)}}(2), SU_{I^{(\nu)}}(2), SU_{F^{(p)}}(2)$ or $SU_{F^{(h)}}(2)$, the EIBM and the IBM-2 can be considered as an $SU(4)$-symmetry breaking while one of the mentioned $SU(2)$ algebra becomes the real symmetry for the system. The $I$ spin multiplet members as mentioned in [4, 5] are an example of $SU_{I^{(\pi)}}(2)$ symmetry. By doing so, it is obvious why we then consider $I^{(\nu)} = 0$. The algebraic structure describing these multiplets is then given by :

$$U(24) \supset U^{(\pi)}(12) \supset \begin{array}{c} U_L^{(\pi)}(6) \quad \supset G_\lambda \supset O_L(3) \\ \otimes \\ U_{I^{(\pi)}}(2) \end{array} \tag{22}$$

where the generators of $U(24)$ are [6] $b^\dagger_{\alpha,\beta,lm} b_{\alpha',\beta',l'm'}$ with $\alpha, \alpha' = p, h$ describing the particle or hole character of the bosons, $\beta, \beta' = \pi, \nu$ describing the charge character of the bosons and $l, l' = 0, 2$ for s- and d-bosons, while $m = -l, ..., l$ and $m' = -l', ..., l'$. The generators for $U^{(\pi)}(12), U_L^{(\pi)}(6)$ and $U_{I^{(\pi)}}(2)$ are respectively $b^\dagger_{\alpha,\pi,lm} b_{\alpha',\pi,l'm'}, \sum_\alpha b^\dagger_{\alpha,\pi,lm} b_{\alpha,\pi,l'm'}$ and $\sum_{l,m} b^\dagger_{\alpha,\pi,lm} b_{\alpha',\pi,lm}$. The algebras $G_\lambda$ and $O_L(3)$ are those commonly used in the IBM-1 reduction [1]. One can e.g. describe the complementary symmetry of $SU_{I^{(\nu)}}$ at neutron closed shell nuclei N=50, N=82.

The other symmetries, for example $F^{(p)}$ spin or $F^{(h)}$ spin, can be described using a complete analogous reduction scheme. Each of these symmetries reflects the breaking of the $SU(4)$ symmetry into a symmetry of a specific $SU(2)$ subalgebra.

In conclusion, we have shown how the Extended Proton-Neutron Interacting Boson Model (EIBM-2) allows the multi-particle-multi-hole excitations in ${}^{16}_{8}O_8$ and the regular low-lying bands in ${}^{24}_{12}Mg_{12}$ to be described as members of the same $N = 4$ $SU(4)$ multiplet in the N=Z=8 mass region. Moreover, a consistent way has been established in order to describe both the EIBM and the IBM-2 as resulting from an $SU(4)$ symmetry breaking. Other examples of realization of the $SU(4)$ symmetry will be examined in other mass regions and results will be published in a forthcoming article.


## Aknowledgements

Three of the authors (B.D. C.D.C. and K.H.) thank the IWT, FWO and IIKW for financial support. One of them (KH) gratefully acknowledges financial support from CERN. We also thank P. Van Isacker and J.L. Wood for inspiring discussions.


## References


[1] F. Iachello and A. Arima, The interacting boson model (Cambridge Univ. Press, Cambridge, 1987).

[2] A. Frank and P. Van Isacker, Algebraic Methods in molecular and nuclear structure physics (John Wiley and Sons, Inc. New York, 1994).

[3] C. De Coster, K. Heyde, B. Decroix, P. Van Isacker, J. Jolie, H. Lehmann, J.L. Wood, Nucl. Phys. A600 (1996) 251.

[4] H. Lehmann, J. Jolie, C. De Coster, B. Decroix, K. Heyde, P. Van Isacker, Nucl. Phys. A621 (1997), 767.

[5] C. De Coster, B. Decroix, K. Heyde, J.L. Wood. H. Lehmann, J. Jolie, Nucl. Phys. A621 (1997), 802.

[6] B. Decroix, J. De Beule, C. De Coster, K. Heyde, A. Oros, P. Van Isacker, Phys. Rev. C Vol. 57 (1998), 2329.

[7] S. E. Larsson, G. Leander, S. G. Nilsson, I. Ragnarsson and R. K. Sheline, Phys. Lett. B47 (1973), 422.

[8] S. Aberg, I. Ragnarsson, T. Bengtsson and R. K. Sheline, Nucl, Phys. A391 (1982), 327.

[9] F. D. Becchetti et al., Nucl. Phys. A344 (1980), 336.