N DELTA - TRANSITION FORM FACTORS AT LOW MOMENTUM TRANSFER

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The three complex form factors entering the $\Delta \to N\gamma^*$ vertex are calculated to $\mathcal{O}(\varepsilon^3)$ in the framework of a chiral effective theory with explicit $\Delta(1232)$ degrees of freedom. Furthermore, the role of presently unknown low energy constants that affect the values of EMR and CMR is elucidated.

1 Introduction

In this talk we present a new calculation [1] of the ΔN transition form factors at low momentum transfer $t \equiv q^2$ in an effective chiral lagrangian framework. For a survey of the ongoing experimental efforts regarding this fundamental transition of the nucleon to its first excited state we refer to the contributions of P. Bartsch, M.O. Distler and R.W. Gothe in these proceedings. In our calculation a scale $\varepsilon = \{p, m_\pi, \delta\}$ denoting, collectively, small momenta, the pion mass and the Delta-nucleon mass splitting is used to establish a systematic power-counting, thus telling us precisely which diagrams/vertices have to be included if we want to calculate up to a certain order in ϵ. This approach allows for an efficient inclusion of $\Delta(1232)$ degrees of freedom consistent with the underlying chiral symmetry of QCD and is referred to as the "Small Scale Expansion" (SSE) [2], constituting a phenomenological extension of Heavy Baryon Chiral Perturbation Theory [3]. So far, we have performed the calculation to $\mathcal{O}(\varepsilon^3)$ and in the process have also established the appropriate relations between the non-relativistic microscopic calculation as well as phenomenological parameterizations of the transition current. We find that the q^2 evolution of the three complex form factors is completely determined by the dynamics of the nucleon's pion cloud governed by chiral symmetry. Finally, utilizing previous calculations performed at $q^2 = 0$ we discuss the role of relevant low energy parameters for the sought after multipole ratios EMR(q^2) and CMR(q^2).

2 Non-relativistic Reduction of the $\Delta \to N\gamma^*$ Vertex

Demanding Lorentz covariance, gauge invariance and parity conservation the most general form of the $\Delta \to N\gamma^*$ radiative decay amplitude is described by three form factors $G_i(q^2)$, i=1,2,3,

$$
\begin{aligned}
i\mathcal{M}^{full}_{\Delta N\gamma} &= \frac{e}{2M_N}\bar{u}(p)\gamma_5\left[G_1(q^2)(\not{q}\epsilon_\mu - \not{\epsilon}q_\mu) + \frac{G_2(q^2)}{2M_N}(p\cdot\epsilon\, q_\mu - p\cdot q\,\epsilon_\mu)\right.\\
&\left. + \frac{G_3(q^2)}{2(M_\Delta - M_N)}(q\cdot\epsilon\, q_\mu - q^2\epsilon_\mu)\right] u^\mu_\Delta(p_\Delta)\,. \qquad (1)
\end{aligned}
$$

Here $M_N\,(M_\Delta)$ is the nucleon (Delta) mass, p^μ, p^μ_Δ denotes the four-momentum of the nucleon, Delta and q^μ, ϵ^μ represent the photon four-momentum and polarization vectors, respectively. A SSE calculation of the radiative vertex to $\mathcal{O}(\varepsilon^3)$ entails a restriction to $\mathcal{O}(1/M_N^2)$ accuracy [2]. We must, therefore, compute the most general form of Eq.(1) consistent with this estimate. Our calculation will be performed in the $\Delta(1232)$ rest frame which is convenient for the identification of various multipoles [1]. In this frame, taking into account that to $\mathcal{O}(\varepsilon^3)$ the photon energy obeys $\omega = \delta + \mathcal{O}(1/M_N)$ and assuming that each form factor scales at least as $\mathcal{O}(M_N^0)$ one determines

$$
\begin{aligned}
i\mathcal{M} &= e\;\bar{u}_v(r_N)\left\{(S\cdot\epsilon)q_\mu\left[\frac{G_1(q^2)}{M_N} + \mathcal{O}(1/M_N^3)\right]\right.\\
&+(S\cdot q)\epsilon_\mu\left[-\frac{G_1(q^2)}{M_N} - \frac{\delta G_1(0)}{2M_N^2} + \frac{\delta G_2(q^2)}{4M_N^2} + \frac{q^2 G_3(q^2)}{4M_N^2\delta} + \ldots\right]\\
&+(S\cdot q)(v\cdot\epsilon)q_\mu\left[\frac{G_1(0)}{2M_N^2} - \frac{G_2(q^2)}{4M_N^2} + \mathcal{O}(1/M_N^3)\right]\\
&\left.+(S\cdot q)(q\cdot\epsilon)q_\mu\left[-\frac{G_3(q^2)}{4M_N^2\delta} + \mathcal{O}(1/M_N^3)\right]\right\} u^{\mu,i=3}_{v,\Delta}(0)\;, \qquad (2)
\end{aligned}
$$

where v_μ denotes the rest frame four velocity of the Delta field and S_μ corresponds to the Pauli-Lubanski vector. From Eq.(2) one can already see that to $\mathcal{O}(\epsilon^3)$ one is sensitive to the first two orders in the chiral expansion of G_1, whereas the quadrupole form factors G_2, G_3 only start at $\mathcal{O}(\epsilon^3)$ and therefore only their leading behavior is determined in this calculation. Note that we had to introduce a particular mass dependence proportional to δ^{-1} accompanying the form factor $G_3(q^2)$ in Eq.(1) in order to achieve consistency with the chiral $\mathcal{O}(\epsilon^3)$ calculation [1].

3 The Calculation

Independent of any choice of gauge one finds that even for finite q^2 only two one-loop diagrams—the well known $\Delta \to N\gamma$ triangle diagrams [4] corresponding to the photon been attached to two pions and the intermediate baryon state being a nucleon and a Delta, respectively—contribute to this order. Due to the fact that the $\Delta \to N\gamma^*$ transition starts with a magnetic dipole amplitude M1, one finds no $\mathcal{O}(\varepsilon)$ vertex but has to take into account all $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(\varepsilon^3)$ tree contributions. For the Born contributions one obtains [1]

$$\begin{aligned} i\mathcal{M}^{Born}_{\Delta\to N\gamma^*} &= e\,\bar{u}_v(r_N)\left[S\cdot\epsilon\; q_\mu\left(\frac{-b_1}{2M_N}+\frac{(2E_1-D_1)\delta}{4M_N^2}\right)\right. \\ &\quad +S\cdot q\;\epsilon_\mu\left(\frac{b_1}{2M_N}-\frac{E_1\delta}{2M_N^2}+\frac{(b_1+2b_6)\,\delta}{4M_N^2}\right) \\ &\quad \left.+\epsilon_0\; S\cdot q\; q_\mu\left(\frac{D_1}{4M_N^2}-\frac{b_1+2b_6}{4M_N^2}\right)\right] u^{\mu,i=3}_{v,\Delta}(0), \qquad (3) \end{aligned}$$

where b_1, b_6 correspond to the two leading (*i.e.* $\mathcal{O}(\varepsilon^2)$) $\Delta N\gamma$ couplings [5], whereas D_1, E_1 are new [1] $\Delta N\gamma$ couplings of $\mathcal{O}(\varepsilon^3)$, which also take part in the renormalization of the loop diagrams.

The loop contributions can be formally written as [1]

$$\begin{aligned} i\mathcal{M}^{Loop}_{\Delta\to N\gamma^*} &= \frac{2eg_{\pi N\Delta}}{F_\pi^2}\;\bar{u}_v(r_N)\Big\{(S\cdot\epsilon)q_\mu\left[g_A F_N(t)+\chi g_1 F_\Delta(t)\right] \\ &\quad +(S\cdot q)\epsilon_\mu\left[g_A G_N(t)+\chi g_1 G_\Delta(t)\right]+(S\cdot q)q_\mu\Big(\epsilon_0[g_A J_N(t) \\ &\quad +\chi g_1 J_\Delta(t)](q\cdot\epsilon)\left[g_A H_N(t)+\chi g_1 H_\Delta(t)\right]\Big)\Big\}u^{\mu\, i=3}_{v,\Delta}(0)\,, \qquad (4) \end{aligned}$$

where the functions $I_{N,\Delta}$, $I\in\{F,G,J,H\}$ are given in terms of integrals over a Feynman parameter. The last structure proportional to $q\cdot\epsilon$ is the one which cannot be reproduced by Eq.(2) without the particular mass term accompanying the G_3 form factor in Eq.(1) as discussed above. Comparing Eq.(2) with Eqs.(3,4), the identification of the three form factors is straightforward. *We note that each form factor has a real and an imaginary part as a direct consequence of chiral symmetry*—the explicit q^2-dependence is given below. Finally, gauge invariance of the identification of the form factors can be shown to be satisfied [1].

4 N Delta multipole transitions

With the complete set of $N\Delta$ form factors $G_i(q^2)$, $i = 1, 2, 3$ now known to $\mathcal{O}(\epsilon^3)$ one can easily calculate [1] all $N\Delta$ multipole transitions of interest. At present our knowledge of the four entering couplings b_1, b_6, D_1, E_1 is rather poor, but this situation is bound to improve soon with several SSE calculations in the Delta region well under way. For the moment we have utilized input from the Delta decay width and previous phenomenological calculations [6] to determine the required couplings, at $q^2 = 0$, and present here the q^2-evolution of the electric and coulomb multipole ratios EMR(q^2), CMR(q^2) as the subsequent SSE prediction to $\mathcal{O}(\epsilon^3)$.

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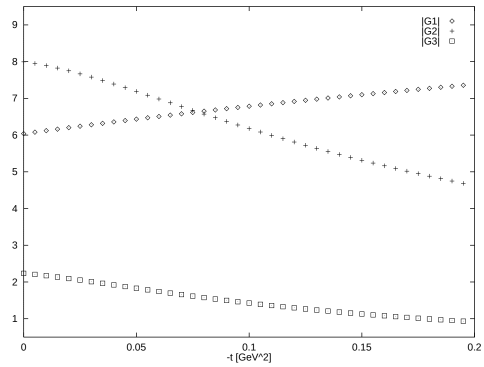

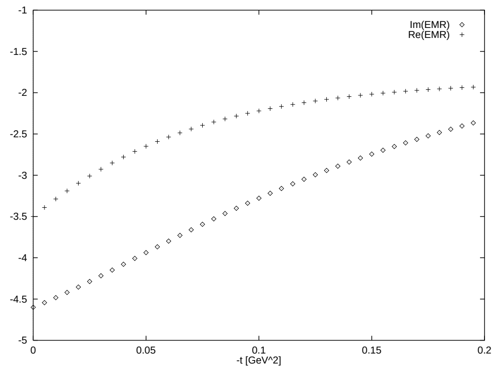
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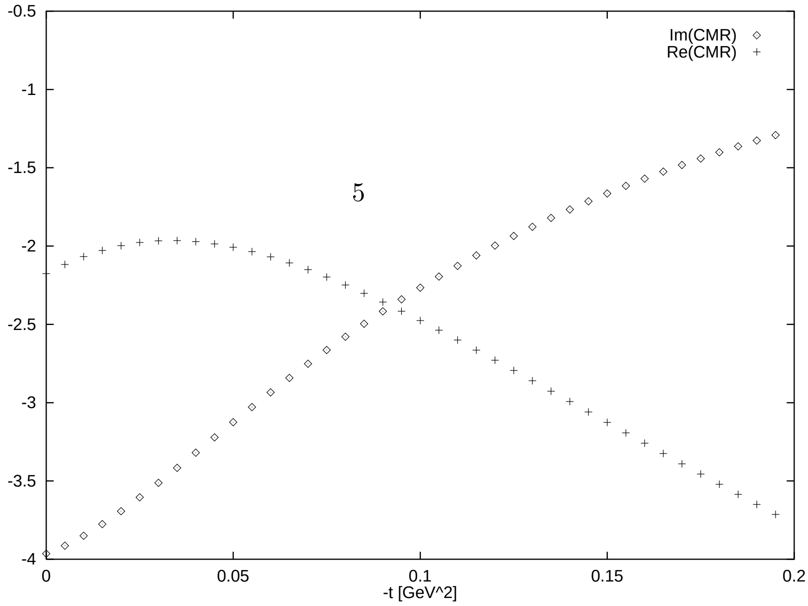
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-t [GeV^2]