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New two-loop contribution to electric dipole moment in supersymmetric theories

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ABSTRACT

We calculate a new type of two-loop contributions to the electric dipole moments of the electron and neutron in supersymmetric theories. The new contributions are originated from the potential CP violation in the trilinear couplings of the Higgs bosons to the scalar-top or the scalar-bottom quarks. These couplings were previously very weakly constrained. The electric dipole moments are induced through a mechanism analogous to that due to Barr and Zee. We find observable effects for a sizeable portion of the parameter space related to the third generation scalar-quarks in the minimal supersymmetric standard model which cannot be excluded by earlier considerations.

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Supersymmetry (SUSY) is considered to be theoretically the most conceivable avenue known so far which may lead to a successful unification of gravity with all other fundamental forces by means of supergravity and superstrings. SUSY also provides the most appealing perturbative solution to the gauge-hierarchy and naturalness problems of the standard model (SM). However, SUSY is not an exact symmetry of nature. Its minimal realization, the minimal supersymmetric SM (MSSM) must break SUSY softly in order to accomplish agreement with experimental observations. The scale of SUSY-breaking should not be much higher than few TeV if one wishes to retain the good property of perturbative naturality mentioned above, namely quantum corrections to the parameters of the theory must not exceed in size the parameters themselves up to energies of the Planck scale. On the other hand, unlike the SM, a serious weakness of supersymmetric theories as well as of the MSSM [1,2] is their failure to explain the smallness of the observed flavour-changing neutral currents (FCNC) involving the first two families of quarks, and the absence of sizeable electric dipole moments (EDMs) of the neutron and electron [3]. The present experimental upper bounds on the neutron EDM d_n and electron EDM d_e are very tight [4]: $|d_n| < 10^{-25}$ $e$cm and $|d_e| < 10^{-26}$ $e$cm.

As a result of the afore-mentioned FCNC and CP crises, some degree of fine-tuning is necessary in generic supersymmetric theories to avoid these problems. A few phenomenologically attractive solutions have been suggested in the literature. For example, Ref. [5] suggests a solution within the context of an effective SUSY model which seems to combine all healthy features of both the MSSM and technicolour theories. The main virtue of the effective SUSY model is that any non-SM source of CP violation and FCNC involving the first two generations is suppressed by allowing their respective soft-SUSY-breaking masses to be as high as 20 TeV, whereas third generation scalar quarks and leptons may naturally be light well below the TeV scale. Another interesting way to suppress CP-conserving FCNC interactions without resorting to a high scale of SUSY-breaking is to impose a kind of universality [1] or alignment [2] condition on the flavour space of all scalar fermions.

Most interestingly, we shall see that the existing bounds on CP-violating operators such as EDMs of the electron and neutron are dramatically relaxed if an approximate alignment between the μ parameter and the gaugino masses exists and the trilinear couplings A_f of the Higgs-boson to the first two generations are very small. Note that these suppression mechanisms for EDMs are designed to suppress only those contributions that do not involve the third generation.

Whichever the suppression mechanism of FCNC and CP violation for the first two families might be, large CP-violating trilinear couplings of the Higgs bosons to the scalar-top and scalar-bottom quarks can lead by themselves to large loop effects of CP noninvariance in the Higgs sector of the MSSM [6]. In this Letter we shall show that the very same source of CP violation can give rise to EDMs of electron and neutron at the observable level by virtue of the two-loop Barr-Zee-type graphs [7,8] shown in Fig. 1. Apart from the MSSM, these novel EDM contributions are present in any supersymmetric theory and as we will see, for a wide range of parameters they may even dominate by several orders of magnitude over all other one-, two- and three-loop contributions discussed extensively in the existing literature [9,10,11,12,13,14,15].

Supersymmetric theories contain many new CP-odd phases that are absent in the SM. However, not all of them are physical. Specifically, in the MSSM [9] supplemented by a universality condition at the grand unification scale, only two of the four complex parameters $\{\mu,\ B,\ m_\lambda,\ A\}$ are phase convention independent. For instance, one can absorb the common phase of the gaugino masses m_λ by a chiral rotation into the λ fields, where λ collectively represents the gauginos $\tilde{g}$, $\tilde{W}$ and $\tilde{B}$ of the $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ gauge groups, respectively. Furthermore, the renormalization condition that the total tadpole contribution to the CP-odd Higgs boson a must vanish [6] relates the phase of the soft bilinear Higgs-mixing mass $B\mu$ to the phase difference of the two Higgs doublets in the MSSM order by order in perturbation theory, where μ is the usual mixing parameter of the two Higgs chiral superfields in the superpotential. In accordance with the above renormalization condition, one may then choose a basis for the Higgs fields in which $B\mu$ is real at the tree level. Consequently, the two non-trivial CP-violating phases in the weak basis under study reside in μ and the universal soft trilinear coupling $A_f = A$ of the Higgs fields to the scalar fermions $\tilde{f}$.

Before we calculate the two-loop Barr-Zee-type diagrams in Fig. 1, we shall briefly review the most significant one-, two- and three-loop contributions to the electron and neutron EDMs in the MSSM. At one loop, the dominant contributions to the EDMs of electron and u-, d- quarks come from the chargino mass eigenstates $\tilde{\chi}_1^+$, $\tilde{\chi}_2^+$ and the gluino $\tilde{g}$, respectively [9,10,11]. Chargino quantum effects give rise to a EDM of a fermion f

$$\Big(\frac{d_f}{e}\Big)^{\tilde{\chi}^+} \approx \frac{\alpha_w}{2\pi}\, R_f\, T_z^f\, \frac{m_{\widetilde{W}}\,\mathrm{Im}\,\mu}{M^2_{\tilde{\chi}_1^+} - M^2_{\tilde{\chi}_2^+}}\, \frac{m_f}{M^2_{\tilde{f}'}}\, \Big[J\Big(\frac{m^2_{\tilde{\chi}_2^+}}{M^2_{\tilde{f}'}}\Big) - J\Big(\frac{m^2_{\tilde{\chi}_1^+}}{M^2_{\tilde{f}'}}\Big)\Big]\,, \qquad (1)$$

where $\alpha_w = g_w^2/(4\pi)$ is the $\mathrm{SU}(2)_L$ fine-structure constant, $R_f = \cot\beta$ $(\tan\beta)$ for $T_z^f = 1/2$ $(-1/2)$, $\tilde{f}'$ denotes a scalar fermion having opposite weak isospin to $\tilde{f}$, *i.e.*, $T_z^f = -T_z^{f'}$, and $J(z) = [3/2 - z/2 + \ln z/(1-z)]/(1-z)^2$ is a one-loop function [10], with $J(1) = -1/3$, $J(z \ll 1) = (3 + 2\ln z)/2$ and $J(z \gg 1) = -1/(2z)$. Correspondingly, the gluino contribution to the EDM of a quark q is given by

$$\Big(\frac{d_q}{e}\Big)^{\tilde{g}} = \frac{2\alpha_s}{3\pi}\, Q_q\, \frac{\mathrm{Im}(A_q + R_q\mu^*)}{M_{\tilde{q}}}\, \frac{m_q}{M^2_{\tilde{q}}}\, \frac{m_{\tilde{g}}}{M_{\tilde{q}}}\, K\Big(\frac{m^2_{\tilde{g}}}{M^2_{\tilde{q}}}\Big)\,, \qquad (2)$$

where $\alpha_s = g_s^2/(4\pi)$ is the strong fine-structure constant, Q_q is the charge of (scalar) quarks in $|e|$ units ($Q_u = 2/3$, $Q_d = -1/3$), and $K(z) = -[1/2 + 5z/2 + z(2+z)\ln z/(1-z)]/(1-z)^3$ [10]; $K(1) = -1/12$; $K(z \ll 1) = -1/2$; $K(z \gg 1) = (5/2 - \ln z)/z^2$. Recently, one-loop chromo-EDM (CEDM) contributions have been studied in [12] and found to be comparable to the EDM ones with related dependence. A fairly good estimate of the combined effect of all one-loop contributions to the electron and u-, d- quark EDMs, including neutralino quantum corrections, may generically be obtained by

$$\Big(\frac{d_f}{e}\Big)^{1-\mathrm{loop}} \sim 10^{-25}\ \mathrm{cm} \times \frac{\{\mathrm{Im}\,\mu,\ \mathrm{Im}\,A_f\}}{\max(M_{\tilde{f}}, m_\lambda)}\, \Big(\frac{1\ \mathrm{TeV}}{\max(M_{\tilde{f}}, m_\lambda)}\Big)^2 \Big(\frac{m_f}{10\ \mathrm{MeV}}\Big)\,. \qquad (3)$$

From Eq. (3) one finds the known result [10] that large CP-violating phases are only possible if scalar quarks of the first two families or gauginos have few TeV masses. Another interesting and, perhaps more natural, way to suppress the one-loop EDM contributions having most of the SUSY particles much below the TeV scale is to require $\arg(\mu) \lesssim 10^{-2}$, a constraint also favoured by cosmological considerations [11], and assume an hierarchic pattern for A_f's: $A_e\,, A_{u,c}\,, A_{d,s} \lesssim 10^{-3}\,\mu$, whereas A_t and A_b are the only large trilinear couplings in the theory with CP-violating phases of order unity.

At two loops, neutron and electron EDMs receive a contribution from the W-boson EDM through a one-loop graph mediating charginos and neutralinos [13]. However, these effects were found to be at least one order of magnitude smaller than the present experimental bounds if μ, $\tilde{m}_2 \sim 100$ GeV, and become more than two-orders suppressed if μ, $\tilde{m}_2 > 300$ GeV. The results are independent of the scalar fermion masses but depend linearly on $\arg\mu$, and therefore, the suppression is substantial if $\arg(\mu) \sim 10^{-2}$. As we will discuss below, apart from the two-loop small effects, there are sizeable contributions to EDMs due to two-loop Barr-Zee-type graphs. Finally, there is a significant three-loop contribution to neutron EDM through Weinberg's three-gluon operator [16,14]. Recent studies have shown [12] that these effects scaling as $1/m^3_{\tilde{g}}$ are well below the experimental neutron EDM bound if gluinos are heavier than about 400 GeV.

To demonstrate explicitly the significance of the new Barr-Zee-type contribution displayed in Fig. 1, we shall adopt a SUSY scenario in which the only large CP-violating phase is contained in $A_\tau = A_t = A_b$. As has been discussed above, such a scenario is also compatible with present experimental upper bounds on EDMs. In this model, CP violation is induced by the interaction Lagrangian having the generic form

$$\mathcal{L}_{\rm CP} \;=\; -\,\xi_f v\, a\,(\tilde{f}^*_1\tilde{f}_1\;-\;\tilde{f}^*_2\tilde{f}_2)\;+\;\frac{ig_w m_f}{2M_W}\,R_f\,a\,\bar{f}\gamma_5 f\,, \qquad (4)$$

where a is the would-be CP-odd Higgs boson, $M_W = \frac{1}{2}g_w v$ is the W-boson mass, and $\tilde{f}_1$, $\tilde{f}_2$ are the two mass-eigenstates of the left-handed and the right-handed scalar fermions of the

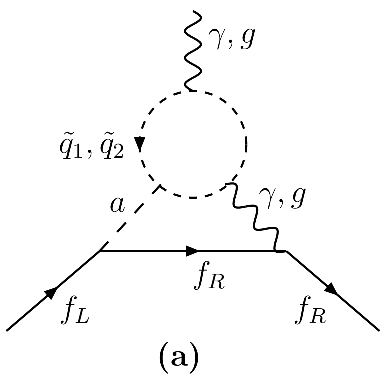

Figure 1: Two-loop contribution to EDM and CEDM in supersymmetric theories (mirror graphs are not displayed.)

third family. Moreover, ξ_f is a CP-violating parameter depending on the supersymmetric model under consideration. From all scalar fermions, only $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ are Yukawa-coupling enhanced, and are therefore expected to give the biggest contributions. Another important point is that CP violation induced by the Lagrangian (4) is proportional to the mass difference of scalar quarks, $m_{\tilde{f}_1} - m_{\tilde{f}_2}$, and is big for the states $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ as their respective mass splitting may naturally be maximal. In the MSSM, the quantities ξ_t and ξ_b are given by [6]

$$\xi_t \;=\; \frac{\sin 2\theta_t m_t \mathrm{Im}(\mu e^{i\delta_t})}{\sin^2\beta \; v^2}\,, \qquad \xi_b \;=\; \frac{\sin 2\theta_b m_b \mathrm{Im}(A_b e^{i\delta_b})}{\sin\beta\ \cos\beta\, v^2}\,, \tag{5}$$

where $\delta_q = \arg(A_q + R_q \mu^*)$.

We shall now consider the $\tilde{t}$- and $\tilde{b}$-mediated two-loop graphs shown in Fig. 1 which may give rise to an EDM and a CEDM for a light fermion. There is in principle an analogous contribution to EDM due to a chargino loop. Such a contribution originating from the CP-violating part of the coupling $a\chi_i^+\chi_i^-$ $(i = 1, 2)$ is proportional to $\arg\mu$, and hence small in our model. We also neglect Barr-Zee-type graphs where the photon is replaced by a Z boson, as the vectorial part of the Z-boson-mediated interaction is suppressed relative to the photonic one by a factor $(1-4\sin^2\theta_w)/4 \approx 2.4\ 10^{-2}$ with $\cos\theta_w = M_W/M_Z$ for the electron case, and roughly 1/4 for the u and d quarks. Under these assumptions, a straightforward calculation of the EDM of a light fermion f induced by Figs. 1(a) and 1(b) at the electroweak scale yields

$$\left(\frac{d_f}{e}\right)^{\tilde{q}}_{\rm EW} = Q_f\,\frac{3\alpha_{\rm em}}{64\pi^3}\,\frac{R_f\,m_f}{M_a^2}\sum_{q=t,b}\xi_q\,Q_q^2\left[F\left(\frac{M^2_{\tilde{q}_1}}{M_a^2}\right)-F\left(\frac{M^2_{\tilde{q}_2}}{M_a^2}\right)\right], \tag{6}$$

where $\alpha_{\rm em} = e^2/(4\pi)$ is the electromagnetic fine structure constant, the subscripts EW indicate that all kinematic parameters must be evaluated at the electroweak scale M_Z, and $F(z)$ is a two-loop function given by

$$\begin{aligned} F(z) &= \int_0^1 dx\,\frac{x(1-x)}{z-x(1-x)}\;\ln\Big[\frac{x(1-x)}{z}\Big] \\ &= 2+\ln z+\frac{z}{x_+-x_-}\Big\{\Big[\ln\Big(-\frac{x_+}{x_-}\Big)-\ln\Big(-\frac{x_-}{x_+}\Big)\Big]\ln z+2{\rm Li}_2\Big(\frac{1}{x_+}\Big)-2{\rm Li}_2\Big(\frac{1}{x_-}\Big)\Big\}, \end{aligned} \tag{7}$$

with $x_\pm = \frac{1}{2}(1\pm\sqrt{1-4z})$ and the dilogarithmic function defined as ${\rm Li}_2(z) = \int_0^1 dt\,\ln t/[t-(1/z)]$; $F(z\ll 1) = \ln z+2$; $F(z\gg 1) = -\frac{1}{6}(\ln z+\frac{5}{3})/z$. In addition, the CEDM at the electroweak scale reads

$$\left(\frac{d^C_f}{g_s}\right)^{\tilde{q}}_{\rm EW} = \frac{\alpha_s}{128\pi^3}\,\frac{R_f\,m_f}{M_a^2}\sum_{q=t,b}\xi_q\left[F\left(\frac{M^2_{\tilde{q}_1}}{M_a^2}\right)-F\left(\frac{M^2_{\tilde{q}_2}}{M_a^2}\right)\right]. \tag{8}$$

The neutron EDM d_n induced by d_u and d_d may be estimated in the valence quark model at the hadronic scale Λ including QCD renormalization effects [8] through the expression

$$\frac{d_n}{e} = \left(\frac{g_s(M_Z)}{g_s(m_b)}\right)^{\frac{32}{23}}\left(\frac{g_s(m_b)}{g_s(m_c)}\right)^{\frac{32}{25}}\left(\frac{g_s(m_c)}{g_s(\Lambda)}\right)^{\frac{32}{27}}\left[\frac{4}{3}\left(\frac{d_d}{e}\right)_\Lambda-\frac{1}{3}\left(\frac{d_u}{e}\right)_\Lambda\right]. \tag{9}$$

Here anomalous dimension factors are explicitly separated out from quantities $(d_d/e)_\Lambda$ and $(d_u/e)_\Lambda$ which are simply given by Eq. (6) with the running couplings and the running masses of u- and d-quarks evaluated at the low-energy hadronic scale Λ. For definiteness, we take $m_u(\Lambda) = 7$ MeV, $m_d(\Lambda) = 10$ MeV, $\alpha_s(M_Z) = 0.12$, and $g_s(\Lambda)/(4\pi) = 1/\sqrt{6}$ [16]. By analogy, the light-quark CEDMs d^C_u and d^C_d give rise to a neutron EDM

$$\frac{d^C_n}{e} = \left(\frac{g_s(M_Z)}{g_s(m_b)}\right)^{\frac{28}{23}}\left(\frac{g_s(m_b)}{g_s(m_c)}\right)^{\frac{28}{25}}\left(\frac{g_s(m_c)}{g_s(\Lambda)}\right)^{\frac{28}{27}}\left(\frac{g_s(M_Z)}{g_s(\Lambda)}\right)^2\left[\frac{4}{9}\left(\frac{d^C_d}{g_s}\right)_\Lambda+\frac{2}{9}\left(\frac{d^C_u}{g_s}\right)_\Lambda\right], \tag{10}$$

where quantities in the last bracket are given by Eq. (8) with the strong coupling constant g_s and the u- and d-quark masses evaluated at the scale Λ.

In Fig. 2 we show the dependence of the EDMs d_e (solid line), d_n^C (dashed line), and d_n (dotted line) on $\tan\beta$ and μ, for three different masses of the would-be CP-odd Higgs boson a, $M_a = 100, 300, 500$ GeV. Since EDMs are mostly dominated by the down-family fermions, i.e., the electron and the down quark, the strong $\tan\beta$ dependence is then expected. The EDMs depend significantly on μ through the $a\tilde{f}^*\tilde{f}$ coupling in Eq. (4). Our analysis in Fig. 2 clearly shows that large $\tan\beta$ scenarios, *i.e.*, $40 < \tan\beta < 60$ with μ, $A > 0.5$ TeV, $M_a \leq 0.5$ TeV, and large CP phases are practically ruled out. On the other hand, we find that for low $\tan\beta$ scenarios, *e.g.* $\tan\beta \lesssim 20$, the two-loop Barr-Zee-type contribution to EDMs is less restrictive for natural values of parameters in the MSSM. Finally, the results exhibit an approximate linear dependence on the mass of the a boson for the range of phenomenological interest and validity of perturbation theory, *i.e.*, for $0.1 < M_a \lesssim 1$ TeV.

In conclusion, we have derived for the first time *direct* limits on the CP-violating parameters related to the third generation scalar-quarks, which originate from the experimental limits on the EDMs of electron and neutron. These novel constraints induced by the SUSY version of the two-loop Barr-Zee graph will have an important impact on possible effects of CP nonconservation at collider experiments, on dark-matter detection rates, and on studies of the baryonic asymmetry of the Universe in SUSY theories.

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There is a normalization factor of 2 missing in the analytic expressions of Eqs. (6) and (8). To be precise, both Eqs. (6) and (8) must be multiplied by a factor of 2. As a result, the numerical predictions for the EDMs are by a factor 2 larger than those plotted in Fig. 2. Finally, there is a typographic error in Eq. (5): phase δ_b should be replaced by $-\delta_b$.

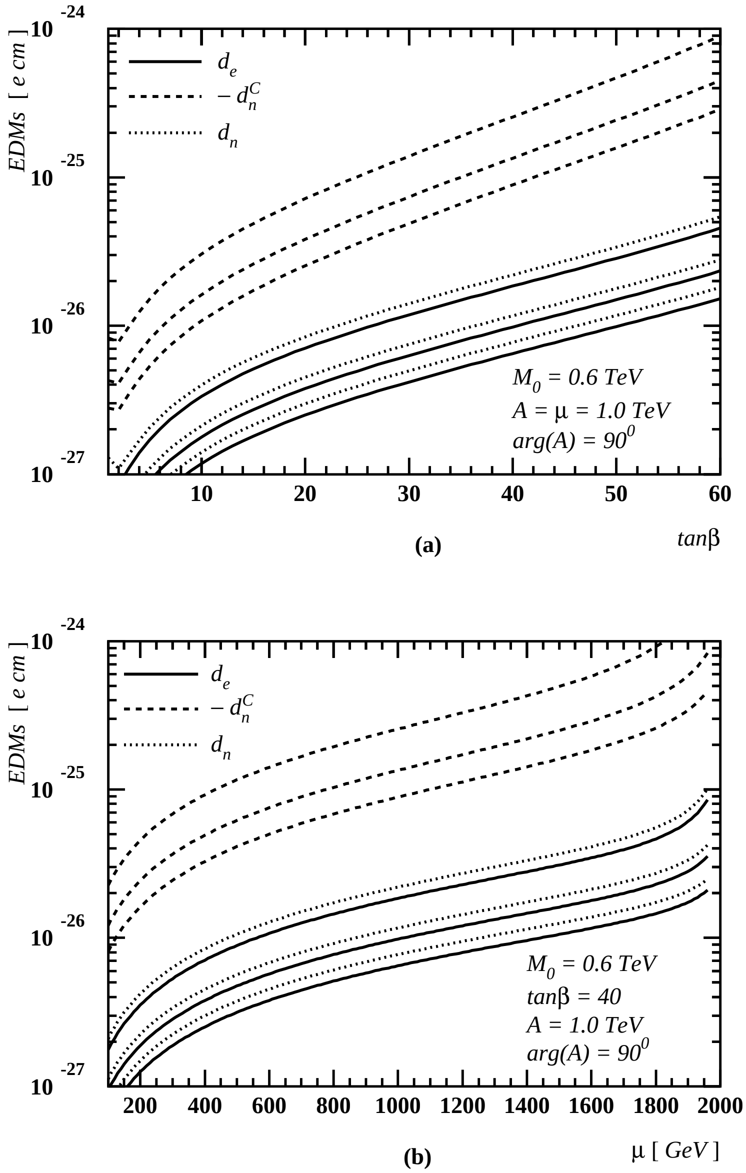

Figure 2: Numerical estimates of EDMs. Lines of the same type from the upper to the lower one correspond to $M_a = 100,\ 300,\ 500$ GeV, respectively.