Time-Dependent Vacuum Energy Induced by D-Particle Recoil

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Abstract

We consider cosmology in the framework of a 'material reference system' of D particles, including the effects of quantum recoil induced by closed-string probe particles. We find a time-dependent contribution to the cosmological vacuum energy, which relaxes to zero as $\sim 1/t^2$ for large times t. If this energy density is dominant, the Universe expands with a scale factor $R(t) \sim t^2$. We show that this possibility is compatible with recent observational constraints from high-redshift supernovae, and may also respect other phenomenological bounds on time variation in the vacuum energy imposed by early cosmology.

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1 Introduction

The possibility that the vacuum, the lowest-energy state, might not actually have zero energy was first raised by Einstein [1], who regarded this proposal as his greatest mistake. Various possible contributions to this vacuum energy are known in field theory, including contributions associated with condensates in QCD and the electroweak theory that are many orders of magnitude larger than the possible physical value of the cosmological constant today, and quantum contributions that are formally highly divergent. The existences of these possible contributions to the vacuum energy mean that the issue of a possible cosmological constant cannot be avoided [2, 3], although this possibility can only be addressed theoretically in a complete quantum theory of gravity.

There have been attempts to address the issue of the cosmological constant in various approaches to quantum gravity [3]. The cogency of many of these approaches was limited by the presence of unrenormalizable quantum divergences, but some interesting ideas emerged, including the suggestion that the Universe might be relaxing towards a zero-energy state during the course of its cosmological expansion [4]. String theory is the only known candidate for a completely consistent theoretical framework for quantum gravity, and has offered several new insights into the cosmological-constant problem. For example, the cosmological constant vanishes in a supersymmetric string theory, ideas have been proposed for concealing supersymmetry in the observable world [5], and suggestions have been made how the cosmological constant might vanish even in the absence of supersymmetry [6].

The issue of possible vacuum energy has been cast in a new light by recent astrophysical observations suggesting that it might indeed be non-zero. The theory of cosmological inflation strongly suggests that the current density of the Universe is close to the critical value: $\Omega_{TOT} = 1$, and this is supported by the location of the first acoustic peak, whose existence is hinted at by data on fluctuations in the cosmic microwave background radiation [7]. On the other hand, the matter density inferred from data on large-scale structures [8] in the Universe does not rise above $\Omega_M \sim 0.3$. This includes the baryonic density, which is believed on the basis of cosmological nucleosynthesis arguments to be much smaller: $\Omega_B < 0.1$. Most of the matter density is thought to consist of cold dark matter, but this is not sufficient by itself to explain all the data on microwave background fluctuations and large-scale structure formation [9]. Hot dark matter was for some time a favoured epicycle for cold dark matter, but this would also need to be included within the $\Omega_M \sim 0.3$ inferred from observations of large-scale structure. Moreover, the recent data on atmospheric and solar neutrino oscillations do not suggest a neutrino mass large enough to contribute significantly to Ω_M [10]. Taken together, these arguments reopen the possibility of a cosmological constant Λ: $\Omega_\Lambda \sim 0.7$.

This possibility has recently received dramatic support from an unexpected source, namely observations of high-redshift supernovae [11]. These indicate that the large-scale geometry of the Universe is not that of a critical matter-dominated cosmology, and that its expansion may even be accelerating. The supernova data

are consistent with $\Omega_\Lambda \sim 0.7$, if the Universe is indeed close to critical as suggested by inflation. It should be stressed that these observations are entirely independent of the earlier arguments given in the previous paragraph, encouraging us to take seriously the possibility that the vacuum energy density may be non-zero.

This presents theoretical physics with a tremendous opportunity: a number to be calculated within one's quantum theory of gravity, that can be confronted with measurement. Many of the previous discussions of the cosmological constant had included attempts to show why it vanishes. Maybe it does not? The known exact symmetries are not adequate to derive $\Lambda = 0$, and it may be small because of some approximate symmetry: for example, the value allowed by the supernova data corresponds to $\Lambda \sim (M_W/M_P)^8$ in natural units. Alternatively, perhaps the vacuum energy is relaxing towards zero [4]: for example, the present age of the Universe $t_0 \sim 10^{60}$ in natural units, so perhaps the vacuum energy is decreasing as $\Lambda \sim (1/t_0)^2$? We present just such a scenario in this paper.

Our starting-point is the expectation that the vacuum contains Planck-scale quantum fluctuations in topology, on the Planck time scale and with Planckian density. Working in the context of string theory, in which this space-time foam may be described [12, 13] using D-brane technology [14], we are not in a position to calculate absolutely the limiting value of the vacuum energy density. However, we are able to isolate a contribution to the vacuum energy that decreases like $(1/t)^2$, providing a mechanism for relaxation towards a limiting value that may well vanish. This contribution is due to the quantum recoil of D branes in the space-time foam, which exhibit energy excitations that are interpreted classically as a time-dependent energy density. We show that the time-dependence we find is compatible with the constraints imposed by the high-redshift supernova data [11], as well as with the values of Λ allowed earlier in the history of the Universe.

2 Material Reference Frame of D Particles, Recoil and Anti-de-Sitter Space

The use of a material reference system (MRS) in General Relativity has a long history. First conceived by Einstein [1] and Hilbert [15] in the form of a system of rods and clocks, MRS have been subsequently used as a general tool to specify events in space time and to address conceptual questions in General Relativity and later in Quantum Gravity [16], particularly in connection with the implications of the uncertainty principle for measurements of the gravitational field. In this latter respect, we mention arguments [17] that the quantum properties of the bodies that form a MRS are responsible for making physical operators in Quantum Gravity well defined.

A useful example of a MRS is that of a relativistic elastic medium considered by DeWitt [16]. Its action is :

$$S_{mrs}[x^a; g_{ab}] = \int d\sigma \int_{S^{(3)}} d^3\zeta \{-(nM + w)\sqrt{-\dot{x}^a \dot{x}^b g_{ab}(x)}\} \tag{1}$$

where $S^{(3)}$ is the 'matter' spatial manifold, whose points $\zeta \in S^{(3)}$ label the particle world lines, the variables x^a denote the coordinates of a relativistic particle probe of mass M moving in the MRS, which, together with the background metric g_{ab}, are considered functions of σ and ζ^i. The quantity n denotes the particle-number density, whilst w is the interaction-energy density in the comoving frame. The system described by (1) is reparametrization invariant, i.e., it is invariant under the infinitesimal transformations $\delta x^a = -\epsilon \dot{x}^a$ induced by reparametrizations $\sigma \to \epsilon(\sigma, \zeta)$ of the particle world lines.

The above example is a prototype for our case, where we consider an ensemble of Dirichlet D branes [14] as a MRS through which closed-string matter propagates. We assume the existence of a suitable conformal closed-string theory in $D = 10$ or 11 dimensions [1] that admits D-brane solutions. These solitonic objects are located at fixed points in target space, and hence are suitable for defining a MRS.

We now consider a configuration combining a closed-string state (matter) and a D particle, which induces a recoil distortion of the D brane describable within a conformal field theory setting as in [19]. The recoil is best described by the splitting of the closed-string matter state into two open-string states with their ends attached on the D brane. In the world-sheet formalism, the recoil is described [19] by a suitable pair of logarithmic operators [20], corresponding to the collective coordinates y^i and velocities u_i of the recoiling D particles. Such a scattering procedure constitutes a generalization of the Heisenberg microscope approach, where the rôle of Heisenberg's photon is played by the closed-string state, whilst the system of D branes plays the rôle of the detector (or measuring apparatus).

As already mentioned, we concentrate on a single scattering event, namely the scattering of a single closed-string state by a single defect. We are unable at present to treat fully the more realistic case of an ensemble of defects with Planckian density, due to our limited understanding of the underlying microscopic dynamics. Instead, we interpret the single-scattering approach as the first step in a dilute-gas approximation for the D particles, which should be sufficient to describe qualitatively the leading behaviour of the vacuum energy of the Universe.

The combined system is characterized by a homotopic 'evolution' parameter $\mathcal{T}$. We look for a consistent description of the coupled system in a maximally-symmetric background space, which includes the pair of logarithmic deformations that correspond to the D-dimensional location y_i of the recoiling D brane and the homotopic 'velocity' $u_i \equiv \partial_\mathcal{T} y_i$ [19, 21]. These two operators are slightly relevant [19], in a world-sheet renormalization-group sense, with anomalous dimensions $\Delta = -\epsilon^2/2$ where $\epsilon \to 0^+$ is a regularization parameter. This is independent of the homotopic 'velocity' u_i, but is related [19] to the world-sheet size L and a world-sheet short-distance cut-off a via

$$\epsilon^{-2} \sim \eta \ln(L/a)^2, \tag{2}$$

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[1] An eleven-dimensional manifold arises naturally when one incorporates world-sheet defects [18]. For our purposes in this paper, the initial dimension of the string theory is not relevant, as long as it is at least ten.

where $\eta = \pm 1$ for a Euclidean- (Minkowski-)signature homotopic parameter $\mathcal{T}$. Thus, the recoiling D brane is no longer described by a conformal theory on the world sheet, despite the fact that the theory was conformally invariant before the encounter that induced the recoil.

To restore conformal invariance, we invoke Liouville dressing [22] by a mode φ that can be identified [12, 13, 23] with a *time-like* homotopic variable $\mathcal{T}$. This Liouville field restores conformal invariance in an initially critical string theory. The dressing by such a time-like Liouville mode $\varphi \equiv \mathcal{T}$ leads to an effective curved space-time manifold in $D+1$ dimensions. We find a consistent solution of the world-sheet σ-model equations of motion which is described [13] by a metric of the form:

$$G_{00} = -1 \,, \; G_{ij} = \delta_{ij} \,, \; G_{0i} = G_{i0} = f_i(y_i, \mathcal{T}) = \epsilon(\epsilon y_i + u_i \mathcal{T}) \,, \; i,j = 1, ..., D \quad (3)$$

We restrict ourselves to the case where the recoil velocity $u_i \to 0$, as occurs if the D brane is very heavy. This is formally justified in the weak-coupling limit for the string, since the D-brane mass $M \propto 1/g_s$, where $g_s \to 0$ is the string coupling. From the world-sheet point of view [24, 18], such a very heavy D brane corresponds to a strongly-coupled defect, since the coupling g_v of the world-sheet defect is related to the string coupling g_s by

$$g_v \propto \frac{1}{\sqrt{g_s}} \quad (4)$$

This is a manifestation of world-sheet/target-space strong/weak-coupling duality.

In the limit $u_i \to 0$, the only non-vanishing components of the D-dimensional Ricci tensor are [13]:

$$R_{ii} \simeq \frac{-(D-1)/|\epsilon|^4}{(\frac{1}{|\epsilon|^4} - \sum_{k=1}^{D} |y_i|^2)^2} + \mathcal{O}(\epsilon^8) \quad (5)$$

where we have taken (2) into account, for the appropriate Minkowskian signature of the Liouville mode $\mathcal{T}$. In this limiting case, the Liouville mode decouples when $\mathcal{T} >> 0$, and one is effectively left with a maximally-symmetric D-dimensional manifold. Hence, we may write (5) as

$$R_{ij} = \frac{1}{D}\mathcal{G}_{ij}R \quad (6)$$

where $\mathcal{G}_{ij}$ is a diagonal metric corresponding to the line element:

$$ds^2 = \frac{|\epsilon|^{-8} \sum_{i=1}^{D} dy_i^2}{(\frac{1}{|\epsilon|^4} - \sum_{i=1}^{D} |y_i|^2)^2} \quad (7)$$

This metric describes the interior of a D-dimensional ball, which is the Euclideanized version of an anti-de-Sitter (AdS) space time. In its Minkowski version, one can easily check that the curvature corresponding to (7) is

$$R = -4D(D-1)|\epsilon|^4, \quad (8)$$

which is *constant* and *negative*. The radius of the AdS space is $b = |\epsilon|^{-2}$.

The Ricci tensor (5) corresponds to the low-energy: $\mathcal{O}(\alpha')$, $\alpha' << 1$ equation of motion for a world-sheet σ model, as obtained from the vanishing of the β function in this background. The Ricci tensor (5) cannot be a consistent string background compatible with conformal invariance to order α' if only tree-level world-sheet topologies are taken into account. However, as shown in [25], this conclusion no longer holds when one includes string-loop corrections. These induce a target-space cosmological constant, corresponding to a dilaton tadpole, which renders the backgrounds (5) consistent with the conformal-invariance conditions.

Alternatively, as discussed in [13], the cosmological vacuum energy may be considered as being obtained from an effective tree-level non-critical Liouville string with central-charge deficit

$$Q^2 = \Lambda \propto -2(\alpha')^2(D-1)(D-2)|\epsilon|^4 + \mathcal{O}(\epsilon^6) \tag{9}$$

As we argue in the next section, this leads to a non-trivial time-dependent vacuum energy when we identify ϵ^2 with a temporal evolution variable, after appropriate analytic continuation to imaginary values. The analytic continuation restores positivity of the deficit Q^2, as is appropriate for supercritical string models [26].

3   Interpretation as Physical Vacuum Energy

In order to discuss the physical interpretation of the above analysis, we consider the D-dimensional components G_{ij} of the σ-model metric (7) to be purely *spatial*. In that case we may identify the analytically continued $i\epsilon^2$ as a Liouville 'physical' time t,

$$i\epsilon^{-2} \to t \tag{10}$$

By construction [22], the resulting Universe is of Friedmann-Robertson-Walker (FRW) type, since the σ-model kinetic terms for the Liouville field ϕ are of the form $\int d^2\sigma(-\partial\phi\overline{\partial}\phi)$, corresponding to a time-like component of the metric of the form:

$$G_{00} = -1 \tag{11}$$

This Minkowskian signature is a consequence of the fact that the original string was *supercritical* [26].

The spatial part of the σ-model metric (7) may then be written in the form:

$$G_{ij} = e^{-\ln(t^2+|y_i|^2)}t^4\delta_{ij} \ , i,j = 1,\ldots D \text{ space} - \text{like} \tag{12}$$

There is a unique way in which this metric can become a solution of standard Einstein's equation in a $D+1$ Universe, with time (10) and time-like metric component (11). One should redefine the spatial part of the metric by:

$$G_{00}^{ph} = G_{00} \ , \qquad G_{ij}^{ph} = e^{-\Phi(y_i,t)}G_{ij} = t^4\delta_{ij}, \tag{13}$$

with

$$\Phi(y_i, t) \equiv -\ln(t^2 + |y_i|^2) \tag{14}$$

where $\Phi(y_i, t)$ may be regarded as a dilaton contribution. For the purposes of the present work, we assume that such a dilaton configuration is consistent with the world-sheet conformal invariance of the Liouville-dressed σ model. At present an explicit check of this is beyond our control.

The consistency of the resulting metric $G^{ph}_{\mu\nu}$, $\mu, \nu = 1, \ldots D + 1$, with Einstein's equations has non-trivial consequences. Using (13), we seee that the physical Universe is of FRW type with a scale factor

$$R(t) = t^2 \tag{15}$$

This can be contrasted with the tree-level cosmological model of [26], where a linear expansion was found as $t \to \infty$. We see from (9) and (10) that the Universe (13, 15) has a time-dependent vacuum energy $\Lambda(t)$ which relaxes to zero as:

$$\Lambda(t) = \Lambda(0)/t^2 = 1/R(t) \tag{16}$$

In accordance to the standard Einstein's equation, this time-varying *positive* vacuum energy drives the cosmic expansion:

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{1}{3}\Lambda(t) \tag{17}$$

where the dot denotes a derivative with respect to the physical (Einstein) time t. From the point of view of the stringy σ model, this result should be interpeted as meaning that the dilaton configuration and the rest of stringy matter act together in such a way that the conformal invariance conditions are satisfied, and the contribution of other of the fields does not alter the low-energy Einstein dynamics at late stages of the evolution of the Universe, the only remnant of string matter being the time-dependent vacuum energy.

4   Comparison with Observations

In this section we compare the above result (15),(16), with observational constraints on the cosmological constant. As was already mentioned in the introduction, data on large-scale structure formation [8] favour the existence of some form of vacuum energy, as well as conventional matter. However, these data do not discriminate between a time-dependent contribution to the vacuum energy, as derived in the previous section, and a true cosmological constant. Some such discrimination is provided by recent studies of high-redshift supernovae [11]. These measure the evolving geometry of the Universe over most of its history, and hence constrain the cosmic equation of state from the era corresponding to redshift $z \simeq 1$ to the present. The question arises, therefore, whether these observations may distinguish

in principle or in practice between a true cosmological constant and the variety of time-dependent vacuum energy derived above within our Liouville approach to D brane recoil [12, 13].

We first review briefly the parametrization of [11], which is used in their analysis. The experimentally measured quantities are redshifts z defined as $\lambda/\lambda_0 \equiv 1/(1+z) = R/R_0$ (where λ denotes wavelength and R scale factor, with the subscript 0 denoting quantities at the present epoch), angular diameter distances $d_A = D/\theta$ (for astrophysical objects of proper sizes D that are assumed to be known), proper motion distances $d_M = u/\dot{\theta}$ (where u is a transverse proper velocity and $\dot{\theta}$ an apparent angular motion), and luminosities d_L. There is a relation [2] between these observables that is model-independent:

$$d_L = (1 + z)d_M = (1 + z)^2 d_A \qquad (18)$$

which allows one to make a fit with only two of these quantities, conveniently chosen to be the redshift z and the luminosity d_L. Using Einstein's equations in a FRW Universe, the luminosities can be related [2] to the energy densities Ω_X for different material components X:

$$d_L = \frac{c(1+z)}{H_0\sqrt{\Omega_k}}\mathrm{sinn}\{\sqrt{\Omega_k}\int_0^z dz'[\sum_i \Omega_i(1+z')^{3(1+\alpha_i)} + \Omega_k(1+z')^2]^{-1/2}\} \qquad (19)$$

where the Ω_i denote the normalized energy densities of the various energy components, excluding the one corresponding to the spatial curvature, and $\Omega_k = 1 - \sum_i \Omega_i$ denotes the effects of the spatial curvature of the FRW Universe. The function $\mathrm{sinn}(x)$ is defined by

$$
\begin{aligned}
\mathrm{sinn}(x) &= \sinh(x) \text{ for } \Omega_k > 0 \\
&= x \text{ for } \Omega_k = 0 \\
&= \sin(x) \text{ for } \Omega_k < 0
\end{aligned}
\qquad (20)
$$

and the scaling exponents α_i are defined in terms of the pressure P_i. Specifically, for an energy component ρ_X which scales like:

$$\rho_X \sim R^{-n} \qquad ; \qquad n = 3(1 + \alpha_X) \qquad (21)$$

where R is the cosmic scale factor in a FRW Universe. The analysis is based on an equation of state, derived from Einstein's equations, which defines α_X in terms of the pressure P_X:

$$\alpha_X = P_X/\rho_X \qquad (22)$$

In the case of ordinary matter without a cosmological constant, $\alpha_{X=m} = 0$, since the energy density of ordinary matter scales with the inverse of the spatial volume of the Universe. On the other hand, in the case of a true cosmological constant that does not vary with time, the constancy of the corresponding component of the energy density as the universe expands corresponds to $\alpha_{X=\Lambda} = -1$.

The observational analysis of [11] constrained the cosmological equation of state of any unknown energy component $\Omega_{X \neq m}$ that may contribute to the expansion of the Universe:

$$\alpha_X < -0.55 \quad \text{for any value of } \Omega_{\mathrm{m}}$$
$$\alpha_X < -0.60 \quad \text{for } \Omega_{\mathrm{m}} \geq 0.1$$

$$(23)$$

The scaling of the vacuum energy density given in (16), which is inversely proportional to the scale factor, implies in the parametrization (21) of [11]:

$$\alpha_\Lambda = -2/3 \tag{24}$$

which is *consistent* with the observational high-redshift supernova constraint (23) [2]. It is encouraging that the time dependence we find is close to the range already excluded by the supernova observations. This suggests that it may soon be possible to exclude our speculative proposal.

A vacuum energy that relaxes to zero according to a general power law

$$\Lambda = \Lambda_0/t^\lambda \tag{25}$$

is restricted by several phenomenological constraints. Here we review some relevant considerations, with particular emphasis on the specific features that are most relevant to the recoil model described above. We emphasize that our calculation is not a complete one, and the contribution whose functional form we have discussed above may not be the only contribution to the vacuum energy, and may not even be the dominant one. However, for the purposes of this discussion we assume that the recoil contribution is indeed dominant.

Being inspired by the superstring approach, which underlies our D-brane analysis, we focus on theories which reduce to supergravity at large distances. If supersymmetry were unbroken, the vacuum energy would be zero, and one would expect a zero cosmological constant. However, in all physically relevant theories, supersymmetry is broken in the observable sector, so a non-zero vacuum energy is to be expected. In generic supergravity models, one has a maximum value

$$\Lambda \sim M_W^2 M_P^2 \tag{26}$$

where $M_W \sim 100$ GeV represents the electroweak scale. We consider this maximal Λ_0 as a possible initial value at small t before the relaxation mechanism kicks in. We further assume that supersymmetry breaking occurs at a characteristic temperature

$$T_a \sim \sqrt{M_W M_P} \tag{27}$$

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[2] We also note that this is consistent with the null energy condition [27], which requires $\rho_X + P_X \geq 0$ and hence $\alpha_X \geq -1$.

Alternatively, in certain no-scale models [28] one has that

$$\Lambda_0 \sim M_W^4 \tag{28}$$

and the temperature at which supersymmetry breaking occurs is

$$T_b \sim M_W \tag{29}$$

Thus we consider two possible sets of initial conditions for the relaxation of the vacuum energy: either (26, 27) or (28, 29).

The constraints coming from early cosmology are easily satisfied if one assumes that the matter energy density dominates over the vacuum energy density

$$G_N \rho_m \geq G_N \Lambda_0 / t^\lambda \tag{30}$$

We first analyse the constraint (30) in case of generic supergravity models (26, 27). We assume that the matter energy density scales with temperature as T^4 at early epochs, and hence that $t \sim T^{-2}$ in the Einstein frame, in natural units. From this and (30, 25) we find

$$T^{2-\lambda} M_P^\lambda \geq M_W M_P \tag{31}$$

It is clear that if we had $\lambda = 1$ we would need $T \geq M_W$ for the temperature in late Universe, which is clearly unacceptable. Fortunately, this is not the relaxation rate we found above, which was $\lambda = 2$. For this case, the inequality (31) is always respected.

In the case of no-scale models [28], the constraint (30) leads to

$$T^{2-\lambda} M_P^\lambda \geq M_W^2 \tag{32}$$

The case $\lambda = 1$ leads to $T \geq 0.1$ K, whilst the case $\lambda = 2$ again always satisfies the constraint (32).

We conclude that our model of a relaxing vacuum energy is compatible with all the relevant observational and phenomenological constraints.

5   Conclusions

We have presented in this paper a heuristic calculation of a contribution to cosmological vacuum energy $\Lambda \sim 1/t^2$. This calculation is incomplete and unsatisfactory in many respects. For example, we are unable to control all other possible string- (or M-) theory contributions to the vacuum energy, and hence cannot be sure that the contribution we have identified here cannot be cancelled or modified by some other effect. Even within our approach, the calculation presented here may well be invalid because our dilute-gas approximation is unjustified or inadequate. Nevertheless, we think that our result has several interesting features.

It exemplifies the possibility that the vacuum energy may be neither zero nor a non-zero constant, but may instead be relaxing towards an asymptotic value.

This calculation reflects the philosophy that the vacuum should be regarded as a dynamical medium in constant interaction with the matter propagating through it, which induces recoil effects that should not be neglected. The energy of quantum space-time foam is increased by this recoil excitation, which vanishes only when the Universe becomes empty at large times.

We leave to future work the tasks of justifying such a calculation more formally, of searching for possible cancelling contributions to the vacuum energy, of determining the possible asymptotic value of the vacuum energy, of going beyond the dilute-gas approximation, incorporating features of realistic string- (M-) theory models such as supersymmetry, etc.. However, we are not discouraged by the fact that this simple-minded calculation produces a result that is not in obvious contradiction with observational data. If nothing else, perhaps our calculation will stimulate attempts to pin down more accurately the equation of state of the vacuum, which may not be trivial.

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