EVALUATION OF THE LEP CENTRE-OF-MASS ENERGY ABOVE THE W-PAIR PRODUCTION THRESHOLD

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Knowledge of the beam energy at LEP2 is of primary importance to set the absolute energy scale for the measurement of the W-boson mass. The beam energy above 80 GeV is derived from continuous measurements of the magnetic bending field by 16 NMR probes situated in a number of the LEP dipoles. The relationship between the fields measured by the probes and the beam energy is calibrated against precise measurements of the average beam energy between 41 and 55 GeV made using the resonant depolarisation technique. The linearity of the relationship is tested by comparing the fields measured by the probes with the total bending field measured by a flux loop. Several further corrections are applied to derive the the centre-of-mass energies at each interaction point. The beam energy has been determined with a precision of 25 MeV for the data taken in 1997, corresponding to a relative precision of 2.7×10^{-4}. Prospects for improvements are outlined.

1 Introduction

The large electron-positron (LEP) collider has been running above the W-pair production threshold since 1996, when the LEP2 programme began. The beam energy at LEP2 is used to set the absolute energy scale for the measurement of the W-boson mass, resulting in a relative uncertainty of $\Delta M_{\mathrm{W}}/M_{\mathrm{W}} \approx \Delta E_{\mathrm{beam}}/E_{\mathrm{beam}}$. The statistical uncertainty on the W mass, $M_{\mathrm{W}} \approx 80.4$ GeV is expected to be around 25 MeV with the full LEP2 data sample, which sets a target for a relative precision of around 10^{-4} for the beam energy to avoid this leading to a significant contribution to the error. For a beam energy of around 90 GeV, the aim is for a beam energy uncertainty at the 10 to 15 MeV level. This contrasts with LEP1, where the Z mass was measured with an uncertainty of about 2 MeV (2×10^{-5} relative), corresponding to beam energy uncertainties of typically 1 MeV.

The centre-of-mass energy is derived by first determining the average beam energy around the ring. In practice this involves normalising the overall energy scale with respect to a precise reference, and in addition tracking time variations of the beam energy. Further corrections are applied to obtain the e^+ and e^- beam energies at the four interaction points, and the centre-of-mass energy in the e^+e^- collisions.

At LEP1, the average beam energy was measured directly at the physics operating energy with a precision of better than 1 MeV by resonant depolarisation (RD)[2]. The spin tune, ν, determined by RD, and the average beam energy, are both proportional to the total integrated vertical magnetic field, B, along the beam trajectory, ℓ:

$$\nu \propto E_{\mathrm{beam}} \propto \oint B \cdot \mathrm{d}\ell. \tag{1}$$

Unfortunately, the RD technique can not be used at LEP2. A sufficient level of transverse polarisation was not achieved above a beam energy of 55 GeV in 1997, due to the sharp increase of depolarising effects with beam energy. Instead, the beam energy is determined from an estimate of the field integral derived from continuous magnetic measurements by 16 NMR probes situated in some of the LEP dipoles. The relation between their readings and the beam energy can be precisely calibrated in the beam energy range 41 to 55 GeV.

This relation is assumed to be linear, and to be valid at physics energies. The linearity can only be tested over a limited range with the RD data themselves. A second comparison of the NMR readings with the field integral, which is proportional to the beam energy, is made using the flux loop. The flux loop experiments provide a measurement of almost the complete field integral (97%). The local bending fields measured by the NMR probes are read out over the full range from RD to physics energies, allowing an independent constraint on the linearity of the relation between the probe fields and the total bending field.

The use of the NMR probes to transport the precise energy scale determined by RD to the physics operating energy is the main novelty of this analysis. The systematic errors on the NMR calibration are evaluated from the reproducibility of different experiments, and the variations from probe to probe. The dominant uncertainty comes from the flux-loop test.

In addition, the other effects that were extensively studied at LEP1 [2] which can modify the beam energy, such as earth tides, leakage currents from trains, *etc.*, must be taken into account when comparing the NMR measurements with the RD beam energies, and in deriving the centre-of-mass energy of collisions as a function of time.

The exact beam energy along the ring differs from the average because of the loss of energy by synchrotron radiation in the arcs and the gain of energy in the RF accelerating sections, which must be taken into account in determining the centre-of-mass energy at each IP. The centre-of-mass energy at each collision point can also be different from the sum of the beam energies due to the interplay of collision offsets and dispersion.

A more detailed version of this contribution can be found in [1].

2 Data samples

2.1 Luminosity delivered by LEP2

LEP has delivered about 10pb^{-1} at each of two centre-of-mass energies, 161 and 172 GeV, in 1996, and over 50pb^{-1} at a centre-of-mass energy of around 183 GeV in 1997. Combining the data from all four LEP experiments, these data give a measurement of the W mass with a precision of about 90 MeV [3]. This paper emphasises the 1997 energy analysis.

2.2 Polarisation measurements

		Beam Energy [GeV]				
Date	Fill	41	44	50	55	Optics
19/08/96	3599			yes		90/60
31/10/96	3702		yes			90/60
03/11/96	3719		yes	yes		90/60
17/08/97	4000		yes			90/60
06/09/97	4121		yes	yes		60/60
30/09/97	4237		yes	yes		60/60
02/10/97	4242	yes	yes	yes	yes	60/60
10/10/97	4274		yes			90/60
11/10/97	4279	yes	yes	yes	yes	60/60
29/10/97	4372	yes	yes			60/60

Table 1: Fills with successful polarisation measurements in 1996 and 1997. Successful measurements are marked "yes".

The successful RD experiments in 1996 and 1997 are listed in table 1. To reduce uncertainties from fill-to-fill variations, an effort was made to measure as many beam energies as possible with RD during the same LEP fill. In 1997, improvments in the orbit quality and reducing depolarising effects resulted in 5 fills with more than one energy point, and 2 fills with 4 energy points, which allow a check of the linearity assumption.

2.3 Magnetic measurements

The LEP dipole fields are monitored in two ways. There are 16 NMR probes positioned inside some selected main bend dipoles. The probes measure the local magnetic field with a precision of around 10^{-6}, and can be read out continually, but each probe only samples the field in a small region of one out of 3200 dipoles. The probes are read out during normal physics running, and also during RD and flux-loop measurements.

In contrast to the NMR probes, the flux loop measures 96.5% of the total bending field of LEP, including 98% of the main bend dipole field.

The flux loop measures the change in field during a dedicated demagnetisation cycle, outside physics running. This possibility to cross-calibrate the field measured by the flux loop and by the NMR probes is crucial to the analysis.

3 The beam energy model

The LEP beam energy is calculated as a function of time according to the following formula:

$$\begin{aligned} E_{\text{beam}}(t) = {} & (E_{\text{initial}} + \Delta E_{\text{dipole}}(t)) \\ & \cdot(1 + C_{\text{tide}}(t)) \cdot (1 + C_{\text{orbit}}) \cdot (1 + C_{\text{RF}}(t)) \\ & \cdot(1 + C_{\text{h.corr.}}(t)) \cdot (1 + C_{\text{QFQD}}(t)). \end{aligned} \tag{2}$$

The first term, E_{initial}, is the energy at the start of the fill derived from the estimate of the integral dipole bending field at the end of the LEP ramp up to operating energy, and $\Delta E_{\text{dipole}}(t)$ is the shift in energy caused by changes in the bending dipole fields during a fill. Both are averages over the energies predicted by each functioning NMR probe. This is a simplification over the treatment at LEP1, since the energy is taken from the average of the measured magnetic fields, rather than a model of the field evolution.

The remaining terms correct for other contributions to the integral bending field. They are discussed in more detail in references [2,1]. The effect of earth tides which move the quadrupole magnets with respect to the fixed length beam orbit is accounted for by $C_{\text{tide}}(t)$. Distortions of the ring geometry on a longer time scale are corrected for by the measured average horizontal orbit displacement, C_{orbit} which is evaluated for each LEP fill. Regular changes in the RF frequency away from the nominal central frequency are made to optimise the luminosity, and are accounted for by $C_{\text{RF}}(t)$. The term $C_{\text{h.corr.}}(t)$ accounts for changes in the beam steering by horizontal orbit corrector magnets, and the term $C_{\text{QFQD}}(t)$ takes into account the effects of stray fields due to different excitation currents in the focussing and defocussing quadrupoles in the LEP lattice.

4 Calibration of NMR probes

4.1 Calibration of NMR probes with RD measurements

The magnetic fields $B^i_{\rm NMR}$ measured by each NMR $i = 1,16$, are converted into an equivalent beam energy. The relation is assumed to be linear, of the form

$$E^i_{\rm NMR} = a^i + b^i B^i_{\rm NMR}. \quad (3)$$

In general, the beam energy is expected to be proportional to the integral bending field. The offset allows remnant fields or stray constant fields such as the earth's field to be absorbed. The two parameters for each probe are determined by a fit to the energies measured by resonant depolarisation. The NMR probes give an estimate of the dipole contribution to the integral bending field, so the energy measured by RD must be corrected using equation 2 for the effects of coherent quadrupole motion etc.

The residuals, $E_{\rm pol} - E^i_{\rm NMR}$, are examined for each NMR. The residuals evolve with beam energy in a different way for different probes, but for a particular probe this behaviour is reproduced from fill to fill. The residuals averaged over NMR probes at each polarisation point are shown in figure 1, in which the errors are displayed as the rms/$\sqrt{N}$, where $N \leq 16$ is the number of NMR probes functioning for the measurement. This figure shows the average residuals with respect to the simultaneous fit to all polarisation fills in 1997, which was used to calibrate the NMR probes. The residuals show a reproducible small but statistically significant deviation from zero, with the 45 and 50 GeV points being a few MeV higher than those at 40 and 55 GeV.

4.2 Test of NMR calibration using the flux loop

The total change of field measured by the flux loop can also be fit to a straight line as a function of NMR magnetic field for each probe:

$$B_{\rm FL} = c^i + d^i B^i_{\rm NMR} \quad (4)$$

From octant-to-octant and magnet-to-magnet the probes show different behaviour; the slopes are not constant with energy. However, the behaviour is reproduced from fill-to-fill.

As would be expected if the RD measured energies, $E_{\rm pol}$, are proportional to the total bending field measured by the flux loop, $B_{\rm FL}$, there is a strong correlation between the parameters of the polarisation and the flux loop fits, when the NMR/flux-loop comparison is restricted to the range corresponding to 40–55 GeV:

$$E_{\rm pol} = a^i + b^i B^i_{\rm NMR} \text{ and} \quad (5)$$
$$B_{\rm FL} = c'^i + d'^i B^i_{\rm NMR} \text{ fit restricted to 40–55 GeV.} \quad (6)$$

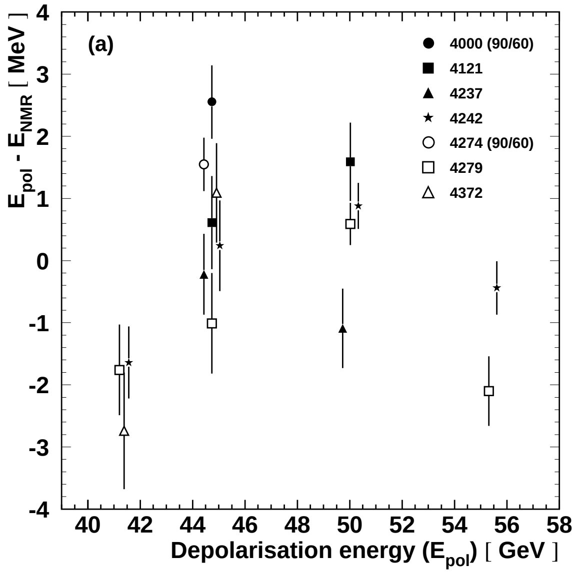

Figure 1: Residuals of the fit comparing RD energies to the energies predicted by the model for a simultaneous fit to all fills

Having established this correlation, which is shown in figure 2, the flux loop can now be used to test the extrapolation method. From the fit in the 40–55 GeV region, the NMR probes can be used to predict $B_{\rm FL}$ at physics energy, and this prediction can be compared with the flux loop measured $B_{\rm FL}$. Any difference between the prediction and the direct measurement of the field can then be interpreted as a possible bias in the predicted energy.

The deviations at physics energy for all five flux loop measurements are shown in figure 3. The bias at physics energy is up to 20 MeV, with an rms over the probes of 30–40 MeV, corresponding to an uncertainty of 10–20 MeV depending on the number of working probes. The bias tends to decrease during the year. No cause for this change has been identified so far.

The tests with the flux loop are not used to correct the NMR calibration from polarisation data, but allow an independent estimate of the precision of the method. The difference between the NMR and flux-loop measured field increase is taken to assign a systematic uncertainty of 20 MeV. This covers the maximum difference seen during the year.

5 Evaluation of the centre-of-mass energy at each IP

As at LEP1, corrections to the centre-of-mass energy arise from the non-uniformity of the RF power distribution around LEP and from possible offsets of the beam centroids during collisions in the presence of opposite-sign vertical dispersion[2].

5.1 Corrections from the RF System

Since the beam energy loss due to synchrotron radiation is proportional to $E^4_{\rm beam}$, operation of LEP2 requires a large amount of RF acceleration to maintain stable beam orbits. This implies that the energy variation in the beams (the "sawtooth") as they circulate around LEP is quite large, which increases the sensitivity of the centre-of-mass energy to non-uniformities in the energy loss arising from differences in the local magnetic bend field, machine imperfections, etc.

The average corrections for the 1996 and 1997 running are about 20 MeV in IPs 2 and 6, and -5 MeV at IPs 4 and 8.

As at LEP1, the errors on the energy corrections are evaluated by a comparison of those quantities (the synchrotron tune Q_s, the orbit sawtooth, and the longitudinal position of the interaction point) which can be calculated in the RF model and can be measured in LEP. In addition, uncertainties from the inputs to the model, such as the misalignments of the RF cavities and the effects of imperfections in the LEP lattice, must also be considered. This results in a systematic error of 4 MeV per beam.

5.2 Opposite sign vertical dispersion

In the bunch train configuration, beam offsets at the collision point can cause a shift in the centre-of-mass energy due to opposite sign dispersion [2]. The change in energy is evaluated from the calculated dispersion and the measured beam offsets from beam-beam deflections scans.

Beam offsets were controlled to within a few microns by beam-beam deflection scans. The resulting luminosity weighted correction to the centre-of-mass energies are typically 1 to 2 MeV, with an error of about 2 MeV. No corrections have been applied for this effect, and an uncertainty of 2 MeV has been assigned.

6 Summary of systematic uncertainties

The contributions from each source of uncertainty described above are summarised in table 2. The first groups describe the uncertainty in the normalisation derived from NMR-polarisation comparisons, NMR-flux-loop tests and the part of the bending field not measured by the flux-loop. These extrapolation uncertainties dominate the analysis. The following errors concern the polarisation measurement, specifically its intrinsic precision (which is less than 1 MeV), the possible difference in energy between electrons and positrons, and the difference between optics. None of the additional uncertainties from time variations in a fill, and IP specific corrections contribute an uncertainty greater than 5 MeV.

Source	Error [MeV]
Extrapolation from NMR–polarisation:	
NMR rms/sqrt(N) at physics energy	10
Different $E_{\rm pol}$ fills	5
Flux-loop test of extrapolation:	
NMR flux-loop diff. at physics energy	20
Field not measured by flux loop	5
Polarisation systematic	1
e^+e^- energy difference	2
Optics difference	4
Corrector effects	3
Tide	1
Initial dipole energy	2
Dipole rise modelling	1
IP specific corrections ($\delta E_{\rm CM}/2$):	
RF model	4
Dispersion	2
Total	25

Table 2: Summary of contributions to the beam energy uncertainty.

6.1 Uncertainty for data taken in 1996

The analysis of the 1996 data was largely based on a single fill with RD measurements at 45 and 50 GeV. The apparent consistency of the flux-loop and NMR data compared to RD data was about 2 MeV over this 5 GeV interval, i.e. a relative error 4×10^{-4}, which using a naive linear extrapolation would give an uncertainty of 13.5 (15) MeV at 81.5 (86) GeV. These errors were inflated to 27 (30) MeV before the 1997 data were available, since there was no test of reproducibility from fill to fill, there was no check of the non-linearity possible from a fill with two energy points, and the field outside the flux loop had not been studied. It can be assumed that the 25 MeV uncertainty of 1997 data is common to the 1996 data.

7 Conclusions and outlook

The method of energy calibration by magnetic extrapolation of resonant depolarisation measurements at lower energy has made substantial progress with the 1997 data. The success in establishing polarisation above the Z has allowed a robust application of the method, and the mutual consistency of the resonant depolarisation, NMR and flux-loop data has been established at the 20 MeV level at physics energy, with a total systematic uncertainty in the beam energy of 25 MeV. The precision is limited by the understanding of the NMR/flux-loop comparison.

As LEP accumulates more high energy data, the experiments themselves will be able to provide a cross-check on the centre-of-mass energy by effectively measuring the energy of the emitted photon in events of the type $e^+e^- \to Z\gamma \to f\bar{f}\gamma$, where the Z is on-shell. This can be done using a kinematic fit of the outgoing fermion directions and the precisely determined Z-mass from LEP1. The ALEPH collaboration have shown[4] the first attempt to make this measurement in the $q\bar{q}\gamma$ channel, where they achieve a precision of $\delta E_{\rm beam} = \pm 0.110({\rm stat}) \pm 0.53({\rm syst})$. With 500 pb^{-1} per experiment, the statistical precision on this channel should approach 15 MeV. Careful evaluation of systematic errors will determine the usefulness of this approach.

In future, a new apparatus will be available for measuring the beam energy. The LEP Spectrometer Project[5] will measure the bend angle of the LEP beams using beam pick ups with new electronics to measure the position to the order of a micron precision as they enter or leave a special dipole in the LEP lattice whose bending field has been surveyed with high precision. A first phase of the spectrometer is already in place for the 1998 running, with the aim of checking the mechanical and thermal stability of the position measurement. In 1999, the new magnet will be installed, and the aim is to use this new, independent method to measure the beam energy to 10 MeV at high energy. It should be possible to propagate any improvement in the beam energy determination back to previous years by correcting the extrapolation and correspondingly reducing the uncertainty.

References

1. LEP Energy Working Group, "LEP Energy Calibration Above the W-Pair Production Threshold", Contributed paper 352 to this conference.
2. LEP Energy Working Group, "Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties", CERN-EP/98-040, CERN-SL/98-012, submitted to Eur. Phys. J. C.
3. See for example the papers on the W mass measurement at LEP presented by Helenka Przysiezniak and Mark Thomson, these proceedings.
4. The ALEPH Collaboration, "Preliminary Evaluation of the LEP Centre-of-Mass Energy Using $Z\gamma$ Events", Contributed paper 1038 to this conference.
5. See for example the talk by Massimo Placidi in the proceedings of the LEP performance workshop in Chamonix, 1998.

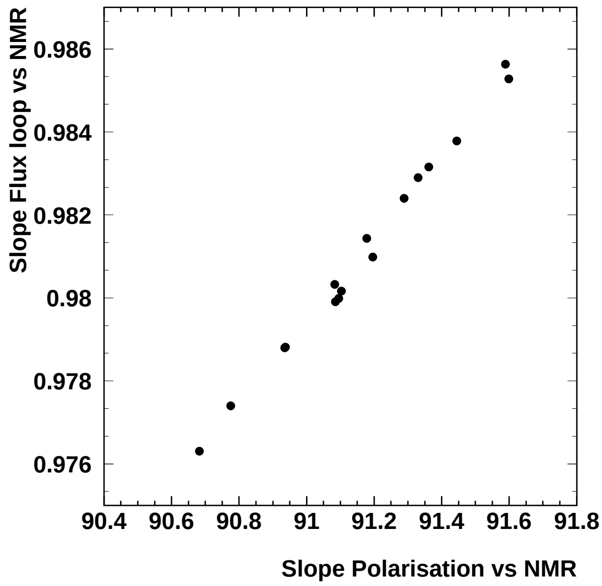

Figure 2: The slopes of equations 5 and 6, showing good correlation.

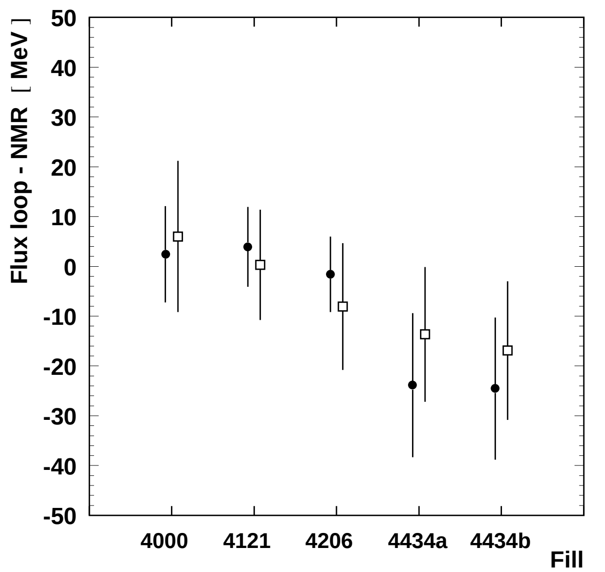

Figure 3: The difference in energy between flux loop and NMR probes at the equivalent of physics energy for each flux loop measurement. The solid points are for all working probes for each fill, while the empty points are for the common set of probes working for all fills.