LONG-TERM STABILITY IN HADRON COLLIDERS IN PRESENCE OF SYNCHROTRON OSCILLATIONS AND TUNE RIPPLE

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Abstract

The problem of long-term losses in hadron colliders such as the Large Hadron Collider (LHC) is considered. A previous formula that provides the reduction of dynamic aperture with the number of turns is generalized to include also the relevant cases of off-momentum and tune ripple. The dynamic aperture turns out to shrink with a power of the inverse logarithm of the number of turns. Long-term tracking data of the LHC are analysed in this framework. The formula proves to hold in all cases, and the possibility of using its extrapolation to predict long-term losses are explored.

1 INTRODUCTION

In this paper we analyse the long-term stability in large hadron accelerators. In the LHC [1], one has to estimate the stability for at least 10^7 turns, corresponding to the injection plateau before energy ramping. Long-term stability is determined by intricate relations between nonlinearities, resonances, and tune dependence on amplitude and momentum; its evaluation cannot be worked out analytically, and is based on tracking codes. The resulting particle losses may also occur after many millions of turns [2, 3, 4, 5, 6, 7]. Long-term losses are drastically enhanced if the betatron tune is modulated by some external causes, such as the power supply ripple, or by synchro-betatron coupling, via the residual uncompensated chromaticity [6, 7, 8, 9]. For a realistic model of the LHC lattice, simulations up to 10^6 turns are very onerous.

A pragmatic approach is based on plotting survival times provided by tracking at $10^5 - 10^6$ turns versus the initial amplitude (survival plots [2, 3, 5, 7]). In a previous paper [5] we shown for the purely four-dimensional case that if the initial amplitude is averaged over the phase space [10], survival plots can be interpolated by a two-parameter formula. Here we review the results of analysing realistic models of the LHC also in presence of tune modulation. It turns out that the dynamic aperture is well interpolated by the three parameters formula:

$$D(N) = A + \frac{B}{\log^{\kappa} N}. \qquad (1)$$

In the followings, we discuss the validity and the dynamical model underlying the above formula, and we extrapolate it to predict long-term stability.

2 LHC MODEL

The lattice of the LHC is described in Ref. [11]. The field-shape errors are described by thin-lens multipoles up to order eleven. For every magnet, each multipolar component is determined using a random number generator with gaussian distribution, truncated at 3 r.m.s. deviations. The selected realisation of the random imperfections has a dynamic aperture at 10^5 turns close to the average value in a set of 64 random realisations. A set of sextupoles and decapoles is used to correct the non-linear imperfections. Two additional families of sextupoles are used to correct the chromaticities. To partially take into account the operational difficulty of this correction in a real machine, we decided, somehow arbitrarily, to set $Q' = 2$. We disregard linear imperfections.

The tune modulation is obtained by summing up seven sine-waves, with the same frequencies Ω_k and amplitudes ϵ_k observed in the SPS spectrum (see Table 1). The amplitude ϵ_1 of the main frequency is set to 10^{-4}, and all the amplitudes ϵ_k are varied by a multiplicative factor ϵ that ranges from 1 to 8. The horizontal and the vertical tunes are affected by a synchrotron modulation of the same order of magnitude (i.e., $\Delta p/p = 10^{-4}$).

k	Ω_k	ϵ_k
1	$2\pi/868.12$	$1.000 \cdot 10^{-4}$
2	$2\Omega_1$	$0.218 \cdot 10^{-4}$
3	$3\Omega_1$	$0.708 \cdot 10^{-4}$
4	$6\Omega_1$	$0.254 \cdot 10^{-4}$
5	$7\Omega_1$	$0.100 \cdot 10^{-4}$
6	$10\Omega_1$	$0.078 \cdot 10^{-4}$
7	$12\Omega_1$	$0.218 \cdot 10^{-4}$

Table 1. Parameters of the tune modulation frequencies

3 DYNAMIC APERTURE EVALUATION

In a previous work [10] we have proposed a definition of dynamic aperture $D(N)$ as a function of the number of turns N as the first amplitude where particle loss occurs before N turns, averaged over the phase space. Particles are started along a 2D polar grid (ρ, θ) in the coordinate space $x = \rho\cos\theta$, $y = \rho\sin\theta$: with respect to the approach used in several long-term simulations (see for instance [2, 4]), where a fixed value of θ is considered in order to speed up simulations, this definition provides a smoother dependence of D on N, thus allowing to derive interpolating formulae and to extrapolate them to predict long-term particle loss. An error estimate due to the finite-size grid used to start initial conditions can be worked out (see Ref. [7] for more details); this estimate is crucial to determine the reliability of the fit.

4 DYNAMIC APERTURE PREDICTION

We propose to interpolate the long-term dynamic aperture $D(N)$ versus the number of turns N according to the following empirical formula:

$$D(N) = A + \frac{B}{\log^{\kappa} N}. \quad (2)$$

This equation can be justified [5] for a four dimensional mapping in terms of the KAM (Kolmogorov-Arnol'd-Moser) and of the Nekhoroshev theorems, using a phase space model that features two regions: an inner KAM domain where almost all the initial conditions give rise to regular orbits, stable for infinite times, and an outer chaotic region where particles diffuse according to the Nekhoroshev exponential estimate. Numerical simulations based on long-term tracking and frequency analysis have confirmed this scenario for 4D mappings [5]. In the case of tune modulation, this theoretical justification does not hold any more. Nevertheless, we used the above formula to interpolate tracking data, with good results.

The fitting procedure has been carried out using the standard approach based on least-squares minimization. Some care has to be taken since the fit is nonlinear. The procedures used to work out the confidence limits for the three parameters A, B and κ are described in [7]. The extrapolation is carried out by using the best parameters; the error is obtained by extrapolating through all the parameters inside the confidence limits, and selecting the maximum and the minimum extrapolation value.

5 NUMERICAL RESULTS

The dynamic aperture is given in mm normalized at $\beta_{max} = 182m$. Onerous simulations were carried out up to 10^6 turns, with a scan over 17 angles and 100 steps in the radius. The relative error in the dynamic aperture is of the order of 2%.

We interpolated the dynamic aperture versus the number of turns according to Eq. (2). The value of χ^2, and of the parameters κ and A, with the error estimated with a confidence level of 90%, are given in Table 2. The dynamic aperture estimate through tracking with the associated errors (bars), the best fit through Eq. (2) (solid line), and the extrapolation to infinity (dotted line) are shown in Figs. 1-3 for three different cases. The off momentum is $\Delta p/p = \delta 10^{-4}$.

The interpolation is very good: the best fit has a χ^2 that ranges from 0.4 to 2.0. Both κ and A decrease as the modulational amplitude ϵ gets larger: the effect of the modulation is to shrink the stable core and to slow down the escape rate of the initial conditions in the outer region. For $\epsilon = 1$, A becomes negative and therefore according to the extrapolation all initial conditions will be lost sooner or later. The exponent κ is determined within $0.5 - 1.0$. The errors on A and B become larger when the modulation is increased: A, that denotes the extrapolation of the dynamic aperture for infinite number of turns when κ is positive, is rather sharply defined for $\epsilon = 0$, but becomes rather loose when ϵ is increased. When κ change sign in the interval of 90% confidence level (i.e., $\epsilon = 1$), it becomes impossible to associate an error to A and B since our formula contains a singularity for $\kappa = 0$.

ϵ	δ	χ^2	κ	A
0	0	0.4	$1.9^{+1.1}_{-1.2}$	$12.0^{+0.3}_{-1.7}$
0	1	1.0	$0.8^{+1.0}_{-1.1}$	9.6
1	1	1.4	$0.3^{+0.9}_{-1.0}$	3.4
2	1	2.0	$-0.1^{+0.9}_{-0.8}$	42
4	1	1.0	$-0.1^{+0.8}_{-0.7}$	47
8	1	1.3	$-0.2^{+0.5}_{-0.5}$	33

Table 2. Main fitting parameters of Eq. (2) for the LHC

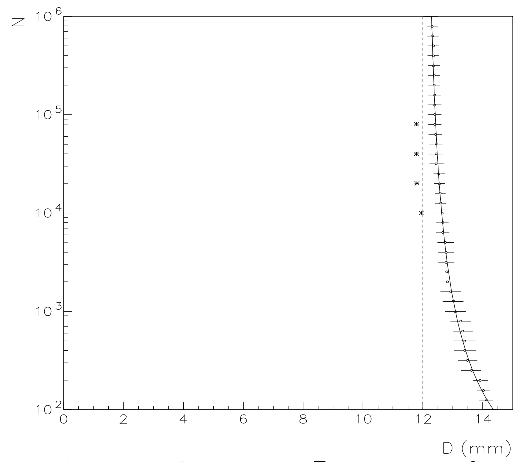

Figure 1: Dynamic aperture D versus number of turns N for the LHC on momentum, without modulation ($\epsilon = 0$). Tracking data (error bars), interpolation according to Eq. (2) (solid line) and extrapolation at infinity (vertical dotted line), prediction through Lyapunov exponent (squares).

We use tracking data from 10^2 up to 10^5, to evaluate the three parameters of Eq. (2), and then we extrapolate at 10^6. The results (see Table 3) are good: all the extrapolations are in agreement with direct tracking, and rather precise (within 5%); there are some indirect indications that data up to 10^5 can be safely extrapolated at 10^7, even though

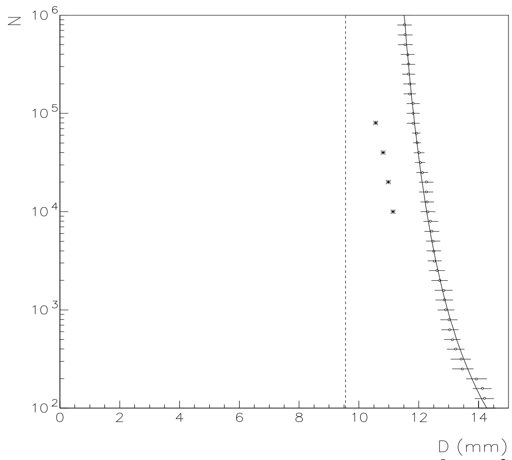

Figure 2: Dynamic aperture D versus number of turns N for the LHC off momentum ($\delta = 1$), without modulation ($\epsilon = 0$). Tracking data (error bars), interpolation according to Eq. (2) (solid line) and extrapolation at infinity (vertical dotted line), prediction through Lyapunov exponent (squares).

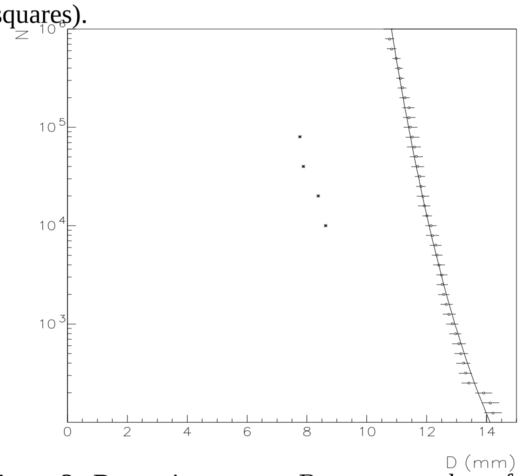

Figure 3: Dynamic aperture D versus number of turns N for the LHC off momentum ($\delta = 1$), with modulation ($\epsilon = 2$). Tracking data (error bars), interpolation according to Eq. (2) (solid line) and prediction through Lyapunov exponent (squares).

no direct check has been carried out for lack of computing power (see Ref. [7] for more details).

In comparison with the results of Ref. [5], where the case without modulation was analysed, we note that a larger number of turns is required to obtain a reliable prediction. This is due to the fact that the effect of the modulation on the beam stability requires a longer time to become evident.

We also give the comparison with the long-term estimate provided by a method based on the Lyapunov exponents (see Refs. [7, 5]). The prediction of the Lyapunov exponent are marked as stars in Figs. 1-3; one can observe that without modulation the extrapolation of the dynamic aperture at infinity A agrees with the Lyapunov exponent prediction (see Fig. 1), thus supporting previous results on simplified models [7]. When a tune modulation is considered (see Figs. 2 and 3), it is hard to say whether the Lyapunov exponent predicts a finite stability domain or not and it seems very hard to extract quantitative information on the long-term stability.

ϵ	δ	Extrap.	Track.
0	0	$12.3^{+0.1}_{-0.4}$	$12.3^{+0.2}_{-0.2}$
0	1	$11.7^{+0.2}_{-0.4}$	$11.5^{+0.2}_{-0.2}$
1	1	$11.4^{+0.4}_{-0.4}$	$11.1^{+0.2}_{-0.2}$
2	1	$11.1^{+0.5}_{-0.5}$	$10.7^{+0.2}_{-0.2}$
4	1	$10.6^{+0.5}_{-0.5}$	$10.4^{+0.2}_{-0.2}$
8	1	$10.0^{+0.5}_{-0.7}$	$10.1^{+0.2}_{-0.2}$

Table 3. Comparison between extrapolation of dynamic aperture at 10^6 and tracking for the LHC

6 CONCLUSIONS

We have proposed an empirical formula to analyse survival plots. Using a definition of dynamic aperture that involves averages in phase space, the dynamic aperture turns out to shrink with an inverse power of the logarithm of the number of turns. This numerical evidence confirms a scenario that features a hard core of phase space stable for infinite times and an outer chaotic region where the escape rate can be evaluated. When modulation is added, one reaches a limit where all the phase space becomes unstable. Besides giving a phenomenological framework to interpret tracking data, this formula allows one to extrapolate the dynamic aperture of at least one order of magnitude in the number of turns to predict long-term stability.

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