

# LONG TERM ESTIMATES FOR SORTING STRATEGIES OF THE LHC DIPOLES

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## Abstract

Sorting strategies are investigated in view of improving the dynamic aperture of the CERN-LHC. Local and quasi-local compensation of the random field shape imperfections are discussed and applied to simplified model of the LHC lattice. The most promising strategies are further investigated on a realistic LHC model with particular emphasis on the analysis of the robustness of the dynamic aperture improvements including long term effects. First results on the application of the recently developed extrapolation law for the prediction of the dynamic aperture to the sorting problem are presented.

## 1 INTRODUCTION

Unavoidable field-shape imperfections are present in the superconducting magnets of modern hadron colliders, due to mechanical tolerances, persistent currents, design imperfections, coil deformations and iron saturation. The residual multipolar errors, typically of the order of a few units in  $10^{-4}$  at the usual reference radius of 1 cm, have a detrimental effect on the stability of the particle orbits. A sound design of the machine layout, with well focused orbit functions and a suited set of multipolar correctors, may help in improving the beam stability. However, random fluctuations of the magnetic imperfections due to construction tolerances, affect substantially the stability of the particle motion and may significantly reduce the dynamic aperture (hereafter DA) available for the beam. In such a case, sorting strategies are considered as an appropriate tool to compensate the destabilising effect of the random errors of the superconducting magnets in large hadron colliders [1, 2, 3]. It has been shown that a suitable disposition of the magnetic elements carrying random errors, along the azimuthal length of the lattice, can provide a self compensation of the random errors thereby improving the DA [4]. This procedure is quite demanding, therefore it is crucial to evaluate in detail the expected beneficial effects on beam dynamics. In fact, all the magnets have to be carefully measured as they are manufactured and qualified in terms of the random imperfections. In addition, a sizeable number of magnets must be available for sorting before the final installation.

The techniques developed were first tested on a simplified model of the LHC where only random errors were included in the main dipoles and tracking was performed considering only the short term DA (1000 turns) in the 4D motion. The most promising techniques were then applied

to a realistic model of the LHC where all multipolar errors were included in the dipoles and the DA is computed including the synchrotron motion, power supplies ripple in the main quadrupoles. Long term effects in beam stability were considered by tracking particles up to  $10^5$  turns and analysing the survival plots by means of the extrapolating laws for the long term estimates of the DA [5, 6].

In Section 2 we present in detail the sorting strategies. In Section 3 we apply them to a simplified LHC model while long term verification of the improvement in the DA are presented in Section 4. Conclusions are drawn in Section 5.

## 2 SORTING STRATEGIES

The techniques proposed for the sorting of the main LHC dipoles are based on local or quasi-local cancellation of the random errors by pairing the magnets with similar errors in magnitude and sign and placing each element of a pair along the azimuth of the accelerator separated by an opportune phase advance.

Pairing at zero phase advance. Taking into account that two adjacent dipoles have similar optical functions and the phase advance between them is almost negligible, one can obtain a local compensation scheme by placing in adjacent position two errors equal in strength but with opposite signs. Indeed this scheme is used for the correction of the systematic  $b_3$  and  $b_5$  errors in the LHC dipoles by means of spool pieces correctors. In the LHC the phase advance between two dipoles is approximately 15 degrees and the optical function are significantly different so the neutralisation of the errors is only partial.

Pairing at 180 and 360 degrees. In absence of strong deviation from the linear motion, a better cancellation scheme of equal and opposite random errors can be obtained by placing the magnets at position separated by 360 degrees or, equivalently, 180 degrees for errors also equal in sign. In the LHC each cell contains 6 dipoles and the phase advance is 90 degrees per cell, so that positions separated by 24 (12) magnets correspond to a phase advance of 360 (180) degrees. Furthermore owing to the regularity of the cells, these positions correspond to the same location in the cell therefore the optical functions are equal.

Mixed techniques. It is possible to define sorting procedures based on more than one of the previous strategies. The pairing of two adjacent magnets can be improved by a compensation at 360 or 180 degrees. In this case 4 dipoles are paired.

Order	Normal	Skew
1	-	-
2	0.372	1.227
3	0.882	0.186
4	0.055	0.186
5	0.083	0.041
6	0.014	0.022
7	0.012	0.011
8	0.005	0.005
9	0.003	0.004
10	0.002	0.002
11	0.001	0.001

Table 1: Random errors in the LHC dipoles at injection, in unit  $10^{-4}$  referred to a radius of 1 cm.

Further improvement of these technique can be obtained by the minimisation of dynamical quantities such as perturbative estimations of tune shifts or resonance driving terms by means of random permutations of the previously generated pairs of dipoles<sup>[4]</sup>. In the following we present the results of the application of the mixed technique based on pairings at 360 degrees to different LHC models of increasing complexity.

### 3 4D ANALYSIS OF THE LHC MODEL

A simplified model of the LHC was used to gain a preliminary idea on the amount of the improvement in DA given by the different techniques. In particular we used first the old LHC version 2, with the injection optics: it is made of 8 octants, each of them carrying 16 dipoles in the dispersion suppressor region and 144 dipoles in the arcs. Each arc is composed of 24 FODO cells each carrying 6 dipoles. The overall number of dipoles is 1280. The set of 376 chromatic sextupoles is considered in the simulations. The distribution of the random errors is assumed to be Gaussian truncated at  $3\sigma$  and the  $\sigma$  of the multipolar coefficients used in the simulation are given in Table 1. We assume that the magnets will be installed as the production goes on, therefore only a limited number of dipoles will be stored and available for sorting. We applied the sorting strategies on groups of 144 dipoles. Two extreme cases were analysed in detail: dipoles with only random  $b_3$ , and dipoles with the full set of random errors given in Table 1.

#### 3.1 Random $b_3$

We applied the sorting strategies to a set of 100 realizations of the random errors. The characteristics of the distribution of the DA and the results of the sorting strategies are reported in Table 2. The average value over 100 realizations is denoted by  $D$  and the R.M.S. by  $\sigma_D$ . The improvement of the DA due to the sorting of the cases with an initially small value of the DA are denoted as 'Worst Cases' in Table 2. The effect of sorting on the DA can be put in evidence by plotting the relative gain in DA as a function of the DA of the unsorted realizations of the errors, as shown

in Fig. 1. It appears clearly that the realizations with an initially small value of the DA are more efficiently corrected and in the worst cases the DA is more than doubled.

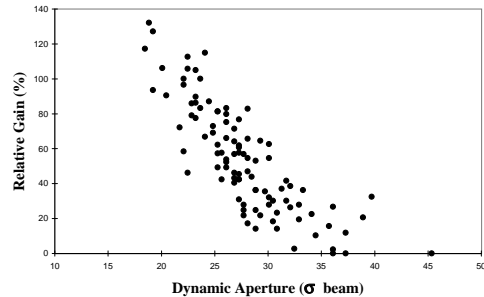


Figure 1: Relative gain as a function of the DA of the unsorted sequence of random errors for 100 realizations. Only random  $b_3$ .

#### 3.2 All random errors

The sorting strategies were then applied to the case where all the random errors up to the 11-th order are considered in the dipoles. The effects on the DA were investigated on a set of 100 random realizations of the errors and the results are given in Table 2 and Fig. 2. The improvement of the DA is still non negligible especially for the worst cases. The results of these studies show that the sorting strategies

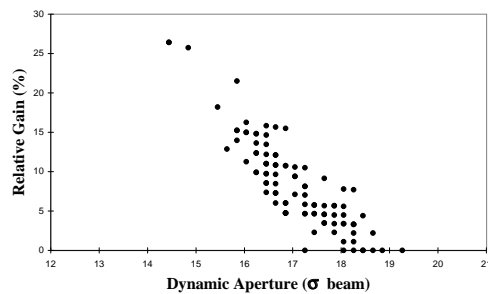


Figure 2: Relative gain as a function of the DA of the unsorted sequence of random errors for 100 realizations. All random errors.

we developed are very effective in the case where there is a dominant multipole, e.g. the case with only random  $b_3$ . In the general case the improvement in DA, although reduced, is still significant at least for the realization of the random errors which have a small DA before sorting: these realizations are indicated as worst cases and they are characterised by a value of the DA that is  $2\sigma_D$  below the average value of the DA distribution before sorting. These results are summarised in Table 2.

## 4 LONG TERM AND 6D CHECKS

The robustness of the sorting strategies developed was tested on a realistic and more recent model, the LHC version 4.3. Extensive tracking simulations were performed

	$D$	$\sigma_D$	Gain	Worst Cases
$b_3$ random errors.				
unsorted	2.57	0.44	-	
sorted	3.78	0.42	48%	91%
Random errors up to 11-th order.				
unsorted	1.57	0.09	-	-
sorted	1.68	0.05	7%	17%

Table 2: Characteristics of the DA distribution over 100 random realizations of the multipolar errors. 4D motion LHC v2.

$D(10^3)$	$\sigma_{D(10^3)}$	$D(10^5)$	$\sigma_{D(10^5)}$
13.36	0.49	11.89	0.49

Table 3: Characteristics of the DA distribution over 30 random realizations of the multipolar errors. 6D motion LHC v4.3.

including all multipolar errors in the main dipoles and quadrupoles, both systematic and random, coupling with the longitudinal motion and power supplies ripple in the main quadrupoles. Furthermore a more realistic value of 48 dipoles was assumed to be available from the storage areas. The characteristic of the corresponding DA distribution before sorting are given in Tab. 4. Owing to the large amount of CPU time needed, sorting strategies were applied only to the five worst cases. Although the absolute gain in DA is small, in all cases the value obtained after sorting is close to the average value of the distribution of DA for the unsorted realization which accounts to say that the worst realizations can at least be corrected to average realizations.

The extrapolation of the survival plots allows to estimate the long term behaviour to a number of turns as large as  $10^6$ . The plots of the DA vs the number of turns  $N$  obtained from tracking up to  $10^5$  turns are interpolated using the empirical formula <sup>[6]</sup>

$$D(N) = A + \frac{B}{\log^k N} \quad (1)$$

to obtain DA estimates at  $10^6$  turns.

This analysis shows that the gain in DA persists and an example of the comparison of the survival plot for a realization (seed #1) of the random errors is shown in Fig. 3. The values of the DA for these 5 cases, before and after sorting, is given in Table 4.

## 5 CONCLUSIONS

We presented several sorting strategies and their application to different LHC models of increasing complexity. The effect of sorting on the dynamic aperture is very good in the case where there is a dominant random multipolar error. In more realistic cases the gain is significantly reduced: only the worst cases are appreciably improved and long term checks show that the expected gain is of the order of 1

	$D(10^3)$ tracking	$D(10^5)$ tracking	$D(10^6)$ extrapolation
unsorted			
# 1	12.20	10.37	9.61
# 4	12.35	11.22	10.93
# 9	12.33	11.23	10.81
#19	12.26	10.90	10.35
#29	12.02	10.57	9.71
sorted			
# 1	13.11	11.59	11.02
# 4	13.36	12.08	11.69
# 9	12.91	11.76	11.29
#19	12.74	11.70	11.42
#29	13.23	11.42	10.89

Table 4: DA values calculated at  $10^3$ ,  $10^5$ ,  $10^6$  turns for the 5 worst realizations of the random errors before and after sorting.

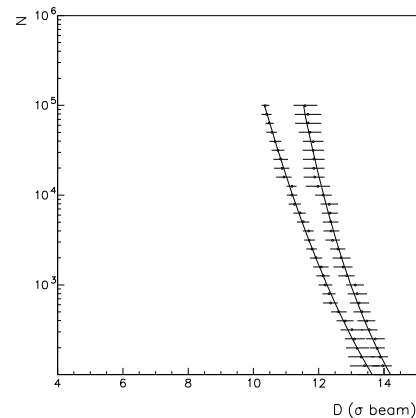


Figure 3: Interpolation of survival plot for the realization seed #1. The leftmost curve is the survival plot before sorting, the rightmost after sorting.

$\sigma$ . These tests were performed assuming a single aperture LHC model: the analysis of the 2-in-1 scheme of the LHC dipoles is in progress.

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