N=6 Supergravity on AdS_5 and the $SU(2,2/3)$ Superconformal Correspondence

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ABSTRACT

It is argued that N=6 supergravity on AdS_5, with gauge group $SU(3) \times U(1)$ corresponds, at the classical level, to a subsector of the "chiral" primary operators of N=4 Yang-Mills theories. This projection involves a "duality transformation" of N=4 Yang-Mills theory and therefore can be valid if the coupling is at a self-dual point, or for those amplitudes that do not depend on the coupling constant.

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1   Introduction

The recent understanding of many features of the AdS_{d+1}^{2N}/CFT_d^N correspondence [1, 2, 3], where d is the dimension of the boundary conformal field theory and N the number of (boundary) Poincaré supersymmetries, naturally points to investigate more general supergravity theories in AdS_d^N, that have lower supersymmetry, as well the cases when these theories have no obvious interpretation in terms of standard compactifications. Among the latter is the class of theories for which the number of supersymmetries N is not a power of two, and that can not, therefore, be obtained by standard compactifications of superstring theories.

The most familiar examples are $O(N)$ AdS_4 supergravities with $N = 5, 6$, corresponding to three-dimensional superconformal algebras $OSp(N/4)$ [4].

Another example, which is the one considered in this note, is $N = 6$ AdS_5 supergravity[3] in five dimensions, associated to the $N = 3$ superconformal algebra in $d = 4$ dimensions.

An immediate puzzling feature of this particular case is, of course, the fact that $N = 3$, $d = 4$ Yang-Mills theory is known to be the same as $N = 4$ Yang-Mills theory, although the corresponding superalgebras ($SU(2, 2/N)$ [4]) are different. Multiplets with spin greater than one are however different in the two theories, and $N = 6$ supergravity is not the same as $N = 8$ [7].

In this note, we show that the existence of $N = 6$ supergravity, at least classically, may be related to some properties of the OPE of $N = 4$ Yang-Mills theory, at least in a regime in which a certain symmetry is supposed to hold. This symmetry truncates the $N = 4$ "chiral" operators to a subset of $N = 3$ operators, which do not contain the additional AdS_5 representations which complete $N = 6$ to $N = 8$, AdS_5 supergravity.

2 $N = 6$ Supergravity and its Symmetries

In their several papers on supergravity on AdS$_5$, Günaydin, Romans and Warner discussed $N = 6$ supergravity [8] as a consistent truncation of $N = 8$ supergravity [9]. The two $N = 6$ multiplets are the graviton multiplet (containing three complex gravitinos), and the gravitino multiplet. The components of these multiplets are given in table 3 in [8], together with their quantum number under the $SU(3) \times U_D(1)$ subgroup of the $USp(6)$ hidden symmetry of the $N = 6$ theory. The $N = 8$ graviton multiplet decomposes in an $N = 6$ graviton and an $N = 6$ gravitino multiplet. $N = 6$ supergravity is obtained by consistently truncating the $N = 8$ supergravity to the $N = 6$ graviton multiplet. We notice that, in this truncation, the original gauge group $SU(4)$ is broken to $SU(3) \times U(1)$, and that the $N = 8$ dilaton field, belonging to the $N = 6$ gravitino multiplet, disappears from the spectrum.

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[3]Stringy constructions of $N = 6$ 5D supergravity in flat space M_5 were given in [5, 6]

A crucial ingredient in the truncation is the fact that in the original $N = 8$ theory there is a $U(1)$ symmetry commuting with the gauge group $SU(4)$. This comes from the fact that [8]

$$E_{6(6)} \to SL(2; R) \times SL(6; R) \to U_S(1) \times SU(4). \tag{1}$$

On the other hand, the maximal compact group $USp(8)$ in $E_{6(6)}$ can be decomposed in two different ways down to $SU(3) \times U(1)$:

$$\mathrm{I}: \qquad USp(8) \to SU(4) \times U_S(1) \to SU(3) \times U_R(1) \times U_S(1). \tag{2}$$

Here $U_S(1)$ is a subgroup of the $SL(2; R)$ symmetry of the theory whose $SL(2; Z)$ subgroup can be identified with the S-duality group of both the underlying type IIB theory and the N=4 boundary Yang-Mills theory.

$$\mathrm{II}: \qquad USp(8) \to USp(6) \times SU(2) \to SU(3) \times U_D(1) \times U_1(1). \tag{3}$$

Under $USp(6)$, the 15 original ungauged vectors transform as 14+1, and the $SU(3) \times U(1)$ gauge bosons come from the octect in the $14 \to (8, 0) + (3, 1) + (3, -1)$ decomposition of $USp(6) \to SU(3) \times U_D(1)$, as well as from the $USp(6)$ singlet in the original decomposition of $USp(8) \to USp(6) \times SU(2), 27 \to (14, 1) + (6, 2) + (1, 1)$.

Clearly, there are only two independent $U(1)$ factors: the pair of $U(1)$s in the decomposition II are linear combinations of the $U(1)$s in the decomposition I, and viceversa. The $U(1)$s in decomposition I correspond to symmetries of the original $N = 4$ Yang-Mills theory, or type IIB theory on $AdS_5 \times S^5$, namely, a $U(1)$ subgroup of the R-symmetry $SU(4)$ and a (discretized) subgroup of $SL(2; Z)$. The $U(1)$s in decomposition II are more suitable for discussing the structure of N=6 representations and the consistent truncation from $N = 8$, as we shall see shortly.

The $U_S(1)$ transformation is an automorphism of the $SU(2, 2/4)$ superalgebra, acting as $e^{i\frac{3}{4}\alpha}Q_L, e^{-i\frac{3}{4}\alpha}S_L$ on Poincaré and conformal left supercharges [10], respectively. It corresponds to a γ_7 transformation in the $O(4, 2)$ covariant formulation [11, 12].

3   The $N = 3$ Truncation

The very fact that the $N = 8$ supergravity admits a consistent truncation to $N = 6$ seems to suggest that, in a certain regime, we can define a closed $N = 3$ subalgebra of gauge singlet operators in $N = 4$ Yang-Mills theories.

In complete analogy with the $N = 4$ case [13], we can construct the $N = 3$ "chiral" spectrum of operators (or, equivalently, $N = 6$ KK excitations) by considering tensor products of the fundamental $N = 3$ singleton representation that can be obtained by considering the $N = 3$ pure Yang-Mills theory in $N = 3$ superspace.

Following [14], such singleton representation is described in $N = 3$ superspace by a superfield-strength, which is a Lorentz scalar and an $SU(3)$ triplet, $W_i(x, \theta)$. This W_i satisfies some constraints, which can be found in [14], however its physical components lie in just the first few terms of the θ expansion,

$$W_i(x, \theta) = \phi_i + \theta_{iL}\lambda_L + \theta_R^l \lambda_R^m \epsilon_{lmi} + \theta_R^l \theta_R^m \epsilon_{lmi} F_R^+ + \ldots \quad . \tag{4}$$

Notice that both θ_L and θ_R contribute to the physical components of W_i. Under $N = 1$ supersymmetry, W_i decomposes into three "chiral" multiplets containing the physical fields (ϕ_i, λ_i) and a vector multiplet containing (F, λ).

Let us describe the U_D and U_1 quantum numbers of the W_i components. We see from eq. (3) that U_1 commutes with $USp(6)$ while U_D must act as a R-symmetry. We can assign $U_D(1)$ charge $-1/2$ to ϕ_i and $+1$ to the θs, and $U_1(1)$ charge $+3/2$ to all the components of the multiplet. The transformation rules:

$$U_D(1) \quad : \qquad W_i(\theta) \to e^{-\frac{1}{2}i\alpha}W_i(e^{i\alpha}\theta), \tag{5}$$

$$U_1(1) \qquad : \qquad W_i(\theta) \to e^{\frac{3}{2}i\alpha}W_i(\theta), \tag{6}$$

give for the components:

$$U_D(1) \quad : \qquad \phi_i\left(-\tfrac{1}{2}\right), \lambda_{iL}\left(-\tfrac{1}{2}\right), F_L^-\left(-\tfrac{3}{2}\right), \lambda_L\left(-\tfrac{3}{2}\right), \tag{7}$$

$$U_1(1) \qquad : \qquad \phi_i\left(\tfrac{3}{2}\right), \lambda_{iL}\left(-\tfrac{3}{2}\right), F_L^-\left(-\tfrac{3}{2}\right), \lambda_L\left(\tfrac{3}{2}\right). \tag{8}$$

Notice that since the above symmetries act on F_L as a duality rotation, they can only be realized in free-field theory as continuous invariances, and in non-perturbative Yang-Mills theory at the self-dual point as discrete subgroups. Notice also that U_D commutes with $N = 1$ supersymmetry.

We give also the quantum numbers of the gauge group $U_R(1)$. They can be easily obtained by decomposing $SU(4) \to SU(3) \times U_R(1)$,

$$U_R(1) : \qquad \phi_i(-2), \lambda_{iL}(1), F_L^-(0), \lambda_L(-3). \tag{9}$$

This symmetry is known to be a continuous symmetry of perturbative Yang-Mills theory.

As for the $U_S(1)$ symmetry, that commutes with $SU(4)$ and corresponds to the linearly realized $U(1)$ subgroup of $SL(2; R)$, we can define it directly in terms of the $N = 4$ superfield:

$$W_{[AB]} \to W_{[AB]}(e^{i\frac{3}{4}\alpha}\theta_a), \qquad A, a = 1, .., 4. \tag{10}$$

This gives for the components:

$$U_S(1) = U_D(1) - \frac{1}{4}U_R(1) : \qquad \phi_i(0), \lambda_{iL}, \lambda_L\left(-\frac{3}{4}\right), F_L^-\left(-\frac{3}{2}\right). \tag{11}$$

It is clear that several relations among the various $U(1)$s hold, since only two of them are independent. For example, we have

$$U_1(1) = U_D(1) - U_R(1) = -\frac{3}{4}U_R(1) + U_S(1). \tag{12}$$

It is crucial for us that there exists $U_1(1)$, which commutes with $USp(6)$. We can use this $U(1)$ to define the truncation of the $N = 8$ supergravity theory, or, equivalently, the truncation of the $N = 4$ Yang-Mills theory. We can express $U_1 = -\frac{3}{4}U_R + U_S$ as a linear combination of a $U(1)$ subgroup of the $SU(4)$ R-symmetry of $N = 4$ SYM and a (discretized) $U(1)$ subgroup of the S-duality group $SL(2; Z)$. By choosing an appropriate element of $SL(2; Z)$, the discretized U_1 is a (non-perturbative) symmetry of the $N = 4$ SYM theory at the self-dual point.

The two spin-2 and spin-3/2 $N = 6$ multiplets, which will be denoted G and g, respectively, can be written as bilinears of the singleton fields:

$$G_i^j = \text{Tr}\left(W_i\bar{W}^j - \tfrac{1}{3}\delta_i^j W_k\bar{W}^k\right) \tag{13}$$

$$g_{ij} = \text{Tr}\left(W_iW_j\right). \tag{14}$$

The θ expansion of these superfields exactly reproduces the structure and $SU(3) \times U_D(1)$ quantum number of the graviton and gravitino $N = 6$ multiplet, as shown in table 3 of [8].

At the bilinear level, the other multiplet, corresponding to the radial mode, is the Konishi multiplet $\text{Tr}\,W_i\bar{W}^i$, which is not a "chiral" operator [15].

It is obvious that, under U_1, the two superfields transform as,

$$G_i^j \rightarrow G_i^j \tag{15}$$

$$g_{ij} \rightarrow e^{3i\alpha}g_{ij} \tag{16}$$

We see that, as promised, U_1 can be used to eliminate the unwanted gravitino multiplet and to truncate $N = 8$ supergravity to $N = 6$. Note that the Konishi multiplet is not projected out by the truncation as required by consistence of the OPE of two stress-energy tensors in Yang-Mills theory [16].

This analysis can be extended to the whole set of $N = 4$ "chiral" operators. We can use the U_1 projection to define an $N = 3$ subset of "chiral" operators. For N-extended supersymmetry, a long-multiplet has maximum spin $(N/2, N/2)$. Therefore, long multiplets of $N = 6$ have maximum spin at least equal to 3. It then follows that the "chiral" $N = 4$ primary fields, having at most spin 2, are also (reducible) short multiplets of the $SU(2, 2/3)$ superalgebra. So, the quantization of the spectrum is, also in the $N = 3$ case, a consequence of supersymmetry.

The $N = 4$ "chiral" operators –which can be written as $\text{Tr}\,W^p$ [17], where W is the $N = 4$ singleton, as defined in harmonic superspace [18]– decompose in $N = 3$ operators that are

products of W and $\bar{W}$, with suitable symmetrizations and removal of traces. The U_1 projection eliminates all the products that do not involve an equal number of W and $\bar{W}$. The expected $N = 3$ "chiral" operator has, therefore, the general form

$$W^{2p} = \text{Tr}\,(W_{i_1}\bar{W}_{i_2}...W_{i_{2p-1}}\bar{W}_{i_{2p}}). \tag{17}$$

Of course, the U_1 projection involving an element of $SL(2;Z)$ is discrete. This implies that particular powers of W and $\bar{W}$ may survive the projection. If the charge of W_i is $2\pi/k$, strings of operators of the form $W_{1_1}....W_{i_k}$ or $\bar{W}_{1_1}....\bar{W}_{i_k}$ are allowed in eq. (17).

4 The Truncation on the Supergravity Side.

The argument in the previous section can be retrieved by reasoning in the $N = 8$ supergravity context.

$N = 8$ supergravity on AdS_5 can be regarded as the supersymmetric completion of a gauged σ-model with $G/H = E_{6(6)}/USp(8)$ and gauge group $SU(4)$.

The relevant decomposition to obtaining $N = 6$ supergravity is

$$E_{6(6)} \to SU^*(6) \times SU(2), \tag{18}$$

with the following embedding of the previous defined $U(1)$'s: $U_1(1) \subset SU(2)$ and $U_D(1) \subset SU(3) \in SU^*(6)$. The truncation defining $N = 6$ gauged supergravity is obtained by retaining only $U_1(1)$ singlets. This leads to the identification of $U_D(1)$ with $U_R(1)$ on the singlet modes, according to eq. (12). As a result, $G/H = E_{6(6)}/USp(8)$ with gauge group $SU(4)$ is truncated to $SU^*(6)/USp(6)^4$ with gauge group $SU(3) \times U(1)$ [8].

The extension of this argument to the massive states requires that one can define the $N = 6$ truncation directly in 10 dimensions. No *geometrical* symmetry can be used to truncate $N = 8$ to $N = 6$ on $AdS_5 \times S^5$, but a combination of isometries and a duality transformation preserves the right number of Killing spinors. This is easily seen by recalling that the Killing spinor transforms as a 4 of $SU(4)$, the isometry group of $S^5 = SO(6)/SO(5) \approx SU(4)/USp(4)$ [21]. Using the harmonic expansion on S^5, the Killing spinor reads [22]

$$\epsilon_i(x) = D^4_{ij}[L^{-1}(x)]c_j, \quad x \in S^5. \tag{20}$$

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[4]Note that the rank 6 coset $E_{6(6)}/USp(8)$, as a solvable Lie algebra [19], decomposes as

$$\text{Solv}(E_{6(6)}/USp(8)) = \text{Solv}(SU^*(6)/USp(6)) + \text{Solv}(F_{4(4)}/USp(6) \times SU(2)) \tag{19}$$

where the rank 2 and rank 4 cosets above correspond to the following decomposition $42 = (14,1) + (14',2)$ of the original $N = 8$ scalars with respect to $USp(6) \times SU(2)$.

Here $D^4(g)$ is the 4×4 matrix transforming in the 4 of $SU(4)$; $L(x)$ is the coset representative of x in $SU(4)$, and c_j are arbitrary constants. Isometries of S^5 act on D^4 as right multiplications, while the linearly-realized $U_S(1)$ in $SL(2, R)$ acts as multiplication by a phase:

$$\begin{aligned} g \in SU(4) \quad &: \quad D^4_{ij}[L^{-1}(x)] \to D^4_{il}[L^{-1}(x)]D^4_{lj}[g^{-1}], \\ h \in U(1)_S \quad &: \quad D^4_{ij}[L^{-1}(x)] \to \exp(i\theta)D_{ij}[L^{-1}(x)]. \end{aligned} \tag{21}$$

By choosing a projection acting as an $SU(4)$ isometry $g = \exp(i\theta\lambda)$, $\lambda = \text{diag}\,(1, 1, 1, -3)$, combined with a $U_S(1)$ $h = \exp(i\theta)$, one can project away the Killing spinors with $c_4 \neq 0$, and preserve only 6 of the original 8 supersymmetries of the background.

5 Hidden Symmetry in $N = 4$ Yang-Mills Theory

Let us examine the consequence of the truncation defined in the previous section on the Green functions of the $N = 4$ Yang-Mills theory.

The vanishing of amplitudes with unequal number of W and $\bar{W}$ $N = 3$ field strengths implies a certain set of relations among $N = 4$ correlators.

The $N = 3$ correlators with only one gravitino, $\langle G_1...G_n g \rangle$, vanish, as any amplitude with an odd number of g. However, amplitudes of the form $\langle G_1...G_n g_1...g_m \bar{g}_1...\bar{g}_m \rangle$ may not vanish. As an example, in the decomposition of $SU(4)$ into $SU(3)$, the scalars in the $N = 4$ graviton multiplet decompose as

$$20_R \to 8 + 6 + \bar{6}, \qquad 10 \to 1 + 3 + 6, \qquad 1 \to 1. \tag{22}$$

The $N = 3$ supergravity only contains the 14 scalars in $8 + 3 + \bar{3}$. Therefore, the dilaton-axion belongs to $g = \text{Tr}\,W^2$ and (partially) decouples from $N = 3$ amplitudes.

The non-perturbative projection U_1 is in general discrete. If it acts as a π phase on the gravitino g, then the relation $\langle G_1...G_m g \rangle = 0$ is still valid, but $\langle g^2 \rangle$, for example, is allowed as a composite operators, at the non-perturbative level. In the sequence $\langle \text{Tr}\,W^{4n}W\bar{W}...W\bar{W} \rangle$ this gives extra allowed "chiral" operators which show up only at the $p = 4$ level.

In the free-field theory, however, the $U_1(1)$ symmetry $W_i \to e^{i\beta}W_i$ can be used to give a stronger selection rule: all correlators with an odd number of W_i must vanish. Let us analyse the consequence of this rule for the free-field theory, and, therefore, for all amplitudes that do not depend on the coupling constant. Composites like $\text{Tr}\,W^2$ can only have non-vanishing even n-point functions. Moreover, OPEs involving $\text{Tr}\,W\bar{W}\,\text{Tr}\,W\bar{W}$ can never produce $\text{Tr}\,W^2$ operators, since the three-point function $\langle \text{Tr}\,W\bar{W}\,\text{Tr}\,W\bar{W}\,\text{Tr}\,W^2 \rangle$ vanishes. In the $N = 3$ language, the dilaton is a high component of the superfield $\text{Tr}\,W_i W_j$. The dilaton 3-point function vanishes, but the four-point function does not. The dilaton has no mixed 3-point functions

$\langle \text{Tr}\, W_i W_j \,\text{Tr}\, W_l \bar{W}_k \,\text{Tr}\, W_m \bar{W}_p \rangle$. It should be noticed that this symmetry is violated by perturbative corrections. It may also be a symmetry at the self-dual coupling, as the consistent truncation of the $N = 8$ supergravity seems to indicate, up to corrections of order $O(1/N)$.

We want to check these selection rules directly in the CFT. In free-field theory, a consistent subset of "chiral" operators is the irreducible $SU(3)$ components of $\text{Tr}\,(W_{i_1} \bar{W}_{i_2} ... W_{i_{2p-1}} \bar{W}_{i_{2p}})$. Using $N = 2$ techniques, it can be shown that any correlation function with different number of W and $\bar{W}$ does in fact vanish. This because the $N = 2$ hypermultiplet in W has vanishing $\langle \phi\phi \rangle$ propagator, but not vanishing $\langle \phi\bar{\phi} \rangle$ propagator. This vanishing is not surprising because U_1 is a symmetry of the free-field theory. We conclude that, in free-field theory and for those amplitudes that are independent of the coupling constant, the selection rules are valid. In the fully interacting theory, however, only a discrete subgroup of U_1 can be a symmetry, and only at a self-dual value of the coupling constant. This suggest that the selection rules may be valid for $N = 4$ Yang-Mills at the self-dual point.

We can also use the CFT/AdS correspondence to give further support to the validity of the selection rules at the self-dual point. We can start with $N = 4$ SYM and its dual description as type IIB string theory on $AdS_5 \times S^5$ at the self-dual value of the coupling, and perform a projection with the symmetry U_1, which is a combination of an element of $SL(2; Z)$ and a discrete element of the isometry group of S^5. This consistently truncates to an $N = 6$ theory, which has the $N = 6$ gauged supergravity as the effective action for the massless modes. Since the coupling constant is fixed, it is clear that we are not exploring the t'Hooft large-N limit. Maldacena's conjecture, however, applies whenever we can trust the supergravity approximation, i.e. whenever we can ignore higher-dimension operators in the expansion of the effective action of type IIB superstrings. One such limit is the large-N limit at fixed string coupling constant (see for instance [20]). In this limit, the α' expansion corresponds to the 1/N expansion of the theory. We conclude that the supergravity description supports the existence of selection rules for the $N = 4$ theory at the self-dual point at least in the large-N limit.

The selection rules clearly extend to arbitrary value of the coupling constant for all amplitudes that do not depend on the coupling constant in the large-N limit. According to a conjecture in [23], all three point functions of "chiral" operators belong to this class of amplitudes, and the result is valid also for finite N. A perturbative computation that agrees with this conjecture was performed in [24, 25]. It is known instead that the four-point functions of "chiral" operators do depend on the coupling constant [20][5].

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[5]Partial results on the four-point functions, based on the CFT/AdS correspondence, can be found in the literature [26].

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