

$U(1)$ -textures and Lepton Flavor Violation

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Abstract

$U(1)$ family symmetries have led to successful predictions of the fermion mass spectrum and the mixing angles of the hadronic sector. In the context of the supersymmetric unified theories, they further imply a non-trivial mass structure for the scalar partners, giving rise to new sources of flavor violation. In the present work, lepton flavor non-conserving processes are examined in the context of the minimal supersymmetric standard model augmented by a $U(1)$ -family symmetry. We calculate the mixing effects on the $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ rare decays. All supersymmetric scalar masses involved in the processes are determined at low energies using two loop renormalization group analysis and threshold corrections. Further, various novel effects are considered and found to have important impact on the branching ratios. Thus, a rather interesting result is that when the see-saw mechanism is applied in the 12×12 sneutrino mass matrix, the mixing effects of the Dirac matrix in the effective light sneutrino sector are canceled at first order. In this class of models and for the case that soft term mixing is already present at the GUT scale, $\tau \rightarrow \mu\gamma$ decays are mostly expected to arise at rates significantly smaller than the current experimental limits. On the other hand, the $\mu \rightarrow e\gamma$ rare decays impose important bounds on the model parameters, particularly on the supersymmetric scalar mass spectrum. In the absence of soft term mixing at high energies, the predicted branching ratios for rare decays are, as expected, well below the experimental bounds.

* Research partially supported by the TMR contract ERBFMRX-CT96-0090.

1 Introduction

In the last years there has been a lot of interest in lepton flavor violation, which can be a powerful tool in any attempt to classify different extensions of the Standard Model (SM) [1-4]. Indeed, there are various ways to enlarge the known particle spectrum in a manner that lepton number violation is generated. The new states may be right-handed neutrinos, additional vector-like heavy fermions, supersymmetric particles, or even all the above together as this is the usual case in supersymmetric grand unified theories (SUSY GUTS) and their string versions.

A well known result in the context of the non-supersymmetric standard model is the conservation of lepton flavor in the case of zero neutrino masses, while in the case of massive, non-degenerate light neutrinos, the amount of lepton flavor violation is proportional to the factor $\eta_\nu = \Delta m_\nu^2/M_W^2$ [5]. However, non-observation of double beta decay as well as other well known neutrino data, imply severe bounds on the light neutrino mass-squared differences Δm_ν^2 , leading to a high suppression of the ratio η_ν and therefore to all flavor violating processes depending on it. An analogous smaller suppression occurs if the process is mediated by heavy neutrinos. When supersymmetry enters in the game, the whole scene changes completely. Even in the absence of right handed neutrinos, flavor violations could occur via the exchange of supersymmetric particles. A large number of new parameters (sparticle masses, mixing angles, e.t.c.) appear in the calculations, therefore enlarging the number of possible decays, while the predicted branching ratios are now comparable to the present experimental bounds. Moreover, the soft breaking terms may violate the individual lepton numbers L_e, L_μ and L_τ by large amounts. Therefore, in the context of unification and low energy phenomenology scenarios, flavor-violating processes can provide useful constraints on the parameter space of a given model. Even in models with universal initial conditions for the scalar masses, the renormalization group runs of the slepton masses will give rise to lepton flavor violation that can be significant.

Thus, there are various sources of Lepton Flavor Violation (LFV) in SUSY models that may be tested in future searches [6]. Then, it is interesting to investigate them in the framework of flavor symmetries, which will give us a guideline on the amount of flavor violation that we may expect at the GUT scale. Indeed:

- 1) Unified supersymmetric theories with $U(1)$ family symmetries generate successfully the observed low energy hierarchy of the fermion mass spectrum and the quark mixing in terms of a minimum number of arbitrary parameters at the unification scale. In addition, a non-diagonal structure of the sparticle mass matrices comes out as a prediction of the theory. Rare processes are sensitive to the scalar mass matrix structure and non-diagonality of the latter usually, in a basis in which the

fermions are diagonal, leads to hard violation of flavor.

- 2) These flavor violating effects are enhanced in particular when the higgs vacuum expectation value (vev) ratio ($\tan \beta$) is large. In fact, many models based on a single unified gauge group (like $SO(10)$ theory) predict equality of the top and bottom Yukawa couplings at the GUT scale, and therefore a large value of $\tan \beta$ is implied. As a result, the 6×6 structure of the slepton mass matrix enhances further the lepton mixing effects.
- 3) When right-handed neutrinos enter in the model, the theory faces another challenge. Firstly, Dirac mass matrices arise of the order of the up-quark masses. In practice the majority of unified theories imply a Dirac matrix at the unification scale equal to the up quark matrix. Charged leptons and neutrinos are no longer diagonal in the same basis and a leptonic mixing matrix, similar to the Cabibbo Kobayashi Maskawa matrix V_{CKM} for the quarks, is unavoidable. Secondly, the Dirac mass matrix itself has an even more intriguing role, since, due to renormalization effects on the supersymmetric scalar sector, it modifies the slepton mass matrix at low energies. Moreover, it enters in the construction of the 12×12 - sneutrino mass matrix which in principle would have the potential to give rise to additional flavor-violating effects. Nevertheless, since only the effective light sneutrino mass matrix is relevant in the calculation, we will show in this work that the m_D effects are canceled at first order, when the see-saw mechanism is applied.
- 4) The contributions of the trilinear scalar mass parameter of the potential, i.e, A -term contributions, are also discussed in this work in some detail. Due to the absence of direct experimental information about its values, sometimes its effects are not considered at all. However, imposing even a zero value for initial condition at the GUT scale, its low energy value will not be unmarked. Renormalization group running effects will drive its value to magnitudes comparable with those of scalar masses, whereas its contribution to the branching ratios is of particular importance.

In this work, we calculate the branching ratios for lepton flavor violating decays in models with abelian flavor symmetries which emulate the situation in many string constructions. We concentrate in particular in the class of models for fermion masses with one family symmetry which is the simplest possibility and have been firstly proposed in [7]. This calculation offers an important test with regard to the viability of these particular models, but also gives further insight on the general predictions of family symmetries. The motivation for their introduction is to explain the observed fermion mass hierarchy yet they have further consequences as is the case of flavor violations. The lepton sector is

ideal for such checks (in the quark sector, large uncertainties may enter in the calculations, due to poor knowledge of hadronic matrix elements).

The analysis follows the lines of [8], where some first estimates for flavor-violation in this class of models have been presented. This paper is organized as follows:

In section two we briefly analyze the basic features of a generalized class of SUSY models with one $U(1)$ -family symmetry. We give the forms of fermion and scalar mass matrices and discuss their role in flavor violating parameters.

In section three, we derive the 12×12 sneutrino mass matrix and show that the effective 3×3 light sneutrino matrix entering the flavor violating decays does not involve Dirac neutrino mass contributions.

In sections four and five we give the loop calculations for the various amplitudes and analyze the procedure for our numerical investigations.

In section six we present the results treating separately the cases with and without scalar mass mixings at M_{GUT} . Finally, in section 7 we present our conclusions.

2 Mass matrices

As has been stressed in the introduction, one of the main advantages of the $U(1)$ -family symmetries ($U(1)_f$) is the determination of the hierarchy of the mass spectrum and mixing angles and the prediction of many other parameters of the minimal supersymmetric standard model (including the scalar matrices) with only a minimal set of arbitrary parameters. In the simplest case of only one $U(1)_f$ symmetry, the fermion mass hierarchy is successfully obtained using an additional singlet field ϕ which develops a vev one order of magnitude below the string scale M_U . Throughout our calculations, we will further assume the existence of a grand unified symmetry at a scale M_{GUT} without specifying the gauge group. (We have in mind models with intermediate gauge symmetry groups[9] where the gauge couplings run together from M_U down to M_{GUT} .) Below the unification scale, only the minimal supersymmetric spectrum is assumed, therefore the unification point will be taken at $\sim 10^{16}$ GeV. In this scheme, the low energy parameters involved in our subsequent calculations depend on M_{GUT} , the common value of the gauge coupling g_{GUT} and the ratio of the singlet vev over the string scale, $\epsilon \sim \langle \phi \rangle / M_U$. When the $U(1)_f$ symmetry is exact, only the third generation has a Yukawa term in the superpotential, whilst all mixing angles are zero and lighter generations remain massless and uncoupled. When ϕ acquires a vev, the $U(1)_f$ -symmetry is broken and mass terms fill in the rest of the mass matrix entries with Yukawa terms suppressed by powers of the ratio ϕ/M_U . In simple supergravity models, a similar situation occurs also in the scalar sector; at tree

level, however, there appear three instead of one, diagonal mass terms, one for each generation. Mixing (off-diagonal) terms in the scalar sector shows up when higher NR-terms are included in the Kähler potential. Thus, $U(1)_f$ -symmetries imply also a non-trivial structure for the corresponding scalar mass matrices. This additional structure may be responsible for new hard flavor violations. The existing experimental bounds on flavor-violating processes are therefore going to give us an indication as to which structures are viable for the scalar mass matrices.

Which are the elements that determine the form of the scalar mass matrices? Clearly, the charges of the various fields under the flavor symmetry will play a dominant role. Moreover, as we discussed in the previous paragraph, the mass and mixing hierarchies depend on expansion parameters that are generated when singlet fields acquire vev's. What these vev's can be, depends on the flat directions of a given theory. Once the flat directions and the $U(1)_f$ charges of a particular model have been fixed, the scalar mass matrix structure may be easily computed through the Kähler function $\mathcal{G} = \mathcal{K} + \log |\mathcal{W}|^2$ where \mathcal{W} is the superpotential and \mathcal{K} has the general form

$$\mathcal{K} = -\log(S + S^*) - \sum h_n \log(T_n + T_n^*) + Z_{ij^*}(T_n, T_n^*) Q_i Q_j^* + \dots \quad (1)$$

with Q_i being the matter fields, S the dilaton, whereas T_n are the other moduli fields. The scalar mass matrices are determined by Z_{ij^*} and \mathcal{W} . The form of the Z_{ij^*} function is dictated by the modular symmetries and depends on the moduli and the modular weights of the fields. Thus, at the tree level, the diagonal terms are the only non-zero entries in the scalar mass matrices. Higher order terms allowed by the symmetries of the specific model fill in the non-diagonal entries.

The lepton Yukawa interactions which appear in the superpotential in the presence of the right handed neutrino are

$$\mathcal{W}_{lep} = e^{cT} \lambda_e \ell H_1 + N^c \lambda_D \ell H_2 + \lambda_N \chi N^c N^c. \quad (2)$$

Here ℓ is the left lepton doublet, e^c is the right singlet charged lepton, N^c is the right-handed (RH) neutrino and $\lambda_{e,D,N}$ represent Yukawa coupling matrices in flavor space. Also, H_1 and H_2 are higgs doublets and χ stands for an effective singlet which may acquire a vev at a large scale.

In addition, soft supersymmetry breaking terms generate mass matrices for the charged slepton fields, denoted by \tilde{m}_ℓ , \tilde{m}_{e_R} . Denoting the various fields collectively with z , in supergravity the scalar potential is given by

$$V = e^{G(z)} \left(G_I G_{I\bar{J}}^{-1} G_{\bar{J}} - 3 \right) + |D|^2 \quad (3)$$

where $|D|^2$ represents the contribution of the D -terms in the potential. Also, with G_I , we denote the derivatives of G with respect to the fields z_I , i.e.,

$$G_I \equiv \frac{1}{\mathcal{W}} \mathcal{D}_I \mathcal{W} \quad (4)$$

where $\mathcal{D}_I \mathcal{W} = \partial_I \mathcal{W} + \mathcal{W} \partial_I \mathcal{K}$ is the Kähler derivative. Writing explicitly the various fields, in the low energy limit one encounters the following scalar mass terms in V

$$\begin{aligned} V = & m_1^2 H_1^* H_1 + m_2^2 H_2^* H_2 + m_q^2 \tilde{q}^* \tilde{q} + m_{\tilde{u}^c}^2 \tilde{u}^{c*} \tilde{u}^c + m_{\tilde{d}^c}^2 \tilde{d}^{c*} \tilde{d}^c \\ & + m_{\tilde{\ell}}^2 \tilde{\ell}^* \tilde{\ell} + m_{\tilde{e}^c}^2 \tilde{e}^{c*} \tilde{e}^c + m_{\tilde{N}^c}^2 \tilde{N}^{c*} \tilde{N}^c \\ & + \{ \epsilon_{ab} (m_3^2 \tilde{H}_1^a \tilde{H}_2^b + A_u \lambda_u \tilde{q}^a \tilde{u}^c \tilde{H}_2^b + A_d \lambda_d \tilde{q}^a \tilde{d}^c \tilde{H}_1^b \\ & + A_l \lambda_e \tilde{\ell}^a \tilde{e}^c \tilde{H}_1^b + A_\nu \lambda_D \tilde{\ell}^a \tilde{N}^c \tilde{H}_2^b) + A_N \lambda_N \chi \tilde{N}^c \tilde{N}^c \} + \text{h.c.} + \text{q.t.} + \dots, \quad (5) \end{aligned}$$

In the above equation a and b are $SU(2)$ indices. The dots stand for scalar trilinear F -terms and q.t. denotes quartic terms in the scalar fields. Also terms proportional to B parameter are not shown explicitly here. The Higgs mass terms contain two contributions: the first arising from the superpotential and the second from the soft supersymmetry breaking parameters: $m_i^2 = \mu^2 + m_{H_i}^2$, with $i = 1, 2$.

We present here the relevant mass matrices of a model whose successful fermion mass hierarchy is predicted by $U(1)_f$ symmetries. We work in the low energy effective model based on the $SU(3) \times SU(2) \times U(1)$ gauge group with an additional $U(1)_f$ symmetry [7]. After the implementation of this symmetry, the fermion matrix for charged leptons in this model is given by

$$m_\ell \approx \begin{pmatrix} \tilde{e}^{2|a+b|} & \tilde{e}^{|a|} & \tilde{e}^{|a+b|} \\ \tilde{e}^{|a|} & \tilde{e}^{2|b|} & \tilde{e}^{|b|} \\ \tilde{e}^{|a+b|} & \tilde{e}^{|b|} & 1 \end{pmatrix} m_\tau \quad (6)$$

where the parameter \tilde{e} is some power of the singlet vev scaled by the unification mass, while a, b are certain combinations of the lepton and quark $U(1)_f$ -charges. Order one parameters c_{ij} in front of the various entries (not calculable in this simple model) are assumed, to reproduce the fermion mass relations after renormalization group running. These parameters c_{ij} are usually left unspecified, here however their exact values are necessary for a reliable calculation of the lepton violating processes.

A phenomenologically viable choice for the charges is to take $a = 3$ and $b = 1$. A successful lepton mass hierarchy in this case is obtained for the choice $\tilde{e} = 0.23$. In this case, a possible choice of the coefficients c_{ij} is given by $c_{12} = c_{21} = 0.4$, $c_{22} = 2.2$, with the rest of the coefficients being unity. We point out that the choice of the parameter b is not completely determined by the lepton mass texture. In fact there is a second possibility, with $b = 1/2$ where the fermion mass hierarchy is also consistent with the low energy data. On the contrary, the choice of b , as well as the choice of coefficients, will have a significant

impact on the magnitude of the rare processes. Thus, flavor violation is a powerful ‘tool’ and an invaluable criterion of the viability of a certain choice. A detailed discussion on this question is one of the main points of the present analysis and will be given after the calculations on the branching ratios will be presented in the subsequent sections.

The Dirac mass matrix in the above model has a similar structure. Due to the simple $U(1)$ structure of the theory, the powers appearing in its entries are the same as the lepton mass matrix; however, the expansion parameter is in general different[10]. Thus, its form is given by

$$m_{\nu_D} \approx \begin{pmatrix} \epsilon^{2|a+b|} & \epsilon^{|a|} & \epsilon^{|a+b|} \\ \epsilon^{|a|} & \epsilon^{2|b|} & \epsilon^{|b|} \\ \epsilon^{|a+b|} & \epsilon^{|b|} & 1 \end{pmatrix} m_{top} \quad (7)$$

The choice of charges $a = 3, b = 1$ allows to identify the Dirac mass matrix with the up-quark mass matrix. Again, order one coefficients have to be introduced in the quark mass matrix in order to obtain consistency with the experimentally determined masses. We denote here the corresponding coefficients multiplying the entries of (7) with d_{ij} . A choice of coefficients leading to correct up-quark masses is obtained for $d_{12} = d_{21} = .5, d_{32} = d_{23} = 1.5$, with the rest of the coefficients being unity. In the case of the up-quark matrix a second expansion parameter[7] is introduced with the value $\epsilon = .053$.

The RH-Majorana mass matrix is constructed from terms of the form $\chi N^c N^c$ where χ is an effective singlet. In GUT models, χ is a combination of scalar (Higgs) fields. Obviously, the structure of M_N depends on the origin of the singlet χ as well as its charge[10, 11]. Thus, in a class of models this singlet may arise from the combination $\tilde{N}^c \tilde{N}^c$ where \tilde{N}^c is the scalar component of the RH-antineutrino supermultiplet. There are therefore various structures of the Majorana matrix, depending on the specific choice of the χ -charge. However, as we are going to show analytically, due to cancellations, the results are not sensitive to the structure of M_N ¹.

The scalar mass matrices of this model are built using the potential mentioned in the beginning of this section. In particular, for the sleptons we obtain at the GUT scale

$$\tilde{m}_{\ell, e_R}^2 \approx \begin{pmatrix} 1 & \tilde{\epsilon}^{|a+2b|} & \tilde{\epsilon}^{|a+b|} \\ \tilde{\epsilon}^{|a+2b|} & 1 & \tilde{\epsilon}^{|b|} \\ \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix} m_{3/2}^2 \quad (8)$$

At this point, we have determined all the necessary ingredients in order to build the basic quantity which determines the flavor violations in SUSY theories. This is the 6×6

¹This, we checked numerically, by picking particular forms of M_N , which also fit the light neutrino data. For example, a zero singlet charge leads to a M_N form similar to that of the Dirac mass matrix. A singlet charge $Q = -1$ leads to an interesting form with large mixing in the 2-3 generations and a two-fold degeneracy, suggesting a solution of the atmospheric neutrino puzzle through the $\nu_\mu \nu_\tau$ oscillations [12].

slepton mass matrix which takes the form

$$\begin{pmatrix} \tilde{m}_\ell^2 & ((A_\ell + \mu \tan \beta)m_\ell)^\dagger \\ (A_\ell + \mu \tan \beta)m_\ell & \tilde{m}_{e_R}^2 \end{pmatrix}. \quad (9)$$

\tilde{m}_{ℓ, e_r}^2 are scale dependent, their value being given by the renormalization group equations. Their initial conditions at M_{GUT} are defined by the matrices (8) and have a significant effect in flavor violations. In addition, as it has been extensively discussed in the literature, due to RGE running slepton mass matrices receive corrections proportional to the Dirac mass matrix. Charged lepton and Dirac mass matrices are not simultaneously diagonalized; thus in the basis where m_ℓ is diagonal, the 3×3 slepton mass matrix acquires a non-diagonal contribution of the form

$$\delta \tilde{m}_\ell^2 \propto \frac{1}{16\pi^2} (3 + a^2) \ln \frac{M_{GUT}}{M_N} \lambda_D^\dagger \lambda_D m_{3/2}^2 \quad (10)$$

where λ_D is the Dirac Yukawa coupling and the proportionality factor depends on the scalar mass parameters squared while the parameter a is related to the trilinear mass parameter $A_l = am_{3/2}$. Thus, even if one starts with a diagonal slepton mass matrix, there are important off-diagonal contributions due to the existence of N^c . This effect, however, will prove less important in theories where the scalar mass matrix textures are also determined by $U(1)$ symmetries.

3 Sneutrino mass matrix

The sneutrino mass matrix is also determined similarly. It is a 12×12 structure given in terms of the 3×3 Dirac, Majorana and slepton mass matrices. It is generally expected that –as in the case of charged sleptons– the Dirac term induces considerable mixing effects. We will show here that this is *not* the case in the sneutrino mass matrix.

This 12×12 matrix is rather complicated and not easy to handle. Vastly different scales are involved and numerical investigations should be carried out with great care. Its form

is as follows:

$$\begin{array}{c|cccc}
& \tilde{\nu} & \tilde{\nu}^* & \tilde{N}^c & \tilde{N}^{c*} \\
\hline
\tilde{\nu}^* & m_{\tilde{l}}^2 + m_D^* m_D^T & 0 & m_D^* M^T & (A_\nu^* + \mu \cot \beta) m_D^* \\
\tilde{\nu} & 0 & m_{\tilde{l}}^2 + m_D m_D^\dagger & (A_\nu + \mu \cot \beta) m_D & m_D M^+ \\
\tilde{N}^{c*} & M^* m_D^T & m_D^\dagger (A_\nu^* + \mu \cot \beta) & m_N^2 + M^* M^T + m_D^* m_D^\dagger & A_N^* M^* \\
\tilde{N}^c & m_D^T (A_\nu + \mu \cot \beta) & M m_D^\dagger & A_N M & m_N^2 + M M^+ + m_D m_D^\dagger
\end{array} \quad (11)$$

One can construct an effective 6×6 matrix for the light sector, by applying matrix perturbation theory, similar to the see-saw mechanism. The result up to second order is:

$$(m_{\tilde{\nu}}^2)_{eff} = \begin{pmatrix} m_{\tilde{l}}^2 - (A_\nu + \mu \cot \beta)(A_\nu - 2A_N) \cdot & ((2A_\nu + A_N) + 2\mu \cot \beta) \cdot \\ (m_D M^{-2} m_D^\dagger) & (m_D M^{-1} m_D^\dagger) \\ ((2A_\nu + A_N) + 2\mu \cot \beta) \cdot & m_{\tilde{l}}^2 - (A_\nu + \mu \cot \beta)(A_\nu - 2A_N) \cdot \\ (m_D M^{-1} m_D^\dagger)^* & (m_D M^{-2} m_D^\dagger) \end{pmatrix} \quad (12)$$

The first and second order terms are obtained assuming all parameters as real and the A -matrices proportional to the identity. Notice that the second order terms along the diagonal can be neglected. The first order off-diagonal terms must be retained, since they lead to complete mixing of the pairwise degenerate states. This, however, does not affect the flavor-violating branching ratio.

The simplicity of this result is rather astonishing. We note that after the ‘see-saw’ mechanism is applied, the Dirac neutrino mass matrix contribution in the effective light sneutrino mass sector is essentially negligible. Moreover, there is an additional benefit, since the complication of the initial 12×12 mass matrix can now be avoided. A direct numerical calculation of mass eigenstates and mixing angles would be a hard task, due to the vastly different scales.

4 Amplitudes for flavor violating processes

Figure 1 shows the one-loop diagrams relevant to the $\mu \rightarrow e\gamma$ process. The corresponding $\tau \rightarrow \mu\gamma$ -decay is represented by an analogous set of graphs. There are also box-diagrams

contributing to this process; they are however relatively suppressed.

The electromagnetic current operator between two lepton states l_i and l_j is given in general by

$$\begin{aligned}\mathcal{T}_\lambda &= \langle l_i(p-q) | \mathcal{J}_\lambda | l_j(p) \rangle \\ &= \bar{u}_i(p-q) \{ m_j i \sigma_{\lambda\beta} q^\beta (A_M^L P_L + A_M^R P_R) \} u_j(p)\end{aligned}\quad (13)$$

where q is the photon momentum. The A_M 's have contributions from neutralino-charged slepton (n) and chargino-sneutrino (c) exchange

$$A_M^{L,R} = A_{M(n)}^{L,R} + A_{M(c)}^{L,R}\quad (14)$$

The amplitude of the process is then proportional to $\mathcal{T}_\lambda \epsilon^\lambda$ where ϵ^λ is the photon polarization vector. An easy way to determine the loop momentum integral contribution to the A_M 's is to search, in the corresponding diagram, for terms of the form $(p \cdot \epsilon)$ and make the replacement $2(p \cdot \epsilon) \rightarrow i \sigma_{\lambda\beta} q^\lambda \epsilon^\beta$. Defining the ratio $x = M^2/m^2$, where M is the chargino (neutralino) mass and m the sneutrino (charged slepton) mass, the following functions appear in the A_M term

$$\begin{aligned}A_{M(n)} : & \quad \frac{1}{6(1-x)^4} (1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x) & \text{(L - L amplitude)} \\ & \quad \frac{1}{(1-x)^3} (1 - x^2 + 2x \log x) \sqrt{x} & \text{(L - R amplitude)} \\ A_{M(c)} : & \quad \frac{1}{6(1-x)^4} (2 + 3x - 6x^2 + x^3 + 6x \log x) & \text{(L - L amplitude)} \\ & \quad \frac{1}{(1-x)^3} (-3 + 4x - x^2 - 2 \log x) \sqrt{x} & \text{(L - R amplitude)}\end{aligned}\quad (15)$$

where m_{l_j} is the mass of the l_j lepton.

Notice in the L-L amplitudes the lack of terms proportional to the gaugino mass M which cancel. The Branching Ratio (BR) of the decay $l_j \rightarrow l_i + \gamma$ is given by

$$BR(l_j \rightarrow l_i \gamma) = \frac{48\pi^3 \alpha}{G_F^2} \left((A_M^L)^2 + (A_M^R)^2 \right)$$

5 Inputs and Procedure

The branching ratio formulae for the $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays involve the masses of most of the supersymmetric particles. It is important therefore for any given set of GUT parameters to know precisely all masses and the other low energy parameters. In the present work, this is obtained by numerical integration of the renormalization group equations of the MSSM with right handed neutrinos. The renormalization-group equations can be found in many papers (see for example[13]).

We evaluate the coupling constants, using renormalization-group equations at two loops. Threshold effects are also taken into account, by decoupling every sparticle at the scale of its running mass $Q = m_i(Q)$. Below the scale m_t , we use the SM beta functions..

Our analysis uses as input values the unified coupling constant α_G , at the GUT scale, the third generation Yukawa couplings $\lambda_t, \lambda_{b-\tau}$, the common scalar diagonal scalar masses $m_{3/2}$, the gaugino mass $m_{1/2}$, the (effective) Higgs bilinear coupling μ , and the ratio of the Higgs vev's described by $\tan\beta$ and the flavor-symmetric soft-breaking parameter A_0 .

Our integration procedure consists on iterative runs of the renormalization-group equations from M_{GUT} to low energies and back, for every set of input parameters $m_{1/2}, m_{3/2}, A_0$ and $\tan\beta$, until agreement with experimental data is achieved. The values for α_G and M_{GUT} are obtained consistently with α_{em}, α_3 and $\sin^2\theta_W$ at m_Z . Supersymmetric corrections to $\sin^2\theta_W$ are also considered[14] .

A similar procedure is followed to obtain the GUT values for the third generation Yukawa couplings. In particular, the GUT value for λ_t is adjusted by requiring the top physical mass to be $m_t = 175 \pm 5GEV$. Similarly, we obtain the value of the unified $\lambda_{b-\tau}$, by requiring the correct prediction for $m_\tau = 1.778GEV$. In all the cases that we analyzed, values for m_b consistent with experiments are found once QED and QCD corrections are taken into account [15].

The value of the μ parameter (up to its sign) can be expressed in terms of the other input parameters by means of symmetry breaking conditions. To this end, we use the semi-analytic formulae including one loop corrections, as given in ref.[16]. Finally, the chargino and neutralino masses are obtained by diagonalization of the 4×4 neutral and the 2×2 charged matrices as described in [17]. The RG evolution of the eigenstates, starting from universal initial conditions at the GUT scale, is properly taken into account.

We then explore the values of the BR for all significant values of the input parameters $m_{1/2}, m_{3/2}, A_0$ and $\tan\beta$.

If we consider common scalar masses and trilinear terms at the GUT scale, leptons and sleptons will be diagonal in the same superfield basis. However, due to the presence of (a) the non diagonal GUT terms Δ at the GUT scale, and (b) the appearance of λ_D in the RG equations, the lepton Yukawa matrix and the slepton mass matrix can not be brought simultaneously to a diagonal form at the scale of the heavy Majorana masses. Therefore, lepton number will be violated by the one loop diagrams of fig. 1.

We define the unitary matrices diagonalizing the Yukawa mass textures λ_D and λ_e , as follows

$$\lambda_D^\delta = T_R^T \lambda_D T_L \tag{16}$$

$$\lambda_e^\delta = V_R^T \lambda_e V_L \quad (17)$$

Here, the index δ indicates a diagonal form. Then, the mixing matrix K in the lepton sector, defined in analogy to V_{CKM} is given by the product

$$K = T_L^\dagger V_L \quad (18)$$

The charged slepton masses are obtained by numerical diagonalization of the 6×6 matrix

$$\tilde{m}_e^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{RL}^2 & m_{RR}^2 \end{pmatrix} \quad (19)$$

where all entries are 3×3 matrices in the flavor space. In the superfield basis where λ_e is diagonal, it is convenient for later use to write the 3×3 entries of (19) in the form:

$$m_{LL}^2 = (m_l^\delta)^2 + \delta m_N^2 + \Delta_L + m_l^2 + M_Z^2 \left(\frac{1}{2} - \sin^2 \theta_W \right) \cos 2\beta \quad (20)$$

$$m_{RR}^2 = (m_{e_R}^\delta)^2 + \Delta_R + m_l^2 - M_Z^2 \sin^2 \theta_W \cos 2\beta \quad (21)$$

$$m_{RL}^2 = (A_e^\delta + \delta A_e + \mu \tan \beta) m_l \quad (22)$$

$$m_{LR}^2 = m_{RL}^{2\dagger} \quad (23)$$

Each component above has a different origin and gives an independent contribution in the Branching Ratios. We further wish to emphasize the following:

- $(m_l^\delta)^2, (m_{e_R}^\delta)^2, A_e^\delta$ denote the scalar diagonal contribution of the corresponding matrices; their entries are obtained by numerical integration of the RG equations as described before. We consider $m_{3/2}^2$ as the common initial condition for the masses at the GUT scale, while the trilinear terms scale as $am_{3/2}$. Since in the RGEs we consider only third generation Yukawa couplings and common initial conditions at the GUT scale for the soft masses, our treatment is equivalent to working in superfield basis, such that: (i) λ_D is diagonal from the GUT scale to the intermediate scale and (ii) λ_e is diagonal from the intermediate scale to low energies. The change of bases will produce a shift in the diagonal elements of the soft mass matrices at the GUT and at the intermediate scale. This effect is negligible (less than one percent).
- δm_N^2 and δA_l stand for the off-diagonal terms which appear due to the fact that λ_D and λ_e may not be diagonalized simultaneously. The intermediate scale that enters in the calculation (which is the mass scale for the neutral Majorana field M_N) is defined by demanding that neutrino masses $\approx 1eV$ are generated via the “see-saw” mechanism. This sets the M_N scale to be around the value 10^{13} GeV. Then, the following values are obtained:

$$\delta m_N^2 = K^\dagger \left[m_l^2(m_N) \right] K|_{nondiagonal} \quad (24)$$

$$\delta A_l = V_L A_l(m_N) V_L^\dagger|_{nondiagonal} \quad (25)$$

- The following values for Δ_L and Δ_R are defined at the GUT scale:

$$\Delta_L = V_L^\dagger \Delta V_L \quad (26)$$

$$\Delta_R = V_R^\dagger \Delta V_R \quad (27)$$

The effective 3×3 sneutrino mass matrix squared has the same form as the m_{LL}^2 part of the 6×6 charged slepton one, with the difference that now Dirac masses are absent (in consistency with what we have shown in the analysis of the 12×12 sneutrino matrix). Thus,

$$\tilde{m}_\nu^2 = (m_i^\delta)^2 + \delta m_N^2 + \Delta_L + \frac{1}{2} M_Z^2 \cos 2\beta \quad (28)$$

It is illustrative to write our results as an approximate function of the input parameters. Below we give the numerical range for the sneutrino and the A parameter as these are defined in (24),(25):

$$\delta m_N^2 \approx \begin{pmatrix} 0 & (4.2 - 6.3) \times 10^{-5} & (2.3 - 3.3) \times 10^{-4} \\ (4.2 - 6.3) \times 10^{-5} & 0 & (0.7 - 1.1) \times 10^{-2} \\ (2.3 - 3.3) \times 10^{-4} & (0.7 - 1.1) \times 10^{-2} & 0 \end{pmatrix} (3 + a^2) m_{3/2}^2 \quad (29)$$

$$\delta A_\ell \approx \begin{pmatrix} 0 & (1.2 - 1.7) \times 10^{-4} & (5.2 - 7.3) \times 10^{-4} \\ (1.2 - 1.7) \times 10^{-4} & 0 & (1.0 - 1.4) \times 10^{-2} \\ (5.2 - 7.3) \times 10^{-4} & (1.0 - 1.4) \times 10^{-2} & 0 \end{pmatrix} A_0 \quad (30)$$

In the last two equations, the ranges in parentheses correspond to $\tan \beta$ values between 14 and 3 (larger contributions are obtained for smaller values of $\tan \beta$).

6 Results

We have seen that, when $U(1)$ -family symmetries are taken into account non-diagonality in the mass matrices is generic in both the fermion and the scalar sector. This may generate unacceptably large flavor-violating effects. It is possible that cyclic permutation symmetries between generations and universal anomalous $U(1)$ -factors may prevent mixing effects in the supersymmetric mass matrices [18]. In our results, we are considering separately two distinct cases: First, we will consider the case where the scalar mass matrices are protected from mixing effects by some kind of symmetry not affecting the fermion mass sector. Second, we will allow mixing effects in both sectors, and recalculate the branching ratios and the new bounds obtained on the sparticle spectrum.

1). Case without scalar mass mixing at the GUT scale, $\Delta = 0$.

We start with the process $\mu \rightarrow e\gamma$, in the absence of mixings at the GUT scale. Let us denote by (n) the contributions from the neutralino-charged slepton exchange, and by (c) the ones from the chargino-sneutrino. Then,

$$A_M^{L,R} = A_{M(n)}^{L,R} + A_{M(c)}^{L,R}. \quad (31)$$

The various amplitudes appear in Fig.2. As we can see, the two contributions to A_M^R (dashed lines) are of the same order of magnitude and opposite signs, while their magnitude decreases with $\tan\beta$. Around a certain value of $m_{3/2}$, there is a partial cancellation of both amplitudes, leading to a decrease of the expected $BR(\mu \rightarrow e + \gamma)$, as we can see in Figs.3, 4 and 5.

The contribution to A_M^L comes almost exclusively from $A_{M(n)}^L$, since the chargino exchange contribution $A_{M(c)}^L$ (Feynman diagram b in Figure 1) arises due to Yukawa interactions and is about three orders of magnitude smaller than the other contributing amplitudes. It is important to make this remark, since when there are no mixings in the right-handed slepton masses, $A_{M(n)}^L$ become relevant due to the presence of non-diagonal mixings from the trilinear terms δA (otherwise, these contributions are of the same order as $A_{M(c)}^L$). The effect of δA can be seen clearly in Fig. 5. For the initial condition $A_0 = 0$, δA is significantly suppressed, and hence a dramatic decrease of the $BR(\mu \rightarrow e + \gamma)$ is observed. On the other hand, the cases with $A_0 \neq 0$ have a remarkable difference with the previous one. All curves now are smoother while there are no particular $m_{3/2}$ -values where BR exhibit large suppression.

Although we start in our case with universal soft masses at the GUT scale, there is an analogy with the situation of the $SU(5)$ model discussed elsewhere [19, 20] in the following sense. Assuming we are in a basis where Yukawa matrices are diagonal, the renormalization of the universal soft mass terms from M_{GUT} down to the RH-neutrino mass scale M_N will split mass parameters of different generations. As a result, flavor universality in the scalar sector at M_N is lost. In the $SU(5)$ case, the deviations from universality arise due to the renormalization group running from the Planck scale M_{Pl} down to the $SU(5)$ scale M_{GUT} . Lepton flavor violation diagrams arise due to the non-universality of the right-handed slepton masses at the GUT scale. In contrast to the case we present here, the main contributions to the $BR(\mu \rightarrow e\gamma)$ arise from the neutralino exchange diagram amplitudes $A_{M(n)}^L$, while there is a similar cancellation to the one described in the above paragraph due to the fact that the two main contributions enter in the calculation with opposite sign. ² In $SO(10)$ unified models [21], LFV arises due to the non-universality of

²In [19], the LL and LR diagrams are discussed separately, while in ours both contributions are included in the single neutralino exchange.

both right and left sleptons, and hence this cancellation does not take place.

We may further compare our results directly with the results of [22], where a similar effect is observed, for the string-embedded version ALR of the Pati-Salam model [24]³. In this work, large values for $\tan\beta$ are considered and the contribution arising from the chargino exchange diagram is the dominant one. Moreover, in this analysis of $\tilde{\nu}_L$ terms, Dirac masses in addition to the soft left lepton masses are also included. For a certain value of $m_{3/2}$, the renormalization-group effects on the soft masses are canceled by the Dirac masses, hence the masses of $\tilde{\nu}_L$ become universal and the contribution from the chargino diagram vanishes. Then, the $BR(\mu \rightarrow e + \gamma)$ decreases dramatically. Although this effect agrees with our results, we would like to emphasize that the small value of the BR for certain values of $m_{3/2}$, arises due to a cancellation of two contributing amplitudes. Note that the contribution of the Dirac matrix to the sneutrino masses is absent in our analysis, as we have shown in our treatment of the full neutral scalar mass.

Figs.3, 4 and 5, show the effects of the changes of the input parameters in the total $BR(\mu \rightarrow e + \gamma)$. The changes in the non-diagonal elements of the scalar matrices can be induced from the approximate formulas that we presented, however the behavior of the BR is not correlated in all cases to the increase in the mixing. More precisely:

- Fig.3 shows the increase of the BR as the gaugino masses decrease. We can see that for a fixed value of $\tan\beta = 7$, and values of $m_{1/2}$ leading to a SUSY mass spectrum inside the experimental limits, the predicted values for the BR are two orders of magnitude lower than the experimental bounds.
- Fig.4 shows the change of the predicted BR for fixed values of $m_{1/2}$. Here, we can see an increase of the BR with $\tan\beta$ (solid lines). The dashed lines are chosen in a way that maximal BR 's are obtained; still we can see that the BR 's stay below the experimental bounds.
- Fig.5 shows the effect of A_0 in the calculation. In this case, the behavior of the BR is directly correlated to the increase in the scalar mixings with A_0 . The sign of A_0 has very little influence in the total result, since it is the square of the A -parameter that enters in the relevant renormalization-group equations.

2). Case with scalar mass mixing at the GUT scale, $\Delta \neq 0$.

Let us now proceed to discuss in detail what happens in the case that mixing of soft terms (arising through $\Delta_{L,R} \neq 0$) is already present at the GUT scale. Here we should note that the contribution from $\Delta_{L,R} \neq 0$ (which is independent of $\tan\beta$ and $m_{1/2}$),

³The phenomenology of these string inspired models has been discussed in [25].

is much bigger than δm_N^2 . This implies that there will be a dramatic increase of all the Branching Ratios and all previously noted effects have to be reconsidered. Note for example that, since m_{RR}^2 is now non-diagonal, additional amplitude contributions destroy the cancellations that we observed in the previous case. Fig. 6 shows that the absolute value of A_M^L is now bigger than A_M^R , (the contribution from $A_M^L(c)$ to A_M^L is again three orders of magnitude smaller than the others, since it involves Yukawa couplings).

The modifications in the case that mixing of soft-terms occurs at the GUT scale, are presented in Figures 7, 8. Changes of the BR with A_0 are less relevant in this case since, as we mentioned before, the dominant mixing terms in the soft mass matrices are independent of it. We have used $A_0 = -1.5m_{3/2}$ in all the calculations of the rest of the section.

Figure 7 indicates the variation of the BR as a function of $m_{3/2}$, for $m_{1/2} = 200, 300, 500$ GeV and $\tan\beta = 7$. As we see, for large values of $m_{3/2}$ the relevant branching ratio exhibits a fixed-point behavior. The reason for this effect is that, while $m_{1/2}$ enters in the calculation via gaugino masses, $m_{3/2}$ multiplies the scalar matrices and thus dominates the flavor-violating processes in the case of non-zero off-diagonal contributions at the GUT scale.

Figure 8 finally, shows the increase of the BR as $\tan\beta$ increases, for fixed values of $m_{1/2} = 300$ GeV and $A = -1.5m_{3/2}$ (solid lines). Then, the experimental bounds impose severe constraints on the allowed range of $(\tan\beta, m_{1/2}, m_{3/2})$ that one may have. For certain combinations of these terms, we predict BR below the experimental limits. This means that the model we analyze, with non-universal soft terms at the GUT scale, is consistent with the current experimental limits for low masses of the sleptons and high masses for gauginos, when low values of $\tan\beta$ are considered (dashed line).

Similar considerations may be done for $\tau \rightarrow \mu\gamma$. This is presented in fig.9. Here, the current experimental bounds are not as strict as in the previous case. Then, in the framework of the models that we are discussing, we find that, for a wide region of the SUSY parameter space, the predicted BR is well below the experimental bounds. However, the prediction exceeds experimental limits for larger values of $\tan\beta$.

Finally, in fig. 10 we compare our results with the choice of parameters made in [8]. The choice of $b = 1/2$ increases the values of the mixing terms of the scalar matrices at the GUT scale as can be seen from (8), hence for this choice of parameters the results obtained for the BR are one order of magnitude bigger than in our case. For completeness, we note that in [8], the choice of the Yukawa coefficients for the lepton matrix 6 were $c_{11} = 4.0$, $c_{12} = c_{21} = 0.9$, $c_{22} = 1.08$, $c_{33} = 1.9$. In our case, we have instead $c_{11} = 1.0$, $c_{12} = c_{21} = 0.4$, $c_{22} = 2.2$, $c_{33} = 1.0$ and $b = 1$.

In retrospect, we can infer from the figures that in the case of non-universality in the scalar sector the allowed ranges of $m_{1/2}$, $\tan\beta$ and $m_{3/2}$ are extremely limited. This is rather evident in particular from our figure 8. Our conclusions are rather generic for this class of models, as far as mixing is also predicted in both the fermion and s-fermion mass textures. This fact naturally raises the question whether the simple $U(1)$ -models are capable of generating a completely realistic low energy theory. We think that these problems cannot find a solution in the present models. Again, as stressed also in the introduction, flavor violations indicate that there should be a kind of ‘mechanism’ in the scalar sector to suppress large mixing effects. In our opinion, string derived models are probably the only realistic ones which may offer new ‘mechanisms’ of additional suppression. For example, in addition to the gauge and $U(1)$ -family symmetries, the trilinear and non-renormalizable terms in string derived models have to respect additional symmetries arising from modular invariance constraints. As a result, a large portion of ($U(1)$ -invariant) mixing terms are eliminated by these string symmetries. Additional suppression of the mixing effects in the scalar sector may also arise by cyclic symmetries as those discussed in ref[18].

In the Figures, we have shown the constraints we can obtain in $m_{1/2}$, $\tan\beta$ and $m_{3/2}$ from $\mu \rightarrow e\gamma$ decays. Let us finally see what this implies for the range of magnitudes of physical masses. To do so, we give the low energy sparticle masses for some indicative values of the input parameters: For a fixed value of $\tan\beta = 7$, the lightest neutralino varies from 80 to 196 GeV as $m_{1/2}$ scales between 200 to 450 GeV. For the same inputs, the lightest chargino varies from 196 to 365 GeV. In both cases the dependence of the results on $m_{3/2}$ is small. For a fixed value $m_{1/2} = 300$ GeV, the masses of the lightest neutralino and chargino change by at most 3 GeV, while the masses of the heavier charginos/neutralinos decrease by up to 50 GeV, as $\tan\beta$ increases from 3 to 14.

What about the charged slepton masses? Here, we find that changes with $\tan\beta$ are negligible for $m_{1/2} \approx 300$ GeV. For $\tan\beta = 7$ and $m_{1/2} = 200$ GeV, when $m_{3/2}$ varies from 100 to 500 GeV, we have: $m_{\tilde{\tau}_R} \approx (125 - 500)$ GeV, $m_{\tilde{e}_L} \approx (160 - 520)$ GeV, while in this case the lightest scalar particle for our initial conditions is $m_{\tilde{b}_R} \approx (100 - 500)$ GeV. For $m_{1/2} = 450$ GeV and the same range of $m_{3/2}$, $m_{\tilde{\tau}_R} \approx (200 - 520)$ GeV, $m_{\tilde{e}_L} \approx (300 - 590)$ GeV, while the lightest scalar particle is again $m_{\tilde{b}_R} \approx (100 - 500)$ GeV.

7 Conclusions

In the present work, we investigated in detail the predictions of $U(1)$ family symmetries for the rare processes $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. We worked in the small $\tan\beta$ regime, and found that in this class of models, $\mu \rightarrow e\gamma$ rare decays may occur at significant rates, particularly in the case of non-zero flavor mixing at the GUT scale. Demanding that the

predicted values do not exceed the current experimental limits, we can put important bounds on the model parameters. On the contrary, $\tau \rightarrow \mu\gamma$ decays are less dangerous and thus do not lead to strong bounds.

Focusing on $\mu \rightarrow e\gamma$, we first discussed the case where no mixing of soft terms at the GUT scale occurs. In this case lepton flavor violation arises from non-zero neutrino masses in the theory. We find that the dominant contributions to the decay rate cancel around a certain value of $m_{3/2}$, leading to very small values for the relevant branching ratio. An important observation is that the mixing effects of the Dirac matrix in the light sneutrino sector are canceled at first order. Here, we should stress that once the accuracy of the experiments is improved, it will be important to have a precise calculation which takes into account the precise form of the sneutrino mass matrix. The branching ratio of the process increases as $m_{1/2}$ becomes smaller and $\tan\beta$ larger, but we are still below the experimental bounds for typical values of the model parameters. Given that the experimental bounds will improve in the future, non-detection of $\mu \rightarrow e\gamma$ events will be associated to low values of gaugino masses as we can see in figure 4.

The situation changes when mixing effects in the soft masses are introduced at the GUT scale. Since the effects of the off-diagonal terms are much larger than those of the massive neutrinos, the cancellation effects that we discussed are no longer present and large lepton number violating effects may be generated. As before, the branching ratio of the decay increases as $m_{1/2}$ becomes smaller and $\tan\beta$ larger, but now strong bounds on the model parameters are derived. In this scenario, the $BR(\mu \rightarrow e\gamma)$ experimental bounds constrain the SUSY parameter space to large values for gaugino masses and small scalar masses as shown in figure 8.

Acknowledgments This work is partially supported by the TMR contract “Beyond the Standard Model”, ERBFMRX-CT96-0090. MEG wishes to acknowledge useful discussions with A. Dedes regarding the numerical integration of the RG equations. MEG and GKL wish to thank the CERN theory division where part of this work has been done, for kind hospitality.

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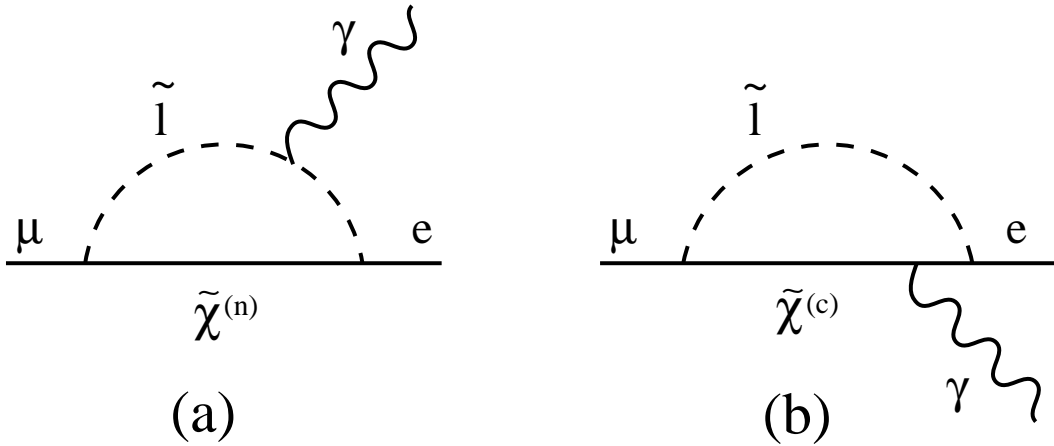


Figure 1: The generic Feynman diagrams for the $\mu \rightarrow e\gamma$ decay. \tilde{l} stands for charged slepton (a) or sneutrino (b), while $\tilde{\chi}^{(n)}$ and $\tilde{\chi}^{(c)}$ represent neutralinos and charginos respectively.

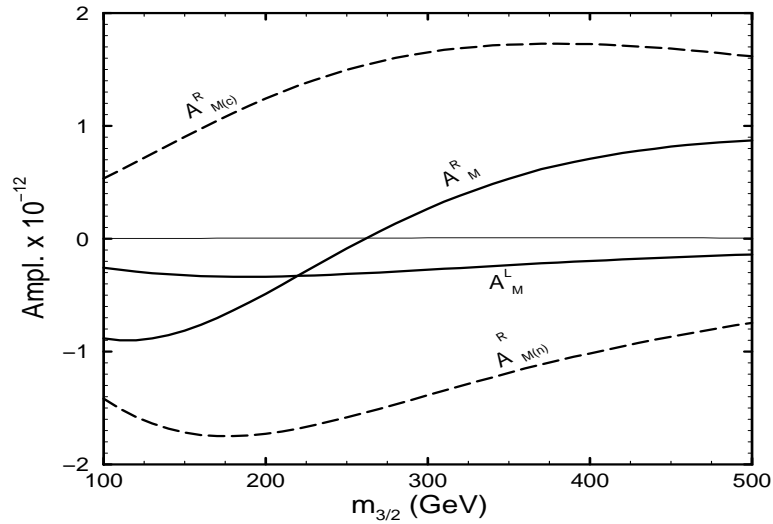


Figure 2: Amplitudes contributing to the $BR(\mu \rightarrow e\gamma)$ when universal soft masses at the GUT scale are considered ($\Delta = 0$). Dashed lines are the two contributions to A_M^R . The curves are obtained using $\tan\beta = 7$, $m_{1/2} = 300$ GeV, and $A_0 = -1.5m_{3/2}$ as input parameters.

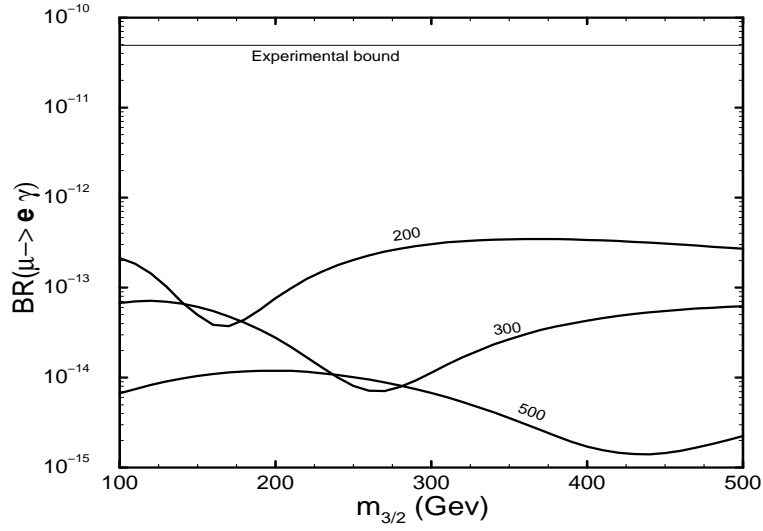


Figure 3: $BR(\mu \rightarrow e\gamma)$ for a range of values of $m_{1/2}$ (labeled above). Universal soft masses at the GUT scale are considered ($\Delta = 0$). The curves are obtained using $\tan\beta = 7$ and $A_0 = -1.5m_{3/2}$ as input parameters.

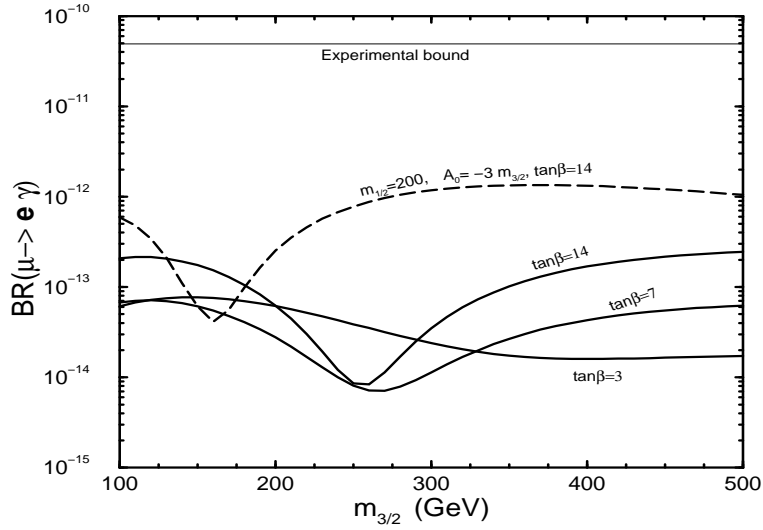


Figure 4: $BR(\mu \rightarrow e\gamma)$ for a range of values of $\tan\beta$ (labeled above). Universal soft masses at the GUT scale are considered ($\Delta = 0$). Solid lines are obtained using $m_{1/2} = 300$ GeV and $A_0 = -1.5m_{3/2}$ as input parameters.

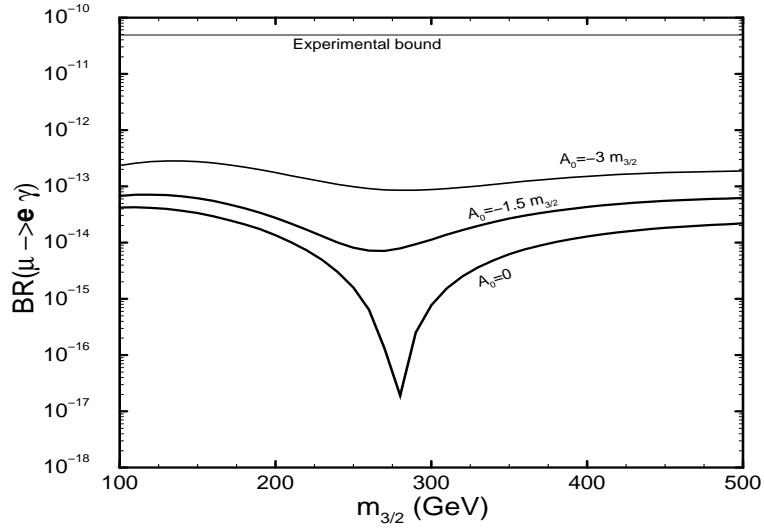


Figure 5: $BR(\mu \rightarrow e\gamma)$ for a range of values of A_0 (labeled above). Universal soft masses at the GUT scale are considered ($\Delta = 0$). The curves are obtained using $\tan\beta = 7$ and $m_{1/2} = 300$ GeV as input parameters.

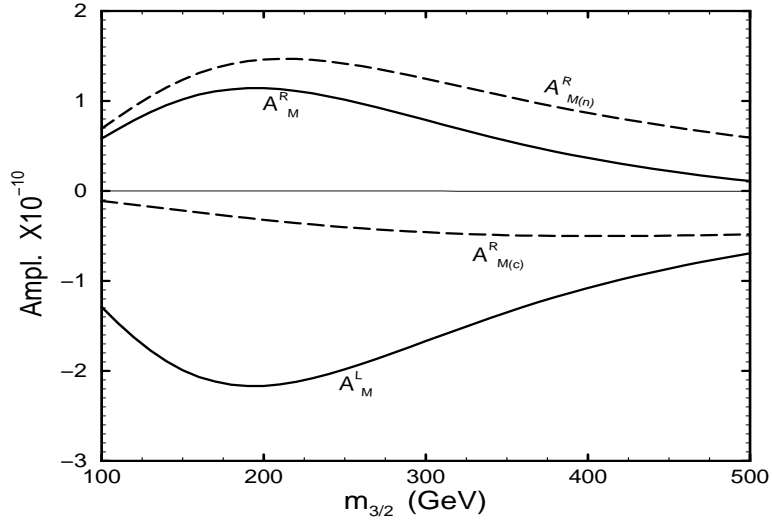


Figure 6: Amplitudes contributing to the $BR(\mu \rightarrow e\gamma)$ when non universal soft masses at the GUT scale are considered ($\Delta \neq 0$). The curves are obtained using $\tan\beta = 7$, $m_{1/2} = 300$ GeV, and $A_0 = -1.5m_{3/2}$ as input parameters.

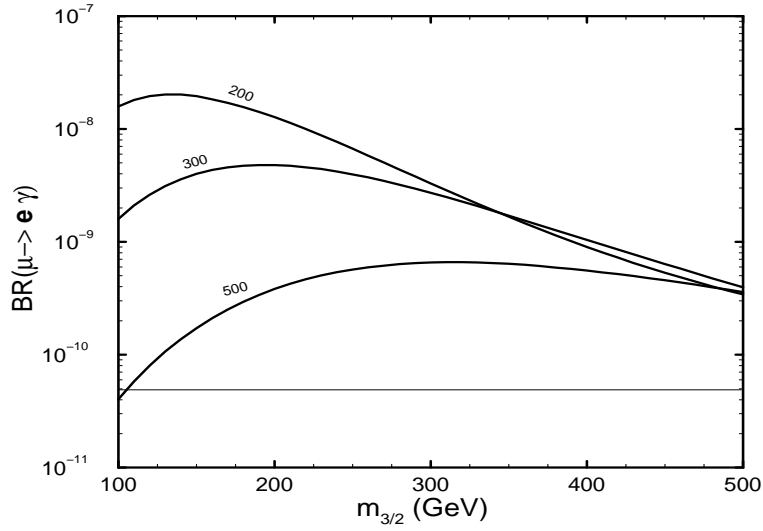


Figure 7: $BR(\mu \rightarrow e\gamma)$ for a range of values of $m_{1/2}$ (labeled above). Non universal soft masses at the GUT scale are considered ($\Delta \neq 0$). The curves are obtained using $\tan \beta = 7$ and $A_0 = -1.5m_{3/2}$ as input parameters.

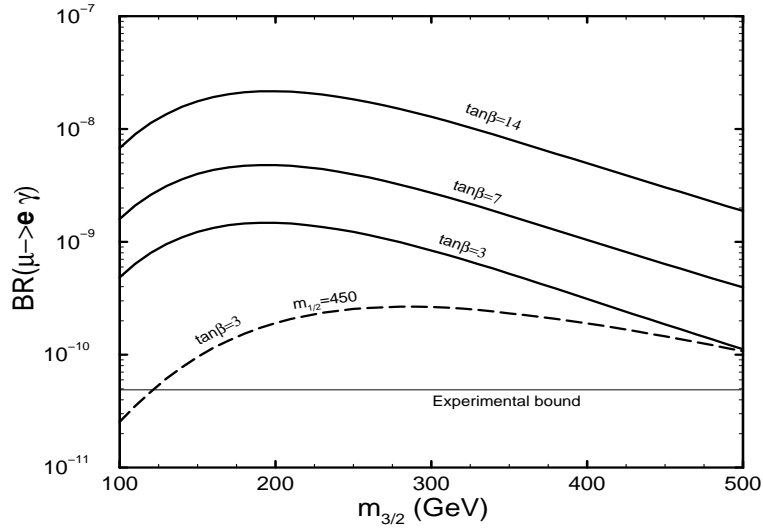


Figure 8: $BR(\mu \rightarrow e\gamma)$ for a range of values of $\tan \beta$ (labeled above). Non universal soft masses at the GUT scale are considered ($\Delta \neq 0$). Solid lines are obtained using, $m_{1/2} = 300$ GeV, and $A_0 = -1.5m_{3/2}$ as input parameters.

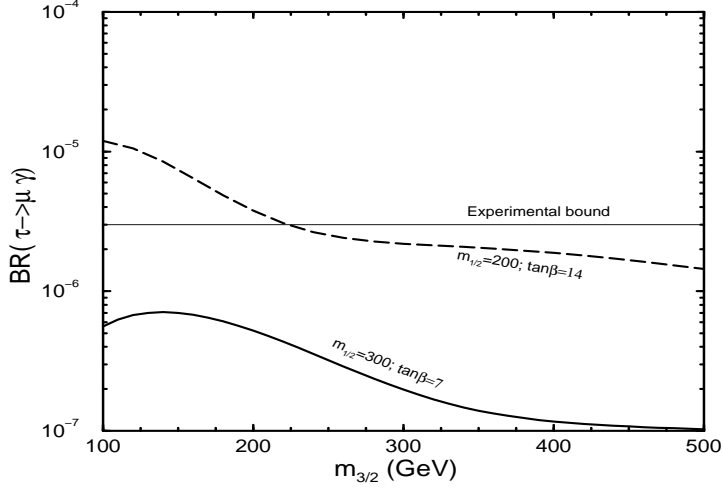


Figure 9: $BR(\tau \rightarrow \mu\gamma)$. Non universal soft masses at the GUT scale are considered ($\Delta \neq 0$). Experimental limits are violated for large $\tan\beta$ and small values of $m_{3/2}$ (dashed line). In both cases $A_0 = -1.5m_{3/2}$.

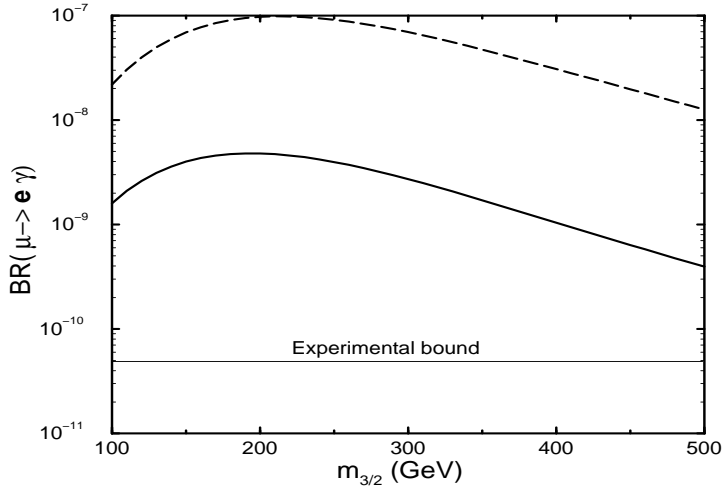


Figure 10: $BR(\tau \rightarrow \mu\gamma)$, for $m_{1/2} = 300$ GeV, $\tan\beta = 7$, and $A_0 = -1.5m_{3/2}$. The dashed line is obtained using the charge and coefficient choices of [8], while the solid line is derived using the choice of parameters presented in this paper. Non universal soft masses at the GUT scale are considered ($\Delta \neq 0$).