Baryogenesis through lepton number violation

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Abstract.
The most promising scenarios of baryogenesis seems to be the one through lepton number violation. Lepton number violation through a Majarana mass of the right-handed neutrinos can generate a lepton asymmetry of the universe when the right-handed neutrinos decay. The left-handed neutrinos get small Majorana masses through see-saw mechanism in these models. A triplet higgs scalar violating lepton number explicitly through its couplings to two leptons or two higgs doublets can also naturally give small Majorana masses to the left-handed neutrinos and also generate a lepton asymmetry of the universe. We review both these models of leptogenesis, where the lepton number asymmetry then gets converted to a baryon asymmetry of the universe before the electroweak phase transition.

1. Introduction

To get the baryon asymmetry of the universe [1] starting from a symmetric universe, one requires [2] three conditions (A) *Baryon number violation*, (B) *C and CP violation*, and (C) *Departure from thermal equilibrium*. In grand unified theories (GUTs) all these conditions are satisfied [3, 4], but the generated asymmetry conserves $(B-L)$. It was then realised that the chiral nature of the weak interaction also breaks the global baryon and lepton numbers in the standard model [5]. At finite temperature these baryon and lepton number violating interactions were found to be very strong in

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the presence of some static topological field configuration - sphalerons [6]. Although the anomalous sphaleron processes conserves $(B-L)$, the GUT $(B+L)$ asymmetry will be completely washed out by these interactions.

Attempts were then made to make use of the baryon number violation of the standard model to generate a baryon asymmetry of the universe. However, in these models one needs to protect the generated baryon asymmetry after the phase transition, which requires the mass of the standard model doublet higgs boson to be lighter than the present experimental limit of 95 GeV. Then the most interesting scenario remains for the understanding of the baryon number of the universe is through lepton number violation [7]–[14], which is also referred to as leptogenesis.

In models of leptogenesis one generates a lepton asymmetry of the universe, which is the same as the $(B-L)$ asymmetry of the universe at some high energy. This $(B-L)$ asymmetry of the universe then get converted to the baryon asymmetry of the universe during the period when the sphaleron fields maintain the baryon number violating interactions in equilibrium. Since lepton number violation is the source of leptogenesis, they are related to models of neutrino masses. In this article we shall review two scenarios of leptogenesis. In the first scenario right handed neutrinos are introduced, which gets a Majorana mass and breaks lepton number [7]. The left-handed neutrinos get small Majorana masses through see-saw mechanism [15]. In the second scenario only a triplet higgs is introduced and the fermion content of the standard model is unaltered [16, 17, 18]. Unlike earlier treatments, lepton number is now broken explicitly at a very high scale [16, 17]. Although the triplet is very heavy, its vev becomes of the order of eV to give very small Majorana mass to the neutrinos naturally [16]. Decays of the triplet higgs generate a lepton asymmetry of the universe at very high scale.

In the next section we shall discuss the electroweak anomalous processes and then how the baryon and lepton numbers of the universe gets related to the $(B-L)$ number of the universe. This will imply that if there is vary fast lepton number violation in the universe during the period when these processes are in equilibrium, that can also wash out the baryon asymmetry of the universe [19, 20]. In the following two sections we present the two scenarios of leptogenesis.

2. Sphaleron processes in thermal equilibrium and relation between baryon and lepton numbers

Anomaly breaks any classical symmetry of the lagrangian at the quantum level. So, all local gauge theories should be free of anomalies. However, there may be anomalies corresponding to any global current. That will simply mean that such global symmetries of the classical lagrangian are broken through quantum effects.

In the standard model the chiral nature of the weak interaction makes the baryon and lepton number anomalous and give us non vanishing axial current [5]

$$\delta_\mu j^{\mu 5}_{(B+L)} = 6[\frac{\alpha_2}{8\pi} W^{\mu\nu}_a \tilde{W}_{a\mu\nu} + \frac{\alpha_1}{8\pi} Y^{\mu\nu} \tilde{Y}_{\mu\nu}]$$

which will break the $(B+L)$ symmetry, while still preserving $(B-L)$, during the electroweak phase transition,

$$\Delta(B+L) = 2N_g \frac{\alpha_2}{8\pi} \int d^4x W^{\mu\nu}_a \tilde{W}_{a\mu\nu} = 2N_g \nu$$

But their rate is very small at zero temperature, since they are suppressed by quantum tunnelling probability, $\exp[-\frac{2\pi}{\alpha_2}\nu]$, where ν is the Chern-Simmons number.

At finite temperature, however, it has been shown that there exists non-trivial static topological soliton configuration, called the sphalerons, which enhances the baryon number violating transition rate [6] and the suppression factor is now replaced by the Boltzmann factor $\exp[-\frac{V_0}{T}\nu]$ where the potential or the free energy V_0 is related to the mass of the sphaleron field, which is about TeV. As a result, at temperatures between

$$10^{12} GeV > T > 10^2 GeV \qquad (1)$$

the sphaleron mediated baryon and lepton number violating processes are in equilibrium. For the simplest scenario of $\nu = 1$, the sphaleron induced processes are $\Delta B = \Delta L = 3$, given by,

$$|vac> \longrightarrow [u_L u_L d_L e^-_L + c_L c_L s_L \mu^-_L + t_L t_L b_L \tau^-_L]. \qquad (2)$$

These baryon and lepton number violating fast processes will wash out any pre-existing baryon or lepton number asymmetry, or will convert any pre-existing $(B-L)$ asymmetry of the universe to a baryon asymmetry of the universe, which can be seen from an analysis of the chemical potential [21].

We consider all the particles to be ultrarelativistic and ignore small mass corrections. The particle asymmetry, *i.e.* the difference between the number of particles (n_+) and the number of antiparticles (n_-) can be given in terms of the chemical potential of the particle species μ (for antiparticles the chemical potential is $-\mu$) as $n_+ - n_- = n_d \frac{gT^3}{6}\left(\frac{\mu}{T}\right)$, where $n_d = 2$ for bosons and $n_d = 1$ for fermions.

In the standard model there are quarks and leptons $q_{iL}, u_{iR}, d_{iR}, l_{iL}$ and e_{iR}; where, $i = 1, 2, 3$ corresponds to three generations. In addition, the scalar sector consists of the usual Higgs doublet ϕ, which breaks the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. In Table 1, we presented the relevant interactions and the corresponding relations between the chemical potentials. In the third column we give the chemical potential which we eliminate using the given relation. We start with chemical potentials of all the quarks $(\mu_{uL}, \mu_{dL}, \mu_{uR}, \mu_{dR})$; leptons $(\mu_{aL}, \mu_{\nu aL}, \mu_{aR}$, where $a = e, \mu, \tau)$; gauge bosons (μ_W for W^-, and 0 for all others); and the Higgs scalars (μ_-^ϕ, μ_0^ϕ).

Table 1. Relations among the chemical potentials

Interactions	μ relations	μ eliminated
$D_\mu \phi^\dagger D_\mu \phi$	$\mu_W = \mu_-^\phi + \mu_0^\phi$	μ_-^ϕ
$\overline{q_L} \gamma_\mu q_L W^\mu$	$\mu_{dL} = \mu_{uL} + \mu_W$	μ_{dL}
$\overline{l_L} \gamma_\mu l_L W^\mu$	$\mu_{iL} = \mu_{\nu iL} + \mu_W$	μ_{iL}
$\overline{q_L} u_R \phi^\dagger$	$\mu_{uR} = \mu_0 + \mu_{uL}$	μ_{uR}
$\overline{q_L} d_R \phi$	$\mu_{dR} = -\mu_0 + \mu_{dL}$	μ_{dR}
$\overline{l_{iL}} e_{iR} \phi$	$\mu_{iR} = -\mu_0 + \mu_{iL}$	μ_{iR}

The chemical potentials of the neutrinos always enter as a sum and for that reason we can consider it as one parameter. We can then express all the chemical potentials in terms of the following independent chemical potentials only, $\mu_0 = \mu_0^\phi$; μ_W; $\mu_u = \mu_{uL}$; $\mu = \sum_i \mu_i = \sum_i \mu_{\nu iL}$. We can further eliminate one of these four potentials by making use of the relation given by the sphaleron processes, $3\mu_u + 2\mu_W + \mu = 0$. We then express the baryon number, lepton numbers and the electric charge and the hypercharge number densities in terms of these independent chemical potentials,

$$B = 12\mu_u + 6\mu_W; \qquad L_i = 3\mu + 2\mu_W - \mu_0$$
$$Q = 24\mu_u + (12 + 2m)\mu_0 - (4 + 2m)\mu_W; \qquad Q_3 = -(10 + m)\mu_W$$

where m is the number of Higgs doublets ϕ.

At temperatures above the electroweak phase transition, $T > T_c$, both $< Q >$ and $< Q_3 >$ must vanish, while below the critical temperature $< Q >$ should vanish, but since $SU(2)_L$ is now broken we can consider $\mu_0^\phi = 0$ and $Q_3 \neq 0$. These conditions and the sphaleron induced $B - L$ conserving, $B + L$ violating condition will allow us to write down the baryon asymmetry in terms of the $B - L$ number density as,

$$B(T > T_c) = \frac{24 + 4m}{66 + 13m}\,(B - L) \qquad B(T < T_c) = \frac{32 + 4m}{98 + 13m}\,(B - L). \quad (3)$$

Thus the baryon and lepton number asymmetry of the universe after the electroweak phase transition will depend on the primordial $(B - L)$ asymmetry of the universe, while all the primordial $(B+L)$ asymmetry will be washed out.

3. Leptogenesis with right-handed neutrinos

To give a small Majorana mass to the left-handed neutrino, right-handed neutrinos were introduced. Although it is most natural to introduce a right handed neutrino in left-right symmetric models [22, 23], in the minimal scenario the standard model is extended with right handed neutrinos $(N_{Ri}, i = e, \mu, \tau)$. In these models neutrino masses come from the see-saw mechanism [15]. The lagrangian for the lepton sector containing the mass terms of the singlet right handed neutrinos N_i and the Yukawa couplings of these fields with the light leptons is given by,

$$\mathcal{L}_{int} = h_{\alpha i} \; \overline{\ell_{L\alpha}} \phi \; N_{Ri} + M_i \; \overline{(N_{Ri})^c} \; N_{Ri} \tag{4}$$

where, $\ell_{L\alpha}$ are the light leptons, $h_{\alpha i}$ are the complex Yukawa couplings and α is the generation index. Without loss of generality we work in a basis in which the Majorana mass matrix of the right handed neutrinos is real and diagonal with eigenvalues M_i, and assume $M_3 > M_2 > M_1$.

Because of the Majorana mass term, the decay of N_{Ri} into a lepton and an antilepton,

$$\begin{aligned} N_{Ri} \;\; &\rightarrow \;\; \ell_{jL} + \bar{\phi}, \\ &\rightarrow \;\; {\ell_{jL}}^c + \phi. \end{aligned} \tag{5}$$

breaks lepton number, which can generate a lepton asymmetry of the universe. There are two sources of CP violation in this scenario :

- (i) vertex type diagrams which interferes with the tree level diagram given by figure 2. This is similar to the CP violation coming from the penguin diagram in $K-$decays.
- (ii) self energy diagrams could interfere with the tree level diagrams to produce CP violation as shown in figure 3. This is similar to the CP violation in $K - \bar{K}$ oscillation, entering in the mass matrix of the heavy Majorana neutrinos.

In the first paper on leptogenesis [7], the vertex type diagram was only mentioned. Subsequently, it has been extensively studied [8] and the amount of CP asymmetry is calculated to be,

$$\delta = -\frac{1}{8\pi} \frac{M_1 M_2}{M_2^2 - M_1^2} \frac{\mathrm{Im}[\sum_\alpha (h^*_{\alpha 1} h_{\alpha 2}) \sum_\beta (h^*_{\beta 1} h_{\beta 2})]}{\sum_\alpha |h_{\alpha 1}|^2} \tag{6}$$

In this expression it has been assumed that the main contribution to the asymmetry comes from the lightest right handed neutrino (N_1) decay, when the other heavy neutrinos have already decayed away.

The heavy neutrinos decay into light leptons and higgs doublets. Because of C and CP violation, the decays of N_{1R} would produce more antileptons than leptons. This will be compensated by an equal amount of asymmetry in phi, so that there is no charge asymmetry.

Initially the self energy diagram was considered for CP violation as an additional contribution [9]. It was then pointed out [10] that this CP violation enters in the mass matrix as in the $K - \bar{K}$ oscillation. Before they decay, the right handed neutrinos were considered to oscillate to an antineutrino and since the rate of $particle \to anti{-}particle \neq anti{-}particle \to particle$, an asymmetry in the right handed neutrino was obtained before they decay. As a result, when the two heavy right handed neutrinos are almost degenerate, *i.e.*, the mass difference is comparable to their width, there may be a resonance effect which can enhance the CP asymmetry by few orders of magnitude [11]. This effect was then confirmed by other calculations [12, 13]. Ref [12] gives a very rigorous treatment based on a field-theoretic resummation approach used earlier to treat unstable intermediate states, which was used earlier in different contexts [24]. This issue has been reviewed in another talk in this meeting [25].

When the mass difference is large compared to the width, the CP asymmetry generated though the mixing of the heavy neutrinos is same as the vertex correction. These two contributions add up to produce the final lepton asymmetry of the universe.

Although the CP asymmetry was found to be non-vanishing, in thermal

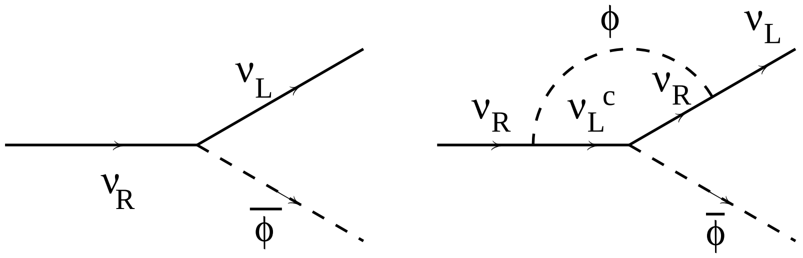


Figure 1. Tree and one loop vertex correction diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

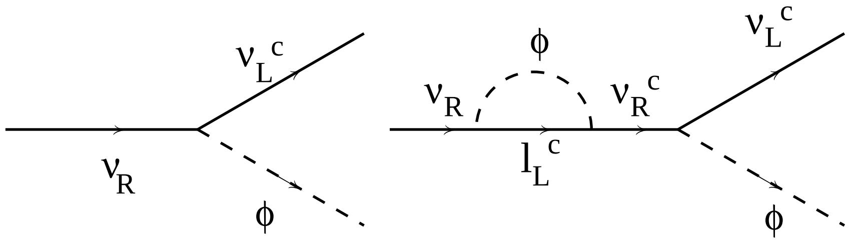


Figure 2. Tree and one loop self energy diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

equilibrium unitarity and CPT would mean that there is no asymmetry in the final decay product. However, when the out-of-equilibrium condition of the heavy neutrinos decay is considered properly, one could get an asymmetry as expected. Consider the decays of K_L and K_S. If they were generated in the early universe, in a short time scale K_S could decay and recombine, but K_L may not be able to decay or recombine. As a result in the decay product there will be an asymmetry in K and $\bar{K}$ if there is CP violation. In the lepton number violating two body scattering processes CP violation in the real intermediate state plays the most crucial role [14], which comes since the decay take place away from thermal equilibrium.

Whether a system is in equilibrium or not can be understood by solving the Boltzmann equations [26]. But a crude way to put the out-of-equilibrium condition is to say that the universe expands faster than some interaction rate. This may be stated as

$$\Gamma < 1.7\sqrt{g_*}\frac{T^2}{M_P} \tag{7}$$

where, Γ is the interaction rate under discussion, g_* is the effective number of degrees of freedom available at that temperature T, and M_P is the Planck scale.

In the case of right handed neutrino decay, the asymmetry is generated when the lightest one (say N_1) decay. Before its decay, the pre-existing lepton asymmetry is washed out by its lepton number violating interactions. So the out-of-equilibrium condition now implies that the lightest right-handed neutrino should satisfy the out-of-equilibrium condition when it decays, which is given by,

$$\frac{|h_{\alpha 1}|^2}{16\pi} M_1 < 1.7 \sqrt{g_*} \frac{T^2}{M_P} \qquad \text{at} \;\; T = M_1 \tag{8}$$

which gives a bound on the mass of the lightest right-handed neutrino to be $m_{N_1} < 10^7 GeV$. Finally the lepton asymmetry and hence a $(B-L)$ asymmetry generated at this scale gets converted to a baryon asymmetry of the universe in the presence of sphaleron induced processes.

4. Leptogenesis with triplet higgs

There are several alternative scenarios to give a small mass to the left-handed neutrinos [16, 17, 18, 27, 28]. However, at present lepton asymmetry could be generated only in models with triplet higgs [16]. In this scenario [16] one adds two complex $SU(2)_L$ triplet higgs scalars ($\xi_a \equiv (1,3,-1); a = 1,2$). The *vevs* of the triplet higgses can give small Majorana masses to the neutrinos [16, 17, 18] through the interaction

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + h.c. \tag{9}$$

If the triplet higgs acquires a *vev* and break lepton number spontaneously, then there will be Majorons in the problem which is ruled out by precision Z–width measurement at LEP. However, in a variant of this model [16] lepton number is broken explicitly through an interaction of the triplet with the higgs doublet

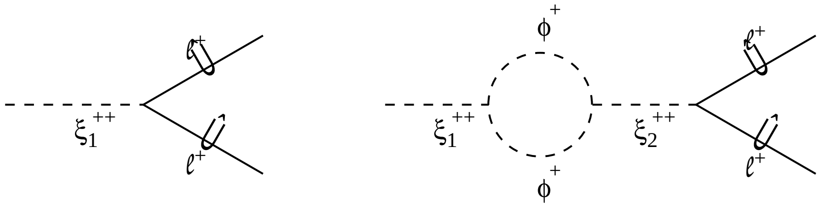
$$V \;\; = \;\; \mu(\bar{\xi}^0 \phi^0 \phi^0 + \sqrt{2} \xi^- \phi^+ \phi^0 + \xi^{--} \phi^+ \phi^+) + h.c. \tag{10}$$

Let $\langle \phi^0 \rangle = v$ and $\langle \xi^0 \rangle = u$, then the conditions for the minimum of the potential relates the *vev* of the two scalars by $u \simeq \frac{-\mu v^2}{M^2}$,, where M is the mass of the triplet higgs scalar and the neutrino mass matrix becomes $-2 f_{ij} \mu v^2 / M^2 = 2 f_{ij} u$.

In this case the lepton number violation comes from the decays of the triplet higgs ξ_a,

$$\xi_a^{++} \rightarrow \left\{ \begin{array}{ll} l_i^+ l_j^+ & (L = -2) \\ \phi^+ \phi^+ & (L = 0) \end{array} \right. \tag{11}$$

The coexistence of the above two types of final states indicates the non-conservation of lepton number. On the other hand, any lepton asymmetry generated by ξ_a^{++} would be neutralized by the decays of ξ_a^{--}, unless CP conservation is also violated and the decays are out of thermal equilibrium in the early universe. In this case there are no vertex corrections which can introduce CP violation. The only source of CP violation is the self energy diagrams of figure 4.



(a) (b)

Figure 3. The decay of $\xi_1^{++} \to l^+ l^+$ at tree level (a) and in one-loop order (b). A lepton asymmetry is generated by their interference in the triplet higgs model for neutrino masses.

If there is only one ξ, then the relative phase between any f_{ij} and μ can be chosen real. Hence a lepton asymmetry cannot be generated. With two ξ's, even if there is only one lepton family, one relative phase must remain. As for the possible relative phases among the f_{ij}'s, they cannot generate a lepton asymmetry because they all refer to final states of the same lepton number.

In the presence of the one loop diagram, the mass matrix ${M_a}^2$ and M_a^{*2} becomes different. This implies that the rate of $\xi_b \to \xi_a$ no longer remains to be same as $\xi_b^* \to \xi_a^*$. Since by CPT theorem $\xi_b^* \to \xi_a^* \equiv \xi_a \to \xi_b$, what it means is that now $\Gamma[\xi_a \to \xi_b] \neq \Gamma[\xi_b \to \xi_a]$. This is a different kind of CP violation compared to the CP violation in models with right handed neutrinos. If we consider that the ξ_2 is heavier than ξ_1, then after ξ_2 decayed out the decay of ξ_1 will generate an lepton asymmetry given by,

$$\delta \simeq \frac{Im\left[\mu_1 \mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^*\right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_1}{\Gamma_1}\right]. \qquad (12)$$

In this model the out-of-equilibrium condition is satisfied when the masses of the triplet higgs scalars are heavier than 10^{13} GeV.

The lepton asymmetry thus generated after the Higgs triplets decayed away would be the same as the $(B-L)$ asymemtry before the electroweak phase transition. During the electroweak phase transition, the presence of sphaleron fields would relate this $(B-L)$ asymmetry to the baryon asymmetry of the universe. The final baryon asymmetry thus generated can then be given by the approximate relation $\frac{n_B}{s} \sim \frac{\delta_2}{3 g_* K (\ln K)^{0.6}}$ To obtain a neutrino mass of order eV or less, as well as the observed baryon asymmetry of the universe, we may choose $M_2 = 10^{13}$ GeV, $\mu_2 = 2 \times 10^{12}$ GeV, and $f_{233} \sim 1$, then $m_{\nu_\tau} \sim 1$ eV, assuming that the M_1 contribution is negligible. Now let $M_1 = 3 \times 10^{13}$ GeV, $\mu_1 = 10^{13}$ GeV, and $f_{1kl} \sim 0.1$, then the decay of $\psi_2^\pm$ generates a lepton asymmetry δ_2 of about 8×10^{-4} if the CP phase is maximum. Using $M_{Pl} \sim 10^{19}$ GeV and $g_* \sim 10^2$, we find $K \sim 2.4 \times 10^3$. Hence $n_B/s \sim 10^{-10}$ as desired.

5. Summary

There are several models of neutrino masses which require lepton number violation. In models with right handed neutrinos, where the left-handed neutrinos get a see-saw mass, lepton number violation is introduced by the Majorana mass term of the right handed neutrinos. In these models the decays of the right handed neutrinos can generate a letpon asymmetry of the universe, which can then get converted to a baryon asymmetry of the universe during the period when the sphaleron induced $(B + L)$ violating processes are in equilibrium. Lepton asymmetry of the universe may also be generated in models with triplet higgs scalars. In these models lepton number is violated explicitly through the coupling of the triplet higgs at very high energy. However, these triplet higgs scalars get a very tiny *vev* through see-saw mechanism in the higgs sector and can naturally produce light left-handed Majorana neutrinos without introducing any right-handed neutrinos. In this model the decay of the triplet higgs can generate a lepton asymmetry of the universe at a very high energy, which can then get converted to a baryon asymmetry of the universe. At present we cannot distinguish these two equivalent models of neutrino masses and leptogenesis with a right handed neutrino or with a triplet higgs scalar from each other.

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References

[1] Kolb E W and Turner M S 1989, *The Early Universe* (Addison-Wesley, Reading, MA).

[2] Sakharov A D 1967, *Pis'ma Zh. Eksp. Teor. Fiz.* **5** 32.

[3] Yoshimura M 1978, *Phys. Rev. Lett.* **41** 281; E 1979: *ibid.* **42** 7461;

[4] Mohapatra R N 1992, *Unification and Supersymmetry* (Springer-Verlag);
Zee A 1982, (ed.) *Unity of Forces in the Universe* **1** (World Scientific).

[5] 't Hooft G 1976, *Phys. Rev. Lett.* **37** 8.

[6] Kuzmin V, Rubakov V and Shaposhnikov M 1985, *Phys. Lett.* **B 155** 36.

[7] Fukugita M and Yanagida T 1986, *Phys. Lett.* **B 174** 45.

[8] Langacker P, Peccei R D and Yanagida T 1986, *Mod. Phys. Lett.* **A 1** 541;
Luty M A 1992, *Phys. Rev.* **D 45** 445;
Mohapatra R N and Zhang X 1992, *Phys. Rev.* **D 45** 5331;
Enqvist K and Vilja I 1993, *Phys. Lett.* **B 299** 281;
Murayama H, Suzuki H, Yanagida T and Yokoyama J 1993, *Phys. Rev. Lett.* **70** 1912;
Acker A, Kikuchi H, Ma E and Sarkar U 1993, *Phys. Rev.* **D 48** 5006;
O'Donnell P J and Sarkar U 1994, *Phys. Rev.* **D 49** 2118;
Buchmüller W and Plümacher M 1996, *Phys. Lett.* **B 389** 73;
Covi L, Roulet E and Vissani F 1996, *Phys. Lett.* **B 384** 169;
Ganguly A, Parikh J C and Sarkar U 1996, *Phys. Lett.* **B 385** 175;
Plümacher M 1997, *Z. Phys.* **C 74** 549;
Faridani J, Lola S, O'Donnell P J and Sarkar U 1998, hep-ph/9804261.

[9] Ignatev A, Kuzmin V and Shaposhnikov M 1979, *JETP Lett.* **30** 688;
Botella F J and Roldan J 1991, *Phys. Rev.* **D 44** 966.
Liu J and Segre G 1993, *Phys. Rev.* **D 48** 4609.

[10] Flanz M, Paschos E A, and Sarkar U 1995, *Phys. Lett.* **B 345** 248.

[11] Flanz M, Paschos E A, Sarkar U and Weiss J 1996, *Phys. Lett.* **B 389** 693.

[12] Pilaftsis A 1997, *Phys. Rev.* **D 56** 5431.

[13] Covi L and Roulet E 1997, *Phys. Lett.* **B 399** 113.

[14] Covi L, Roulet E and Vissani F 1998, *Phys. Lett.* **B 424** 101;
Buchmüller W and Plümacher M 1997, hep-ph/9710460 (revised);
Flanz M and Paschos E A 1998, hep-ph/9805427;
Rangarajan R, Sarkar U and Vaidya R 1998, hep-ph/9809304.

[15] Gell-Mann M, Ramond P and Slansky R 1979, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, ed. by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam);
Yanagida T 1979, in *Proc of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba);
Mohapatra R N and Senjanović G 1980, *Phys. Rev. Lett.* **44** 912.

[16] Ma E and Sarkar U 1998, *Phys. Rev. Lett.* **80** 5716.

[17] Lazarides G and Shafi Q 1998, report no hep-ph/9803397;
Ma E 1998, *Phys. Rev. Lett.* **81** 1171;
Ma E and Sarkar U 1998, hep-ph/9807307;
Sarkar U 1998, hep-ph/9807466.

[18] Gelmini G B and Roncadelli M 1981, *Phys. Lett.* **B 99** 411;
Wetterich C 1981, *Nucl. Phys.* **B 187** 343;
Lazarides G, Shafi Q and Wetterich C 1981, *Nucl. Phys.* **B 181** 287;
Mohapatra R N and Senjanovic G 1981, *Phys. Rev.* **D 23** 165;
Holman R, Lazarides G and Shafi Q 1983, *Phys. Rev.* **D 27** 995.

[19] Fukugita M and Yanagida T 1990, *Phys. Rev.* **D 42** 1285;
Barr S M and Nelson A E 1991, *Phys. Lett.* **B 246** 141.

[20] Fischler W, Giudice G, Leigh R and Paban S 1991, *Phys. Lett.* **B 258** 45;
Buchmüller W and Yanagida T 1993, *Phys. Lett.* **B 302** 240;
Dreiner H and Ross G G 1993, *Nucl. Phys.* **B 410** 188;
Ilakovac A and Pilaftsis A 1995, *Nucl. Phys.* **B 437** 491.
Sarkar U 1997, *Phys. Lett.* **B 390** 97.
Campbell B, Davidson S, Ellis J and Olive K 1991, *Phys. Lett.* **B 256** 457;
Sarkar U 1998, hep-ph/9809209.

[21] Khlebnikov S Yu and Shaposhnikov M E 1988, *Nucl. Phys.* **B 308** 885;
Harvey J A and Turner M S 1990, *Phys. Rev.* **D 42** 3344.

[22] Pati J C and Salam A 1974, *Phys. Rev.* **D 10** 275;
Mohapatra R N and Pati J C 1975, *Phys. Rev.* **D 11** 566;
Mohapatra R N and Senjanovic G 1975, *Phys. Rev.* **D 12** 1502;
Marshak R E and Mohapatra R N 1980, *Phys. Rev. Lett.* **44** 1316.

[23] Pati J C, Salam A and Sarkar U 1983, *Phys. Lett.* **B 133** 330.

[24] Papavassiliou J and Pilaftsis A 1995, *Phys. Rev. Lett.* **75** 3060; 1996 *Phys. Rev.* **D 53** 2128; 1996 *Phys. Rev.* **D 54** 5315;
Pilaftsis A 1996, *Phys. Rev. Lett.* **77** 4996; 1997 *Nucl. Phys.* **B 504** 61; 1990 *Z. Phys.* **C 47** 95;
Pilaftsis A and Nowakowski M 1990, *Phys. Lett.* **B 245** 185; 1991 *Mod. Phys. Lett.* **A 6** 1933.

[25] Pilaftsis A 1998, hep-ph/9810211.

[26] Fry J N, Olive K A and Turner M S 1980, *Phys. Rev. Lett.* **45** 2074; 1980 *Phys. Rev.* **D 22** 2953; 1980 *Phys. Rev.* **D 22** 2977;
Kolb E W and Wolfram S 1980, *Nucl. Phys.* **B 172** 224.

[27] Nandi S and Sarkar U 1986, *Phys. Rev. Lett.* **56** 564;
Joshipura A S and Sarkar U 1986, *Phys. Rev. Lett.* **57** 33;
Masiero A, Nanopoulos D V and Sanda A I 1986, *Phys. Rev. Lett.* **57** 663;
Mann R B and Sarkar U 1988, *Int. Jour. Mod. Phys.* **A 3** 2165;

[28] Farhi E and Susskind L 1979, *Phys. Rev.* **D 20** 3404;
Dimopoulos S 1980, *Nucl. Phys.* **B 168** 69;
Zee A 1980, *Phys. Lett.* **B 93** 389;
Wolfenstein L 1980, *Nucl. Phys.* **B 175** 93;
Nussinov S 1985, *Phys. Lett.* **B 165** 55.