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# Phenomenology of Low Quantum Gravity Scale Models

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## **Abstract**

We study some phenomenological implications of models where the scale of quantum gravity effects lies much below the four-dimensional Planck scale. These models arise from M-theory vacua where either the internal space volume is large or the string coupling is very small. We provide a critical analysis of ways to unify electroweak, strong and gravitational interactions in M-theory. We discuss the relations between different scales in two M-vacua: Type I strings and Hořava–Witten supergravity models. The latter allows possibilities for an eleven-dimensional scale at TeV energies with *one* large dimension below separating our four-dimensional world from a hidden one. Different mechanisms for breaking supersymmetry (gravity mediated, gauge mediated and Scherk-Schwarz mechanisms) are discussed in this framework. Some phenomenological issues such as dark matter (with masses that may vary in time), origin of neutrino masses and axion scale are discussed. We suggest that these are indications that the string scale may be lying in the  $10^{10}$ – $10^{14}$  GeV region.

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# 1 Introduction

One of the fundamental questions of particle physics is about the ultimate structure of particles like quarks and leptons. It is believed that when probing shorter distances one would reach scales where quantum gravitational effects become important. As gravity seems to deal with geometry, these effects may just render invalid our basic notions as shapes and length used to study macroscopic objects. M-theory is supposed to provide us with the formalism necessary to study and formulate the laws governing physics at such small distances. There the fundamental objects of M-geometry are no more points but p-dimensional extended objects: p-branes.

A crucial question is then: At which scales  $M_s$  do quantum gravitational effects become important?. Simple dimensional analysis of the low energy parameters lead to a value of the order of  $M_s^{-1} \sim M_P^{-1} \sim 10^{-33}$  cm. However the structure of space-time might change at much bigger length scale leading to changes of the strength of gravitational interactions for instance in which case  $M_s$  can be much lower. The existence of vacua of M-theory which would allow to decrease this scale has been pointed out by Witten [1]. He suggested that  $M_s$  could correspond to scales of the order of  $10^{16}$  GeV where the three known gauge interactions have been argued to unify [2, 3] in the simplest supersymmetric extension of the standard model: the Minimal Supersymmetric Standard Model (MSSM).

The scale  $M_s$  may in fact lie at much lower values. Experimental bounds on the effects of excitations of standard model particles as higher order effective operators [4] and form factors in the gauge interactions [5] exclude only the region with  $M_s$  less than few TeV. That  $M_s$  lies just above the electroweak scale was proposed by a number of authors<sup>1</sup> [7, 8, 9, 10]. In particular a viable scenario based on some early field theory analysis in [9] was exhibited in [10]. Possible realization of such a scenario in Type I strings has been investigated in [10, 11]. It implies that future accelerators might be able to discover the existence of extra-dimensions [10, 12] and string-like structure of matter [10].

The precise mechanism of unification of coupling constants in these scenarios is an important issue. The scale  $M_s$  and the size of internal dimensions are closely related to the strength of the couplings. The relations between these entities are usually known at  $M_s$ . They involve the values of coupling at much higher energy scales than those where mea-

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<sup>1</sup>The possibility that part of the string spectrum corresponding to the Kaluza-Klein excitation of one or two large dimensions lie at the TeV scale was suggested earlier in [6].

surements are performed. Relating these two values is then necessary before computing the scales. The possibility of using large thresholds to achieve unification has been implemented recently in the case of low  $M_s$  by [13] and [14]. Differences between the two works due to the use of an heterotic string cut-off in the first and a Type I cut-off for the second illustrates the fact that such thresholds have to be computed in a full M-theory framework. Here we will propose that unification might happen naturally in even simpler ways. For instance simple models might unify at intermediary regions, after logarithmic running, either through conventional or rational unification. We discuss these issues in Section 2.

Once the coupling constants are known, the size of the other parameters of the theory can be computed as to fit the observed value for the strength of gravitational interactions. The latter are known to be very weak at low energies. From the point of view of M-theory this can be due to different reasons: (a) the scale  $M_s$  which suppress them is very large, (b)  $M_s$  is low but as the internal space is large, (c) the coupling constant is extremely small at the string scale and gauge couplings grow rapidly below  $M_s$  while the gravitational coupling either grows slowly or remains constant. The case (a) is the conventional one. The case (b) has attracted recently most of the attention [9]. While the case (c) of which a version was proposed in [8] has not been discussed further. The main problem with such a scenario is that one appeals to very large thresholds to drive the gauge couplings from nearly vanishing to order one values in order to comply with the observations. Computations of such thresholds have to be done in a fully M-theoretical framework and such models are not yet available. However this scenario is worth studying as it illustrates the possibility that quantum gravitational effects are never big.

There are two classes of M-vacua that are simple and suitable to discuss Low Quantum Gravity Scale (LQGS) models. The first one is Type I strings [15]. These allow a stringy realization of the proposal of [9] and they offer the advantage that full M-theory computations may be carried on. Another class is the M-theory on  $S^1/Z_2$  of Hořava–Witten [16]. In both cases, the low energy picture is of worlds living on three-branes separated by a bulk where gravitons propagate <sup>2</sup>. All precedent authors claimed that the lowest possible value for the eleven dimensional Planck mass  $M_{11}$  is around  $10^7$  GeV and thus Hořava–Witten compactifications are excluded for TeV-LQGS models. However, these results were deduced with the assumption was made that the six-dimensional Calabi-Yau volumes which determine the gauge and gravitational strength are of the same order. As it was shown in [18] (see also

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<sup>2</sup>Attempts to describe Hořava-Witten models as branes might be found in [17].

[19]) the average volume which determines the Newton constant may be much bigger than the volume on the observable wall. This allows to have scales  $M_{11}$  (hence eleven dimensional physics) at energies as small as few TeV!. We discuss these issues in Section 3.

If the scale  $M_s$  lies much above the TeV region then one may suppose that the theory is supersymmetric at higher energies i.e. supersymmetry is broken in our observable world at scales around the TeV. The most popular mechanisms to achieve the supersymmetry breaking may be put in three categories. The first assumes gravity mediates supersymmetry breaking from a hidden sector to the observable one [22, 23]. The hidden sector might either be on our wall, in the bulk or on another wall. The second scenario assumes that supersymmetry breaking is mediated through gauge interactions [24, 25]. In the latter two cases Kaluza-Klein states contribute to the mediation of supersymmetry breaking. The same remarks hold in the case of gauge mediation [24, 25]. Another possibility is to use the Scherk-Schwarz mechanism where supersymmetry is spontaneously broken at tree level by non-trivial periodicity condition for supersymmetric partners different in some compact internal dimension [26, 27]. We discuss these issues in Section 4 and comment on effects on soft-masses.

In section 6 we discuss some possibilities to have dark matter on the other wall of the universe as suggested for example in the M-theory scenario in [31, 18] and more specifically within the framework of [9] re by [32]. We notice that these might provide candidates for dark matter with variable masses. We comment on neutrino masses and then argue that present experimental data may be taken as indications that a natural value for the string scale is  $10^{10}$ – $10^{14}$  GeV.

In any case the ratio between the Planck mass and the the electro-weak scale needs to be explained, probably through some dynamical mechanism that leads to the necessary values for the moduli (radii and couplings) [33]. We will not address this issue here.

Section 7 gives a summaries our main results.

## 2 Unification of Gauge Couplings in M-theory

By definition M-theory provides a unified theory for all gauge and gravitational interactions. This “unification” might be achieved in many ways which contrast with the historic meaning of the word. Below we will provide a critical view on the different possibilities that have emerged from the study of several  $M$ -theory vacua. In each case we will discuss the advan-

tages and shortcomings when applied to LQGS models. This list might not be exhaustive as the subject is still in development.

In its present form, the “unification” idea is an attempt to explain the low energy parameters of the standard model as predictions of the structure and dynamics of  $M$ -theory. In contrast with early ideas, this does not preclude the existence of a Grand Unified Theory (GUT) group where the standard model symmetry  $SU(3)_c \times SU(2)_w \times U(1)_Y$  is embedded. The simplest set of experimental data one may try to “explain” are the measured values at  $m_Z = 91$  GeV of the strong and electromagnetic couplings  $\alpha_s$  and  $\alpha_{em}$  respectively, as well as the value of  $\sin^2 \theta_w$  where  $\theta_w$  is the weak angle. The best fit of different low energy cross sections corresponds to:

$$\alpha_{em}(m_Z) \sim \frac{1}{128}, \quad \sin^2 \theta_w(m_Z) = 0.231 \pm 0.003, \quad \alpha_s(m_Z) = 0.108 \pm 0.005 \quad (1)$$

These quantities are related to the gauge couplings of the standard model group  $SU(3)_c \times SU(2)_w \times U(1)_Y$  as  $\alpha_s \equiv \alpha_3$ ,  $\alpha_{em} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$  and  $\sin^2 \theta_w = \frac{\alpha_1}{\alpha_1 + \alpha_2}$ .

To proceed further, theoretical inputs which are very much subject to prejudice, are necessary. For instance one specifies a number of additional particles with masses at intermediate scales  $M_{in}^{(i)}$  and given gauge quantum numbers. There are two strategies that might be followed. Either one investigates the existence and the value of possible unification scales for a model (as MSSM) with well defined particle content, or one makes a choice for  $M_s$  then sets of possible spectra of particles necessary to achieve the unification are exhibited.

Climbing up the energies, looking for the unification scale, one makes use of renormalization group equations. The standard model couplings at  $m_Z$  are related to the string scale  $M_s$  through:

$$\frac{1}{\alpha_a(m_Z)} = \frac{k_a}{\alpha} + b_i^{(0)} \ln\left(\frac{M_s}{m_Z}\right) + \sum_{n=1}^N (b_i^{(n)} - b_i^{(n-1)}) \ln\left(\frac{M_s}{M_{in}^{(n)}}\right) + \Delta_a \quad (2)$$

where  $\Delta_a$  contain higher loops and threshold corrections.

The beta-function coefficients  $b_i^{(n)}$  take into account the contribution of new states that appear at each intermediary scale. For  $M_s \gg \text{TeV}$  we assume that the hierarchy between gauge different scales is stable because of the presence of low energy supersymmetry. In this case:

$$b_3^{(0)} = -3 \quad b_2^{(0)} = 1 \quad b_1^{(0)} = 11 \quad (3)$$

is a good approximation (at the level of our discussion). For  $M_s$  close to the TeV region the beta-function coefficients take their standard model values:

$$b_3^0 = -7 \quad b_2^0 = -\frac{10}{3} \quad b_1^0 = 4 \quad (4)$$

The parameters  $k_a$  in (2) account for different normalization or different origin for each of the three couplings. It is natural to discuss the unification as function of the allowed value of  $k_1/k_2$  and  $k_2/k_3$  :

$$\frac{k_1}{k_2} = \frac{1 - \sin^2 \theta_w - \frac{\alpha_{em}}{2\pi} \left[ b_1^{(0)} \ln\left(\frac{M_s}{m_Z}\right) + \sum_{n=1}^N (b_1^{(n)} - b_1^{(n-1)}) \ln\left(\frac{M_s}{M_{in}^{(n)}}\right) + \Delta_1 \right]}{\sin^2 \theta_w - \frac{\alpha_{em}}{2\pi} \left[ b_2^{(0)} \ln\left(\frac{M_s}{m_Z}\right) + \sum_{n=1}^N (b_2^{(n)} - b_2^{(n-1)}) \ln\left(\frac{M_s}{M_{in}^{(n)}}\right) + \Delta_2 \right]} \quad (5)$$

$$\frac{k_2}{k_3} = \frac{\frac{\sin^2 \theta_w}{\alpha_{em}} - \frac{1}{2\pi} \left[ b_2^{(0)} \ln\left(\frac{M_s}{m_Z}\right) + \sum_{n=1}^N (b_2^{(n)} - b_2^{(n-1)}) \ln\left(\frac{M_s}{M_{in}^{(n)}}\right) + \Delta_2 \right]}{\frac{1}{\alpha_s} - \frac{1}{2\pi} \left[ b_3^{(0)} \ln\left(\frac{M_s}{m_Z}\right) + \sum_{n=1}^N (b_3^{(n)} - b_3^{(n-1)}) \ln\left(\frac{M_s}{M_{in}^{(n)}}\right) + \Delta_3 \right]} \quad (6)$$

The unification scenarios might be considered as of two kinds: those who involve only light (four-dimensional) degrees of freedom and those who make use of large threshold corrections generated by states propagating in the compact internal dimensions.

## 2.1 Conventional unification :

This scenario assumes that  $\frac{k_1}{k_2} = \frac{5}{3}$  and  $\frac{k_2}{k_3} = 1$  and the threshold corrections  $\Delta_a$  are either small or universal. In the latter case unification is achieved but the value of the coupling constant receives sizable corrections from unification to string scale. Hence, the unification scale is very close to the string scale  $M_s$ . It is a very popular scenario as low energy data [3] seem to indicate that the MSSM gauge couplings meet at a unification scale  $\sim 10^{16}$  GeV which is close to the Planck scale. Here we want to briefly comment on the application of this scenario to the case of LQGS models.

- *Mechanism:*

With the content of the MSSM the unification scale is around  $10^{16}$  GeV. To get lower values, additional states must be present at intermediary scales such that equations (5) and (6) lead to  $\frac{k_1}{k_2} = \frac{5}{3}$  and  $\frac{k_2}{k_3} = 1$ . For any given value of  $M_s$  many solutions exist. However very few of these spectra are otherwise motivated.

- *Size of couplings:*

In the absence of gauge enhancement at intermediate scales, the coupling of  $U(1)_Y$  increases with energy. This implies that:

$$\alpha > k_1 \alpha_1(M_Z) \sim \frac{k_1}{100} \quad (7)$$

In a minimal scheme  $k_1 = 1$  or  $2$  and  $1/100 < \alpha < 1$ . The upper value is required in order to keep the perturbative approximation of the low energy effective theory valid.

Such a bound might be avoided in the presence of gauge enhancements. In this case the coupling constants might be pulled logarithmically to lower values. Also large threshold corrections (as discussed above) might modify this value.

- *Advantages:*

The relations  $\frac{k_1}{k_2} = \frac{5}{3}$  and  $\frac{k_2}{k_3} = 1$  allow to embed the standard model in a GUT group [34] as  $SU(4) \times SU(2)^2$  or  $SU(3)^3$  (with a discrete  $Z_3$  symmetry). This embedding might explain the value of  $\frac{k_1}{k_2}$  that leads (within the appropriate extension of the standard model) to  $\sin^2 \theta_w(m_Z) \sim 0.231$  as measured at present colliders.

- *Shortcomings:*

Semi-simple GUT groups structure unify quarks and leptons. This makes it hard to exhibit symmetries that would forbid protons from decay. So, these groups are not allowed at scales  $M_s \ll 10^{16}$  GeV. Moreover as new particles have to be introduced in the intermediary region between the electroweak scale and  $M_s$  then one does not predict but fits the values  $\frac{k_1}{k_2} = \frac{5}{3}$  and  $\frac{k_2}{k_3} = 1$ .

In this scenario, the unification might arise just below the string scale, at the scale of the compact internal manifold. In the presence of large ratio of masses (of Kaluza-Klein states or winding modes compare to the string scale), to identify the unification value of the gauge couplings with the string coupling requires that threshold corrections are small. Nontrivial constraints have to be imposed on the string compactification. For instance the excitations of standard model particles might be in (spontaneously broken)  $N = 4$  representations [6].

- *Example*

Take for instance the left-right extension of MSSM with an extra pair of Higgs doublets at a few TeV and a right-handed scale of  $10^8$  GeV [35]. This is a natural candidate to study for low scale unification at  $M_s \sim 10^{11}$  GeV. Further models will be given elsewhere [58].

## 2.2 Rational unification:

This possibility has emerged in heterotic string models where the parameters  $k_a$  are the levels of Kac-Moody algebras on the world-sheet. It constitutes a clear departure from what was previously referred to as unification. In heterotic string derived models, proposal to vary  $k_1/k_2$  was made in [36], while to allow also  $k_2/k_3$  vary was proposed in [37, 38].

- *Mechanism:*

Models with rational unification, i.e. arbitrary  $k_a$ , can be constructed the following way<sup>3</sup>: Consider  $k_a$  copies of a non-abelian group  $G_a$  all with the same gauge coupling constant  $g$ . An appropriate choice of representations allows to break spontaneously this symmetry to its diagonal subgroup. For example in the case of  $k_a = 2$  this can be achieved by using Higgs fields in bi-fundamental representation. The result is a non-abelian factor  $G_a$  with gauge coupling  $g/\sqrt{k_a}$ . If all the non-abelian gauge couplings are related to the same fundamental (string) coupling as  $g_a = g/\sqrt{k_a}$  then we have achieved rational unification. In this way the constants  $k_a$  have to be positive integers for non-abelian groups.

- *Size of couplings:*

The same arguments used for the case of conventional unification hold here.

- *Advantages:*

This scenario offers the possibility of discuss unification without GUTs at the field theory level for models that would have otherwise been thought non-unified (as for left-right models in [37]). The construction of similar models as described above is very simple.

- *Shortcomings:*

From the practical point of view one computes an approximative real value for the ratio of  $k_a$ s. This has to be identified with a rational number. This is awkward in the absence of precise estimate of the higher loops and threshold corrections. If one assumes the latter to be negligible then rational unification requires sometime large values for  $k_a$ s which are not appealing. Moreover the corresponding string constructions lead to additional (undesirable) light states (but this is a curse on all known string models).

- *Example:*

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<sup>3</sup>To classify ways to realize this scenario is an open problem in string theory.



Consider extending the MSSM up to energies of order  $\sim 2.5 \times 10^6$  GeV just below  $M_s$ . Rational unification is obtained at this scale for  $k_2 = 1$ ,  $k_3 = 2$  and  $k_1 \sim 3$ .

### 2.3 Accelerated unification:

In this scenario the thresholds corrections are large and might play an important role in the unification process. In the framework of LQGS models they have been used in [8] and [13].

- *Mechanism:*

The presence of heavy states charged under the standard model gauge group leads to threshold corrections. These might become large if the typical mass scale of the new states is hierarchically smaller or bigger than the string scale and their number is large (infinite). Examples of such states are Kaluza-Klein excitations, winding and massive oscillator modes of strings and other  $p$ -branes.

- *Size of couplings:*

The size of the coupling as determined by the effective field theory running at low energy might be very different from their actual value at  $M_s$ . For instance the latter can be hierarchically smaller or bigger if threshold corrections are very large and negative or positive respectively.

- *Advantages:*

This scenario allows one to change the values of the coupling constant rapidly in a short energy distance<sup>4</sup>. The gauge couplings might then be driven to unification [13].

- *Shortcomings:*

The precise scenario depends on the particular M- (string) theory realization of this mechanism as it involves the knowledge of:

- 1- The threshold corrections involve an infinite number of states and thus *must* be com-

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<sup>4</sup>In [8] it was suggested that in heterotic compactifications the Kaluza-Klein excitations might then drive the coupling constant quickly to very small values. In heterotic string compactifications one consequence of such a scenario is that quantum gravitational quantum effects are weak at the string scale. In more generic vacua of  $M$ -theory the gauge couplings and gravitational couplings are functions of different moduli. Both may be renormalized. Thus at  $M_s$  gauge symmetries may turn to global ones, and quantum gravitational may be strong or extremely weak.

puted in a full M-theory framework. For instance the result of the computation depends on the cut-off which is different for different string types [40].

2- Wilson lines for instance could introduce very light states with exotic gauge quantum numbers [39]. Changing slightly the beta-function coefficient can change dramatically the unification process due to the power law behavior.

3- The spectrum of heavy modes in Calabi-Yau compactifications is generically difficult to compute. If instead one uses orbifold compactifications there are twisted states that are generically charged under the standard model gauge group. These states introduce mixings between different KK levels [4] that could make the gauge coupling behavior with energy different from the one of a purely higher dimensional theory (the theory remembers the boundaries because we are computing the effects of the corresponding states).

- *Example:*

Recently an interesting observation was made in [13] that N=2 supersymmetric multiplets of standard model gauge bosons with or without matter might accelerate conventional unification<sup>5</sup>. This effective field theory study shows that that such scenario might be easily realized. However the precise implementation in a string theory model needs to take care of the shortcomings mentioned above.

## 2.4 Far and close unification

Here we would like to discuss the possibility that logarithmic threshold corrections lead to unification scale  $M_X$  located much above (or below) the string scale. Such a scenario was mentioned in [5] and studied for the case of heterotic strings by [42]. An explicit realization in open string models appeared in [14].

- *Mechanism:*

Threshold corrections might have a logarithmic form:  $\Delta_a \sim b'_a \ln(M_s R)$  where  $b'_a$  is a numerical coefficient and  $R$  is the size associated with a large internal dimension. In equation (2) such a contribution can be seen as a modification of the slope of “running” due to the presence of matter and leads to an apparent unification scale

$$M_{GUT} \sim M_s (M_s R)^{b'_a/b_a^{(N)}} \tag{8}$$

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<sup>5</sup>A heterotic string cut-off [41] was used in [13].

where  $b_a^{(N)}$  is the beta-coefficient in (2) connecting the last intermediary scale and  $M_s$ . We see that depending on the sign of  $b'_a/b_a^{(N)}$  the unification scale might be (further) above or (closer) under the string scale. For instance it has been proposed in [14] for Type I string theories that the unification scale is of the order of Kaluza-Klein states and which might be very heavy leading to  $M_{GUT} \sim 1/R \gg M_s$ . In this picture one has one intermediary scale around  $M_s$  where the couplings run with an  $N = 2$  beta-coefficients as in [43].

- *Size of couplings:*

The discussion of the size of couplings at the string (or physical unification) scale is very much the same as in the accelerated unification as both scenarios rest on large threshold corrections.

- *Advantages:*

In this picture one may perform the computation using low energy effective action below the string scale as if there is gauge coupling constant unification at a higher scale  $M_{GUT} \gg M_s$ . This apparent unification might have a physical origin as a cut-off due to N=2 sector Kaluza-Klein states [14].

- *Shortcomings:*

The thresholds are computable only in a full  $M$  or string theory framework as they are sensitive to the ultraviolet cut-off.

- *Example:*

The scenario proposed by [14].

## 2.5 Hidden Unification

It has been discovered recently that non-perturbative gauge symmetries may arise in string compactifications [44]. The associated couplings are functions of independent moduli fields. Some implications for supersymmetry breaking have been discussed in [45]. For the unification issue the standard model couplings may have arbitrary values at the string scale. The theory (contrary to what was used to in heterotic compactifications) makes no prediction of simple form of unification (if not the framework). From the point of view of the standard model phenomenology this seems quite deceiving as understanding the twenty or so low energy parameters becomes more obscure.

A crucial difference with “traditional” quantum field theories is that in M-theory the couplings are generally vacuum expectations values (vevs) of (moduli) fields. Some moduli that may govern couplings and masses of dark matter (and hidden sectors dynamics) may be decoupled from the observable matter. The large scale dynamics of the universe is then governed by the variation in time and space of such moduli.

As we discussed before, due to large thresholds some of the gauge couplings may evolve to very small values at the string scale resulting in global symmetries. In M-vacua like Type I strings, the Newton constant also gets renormalized [46]. If the threshold corrections<sup>6</sup> are big then they might also drive the strength of gravitational interactions to very small values. M-theory at the scale  $M_s$  could become topological!

### 3 Planck, String and Compactification Scales

We would like to discuss the inter-connections between the four-dimensional Planck scale  $M_{Pl} \sim 1.2 \times 10^{19}$  GeV, of the string scale  $M_s$  and of the volume of the internal space are related to each other and to the “unified” gauge coupling at the string scale. We will focus on two examples: M-theory on  $S^1/Z_2$  and Type I string compactifications.

#### 3.1 M-theory on $S^1/Z_2$

Among the simplest four-dimensional N=1 supersymmetric vacua of M-theory are compactifications on  $S^1/Z_2 \times CY$  [16, 1], where  $S^1/Z_2$  is a segment of size  $\pi\rho$  and  $CY$  is a Calabi-Yau of volume  $V$ . Gauge fields and matter live on the three-branes located at each end of the segment, while gravitons and moduli fields “propagate” in the bulk.

Following [1] one may solve the equations of motion for such configuration as a perturbative expansion in the dimensionless parameter  $\rho M_{11}^{-3}/V^{2/3}$ . At higher orders in this expansion, the factorization in a product  $S^1/Z_2 \times CY$  is lost. The volume of the Calabi-Yau space becomes a function of the coordinate parametrizing the  $S^1/Z_2$  segment. More precisely, the volumes of  $CY$  seen by the observable sector<sup>7</sup>  $V_o$  and the one on the hidden wall

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<sup>6</sup>Their sign depends on the number of hypermultiplets and vector multiplets in the  $N = 2$  sector and may be positive or negative.

<sup>7</sup>We will use the subscripts  $o$  for parameters of the observable sector and  $h$  for those of the hidden sector.

$V_h$  are given by:

$$V_o = V \left( 1 + \left( \frac{\pi}{2} \right)^{4/3} a_o \frac{\rho M_{11}^{-3}}{V^{2/3}} \right) \quad (9)$$

and

$$V_h = V \left( 1 + \left( \frac{\pi}{2} \right)^{4/3} a_h \frac{\rho M_{11}^{-3}}{V^{2/3}} \right) \quad (10)$$

where now  $V$  is the (constant) lowest order value for the volume of the Calabi-Yau manifold and  $a_{o,h}$  are model-dependent constants [18]. Roughly speaking  $a_{o,h}$  count the proportion of instantons and five-branes on each wall.

These formulae were studied for the standard embedding case in [1, 48, 49, 50, 51]<sup>8</sup> and for the non-standard embedding in [18, 19]. In this last case by putting more than half of the instantons on the hidden wall,  $a_o$  becomes negative.

For a given value of  $M_{11}$  we would like to determine the corresponding values of  $V_o$ ,  $V_h$  and  $\rho$  to fit the observed values of a unified gauge coupling  $\alpha_o$  and the Newton constant. In the absence of a precise model, the value of the former is unknown. We will assume that threshold corrections are small enough so that we can take for an approximative value, the one of  $SU(3)_c$ . The relevant relations are:

$$V_o^{-1/6} = (4\pi)^{-1/9} (\alpha_o f_o)^{1/6} M_{11} \quad (11)$$

and

$$\frac{1}{\rho} = 16\pi^2 M_{11}^9 G_N \langle V \rangle \quad (12)$$

Here  $\langle V \rangle$  is the average volume of the Calabi-Yau space on the eleven dimensional segment. The constant  $f_o$  ( $f_h$ ) is a ratio of normalization of the traces of adjoint representation of  $G_o$  ( $G_h$ ) compare to  $E_8$  case [52, 50, 18]. There are three different classes of solutions to consider:

- *Case*  $a_o > 0 \rightarrow M_{11} \sim 10^{16} \text{ GeV}$

Compactifications with standard embedding of the gauge connection fall in this category (see [1]). In these models there is an upper limit on the size of the  $S^1/Z_2$  segment above which the hidden sector gauge coupling blows up. If the observable sector coupling constant is of the order of unity the corresponding lower bound on the string scale is  $M_{11}$  of the order of  $10^{16} \text{ GeV}$ .

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<sup>8</sup>See also [47] for detailed discussion of the derivation of these formulae

This bound might be escaped if there are large threshold corrections that push the unification coupling constant to much smaller values as discussed in section 2.3.

- *Case  $a_o = a_h = 0 \rightarrow M_{11} \gtrsim 10^7 \text{ GeV}$*

In this case the only upper limit on  $\rho$  is from experiments on modification of the Newtonian force at distances of  $\rho \gtrsim \text{mm}$  [5, 53]. Using  $\langle V \rangle = V_o$  and  $\alpha_o \sim 1/10$  one obtained a lower bound on limit  $M_{11}$  of the order of  $4 \times 10^7 \text{ GeV}$ .

Some examples of characteristic size of the radii for different values of  $M_{11}$  are given in table 1.

$M_{11}$ in GeV	$V_o^{-1/6}$ in GeV	$\frac{1}{\rho}$ in GeV
$2 \times 10^{16}$	$1.2 \times 10^{16}$	$2 \times 10^{14}$
$10^{14}$	$5.8 \times 10^{13}$	$1.5 \times 10^8$
$4.2 \times 10^{12}$	$2 \times 10^{12}$	$10^3$
$2 \times 10^{12}$	$8.6 \times 10^{11}$	$10^2$
$2 \times 10^{11}$	$1.1 \times 10^{11}$	0.1
$4 \times 10^{10}$	$1.6 \times 10^{10}$	$10^{-3}$
$4 \times 10^8$	$1.6 \times 10^8$	$10^{-9}$
$4 \times 10^7$	$10^7$	$10^{-12}$

**Table 1.** Examples of values of approximative sizes of the internal space radii in compactifications of  $M$ -theory with  $a_o = a_h = 0$ . We used  $\alpha_o \sim \alpha_3(M_s)$  and  $f_o = 6$ .

- *Case  $a_o < 0 \rightarrow M_{11} \gtrsim \text{TeV}$  with  $\rho^{-1} \ll \text{TeV}$*

The possibility of  $a_o < 0$  has been shown<sup>9</sup> to arise in the non-standard embedding in [18] (see also [19]). In this scenario, as  $\rho$  increases the volume of the internal space on the observable wall is fixed as to fit the desired value of  $\alpha_o$  while the volume on the other end

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<sup>9</sup>For instance an explicit three-generation  $E_6$  model was exhibited in [18] and was found to correspond to  $a_o = -8$ . We take this value as typical order of magnitude in our numerical results.

of the segment increases leading to smaller values of the corresponding coupling constant. Typically,  $\langle V \rangle \sim \frac{V_h}{2} \gg V_o$  for large values of the radius  $\rho$ . Given a value of  $M_{11}$  both  $V_o$  and  $\rho \langle V \rangle$  can be tuned to fit the value of  $\alpha_o$  and  $M_{Pl}$ . The value of  $\rho$  is then extracted from (9).

In table 2 we illustrate the expected sizes of the volume on the hidden wall and the radius of the fifth dimension on some examples.

$M_{11}$ in GeV	$\langle V \rangle^{-1/6}$ in GeV	$\frac{1}{\rho}$ in GeV
$10^{13}$	$7.7 \times 10^{11}$	$5 \times 10^9$
$10^{12}$	$4.8 \times 10^{10}$	$8 \times 10^7$
$10^{11}$	$3 \times 10^9$	$1.2 \times 10^6$
$10^{10}$	$1.6 \times 10^8$	$6 \times 10^4$
$5 \times 10^6$	$2 \times 10^4$	$2 \times 10^{-2}$
$10^4$	12	$3 \times 10^{-7}$
$2 \times 10^3$	1.7	$2 \times 10^{-8}$

**Table 2.** Examples of values of approximative sizes of the internal space radii in compactifications of  $M$ -theory with  $a_o = -a_h = -8$ . We used  $\alpha_o \sim \alpha_3(M_s)$  and  $f_0 = 6$ .

Larger values for  $\rho$  can be obtained the following way: One starts with a symmetric embedding i.e. putting the same number of instantons (five-branes) on both boundaries. Then one moves by very short distances five-branes from the observable wall. To get  $\rho \sim \text{mm}$  one needs to move one five-brane by around an Angstrom away from our wall <sup>10</sup>.

In this case of non-standard embedding, as first discussed in [18], the hidden observer living on the other wall could see the new dimensions at energies ( e.g. GeV) much before the observers on our wall (TeV). This possibility suppose however a better precision of measurements as the interactions are weaker on his side.

Also as mentioned in [18], at energies of the order of  $GeV$  the states in the bulk are not anymore the regular Kaluza-Klein states. Instead, one expects heavier modes localized on our side of the universe which decay to lighter massive modes localized near the other wall before the latter decay to hidden matter.

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<sup>10</sup>One may also see this as a fine-tuning of  $a_o$  as this will take avalue  $(x^{112}/\pi\rho)^2$  where  $x^{11}$  is the position of the five-brane (see [20]).

## 3.2 Type I strings

A simple framework suitable to discuss both gauge and gravitational couplings size is orbifold compactification [54]. In this case the compact space is a product of three tori  $T_1 \times T_2 \times T_3$  divided by a discrete symmetry leading to internal volumes parametrized as  $(2\pi)^2 R_1^2$ ,  $(2\pi)^2 R_2^2$  and  $(2\pi)^2 R_3^2$  respectively.

The four-dimensional Planck mass  $M_P$  and the Newton's constant  $G_N$  are given by

$$G_N^{-1} = M_P^2 = \frac{8M_s^8 R_1^2 R_2^2 R_3^2}{g_s^2} \quad (13)$$

and the gauge couplings of the states on the nine-branes (99) and on the five-branes (55) are given by:

$$g_9^{-2} = \frac{M_s^6 R_1^2 R_2^2 R_3^2}{2\pi g_s}, \quad g_5^{-2} = \frac{M_s^2 R_i^2}{2\pi g_s} \quad (14)$$

where  $g_s$  is the string coupling and the indices  $i$  indicate around which internal torus two of the world-volume directions are wrapped.

In case some volume  $v_i$  is much smaller than the string scale, one performs a T-duality transformations on  $T_i$  which exchanges the role of Newman and Dirichlet boundary conditions (thus Kaluza-Klein and winding modes). This leads to:

$$\begin{aligned} g_s &\rightarrow \frac{g_s}{R_i^2 M_s^2} \\ R_i^2 &\rightarrow \frac{1}{R_i^2 M_s^4} \end{aligned} \quad (15)$$

The string scale  $M_s$  is then given by:

$$M_s = \left( \frac{\sqrt{2}}{\alpha_o M_P} \right)^{1/2} (R_1^2 R_2^2 R_3^2)^{-1/4} \quad (16)$$

For<sup>11</sup>  $\alpha_o \sim 1$ , we see that to get a small value of  $M_s$  we need a large volume. One large dimension corresponds to a  $Z_2$  orbifolds while  $Z_7$  requires all six dimensions to be of the same size.

Some examples of values for the size of the radii are given in Table 3. As pointed out by [10] the tree level relations do not allow a single large dimension while the string scale is lowered to  $M_s \sim \text{TeV}$ .

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<sup>11</sup>In case of large thresholds, the tree level relations need to be modified.



$M_s$ in GeV	$1/R_{(1)}$ in GeV	$1/R_{(2)}$ in GeV	$1/R_{(4)}$ in GeV
$10^{16}$	$8 \times 10^{12}$	$3 \times 10^{14}$	$2 \times 10^{15}$
$10^{14}$	$5 \times 10^6$	$2 \times 10^{10}$	$10^{12}$
$10^{13}$	$5 \times 10^4$	$2 \times 10^8$	$5 \times 10^{10}$
$10^{12}$	3	$10^6$	$10^9$
$10^{11}$	$3 \times 10^{-3}$	$2 \times 10^4$	$4 \times 10^7$
$10^{10}$	$10^{-6}$	200	$10^6$
$10^7$	$2 \times 10^{-15}$	$10^{-4}$	40
$10^3$	$10^{-27}$	$10^{-12}$	$3 \times 10^{-5}$

**Table 3**

Examples of values of parameters of the the compactification of Type I theory with low string scale  $M_s$ . The cases  $1/R_{(1)}$ ,  $1/R_{(2)}$  and  $1/R_{(4)}$  correspond to anisotropic Calabi-Yau with one, two or four dimensions with large radii. We use  $\alpha_o \sim \alpha_3(M_s)$ .

## 4 Mechanisms for supersymmetry breaking:

From a phenomenological point of view, low energy supersymmetry is motivated by the necessity to stabilize the hierarchy of scales present in most of the extensions of the standard model. If the same motivation is invoked to set the string scale to be as low as the  $TeV$ , then it is natural to ask that no supersymmetry is present and there is no need to discuss its breaking. However, one may insists on supersymmetry for other reasons or consider the string scale to lie at much higher energies:  $M_s \gg TeV$ . The absence of observation of any supersymmetric partners of standard model particles, it is natural to demand that supersymmetry is spontaneously broken at energies at least of the order of the electroweak scale. In these section we will investigate the fate of popular mechanisms for to achieve this breaking when applied to LQGS models. In absence of explicit models, our discussion is deliberately made sketchy and remain at a qualitative level. Our main interest is to point

out different scenarios and the challenges behind their implementation in realistic models. The latter goes beyond the scope of this paper .

## 4.1 Gravity Mediated Supersymmetry Breaking:

In this scenario supersymmetry breaking originates in a hidden sector that communicates with the observable sector only through gravitational interactions.

If all the internal dimensions are smaller than the  $\text{TeV}^{-1}$  scale, then the effective theory at the electroweak scale is four-dimensional. The supersymmetry breaking soft terms are given by:

$$m_{soft}^2 \sim \frac{F^2}{M_P^2} \quad (17)$$

where  $F^2$  is the density of energy responsible for supersymmetry breaking. For instance in the case of gaugino condensation [22]  $F \sim \Lambda^3/M_P$  where  $\Lambda^3$  is the vacuum expectation value of the gaugino condensate.

To get soft-terms of the order of TeV the  $F$ -term has to be of the order of:

$$\sqrt{F} \sim 10^{11} \text{GeV} \quad (18)$$

which implies  $M_s \gtrsim 10^{11} \text{ GeV}$ . This bound becomes  $M_s \gtrsim 10^{13} \text{ GeV}$  in the case of gaugino condensation.

If the source of supersymmetry breaking is located on a hidden wall located at the other end of a segment with large size  $R$  separating it from our world, the same relation (17) remains true. The large distance between the two walls constitutes a low infrared cut-off that suppress the contributions of from heavy excitations of bulk fields.

If  $n$  internal space dimensions have sizes below the electroweak scale the situation becomes more difficult. In this case the number of states that contribute increases with energy as  $(ER)^n$  leading to:

$$m_{soft}^2 \sim [1 + \beta(ER)^n] \frac{F^2}{M_P^2} \quad (19)$$

This formula is a simple estimate of orders of magnitude. The factor  $\beta$  for instance reflects the fact that in the bulk there are, in addition to massive excitations of gravitons, excitations of graviphotons and other moduli fields whose massless partners have been projected out

in the process of supersymmetry reduction. These states might contribute with different strength, through both attractive and repulsive interactions [55, 10]. A difficulty in applying this formula is to decide at which scale  $E$  must be taken.

To compute the value of the soft-masses at the electroweak scale one may take  $E$  to be  $\sim \text{TeV}$ . For example the case of  $n = 1$  or 2 dimensions of size  $10^{-3}$  eV the limits on  $\sqrt{F}$  become of the order of  $3 \times 10^6$  GeV and TeV.

If instead  $E$  has to run to the infrared cut-off, then  $E \sim 1/R$  and one recovers the result of the four-dimensional case.

If supersymmetry breaking originates in  $F$ -terms for moduli fields. In general these moduli have non-universal coupling to matter which might lead to non-universal soft terms on the observable sector.

## 4.2 Gauge Mediated Supersymmetry Breaking:

This scenario [24] assumes that supersymmetry is broken in a secluded sector of the theory. Some states are considered to be charged under both the observable and secluded sectors and thus mediate the supersymmetry breaking through gauge interactions.

Within our picture of walls (three-branes) separated by the bulk, we may consider the following three cases:

- *Secluded and observable sectors on the same wall:*

In type I strings, this might for example if on the same point one sector arises from nine-branes (99) while the standard model lives on fivebranes (55) (or seven-branes and three-branes after  $T$ -duality). The sector communicating the supersymmetry breaking would then be the (59) (or (73) after  $T$ -duality) open strings that have one end on the five-branes and another on the nine-branes.

In this case the states in the bulk do not participate to the supersymmetry breaking mediation. The computation and results are very much standard and lead to a mass for gauginos of the order of:

$$m_{1/2_a} \sim k_a \frac{\alpha_a}{4\pi} N M_{ms} \quad (20)$$

and for scalar masses of the order of:

$$m_{o_i}^2 \sim \sum_{a=1}^3 c_a k_a^2 \frac{\alpha_a^2}{(4\pi)^2} [\lambda_a N + \gamma_a N^2] M_{ms}^2 \quad (21)$$

where  $M_{ms}$  and  $N$  are the mass scale and the number of messengers. the coefficients  $c_a, \lambda_a$  and  $\gamma$  are model dependent . For simplicity, we have assumed that their mass splitting is of the order of  $M_{ms}$ . The latter must satisfy  $10 \text{ TeV} \lesssim N M_{ms} \lesssim M_s$  which implies (for low values of  $N$ ) a string scale  $M_s \gtrsim 10 \text{ TeV}$ .

For a string scale  $M_s$  of the order of TeV, a large  $N$  is necessary to not get too small soft terms. This usually enhances the difference of masses between gauginos and scalars. One might speculate that Kaluza-Klein states who became massive due gauge symmetry breaking using Wilson lines would play the role of messengers. however, outside the Scherk-Schwarz mechanism it is not clear how to generate mass splitting for these states.

- *Secluded sector in the bulk and observable sectors on the wall:*

In type I strings, this might arise if the dimension with large size is one of the directions orthogonal to the five-brane where the observable sector resides. The secluded sector arises from nine-branes while the messengers are (59) open strings.

If the distance between the walls is smaller than  $\sim M_{ms}$  then the result is identical to the one in the previous section. However, if the size  $R$  of the separation becomes bigger, then the four-dimensional coupling becomes very small and it is difficult to resort to gauge dynamics to generate supersymmetry breaking of order of  $M_{ms}$ . An alternative would be that supersymmetry is broken by a Scherk-Schwarz mechanism. We discuss this issue in the next section.

- *Secluded and observable sectors on two opposite walls:*

Finally supersymmetry might be broken on the opposite wall and later mediated through additional gauge interactions present in the bulk under which quarks and leptons are charged. This possibility has been studied in [25] in five-dimensions.

The messengers scale  $M_{ms}$  plays the role of a cut-off in the loops responsible of the mediation of supersymmetry breaking. Thus for a distance between the walls  $R < M_{ms}^{-1}$  Kaluza-Klein states are not excited and the result is the same as if the space was four-dimensional.

When the radius of the fifth dimension encreases  $R > M_{ms}^{-1}$  Kaluza-Klein excitations of the gauge bosons are excited. Thus the gauge couplings get contributions from  $(RM_{ms})^n$  states leading to the changement:

$$\alpha_a \rightarrow \frac{\alpha_a}{(RM_{ms})^n} \tag{22}$$

This simple scenario is not appealing for large  $R$  from the phenomenological point of view. The gaugino masses on the observable wall have to be generated at higher orders, and even the scalar masses are small because the four-dimensional gauge coupling in the bulk should be suppressed by the large volume.

Finally, the case of the standard model residing in the bulk is very similar to the case of orbifold compactifications of heterotic strings. One is faced in this case with the problem of power law running of standard model couplings.

### 4.3 Scherk-Schwarz mechanism:

This mechanism requires the existence of a symmetry group  $G_{ss}$  that does not commute with supersymmetry. The members of the same supersymmetric multiplet have different charges  $q_i$  under  $G_{ss}$ . Instead of the usual periodic conditions when going around some direction of the internal space of circle of radius  $R$ , some states transform non-trivially under  $G_{ss}$ . In the simplest case, the result for states with mass:

$$m_n^2 = \frac{n^2}{R^2} + l^2 R^2 M_s^4 \quad (23)$$

is to shift  $n \rightarrow n + q_i$  or  $l \rightarrow l + q_i$ . This creates a splitting inside each multiplet and thus it breaks supersymmetry. The simplest example for  $G_{ss}$  would be R-parity ( $q = 0$  for standard particles and  $q = 1/2$  for sparticles: gauginos, sleptons, squarks and Higgsinos). Another commonly used symmetry is the spin:  $q = s$  which is integer for bosons and half-integer for fermions. In this case fermionic matter, leptons and quarks, have to be identified with “twisted states” living on branes (or orientifolds) orthogonal to the  $z$  direction.

Within our picture of a world made of walls and bulk, the implementation of the Scherk-Schwarz mechanism leads to many different scenarios:

- *Gravity mediated Scherk-Schwarz supersymmetry breaking:*

The first possibility is to consider shift in momenta or winding in a direction orthogonal to world-volume of the brane on which the standard model states live. At tree level only the states propagating in the bulk feel supersymmetry breaking: a mass splitting between supersymmetric partners is generated in the hidden sector.

If there are no large dimensions lying under the TeV scale then supersymmetry breaking is communicated to the observable sector through four-dimensional gravitational interactions. The resulting soft-terms are of the order of [57]  $m_{soft} \sim \frac{1}{R^2 M_{Pl}}$  or  $\frac{R^2 M_s^4}{M_P}$  depending if the shift

was made on the momenta or windings. In the case of presence of  $n$  internal dimensions with larger radii  $r$  i.e.  $r^{-1} \lesssim TeV$  then the strength of gravitational strength changes with energy, leading to a multiplicative factor of order of  $(Er)^n$ .

- *Gauge mediated Scherk-Schwarz supersymmetry breaking:*

A different scenario may be illustrated on the following example: Suppose that the standard model lives on five-branes and a hidden sector arises from the ninebranes. There are (59) strings with one end on the five-branes and one on the nine-branes. The corresponding states are charged under both groups.

If the non-trivial periodic condition is on a direction orthogonal to the fivebrane. Only the ninebranes will feel the supersymmetry breaking at tree level. However this might be communicated to the five-brane. First, the (59) open strings will have splittings due to radiative corrections from (99) sector gauge symmetry. Then the (59) open strings will generate soft breaking in the observable sector. The scale  $1/R$  might lie much higher than the TeV if the gauge coupling in the (99) brane is small.

- *Direct Scherk-Schwarz breaking:*

Another possibility that has been studied in [6] is that the coordinates affected by the Scherk-Schwarz mechanism are parallel to the world-volume of the brane on which the standard particles reside. In this case soft masses generated for the standard particles are of the order of  $1/R$ .

Existing N=1 Type I string compactifications seems to lead only to singlets from twisted states. To avoid giving large masses to standard model fermions and bosons the charge  $q$  could be associated to the  $R$ -parity charge. The boundary conditions are thus universal:

$$m_0 = m_{1/2} = m_{3/2} \sim 1/R \sim TeV \quad (24)$$

where  $m_0$ ,  $m_{1/2}$  and  $m_{3/2}$  denote the scalar, gaugino and gravitino soft-masses.

Other possibilities could be engineered for this kind of compactifications if only a part of the standard model lives on one stack of five-branes orthogonal to the affected direction. For example:

-  $SU(3)$  gauge symmetry might arise on ninebranes parallel to the affected direction. In contrast  $SU(2) \times U(1)$  would arise from fivebranes orthogonal to it. Leptons and Higgs would live in this fivebrane sector and quarks doublets would originate from open strings

stretching between the fivebranes and ninebranes. Only the gluinos would have a tree level soft masses.

- If only  $SU(2)$  arises from the nine-branes then only the corresponding gauginos have soft masses.

- If  $U(1)$  arises from the nine-brane sector then only the bino has a tree level soft mass.

- If  $SU(2) \times U(1)$  arises from nine-branes then the squarks and gluinos have vanishing soft masses.

Finally there is the possibility where the theory arises from an orbifold compactifications (of M-theory) with the matter fields in the twisted sector while the gauge bosons are in the bulk (untwisted sector). The Higgs fields might be chosen in the twisted or untwisted sectors [57]. The first case leads to  $m_0 = m_3 = 0$  and  $m_{1/2} = m_{3/2} \sim 1/R \sim \text{TeV}$ . while the later gives  $m_0 = 0$  and  $m_3 = m_{1/2} = m_{3/2} \sim 1/R \sim \text{TeV}$ .

All these scenarios assume a large dimension  $1/R \sim \text{TeV}$ . The supersymmetry breaking is communicated from one set of the fields to the other one through gauge interactions.

The computation of the sparticle spectrum assumes that one can evolve the the coupling constants above the TeV scale. In which case for low  $M_s$  the running introduces small non-universalities. This is possible if the KK states associated with the large radius do not contribute to the running. This was argued to be the case if the KK states are in (spontaneously broken)  $N = 4$  multiplets [6]. If they are instead in  $N = 2$  representations they generically lead to large (power law increasing) corrections and can not be computed reliably in a field theory framework.

Low energy consequences of these scenarios will be discussed elsewhere [58].

## 5 Other phenomenological implications and the preferred value for the string scale:

We have discussed above scenarios some implications of the LQGS models for the unification and supersymmetry breaking scenarios. Here we would like to comment on other possible phenomenological implications.

- *Dark matter:*

A hidden wall is a candidate to contain an important fraction of dark matter. In the

M-theory context this possibility has appeared<sup>12</sup> (to our knowledge) for the first time in [31] and discussed in some details in [18]. On a simple example of inflation, it was shown in [60] how the dynamics on the two walls, observable and hidden, can be interconnected.

Here we wish to discuss the brane scenario for another issue: that the cosmological constant is simulated by some field with variable mass or “quintessence” [61].

Suppose that we try to fit the expansion rate of the universe can by including dark matter with a variable mass [62]. In the perturbative heterotic string scenario, this typically leads to varying the strength of the gravitational and gauge coupling on the observable world. There are strong constraints on such variation that make this scenario unlikely.

Let’s consider compactifications of Type I on orbifolds. Suppose that there are stacks of fivebranes wrapped around each of the tori. The standard model may arise from a five-brane with two world-volume internal dimensions wrapped around the internal torus  $T_1$  with constant volume  $v_1$ . Now, let’s suppose that the volumes  $v_2$  of  $T_2$  and  $v_3$  of  $T_3$  vary with time such that the product  $v_2v_3$  remains constant. The gauge coupling constants on the observable world depend on  $v_1$  only while the gravitational coupling depends on the product of  $v_1v_2v_3$ , so both are constant in time while dark matter couplings depend on the volume of the internal space and thus vary with time. While probably present, such a scenario has not been found in known heterotic string compactifications [62]. The mass of dark matter is very model dependent is but one expects it to depend on the gauge coupling thus leading to dark matter with variable mass. For instance if the hidden dark matter is made of confined hidden particles, then there mass is governed by the confinement scale. The latter is obviously varying with the strength of the tree level coupling constant. This phenomenon seems to be allowed by Type I string theory.

In the context of Hořava-Witten type of models, dark matter with variable mass might be obtained by taking one or a set of five-brane and arranging that they move in the fifth dimension separating the two boundaries. A judicious choice of five-branes allows the coupling constant on the observable wall to remain constant. For instance one could take a couple of five-branes: one at  $\pi\rho \cos z(t)$  and the other  $\pi\rho \sin z(t)$  where  $z(t)$  is a slowly varying phase.

- *Neutrino masses:*

Recent data from different experiments suggest existence of oscillations of between different neutrinos. Such processes require that the neutrinos are massive In a minimal scenario,

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<sup>12</sup>The phenomenology is similar to the shadow matter that has been studied for instance in [59].



one tries to build a mass matrix with three neutrinos which allows to fit the data from solar and atmospheric neutrinos experiments.

Let us first discuss this issue in the left-right class of models presented in the appendix. The neutrino masses are given by :

$$m_{\nu_i} \sim \frac{m_{Di}^2}{M_R} \quad (25)$$

where  $m_{Di}$  are Dirac neutrino masses and it is a free parameter. For  $M_R \sim 10^8$  GeV, a neutrino mass of  $\sim$  eV corresponds to  $m_{Di} \sim 0.1$  to 1 GeV.

Another possibility<sup>13</sup> is to rely on the violation of global symmetries by quantum gravitational effects [66]. This arises in string theory due to the presence of heavy (oscillators) modes with interactions that violate these global symmetries. For instance the violation of lepton number would lead to operators of the form  $\frac{1}{M_s} LLHH$ . If  $M_s$  is in the region of  $10^{11} - 10^{13}$  GeV then the neutrino masses might be naturally of the order of eV (depending on the precise value of the coefficient of this operator).

Finally, it was proposed that a modulino might play the role of a sterile neutrino [67]. The modulino-neutrino mixing would arise from  $R$ -parity bilinear terms of the form  $\mu LH$  through the dependence of  $\mu$  on the modulus  $S$ . To get light neutrinos one takes  $\mu \sim 1$  to 10 GeV. This values imply that the modulino-neutrino mixing mass will be of the order of eV for  $\langle S \rangle$  of the order of  $\langle S \rangle \sim M_s \sim 10^{11}$  to  $10^{12}$  GeV. For scenarios where the modulino is light enough this might explain the different neutrino anomalies.

- *A preferred value for the string scale?:*

M-theory as known today seems to allow arbitrary values for the string scale. Only experimental limits seem to imply that it is not lower than the TeV. A TeV scale is certainly exciting as it could be probed at future colliders. However there are no experimental indications supporting the existence of such a scale. Three other scales might be considered as more motivated from our observations:  $10^{19}$  GeV which is the natural scale,  $10^{16}$  GeV if one believes that at this scale all interactions should unify (as suggested by LEP) and finally we suggest  $10^{10}$ – $10^{14}$  GeV centered around  $10^{12}$  GeV which is our preferred value. In fact this scale appears naturally when one tries to explain many experimental observations as the neutrino masses discussed above or the scale for axion physics. For instance the breaking

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<sup>13</sup>Relating the neutrino mass to existence of extra-dimensions at scales of the order of  $10^{12}$  GeV is under investigation with J. Ellis [65].

of Peccei-Quinn symmetry<sup>14</sup> is constraint by cosmological and astrophysical bounds to be roughly in the region of  $10^{10}$ – $10^{12}$  GeV. The presence of quantum gravitational effects at this scale due to its identification with  $M_s$  may be responsible of the breaking of the symmetry. Moreover, the observed ultrahigh energy cosmic rays might may just originate at the string scale. One can speculate on their origin as coming from decay of long lived massive string modes, or p-branes wrapped around some internal space direction.

## 6 Conclusions:

In summary, in this paper we have considered many phenomenological aspects of LQGS models and we obtained in our opinion many interesting new results. For instance:

- In contrast with the claims of recent litterature, unification in LQGS models can be achieved in different ways. For certain values of the string scale  $M_s$ , this can be achieved without introduction of ad-hoc exotic matter, and in most cases one does not need to appeal to threshold effects as in accelerated unification. However, if  $M_s$  becomes of the order of the TeV we argue that unification should be studied within a full string theory framework.

- We have exhibited compactifications of Horava–Witten M-vacua that lead to an eleven-dimensional scale of the order of TeV while only one internal dimension has a size in the  $10^{-5}$  to 1 mm region. We illustrated examples for the size of the radii if the internal space dimensions when the string scale varies from TeV to Planckian energies.

- We have studied different scenarios for supersymmetry breaking and pointed out the problems when trying to apply them to phenomenological considerations.

- Finally, we have addressed some phenomenological issues: dark matter, neutrino masses, axion scale and ultra-high cosmic rays. While we believe a string scale at the TeV energies is appealing experimentally, we suggest that the experimental data might seem more natural if  $M_s$  is in the range of  $10^{10}$ – $10^{13}$  GeV.

In this paper, we have began the study of some implications of having a low scale for quantum gravitational effects. In the absence of concrete models, many of the issues were discussed at a qualitative level. We believe that many of them merit to be studied further.

**Note added** When this manuscript was in preparation ref. [69] appeared that overlaps with part of Sections 4.1 and 4.3.

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<sup>14</sup>The proposal to solve the axion problem by decreasing the string scale was made by [68] then more recently by [32]. However they both considered different values of  $M_s$ .

## 7 Acknowledgements:

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