AN INTRODUCTION TO COSMOLOGY

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Abstract
The following is the written version of my two lectures on the standard big bang cosmological model, including a very intuitive description of the scenario of inflation. Due to the limited time, it was only possible to give an overview of the basic structure of the subject, and the exposition is quite elementary. I have not assumed any prior knowledge of general relativity, in accordance with my instructions from the school.

1 INTRODUCTION

Modern cosmology was initiated by the following two papers:

- A. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie*, Ann. Phys. [Leipzig] **49** (1916) 769,
- E. Hubble, *A relation between the distance and radial velocity among extra-galactic nebulae*, Proc. Nat. Acad. Sci. **15** (1929) 168.

In these lectures I am not supposed to assume any prior knowledge of general relativity. Therefore the mentioning of the Einstein paper above is strictly against this rule, since this paper lays the foundation of general relativity. For those without knowledge of this subject, Einstein's paper[1] can still be recommended as an introduction to general relativity, in my opinion surpassing many text books with respect to beauty and clarity.

To understand the significance of the second paper by Hubble, let us mention that in the beginning of this century the universe was still thought to be static. Einstein soon discovered that his general relativity field equations did not allow for a static solution. Actually, the same is the case in Newtonian gravity, as one can easily understand intuitively: If you imagine a static universe consisting of a number of uniformly distributed galaxies, then since gravity is an attractive force, there will always be a collapse, and hence the universe cannot be static[2]. Einstein found that general relativity allowed for another force than the attractive one, behaving like Λr instead of $-GM_1M_2/r^2$. For "the cosmological constant" Λ sufficiently small, the new force is unimportant at "small" distances relevant for our solar system. However, at large distances of cosmological interest this force can provide enough repulsion to stabilize the situation and provide a static universe.

Now we come to Hubble's paper: In the twenties astronomers started to measure the velocities of distant galaxies, and found that they recede from us at high velocities. Using the data available, Hubble then proposed a linear relationship between velocity and distance. This is the famous Hubble law,

$$v = H_0 d, \tag{1}$$

where v is the radial velocity of a galaxy, and d is the distance[3] to the same galaxy. The quantity H_0 is called the Hubble "constant", although from general relativity it turns out not to be a constant.

[1] For an english translation, see [1].
[2] Newton thought that if there was an infinite number of galaxies, the static universe could be maintained. This is, however, not correct.
[3] It is difficult to measure d, and this gives rise to large uncertainties in fitting Eq. (1). The distance problem will not be discussed here.

Table 1: Some known ages

Object	Age
Earth (meteorites)	4.5 By
Oldest rock	3.5 By
Bees [2]	140 My
Flowers [2]	40 My
Homo erectus	1 My
Homo sapiens	350-100 ky

One consequence of Eq. (1) is then that all galaxies recede with a velocity proportional to the distance. If you read this picture backwards in time, it follows that there must have been some initial state, where the universe collapses to a point. This is the famous *big bang*. Taking Eq. (1) completely serious, this will happen for a time

$$t_0 = 1/H_0. \tag{2}$$

This is thus the life time of our universe. According to general relativity, the situation is more complicated, but it turns out that t_0 is indeed an *upper limit* on the life time of our universe.

The Hubble law (1) can be written in different forms. For example, let us consider some light emitted from a galaxy with the wave length λ_e, and subsequently observed on earth with wave length λ_o, then the non-relativistic Doppler law gives

$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \approx \frac{v}{c}. \tag{3}$$

The quantity z is called the red-shift, and using Eq. (3) in the Hubble law (1), we get

$$cz \approx H_0 d. \tag{4}$$

Now it is clear that H_0 has the dimension 1/time. From observations one finds

$$H_0 = 100\, h \frac{\mathrm{km}}{\mathrm{s\ Mpc}}, \ \ \mathrm{with}\ \ 1 \leq h \leq 0.5. \tag{5}$$

Here 1 Mpc=10^6 pc≈ 3.2615 light year≈ 3.1×10^{24} cm. The quantity h is a fuzz factor, which indicates a lack of precise knowledge of the Hubble constant. Alternatively, one can express H_0 as

$$H_0 = \frac{h}{10^{10}\ \mathrm{year}}. \tag{6}$$

This simply expresses that the (upper limit on the) life time, given by the inverse Hubble constant, is of order of 10-20 Billion years.

Hubble actually (over-)estimated H_0, having $h \sim 5$, leading to a life time of order 2 Billion years. One can ask whether this is reasonable? To answer this question we can compare the life time of the universe found from measurements of the radial velocities with other known ages. In Table 1 we give a few of the latter[4].

From this table we see that the life time of the universe obtained by Hubble is considerably smaller than the age of the earth! This was one reason for the emergence of the "steady state models", which I shall not discuss.

The "age problem" was apparently cleared up by Baade and Sandage in 1958, who obtained an age of 10-20 By from Hubble's law. However, recently the problem has popped up again, partly because

[4]Please note that there is an answer as to who were the first, the bees or the flowers? The former were the first by 100 My!

old stars are believed to be of an age which contradicts the Hubble constant obtained from the Hubble space telescope, and partly (and more convincingly) because the space telescope has observed faint (i. e. far away) galaxies with an age close to (or larger than) the age of the universe obtained from the Hubble constant. So the age problem is back!

In principle, it is clear that it is crucial to obtain an age (or, rather, an upper limit of the age, which does not make the problem simpler) of the universe, which respects other known estimates of various relevant ages. Later we shall see that the cosmological constant Λ might be of help in solving this problem.

2 THE COSMOLOGICAL PRINCIPLE

In the last section we discussed how galaxies move away from us. It is clear that this could be interpreted by assuming that we are at the center of the universe, and everybody is moving away from us. This point of view would have been acceptable early in the sixteenth century, but after Copernicus it appears rather unlikely to assume that we are in a special position.

Another possible interpretation of the Hubble law is that every "point" in the universe is physically equivalent to any other "point". This means that the universe is assumed to be homogeneous and rotational invariant around any "point". This is the *cosmological principle*. It states that since we cannot be the center of the universe, nobody else should have this honour[5]. It is an observational question at which scale the cosmological principle actually works. In other words, what is the size of a "point" (apologies to Euclid!)? It is e.g. clear from looking at the sky that the milky way is not a homogeneous structure. Therefore, a point must be taken to be at least of the size of a galaxy[6], but it may even be of the size of a cluster of galaxies. Therefore the cosmological model is a very large scale description of the universe.

Perhaps it is not quite clear that the cosmological principle leads to receding galaxies. Therefore, imagine a three-dimensional coordinate system with units plotted along the axes. At a certain time, two galaxies (points) have some coordinates, (1,2,3) and (-1,2,3) say. Now, at any later time these coordinates are the same, but the units plotted on the axes are enlarged by a *scale factor* $a(t)$. Then the two galaxies have moved away, *not* because their coordinates have changed, but because the units have been enlarged by the scale factor. Also, whether you look at the situation from the point of view of (1,2,3) or (-1,2,3), does not matter. If in doubt, you are encouraged to verify this by drawing a coordinate system, and then scale up the coordinates. Take a number of points, and verify that seen from any of these, it looks as if all the others are moving away.

From this picture we actually get the Hubble law: The relative velocity of two points is clearly proportional to $\dot{a}(t)$, where the dot indicates the time derivative. On the other hand, the distance d is proportional to the scale factor itself, so

$$v \propto \dot{a}(t) = \frac{\dot{a}(t)}{a(t)} a(t) \propto \frac{\dot{a}(t)}{a(t)} d(t), \tag{7}$$

and hence we have, by fixing the units suitably, the Hubble law in the form

$$v = H(t)d(t), \text{ with } H(t) = \dot{a}(t)/a(t). \tag{8}$$

Thus we see that the Hubble "constant" is not really a constant. It can change over cosmological scales[7].

[5] Since we are in Denmark, I would like to mention that we have something called "the Jante law", due to the Danish/Norwegian author Sandemose, who presumably fled this country because of his "law". It states that "Do not think you are anything special" (this is supposed to apply especially in Denmark). The cosmological principle is thus a cosmological Jante law.

[6] The milky way has an extension of the order 10^{23} cm

[7] Please, do not make any jokes about a change in the Hubble constant from 1929 and until now. This time span has no cosmological significance, and the "variation" should therefore be ascribed to changes in the precision of data, the methods used to analyze these, etc.

For the benefit of people with some knowledge of general relativity, I mention that the requirement of isotropy around any point leads to a definite metric,

$$d\tau^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \tag{9}$$

where $d\tau$ is the proper time interval, t is the cosmological time, and $k = +1, 0, -1$ for a closed, flat, and open universe, respectively. The polar coordinates are the fixed ones, and are therefore called the *comoving coordinates*. This metric is called the Robinson-Walker metric.

For a flat universe we have the metric

$$d\tau^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2), \tag{10}$$

so the only thing that happens is that the Euclidean line element is scaled up by the scale factor $a(t)$.

For a closed universe the Robinson-Walker metric has a simple intuitive interpretation: Consider the surface of a sphere where the galaxies are situated. At each point there is, of course, isotropy with respect to the two dimensions spanned by the surface. Now increase the radius of the sphere: Any galaxy then moves away from any other, as expected[8]. This space is clearly closed and two-dimensional. The realistic case is a three-dimensional metric on the surface of a four dimensional hypersphere. In this case the scale factor gives the size of the universe = the radius of the hypersphere.

For an open universe, $k = 0$ or $k = -1$, there is no similar interpretation of the scale factor. The universe is always of infinite extension, except exactly in the big bang singularity where $a(t) = 0$. This is thus a singular point, where the notion of space breaks down.

From Eq. (4), we obtain a new version of the Hubble law[9]

$$\frac{\lambda_e}{\lambda_o} = \frac{a(t_e)}{a(t_o)}, \tag{12}$$

which simply states that the wave length shifts proportionally to the scale factor. From this relation it follows that expansion of the universe produces red shifts, whereas a contraction would produce blue shifts. Since astronomers observe that the spectral lines are red shifted, it follows that our universe expands. We mention that the last form (12) of the Hubble law is exact in the standard general relativistic cosmology. The previous expressions are thus only approximatively valid, for small velocities and distances.

3 THE BASIC EQUATIONS OF STANDARD BIG BANG COSMOLOGY

The basic equations of the standard big bang theory are derived from the Einstein equations, using the Robinson Walker metric, which in turn is a consequence of the cosmological principle (homogenuity and isotropy). Since we have not assumed any knowledge of general relativity, we cannot really derive these results. Instead, let me give a pseudo derivation, which only refers to Newtonian physics, as well as to some imagination from the reader.

We assume that the *large scale* universe can be described as an ideal gas, the "molecules" beeing the (clusters of) galaxies. Therefore we have an energy density ρ and a pressure p. These quantities are independent of space, because of the cosmological principle, but they can of course depend on time.

[8] There exists a special gastronomic version of this picture: Imagine a currant loaf, i.e. a bread with raisins. When this bread is made, yeast is put in, and the bread rises. From the point of view of an arbitrary raisin, all the others are moving away!

[9] In obtaining this expression, we used that the distance $d(t) \approx c(t_o - t_e)$, disregarding the curvature of the universe. Then from (4) we have

$$\lambda_o/\lambda_e \approx 1 + (\dot{a}(t_o)/a(t_o))(t_o - t_e) \approx 1/[1 + (\dot{a}(t_o)/a(t_o))(t_e - t_o)] \approx a(t_o)/a(t_e). \tag{11}$$

Let us consider the universe with the scale factor $a(t)$, expanding with velocity $\dot{a}$, and ask what is the equation for energy conservation in such a universe (if it is open we consider a sphere with radius $a(t)$.). This must be something like[10]

$$\frac{1}{2}\dot{a}^2 - G\left(\frac{4\pi\rho}{3}a^3\right)\frac{1}{a} = -\frac{k}{2}, \tag{13}$$

where it was used that the left hand side is the sum of kinetic and potential energy[11], which should be constant. From general relativity it follows that the constant is related to the parameter k, which determines whether the universe is closed ($k = +1$), flat ($k = 0$), or expanding ($k = -1$). Thus, in a flat universe there is an exact balance between the expansion (the kinetic term) and the gravitational attraction. For an expanding universe, the expansion overwhelms gravitation, whereas for a closed universe the situation is the opposite.

Next, we have the pseudo-Newtonian force law,

$$\ddot{a} = -G\left(\frac{4\pi}{3}(\rho + 3p)a^3\right)\frac{1}{a^2}. \tag{14}$$

Again, cheating is going on, since this time the mass has been replaced by somthing involving energy density *and* pressure. Well, you should learn general relativity! Then you would know that Eq. (13) comes as a consequence of the 00-component of the equation

$$R^{\mu\nu} = -8\pi G(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T^{\alpha}{}_{\alpha}), \tag{15}$$

with the energy-momentum tensor

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^{\mu}U^{\nu} \text{ and } U^{\mu} = (1,0,0,0). \tag{16}$$

Also, Eq. (14) is a consequence of the space-space part of the above equation, combined with Eq. (13).

Finally, we have the thermodynamic condition for adiabatic expansion, $dE + pdV = 0$, which of course translates into

$$d(\rho a^3) + pda^3 = 0. \tag{17}$$

In general relativity this equation is a consequence of the covariant conservation of the energy-momentum tensor $T^{\mu\nu}$, i.e. $T^{\mu\nu}{}_{;\mu} = 0$. In reality there are only two independent equations, but it is convenient to use all three equations (13), (14), and (17).

We end this sections by rewriting the above equations in slightly more streamlined forms,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{18}$$

which is the famous Friedmann equation, and the "force equation"

$$3\ddot{a} = -4\pi G(\rho + 3p)a, \tag{19}$$

as well as

$$d(\rho a^3)/da = -3pa^2. \tag{20}$$

In addition to these equations, we need an "equation of state", which relates ρ and p. This question will be discussed later.

[10] In the following we take $c = 1$.

[11] The careful reader will notice that we are slightly outside Newtonian physics, since we replaced the mass contained inside the radius $a(t)$ by the energy contents, $(4\pi\rho/3)a^3$. Well, energy=mass in special relativity...

4 SOME CONSEQUENCES OF STANDARD COSMOLOGY

Let us start by looking at the "force-equation" (19) under the assumption that

$$\rho + 3p > 0. \tag{21}$$

This condition certainly appears reasonable, since ρ is by definition positive and usually p is also positive (*not* by definition, however!). From (21) and Eq. (19) we get

$$\ddot{a} < 0, \tag{22}$$

which just reflects the fact that gravitation is attractive. Since the second derivative of the scale factor is negative, it follows that $\dot{a}$ is a decreasing function. Thus,

$$\dot{a}(t_{\rm now}) < \dot{a}(t) \;\; \text{with} \;\; 0 \leq t < t_{\rm now}. \tag{23}$$

This allows us to find the upper limit on the age of the universe, which we announced previously, namely

$$\text{Age} = t_{\rm now} = \int_0^{t_{\rm now}} dt = \int_0^{a_{\rm now}} \frac{da}{\dot{a}} < \int_0^{a_{\rm now}} \frac{da}{\dot{a}(t_{\rm now})} = \frac{a(t_{\rm now})}{\dot{a}(t_{\rm now})} = \frac{1}{H_{\rm now}}. \tag{24}$$

As mentioned before, from the measured red-shifts it follows that this upper limit is of order 10-20 By, with hopefully much more precise results coming up soon. It is clearly an important check of the model whether the upper limit agrees with other knowledge on the age of the universe. As an example of such a (negative) check, we remind the reader that in section 1 we have discussed the situation at the time when Hubble proposed his law, where there was a disagreement between the upper limit (24) and the age of the earth, due to rather imprecise observational data.

From (20) it follows that if the pressure is zero or positive, then ρa^3 is either a constant or it decreases. Thus, with increasing a, it follows that ρ itself decreases. Consequently, the Friedmann equation (18) implies that for increasing a, $\dot{a}^2 \to -k$. This is of course only possible in an open universe, with $k = 0$ or $k = -1$. In the latter case $a(t) \to t$ for $t \to \infty$. In a closed universe, the scale factor is limited. After the big bang it expands, but at the point $\dot{a} = 0$ it starts to decrease again, and red-shifts become replaced by blue-shifts. Also, ultimately there will be a time when the universe collapses to $a = 0$ (the big crunch).

5 THE ENERGY DENSITY AND THE AGE PROBLEM

We shall now discuss attempts to obtain information on the universe by using observations in the standard model. For example, it would be nice to know if the universe is open or closed. Let us rewrite the Friedmann equation Eq. (18) as

$$\left(\frac{\dot{a}}{a}\right)^2 = H(t)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \equiv \frac{8\pi G}{3}\rho_{\rm crit}. \tag{25}$$

Here $H(t)$ is the variable Hubble "constant" and $\rho_{\rm crit}$ is the critical density which would exist if the universe is flat ($k = 0$). Using Eq. (6) we can obtain the present value of $\rho_{\rm crit}$ from the Hubble constant today,

$$\rho_{\rm crit} = \frac{3H^2}{8\pi G} \approx 2h^2 \times 10^{-29} \frac{\rm g}{\rm cm^3}. \tag{26}$$

Introducing the density relative to the critical density as a parameter

$$\Omega = \rho/\rho_{\rm crit}, \tag{27}$$

we can rewrite the Friedmann equation in the simple and elegant form

$$\frac{k}{a^2} = H^2(\Omega - 1). \tag{28}$$

In this form the conditions $k = +1, 0, -1$ correspond to $\Omega > 1, = 1, < 1$, respectively. This is easily understood, since if Ω is large, there is a lot of energy, so that the gravitational attraction can counteract the expansion, and the universe is closed. Similarly, a small Ω means little gravitational counteraction of the expansion, and hence an open universe. If $\Omega = 0$ there is an exact balance between expansion (kinetic term in (13)) and gravitational attraction, as discussed in connection with Eq. (13).

To actually determine whether the universe is open or closed, from Eq. (28) we need to know Ω. Observing luminous matter the astronomers get

$$\Omega_{\rm luminous} \approx 0.01, \tag{29}$$

so if this was all, we would conclude that the universe is open, with not enough matter and energy to counteract the expansion. However, as discussed in details at this School, there is also the dark matter: Consider a (spiral) galaxy, where there is a star moving outside the region with luminous matter. We expect Kepler's third law $GM(< r)/r = v^2$, where $M(< r)$ is the mass inside the radius r of the orbit of the star. Observations show that $v \approx$ const., implying that $M(< r) \approx r$. Thus, the mass increases linearly in the region where there is no luminous matter. This shows that there must be *dark matter*. In fact, there must be quite a lot, since it turns out that

$$\Omega_{\rm dark} \approx 0.1 - 0.3. \tag{30}$$

This shows that there is at least 10 times as much dark as luminous matter.

One might still wonder whether some energy has been missed. However, Ω cannot be too large, because then the expansion would be slowed down so much that there is not life time enough to produce old stars. This gives a limit on Ω,

$$0.1 \leq \Omega \leq 3. \tag{31}$$

Thus, we are not able to say whether the universe is closed or open from present day knowledge of Ω.

To solve the basic equations (18), (19), and (20), we need a relation between energy density and pressure. In the very early universe the temperature is high, and hence the average energy is very high, so all particles are relativistic. For such particles we always have that the energy is equal to the length of the momentum vector. Since the particles can move in three directions, this leads to

$$p = \rho/3 \ \ (\text{early universe}). \tag{32}$$

On the other hand, in the late universe temperature is low and the particles are non-relativistic, and hence the pressure is very small,

$$p \approx 0 \ \ (\text{late universe}). \tag{33}$$

From Eq. (20) we then get the density as function of the scale factor,

$$\text{Relativistic} : \rho = D/a^4, \quad \text{Non} - \text{relativistic} : \rho = B/a^3, \tag{34}$$

where D and B are constants. The decrease $1/a^3$ is what one would expect for a density. The additional $1/a$ factor in the relativistic case is due to the fact that relativistic energies are red-shifted by this factor, just like in the case of light (see Eq. (12)).

Using the Friedmann equation and the non-relativistic density given above, one can easily show that the life time is given by

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{dx}{\sqrt{1 - \Omega_0 + \Omega_0/x}} \leq \frac{1}{H_0}, \tag{35}$$

where H_0 and Ω_0 are the present value of the Hubble constant and the density parameter, respectively. From this result we see that the upper limit $t_0 = 1/H_0$ is obtained for $\Omega_0 = 0$, i.e. for a universe without any gravitational repulsion, as expected. Also, for a flat universe we easily get

$$t_0 = \frac{2}{3H_0} \ \text{ for } \ \Omega_0 = 1. \tag{36}$$

Finally, when we have a lot of energy, the life time decreases according to

$$t_0 \to \frac{\pi}{2H_0\sqrt{\Omega_0}} \text{ for } \Omega_0 \to \infty, \tag{37}$$

so if the enegy density was very large, the universe would have a very short life, as expected intuitively from the large gravitational attraction counteracting the expansion.

As indicated before, there are problems with the life time obtained from the most recent measurements of the Hubble constant and other indicators of the life time, like age of old stars and recent observations of faint galaxies. The latter have an age of $\approx$ 13 By (which is also approximately the age of old stars, 13-15 By), whereas there are indications that the Hubble constant corresponds to an age of 11 By or less. Of course, there are relatively large uncertainties in these numbers, and perhaps there is no discrepancy within the error bars. However, the various new results probably indicate that the situation is not so good, and we can therefore ask what to do if we forget about the error bars?

One way to get out of the age problem is to introduce the cosmological constant Λ, which has the effect of giving more expansion. This was explained in the introduction, where we mentioned that this freedom corresponds to having a new "force" Λr, causing repulsion, i.e. extra expansion. In this way the age problem can be solved. Of course, this is obtained at the cost of adding one new parameter.

6 EVIDENCE FOR THE BIG BANG COSMOLOGY

There exist two pieces of evidence for the standard cosmological model, the cosmic background radiation and nucleosynthesis. We discuss these subjects very briefly in the following.

6.1 The Cosmic Background Radiation

Near the big bang we have a relativistic gas, where all particles are effectively massless. As the universe expands, the temperature decreases, and matter and radiation cools correspondingly. At a temperature around 4000^0 K (the "decoupling temperature") matter and radiation decouple, because the free electrons join the nuclei to become bound into neutral atoms. Before, radiation (photons) Compton scattered on the electrons, and there is therefore no way of observing (directly) what happened in the universe before decoupling. After decoupling, we have a black body radiation, consisting essentially of the red-shifted photons left over at the decoupling. Thus, we have Planck's energy density,

$$\rho(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{e^{h\nu/kT} - 1}, \tag{38}$$

where we know how the frequencies red-shift from Eq. (12), $\nu_0/\nu_1 = a(t_1)/a(t_0)$. Assuming thermal equilibrium, we therefore have the important relation

$$T_0/T_1 = a(t_1)/a(t_0), \tag{39}$$

saying that the temperature behaves like the inverse of the scale parameter. Thus expansion means lower temperature, as expected. This relationship was used to predict that today the temperature of the background radiation should be approximately 5^0 K. In 1965 Penzias and Wilson found this radiation as a hiss in their detector, present no matter what direction their antenna pointed. NASA's COBE (the "Cosmic Background Explorer") found a marvellous agreement with the Planck law (38) at a temperature of 2.735 ± 0.06^0 K, which is thus the temperature of the present universe. This is thus a marvellous evidence for the correctness of the big bang scenario.

6.2 Big Bang Nucleosynthesis

From the standard cosmological model we can compute the expansion rate, how fast the universe cools, how fast it is slowed by gravity, etc. etc. Knowledge of nuclear physics then allows a calculation of the rates of the different relevant nuclear reactions in the early history of the universe.

In the very early universe there were no nuclei, only free quarks, gluons, photons, etc. The confinement mechanism for quarks was not yet operative, because of the high temperature. When the nucleons formed after the QCD phase transition, the universe was still so hot that it was not energetically favourable to form nuclei. After a few minutes, protons and neutrons formed nuclei. Then, from the known nuclear reaction rates one can compute the abundancies of the different nuclei. Most of the matter is hydrogen. Around 25% (by mass) is converted to helium. Of course, other nuclei can occur in small amounts. It should be noticed that most of the nuclei observed today were not produced in the early universe, but much later in the interiour of stars and in supernova explosions. However, the primary source of the lightest nuclei is the early universe.

The result of a calculation of the abundancies gives for helium 4 a mass fraction of 25%. For helium 3, deuterium, and lithium 7 one gets mass fractions $\approx 10^{-5}, 10^{-4}$, and 10^{-9}, respectively. These numbers agree well with the known abundancies. Also, it is predicted that the number of light neutrinos should be 3 or 4 (assuming that this number is an integer), and much later this number was found by CERN to be 3 to a very high accuracy.

The calculations of these abundancies depend on the density of protons and neutrons in the universe. In accordance with Eq. (26) this is assumed to be $\approx 10^{-30} - 10^{-31}$g/cm^3, but the results are rather sensitive to this.

The nucleosynthesis is a success for the big bang cosmology, since if we did not have this framework, there is no reason for these particular abundancies.

7 PROBLEMS WITH THE STANDARD BIG BANG MODEL

Having seen in the last section that the standard cosmological model has been very successful, we now turn to some problems with this model.

7.1 How to Get a Huge Number?

We saw that in the early, relativistic universe, the energy density behaves like D/a^4. We can get a lower limit on D by noticing that it contains contributions from all relativistic particles, including the photons. Thus, if we want a lower limit, we can include only the photons. Since we know the Planck spectrum today, we get

$$D \geq D_{\text{background}} \quad \text{with} \quad D_{\text{background}} = (kT_0)^4 \frac{8\pi^5 a_0^4}{15h^3}, \tag{40}$$

where T_0 and a_0 are the temperature and scale factor today, respectively. In natural units this gives

$$D \geq 6 \times 10^{114}, \tag{41}$$

where we used the known temperature today, and an estimate of the scale factor today from the remarks in section 5.

The point about Eq. (41) is that in any fundamental theory, it is impossible to conceive of how to get such a huge number as obtained in Eq. (41). Rather, one would expect a D of order one. There is a long way from 1 to 10^{114}!

7.2 The Flatness Problem

Let us consider the neighbourhood of the value $\Omega = 1$. To this end we rewrite the Friedmann equation (28) in the form

$$k = \dot{a}^2(\Omega - 1), \tag{42}$$

where we used that $H^2 = \dot{a}^2/a^2$ and multiplied both sides of Eq. (28) by a^2. If we go backwards in time, we know from the analysis in section 4 that $\dot{a}$ increases. Thus, from the above equation we see that

$|\Omega - 1|$ should get smaller, as we go backwards in time. Since we know that now Ω is not far from one (it is at least 0.3 and at most 3), it follows that in the past Ω must have been *extremely* close to 1.

Let us give an example: Assume that $\Omega = 0.3$ today. Then, from the variation of $\dot{a}$ it follows that it must have been equal to one to an accuracy of 15 decimal places at the nucleosynthesis, and at the GUT (The Grand Unification of strong, electromagnetic and weak interactions at a temperature of the order 10^{15}) Ω must have been equal to one to an accuracy of 49 decimals. This extreme "fine tuning" of Ω is certainly not explained by the standard model, and since $\Omega = 1$ means a flat universe, this difficulty is called the "flatness problem".

7.3 The Causality (Horizon) Problem

Let us consider the early relativistic universe, and ask the question whether two points can be in causal contact? Obviously, if a light signal can propagate from one point to the other, it means that physical processes in the second point can be influenced by conditions in the first point. In this way one can understand how the universe can be homogeneous and isotropic, provided the universe is "contained" within a causal distance. We shall, however, show that this is not the case in the early universe, and this gives rise to "the causality problem".

For the propagation of light in flat space, one has $dl = dt$, with $c = 1$. However, in cosmology we must distinguish the physical and the comoving coordinates. If dl denotes a comoving distance, then the physical distance is scaled up by $a(t)$, so for light we therefore have $adl = dt$. Integrating this equation, we get

$$l = \int dl = \int dt/a(t), \tag{43}$$

where the integration is taken between the two points. The physical distance is, however, not the comoving l, but $a(t)l$, so

$$\text{Physical distance} = d_{12} = a(t_2) \int_{t_1}^{t_2} dt'/a(t'). \tag{44}$$

Using $\rho = D/a^4$ in the Friedmann equation (18), we get

$$\dot{a}^2 \approx 8\pi G\rho/3 = 8\pi GD/(3a^2), \tag{45}$$

with the solution $a(t) \propto \sqrt{t}$. Hence the physical distance is

$$d_{12} = 2t_2(1 - \sqrt{t_1/t_2}) \approx 2t_2 \text{ for } t_2 \gg t_1, \tag{46}$$

where we used Eq. (44). Now, in the early universe, all times are small, so

$$d_{12} \sim 2t_2 \ll a(t_2) \propto \text{const.}\ \sqrt{t_2}. \tag{47}$$

Since a sets the physical scale, it follows that the early universe is "causally disconnected", since there can be a large number, of order $1/\sqrt{t}$, of causal domains inside a typical scale.

Therefore one may ask how it is possible for two points in the early universe to know that they are supposed to have the same temperature? This problem is especially transparent if we consider photons emitted from the opposite sides of the sky. As already mentioned, there is thermal equilibrium at the *same* temperature to a very high precision, although there was no possibility for these regions of space to have causal contact before the photons were emitted. Certainly, the standard big bang cosmological model does not explain this.

To conclude this section, we see that a closer look at the standard model leads to a number of unsolved problems. There are more problems than those mentioned above, e.g. the famous monopole problem: According to particle physics magnetic monopoles are produced in typical GUT scenarios, but they have never been observed.

8 INFLATION

In the previous discussion we often assumed that the pressure is positive, but in field theory this is not always true. Here we shell discuss this from a rather intuitive point of view.

Let us assume that at an early time the universe is in a metastable state, called the "false vacuum". This simply means that on a sufficiently short time scale the energy cannot be lowered. Thus, the false vacuum is *temporarily* the lowest energy state. Given enough time, however, the false vacuum decays to the "true vacuum", which is the state of the lowest possible energy.

The *inflationary scenario* assumes that it takes a rather long time for the false vacuum to decay to the true one. If you think in terms of a potential, this means that it has a rather long and flat plateau. The false vacuum is then the plateau state.

8.1 The Negative Pressure of the False Vacuum

Let us now find the pressure of the false vacuum. To this end, imagine a cylinder enclosing false vacuum. Outside this cylinder, we have true vacuum. Furthermore, there is a piston in the cylinder, so that we can change the volume of false vacuum by moving the piston. Changing the volume by dV, we need to satisfy $dE + pdV = 0$. Now comes the main point: Since the false vacuum is temporarily the state of lowest energy, the "new" volume dV must also contain the false vacuum. Denoting the constant energy density of the false vacuum by ρ_f, the change in energy is simply given by $dE = \rho_f dV$. Since $dE = -pdV$, we therefore get

$$p = -\rho_f < 0 \tag{48}$$

for the pressure of the false vacuum. It is negative, since by definition the energy density is positive. The extra energy obtained in the above "experiment" comes from whoever is pulling the piston, because it is necessary to do work against the negative pressure. So in the above gedanken-experiment "there is no free lunch". However, in the universe...?

8.2 The Expansion Driven by the Negative Pressure

The relation (48) between negative pressure and the energy density of the false vacuum has rather profound consequences, upsetting the standard result that gravity is attractive. To see this, consider Eq. (19),

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a. \tag{49}$$

Usually this "ensures" that gravity is attractive, since $\ddot{a} < 0$ when the pressure is positive. However, inserting the relation (48), we obtain

$$\ddot{a} = +\frac{8\pi G}{3}\rho_f a. \tag{50}$$

From this we see that *gravity is repulsive* as long as the false vacuum is the energetically accessible state of the universe[12].

Eq. (50) can easily be solved, since ρ_f is constant, and we obtain

$$a(t) = \text{const.}\ \exp\left(t\sqrt{8\pi G\rho_f/3}\right), \tag{51}$$

where we ignored a possible exponentially decreasing term. This strong expansion is clearly driven by the negative pressure.

In natural units any energy density has dimension (mass)^4. Thus we can estimate $\rho_f \sim M^4$, where M should be a typical mass (or energy) in the very early universe. So it is natural to think of the GUT scale, with $M \sim 10^{15}$ GeV. Thus,

$$\rho_f \sim (10^{15}\ \text{GeV})^4. \tag{52}$$

[12]The general condition for this behaviour is, of course, simply $p < -\rho/3$.

This is an enourmous energy density: If the sun should have this density, it should be compressed to the size of a proton!

From Eq. (51) we can compute the Hubble constant,

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3} G\rho_f}. \tag{53}$$

So in inflation the Hubble "constant" is actually a genuine constant! Inserting the value of ρ_f given by (52), we get

$$H \approx 10^{34} \sec^{-1}. \tag{54}$$

This is a correct estimate if inflation is valid up to the GUT scale.

9 THE PROBLEMS OF STANDARD BIG BANG SOLVED BY INFLATION

We shall now see that an inflationary scenario actually solves the problems in the standard cosmological model.

The first problem was related to the lower bound on the constant D in the relativistic expression for the energy density, $\rho = D/a^4$, where $D > 10^{114}$. This can be understood if we assume that in the history of the universe, first there is a relativistic period, with some fundamental theory producing a value $D_1 = O(1)$. So we are far from the lower limit. Next, inflation sets in at time t_1, producing expansion, with the constant energy density ρ_f. Then, when the GUT scale is reached at time t_2, the universe again becomes radiation dominated, with a density $\rho = D_2/a^4$. There must, of course, be continuity in the energy density. Thus, we have the two conditions

$$\rho(t_1) = \rho_f \;\text{ and }\; \rho_f = \rho(t_2). \tag{55}$$

Inserting the expressions for the densities, this leads to

$$\rho_f = D_1/a(t_1)^4 = D_2/a(t_2)^4, \tag{56}$$

so that

$$D_2 = e^{4H(t_2-t_1)}\, D_1. \tag{57}$$

To get $D_2 > 10^{114} \approx \exp(4 \times 66)$ from $D_1 = O(1)$, we need

$$H(t_2 - t_1) \geq 66. \tag{58}$$

If we compare this with the estimate (54), we see that inflation should happen over a time interval

$$t_2 - t_1 \geq 10^{-32} \text{ sec}. \tag{59}$$

This is actually of the order of magnitude expected for time intervals in the early universe.

To see how the flatness problem is solved, we need to consider the equation (42) giving a relation between $\dot{a}$ and Ω, and take into account the exponential expansion of $a(t)$ during inflation. This leads to the conclusion that whatever value Ω has at time t_1 (the beginnig of inflation), it must rapidly approach 1. Thus, the prediction is[13]

$$\Omega = 1. \tag{60}$$

This result does not compare well to the estimates by the astronomers discussed in section 5. However, in principle there may be some uncertainty and surprises in the estimation of the amount of dark matter present. In any case, if there is a cosmological constant Λ the situation is different, because of the energy content of the universe related to Λ. We shall, however, not discuss this point.

[13]There has been proposal for inflationary theories without this result. They appear rather unnatural to me.

The solution of the causality problem is basically quite simple: Suppose we go backwards in time, and start with the present non-relativistic universe with zero pressure. Here there is no problem with causal connections. Moving backwards, we come to the time when inflation stops. Now our rather large universe is *contacted* exponentially to a very small universe, which can be causally connected: The physical distance, exhibited in Eq. (44), is essentially given by the constant $1/H$ (up to exponentially small terms), whereas the scale factor is exponentially decreasing. Going the other way in time, what happens is that one causally connected region contains the present universe, so there is no problem with causality[14]. These remarks are valid only if the inflationary period couples smoothly to the radiation dominated universe. We shall not discuss this difficult question here.

So the final picture is that "the universe" may always be large or infinite, but our part of it started out as a very small domain in the large universe. When it started out, the relevant domain was of the order 10^{-9} of the size of a present day proton.

An important effect of inflation is that it dilutes away any primordial particles like magnetic monopoles (because of the huge exponential expansion), which is the reason why these are not observed. Similar remarks apply to other possible primordial particles.

Finally, after inflation has ended, we would like the universe to return to the standard hot big bang universe, since we certainly want to preserve the successful properties of this model. Therefore the energy released in inflation should be turned into standard particles (quarks, gluons, photons,etc.) without recreating unwanted monopoles. This transition is known as the *reheating*, and one must ensure that the temperature does not reach such a high value that the unwanted make a thermal pop-up!

10 QUALITATIVE DISCUSSION OF OTHER ASPECTS OF INFLATION

The fact that inflation is homoeopathic, i.e. dilutes particles away, can be taken as an explanation of why the universe is homogeneous. This is a very dangerous argument, because in the actually observed universe there are certainly some inhomogeneous domains (like galaxies). Thus, we do not like a fundamental theory to lead to an *exactly* homogeneous universe, this should only be an *approximate* property. Otherwise we would never be able to understand how galaxies are formed.

As long as we consider inflation from a purely classical point of view, the resulting universe would indeed be homogeneous. However, in quantum mechanics there is always fluctuations. This is especially true at a time when the size of the universe is only 10^{-9} times the size of a proton. During the inflation, each dimension is blown up by something like 10^{28}, so that even small fluctuations can acquire large scale properties. This effect would lead to *density fluctuations*, i.e. fluctuations in ρ. In the present universe, the extension of such fluctuations can then be of the order the size of a cluster of galaxies.

Thus, according to inflation the largest structures observed are blown up quantum fluctuations! This is certainly an interesting and unexpected aspect of quantum mechanics, which is usually said to be relevant only for small scale structures like atoms. Not so in cosmology!

The COBE satelite found the density fluctuations in the background radiation, The temperature variations $(\delta T/T)^2$ have the extension expected from inflation, but the magnitude is not so clear[15].

Another aspect of inflation is that it may be eternal. Of course, the decay of the false vacuum is exponential with some life time. On the other hand, when the false vacuum is operative, it drives an exponential expansion. These are two competing mechanisms. So if the expansion is faster than the decay, the total volume of false vacuum increases with time[16]. In this picture pieces of false vacuum domains are decaying constantly, but other domains expand. So the false vacuum never disappears. Of course, for each decaying domain, a universe is born. Thus, there is an infinity of universes, without any causal contact. In this scenario, in a sense there are no initial conditions and no "creation".

[14]For a more detailed discussion and a graphic illustration, see the recent review [3].

[15]In a $\lambda\phi^4-$ approach to inflation this can be translated to the condition $\lambda < 10^{-13}$.

[16]In some formulations of inflation, the parameters can be selected in such a way that this is possible

Inflation is very often described by means of a scalar field ϕ, with a simple Lagrangian (for simplicity written in flat Minkowski space)

$$\mathcal{L} = -\frac{1}{2}\left(\frac{\partial\phi}{\partial x^\mu}\right)^2 - V(\phi), \tag{61}$$

where the "potential" V is taken to be rather flat, in order to ensure that the false vacuum exists long enough, as discussed before. The corresponding energy-momentum tensor is

$$T_{\mu\nu} = \frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} - g_{\mu\nu}V(\phi). \tag{62}$$

Here the metric is (-1,1,1,1). If the derivative terms are small (consistent with a flat potential), this energy-momentum tensor leads to $p = -\rho$, as can be seen by comparison with the energy-momentum tensor of an ideal gas, $T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)U_\mu U_\nu, U^\mu = (1,0,0,0)$.

There are other models of inflation, like having corrections to Einstein's gravity, vector fields, etc. etc. Some of these models lead to power law inflation, $a(t) \propto t^p$. A difficulty is that in some of these models there is no natural way of stopping inflation, a feature which disagrees with observations. Also, the scalar models have the difficulty that they do not explain where the scalar field ("the inflaton") ϕ comes from. So perhaps it is fair to say that the "final model" of inflation has not yet been found. For a recent pedagogic review of inflation, we refer to ref. [3], where several further references can be found.

11 CONCLUSIONS

We are now coming to the end of these two lectures. The conclusion is that the standard big bang cosmology works well after Grand Unification, except perhaps for the age problem, which may anyhow be solved by introduction of a cosmological constant. To understand the very early universe, inflation is a rather convincing scenario or, perhaps, paradigme.

In my lectures I concentrated on what cosmologists call the standard model. To really understand the universe, there are many more aspects to be considered. This is the point where particle physics enters the picture. If we start from the early times, inflation is described by some field teory, which perhaps can be understood from fundamental particle physics. It may be that inflation can be derived from a theory of quantum gravity or a superstring theory.

Similarly, at the GUT scale, we have some, so far unknown, (field) theory, which describes the unification of the strong, electromagnetic and weak interactions. It is generally believed that the GUT theory is supersymmetric. Subsequently this symmetry is broken, and perhaps it is possible to see various traces of the broken symmetry in present day accelerators.

Reaching the electroweak phase transition, at a temperature of approximately 100 GeV, we are on rather firm ground, since we expect the standard electroweak theory to be the right one (perhaps supersymmetric extensions are relevant). Some questions of cosmological interest, which can be raised in connection with this phase transitions, concern the problem of why there is more matter than antimatter in the (present) universe, the generation of primordial magnetic fields (which are seed fields for the magnetic fields observed in galaxies), and other problems. In this connection it is of crucial importance to understand whether this phase transition is of first or higher order. This depends essentially on the value of the Higgs mass, which everybody hopes that the experimental particle physicists will provide. If this mass is larger than $\approx$ 100 GeV, the transition is not of first order, which would have profound consequences for many questions.

After the electroweak phase transition, at a temperature of the order a few hundred MeV, the QCD phase transition occurred, during which quarks became confined in hadrons. This process is, in principle, described by Q(uantum) C(hromo-)D(ynamics), although the exact mechanism for confinement is not quite understood. At still lower temperatures, nuclear and atomic physics became relevant. However, the fundamental structure of the universe is a subject of particle physics, and future discoveries in experimental particle physics can potentially be of profound interest in cosmology.

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