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Three-Flavour Neutrino-Mixing Implications of the LSND Result

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Abstract

The LSND result is shown to fit into a minimal three-flavour neutrino-mixing scenario capable of describing all known experimental facts provided the large $\Delta M^2 = m_3^2 - m_2^2 \sim m_3^2 - m_1^2$ lies in the range $2.5 \times 10^{-1} < \Delta M^2 < 3.0$ eV^2. In this range the value of $P_{\mu\tau}$ is expected to be about 5% or larger.

1 Introduction

In recent years, several unexpected results have appeared in accelerator, atmospheric and solar neutrino physics [1]. Although none of them is really beyond questioning, collectively they represent a fair evidence for the existence of some new phenomenon.

Taken singularly, all these results can be explained in terms of neutrino oscillations [2]. However, this interpretation can only be viable if all existing positive and negative results can be accounted for by a unique set of oscillation parameters.

In this paper we examine whether there exist some conditions under which the LSND result [3], however controversial it may be [4], may fit into a minimal three-flavour neutrino-mixing scenario constrained by all experimental observations. We show that such a global description is possible. Ranges of values of the oscillation parameters for which this occurs are given.

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2 Three-flavour neutrino-mixing phenomenology

In the complete three-flavour approach, the weak eigenstates $|\nu_\alpha\rangle = \nu_e,\ \nu_\mu,\ \nu_\tau$ and the mass eigenstates $|\nu_i\rangle = \nu_1,\ \nu_2,\ \nu_3$ are related by a unitary transformation matrix U. The probability of an initial neutrino ν_α of energy E being equal to another neutrino ν_β at a distance L, can be written as

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2(\Delta_{ij}/2) \quad (1)$$

with $\Delta_{ij} = \Delta m^2_{ij} L/2E$, where $\Delta m^2_{ij} = m^2_i - m^2_j, m_i = m_{\nu_i}$.

Assuming CP-invariance, the U-matrix is real and can be parametrized as

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, where θ_{12}, θ_{13} and θ_{23} are three independent real angles lying in the first quadrant.

Of the three Δm^2_{ij}'s appearing in eq. (1), only two are independent. Therefore, the complete solution of the problem consists in determining five unknowns: two Δm^2_{ij}'s and the three θ_{ij}'s.

Under the additional hypothesis of a natural mass hierarchy $m_1 << m_2 << m_3$, the oscillatory behaviour of eq. (1) is determined by the large $\Delta M^2 = m^2_3 - m^2_2 \sim m^2_3 - m^2_1$ and the small $\Delta m^2 = m^2_2 - m^2_1$. The transition probabilities are then given by the sum of two terms describing, respectively, the fast oscillation (characterised by ΔM^2) and the slow oscillation (characterised by Δm^2).

Lastly, the assumption of the dominance of diagonal terms in the mixing matrix, implying a strong correlation between flavour and mass eigenstates, ensures that $s_{ij} < c_{ij}$. The angles θ_{ij}'s in the mixing matrix are then uniquely defined.

3 Experimental inputs and results

The oscillation analysis of the atmospheric neutrino data implies a δm^2 of about 10^{-3} eV^2 [5, 6]. Such a small δm^2 cannot account for the LSND observation of $(P_{e\mu})_{\rm LSND} = (3.1 \pm 0.09 \pm 0.05) \times 10^{-3}$ at $L/E = 0.7$ m/MeV. Consequently, it must be identified with the smaller Δm^2, thus implying $\Delta m^2 \sim 10^{-3}$ eV^2.

This defines the two ranges $L/E < 10^3$ m/MeV, in which the transition probabilities $(P_{\alpha\beta})_1$ are dominated by the fast component, and $L/E > 10^3$ m/MeV in which the transition probabilities $(P_{\alpha\beta})_2$ depend on both the slow and fast components. The average transition probabilities $\langle P_{\alpha\beta}\rangle_1$ and $\langle P_{\alpha\beta}\rangle_2$ are calculated from eq. (1) for $\sin^2(\Delta_{12}/2) = 0$, $\langle\sin^2(\Delta_{13}/2)\rangle = 0.5$ and $\langle\sin^2(\Delta_{12}/2)\rangle = 0.5$, $\langle\sin^2(\Delta_{13}/2)\rangle = 0.5$, respectively.

$\Delta m^2 \sim 10^{-3}$ eV^2 implies in turn the energy-independence of all oscillation phenomena occurring in solar neutrinos. This is consistent with the present experimental situation.

The energy-independence of the solar neutrino deficit has been long advocated [7, 8, 9, 10, 11]. Using the most recent experimental results [12, 13, 14, 15] and theoretical predictions [16] but neglecting solar model systematic errors [11, 16], a fit for an energy-independent oscillation-induced depletion of the ν_e flux yields the result

$$\langle P_{ee}\rangle_2 = 0.50 \pm 0.06 \tag{2}$$

with a confidence level C. L. = 0.34 %. This is admittedly marginal but, in view also of the many sometimes optimistic approximations, not unacceptably small.

The day- and night-spectra measured by the Super-Kamiokande experiment also show no anomalous behaviour. The fit for the same energy-independent suppression of the β-decay expectations in both spectra yields a confidence level C. L. = 1.6% [15].

The over-all confidence level including all the information from solar neutrino rates, day/night effect and energy spectrum shape is C. L. = 0.26%, largely dominated by the marginal consistency among rates. Thus, even neglecting all caveats [10], a deviation from energy-independence in solar neutrinos has at the most the significance of a 3σ effect.

A second consequence of $\Delta m^2 \sim 10^{-3}$ eV^2 is that for $L/E < 10^3$ m/MeV all oscillation phenomena depend only on the two angles θ_{13} and θ_{23}, the influence of the third angle θ_{12} becoming sizeable only for $L/E = 10^3$ m/MeV or greater.

Two-flavour analyses of transition probability results are normally presented in terms of contours in the $\sin^2(2\theta)$, δm^2 plane. From eq. (1) it is easy to see that, for $L/E < 10^3$ m/MeV, the $\sin^2(2\theta)$'s relative to the transitions ν_e - ν_x, ν_e - ν_μ and ν_μ - ν_τ are in fact, respectively, the three-flavour oscillation amplitudes $(A_{ex})_1 = 4s_{13}^2c_{13}^2$, $(A_{e\mu})_1 = 4s_{23}^2s_{13}^2c_{13}^2$ and $(A_{\mu\tau})_1 = 4s_{23}^2c_{23}^2c_{13}^4$.

Thus, from the knowledge of the maximum values experimentally allowed for the first and the third of these amplitudes as a function of ΔM^2 [17, 18, 19], an upper limit contour for the second can be readily determined.

The limit-curve for $(A_{e\mu})_1 = 4s_{23}^2s_{13}^2c_{13}^2$ as a function of ΔM^2 is shown in Figure 1 together with the 99% C. L. LSND-allowed region. Their compatibility clearly restricts ΔM^2 approximately to the range

$$2.5 \times 10^{-1} < \Delta M^2 < 3.0 \text{ eV}^2.$$

ΔM^2 (eV^2)	3.0	1.8	0.25
s_{13}^2	3.6×10^{-2}	2.0×10^{-2}	9.8×10^{-3}
s_{23}^2	2.0×10^{-2}	1.8×10^{-2}	0.50
$(P_{e\mu})_{\rm LSND}$	1.4×10^{-3}	1.4×10^{-3}	0.94×10^{-3}
$(P_{\mu\tau})_{\rm LSND}$	3.6×10^{-2}	6.8×10^{-2}	4.8×10^{-2}
$\langle P_{ee}\rangle_1$	0.93	0.96	0.98
$\langle P_{e\mu}\rangle_1$	1.4×10^{-3}	0.71×10^{-3}	9.7×10^{-3}
$\langle P_{\mu\mu}\rangle_1$	0.96	0.97	0.50
$\langle P_{\mu\tau}\rangle_1$	3.6×10^{-2}	3.4×10^{-2}	0.49
$r(\Theta = \text{small})$	1.0	1.0	0.50

Table 1: Mixing angles and transition probabilities for the three types of solutions allowed by the LSND result for $L/E < 10^3$ m/MeV. $(P_{\alpha\beta})_{\rm LSND}$ represents a transition probability calculated for the LSND value of $L/E = 0.7$ m/MeV, $\langle P_{\alpha\beta}\rangle_1$ is the same quantity averaged after the onset of the fast oscillation, $r(\Theta = \text{small})$ is the ratio between the observed and expected N_μ/N_e ratio measured in the Superkamiokande experiment at short L.

Within this range, three different situations may occur in an experiment centered around a typical L/E of 0.7 m/MeV. The term $\sin^2(\Delta M^2 L/4E)$ is close to the average value of 0.5 at the upper end of the range, reaches its maximum in the central region and falls to small values at the lower end.

The angles θ_{13} and θ_{23} obviously depend on ΔM^2 and on the amount by which the limit of Figure 1 is accepted to be violated in order to reach the LSND-allowed region. If, in order to avoid exceeding any limit, they are conservatively chosen to coincide with their upper limits, for the three typical choices of ΔM^2 above they take the values reported in Table 1. These solutions are indicated by dots in Figure 1.

In the allowed mass range above, s_{13}^2 is constrained by the tight reactor limits and cannot have large variations whilst s_{23}^2 can swing by as much as a factor of twentyfive. However, the transition rates at the LSND value $L/E = 0.7$ m/MeV are relatively constant around the low-side values $(P_{e\mu})_{\rm LSND} = 1 \times 10^{-3}$ (2σ down relative to the LSND result) and $(P_{\mu\tau})_{\rm LSND} = 5 \times 10^{-2}$.

The Superkamiokande experiment has measured the ratio r between the observed and expected ν_μ/ν_e ratios in cosmic rays as a function of the zenith angle Θ [5, 6]. Small values of Θ correspond to $L/E < 10^3$ m/MeV so that $r(\Theta = \text{small})$ can be calculated from the the average rates $\langle P_{\alpha\beta}\rangle_1$ reported in Table 1 through the relation $r = (P_{\mu\mu} + \rho P_{e\mu})/(P_{e\mu}/\rho + P_{ee})$ where $\rho = 0.47 \pm 0.02$ is the expected ν_e/ν_μ flux ratio in the absence of oscillations [7]. Depending on ΔM^2, $r(\Theta = \text{small})$ varies between 0.5

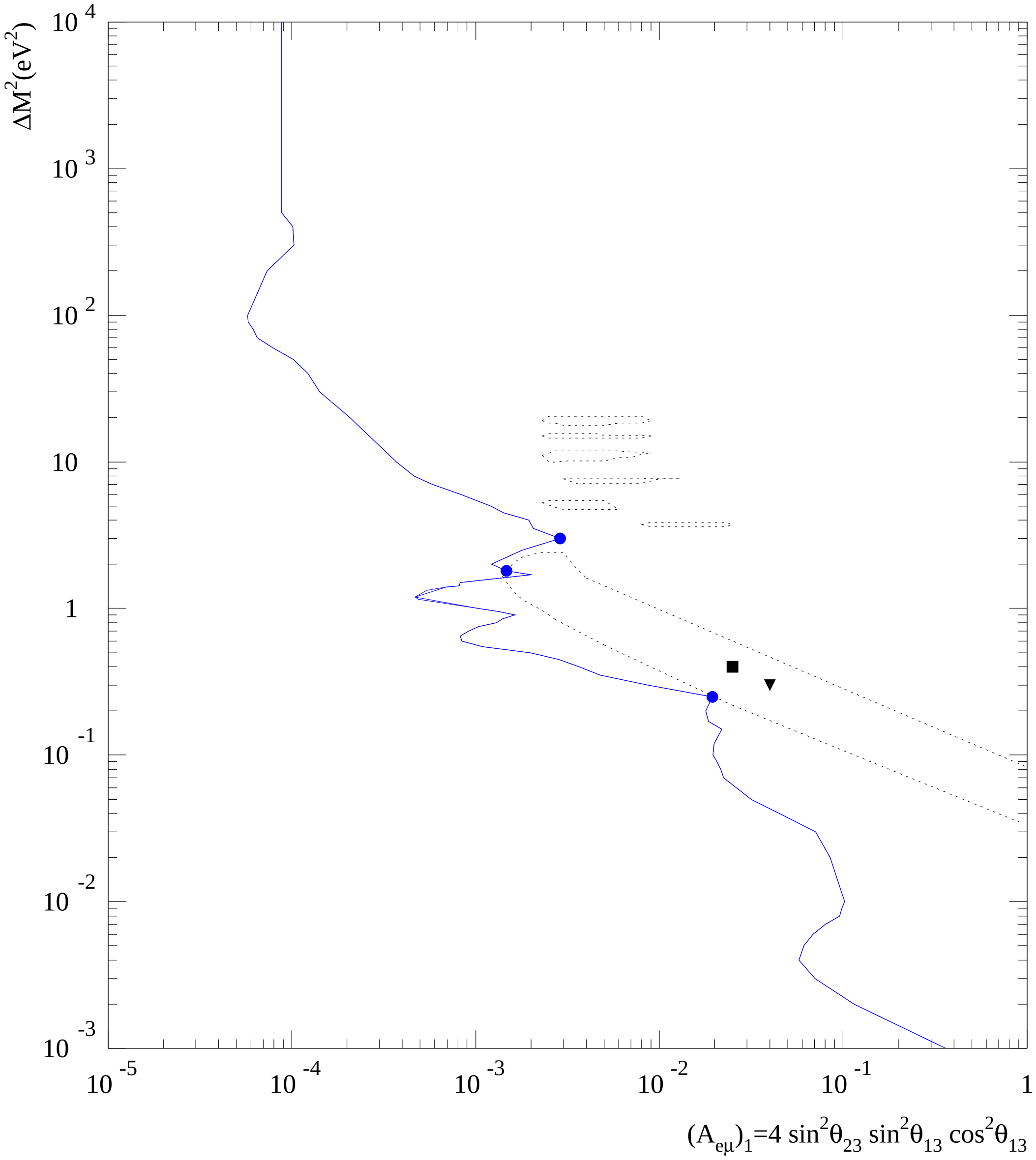

Figure 1: The upper limit on the oscillation amplitude $(A_{e\mu})_1 = 4s_{23}^2 s_{13}^2 c_{13}^2$ calculated from the maximum values experimentally allowed for $(A_{ex})_1 = 4s_{13}^2 c_{13}^2$ and $(A_{\mu\tau})_1 = 4s_{23}^2 c_{23}^2 c_{13}^4$ as a function of ΔM^2 (full line) compared with the 99% C. L. region allowed by the LSND result (dotted lines) . The dots correspond to the choices of the angles θ_{23} e θ_{13} for three typical values of ΔM^2. The square and the triangle represent two solutions recently proposed.

ΔM^2 (eV^2)	3.0	1.8	0.25
s^2_{13}	3.6×10^{-2}	2.0×10^{-2}	9.8×10^{-3}
s^2_{23}	2.0×10^{-2}	1.8×10^{-2}	0.50
s^2_{12}	0.36	0.40	0.43
$\langle P_{e\mu} \rangle_2$	0.44	0.47	0.26
$\langle P_{e\tau} \rangle_2$	0.05	0.03	0.24
$\langle P_{\mu\mu} \rangle_2$	0.51	0.50	0.37
$\langle P_{\mu\tau} \rangle_2$	0.05	0.04	0.37
$r(\Theta = \text{large})$	0.49	0.48	0.47

Table 2: Mixing angles and transition probabilities for the three types of solutions allowed by the LSND and solar neutrino results for $L/E > 10^3$ m/MeV. $\langle P_{\alpha\beta} \rangle_2$ represents a transition probability averaged after the onset of the slow oscillation, $r(\Theta = \text{large})$ is the ratio between the observed and expected N_μ/N_e ratio measured in the Superkamiokande experiment at long L.

and 1.0. The Superkamiokande data indicate a value around 0.8. More precise data could help restricting the range of allowed parameters around the LSND result.

Two most recently suggested solutions $\Delta M^2 = 0.4$ eV^2, $s^2_{13} = 3.2 \times 10^{-2}$, $s^2_{23} = 0.2$ [20] and $\Delta M^2 = 0.3$ eV^2, $s^2_{13} = 5.1 \times 10^{-2}$, $s^2_{23} = 0.21$ [21] are also indicated in Figure 1 by a square and a triangle, respectively.

The third angle θ_{12} can easily be determined from eq. (2). This yields the results reported in Table 2.

The salient feature in the region $L/E > 10^3$ m/MeV is the presence of a large ν_e - ν_μ transition. The value of r for long L, $r(\Theta = \text{large})$, is practically constant at a value slightly below 0.5, in good agreement with the Superkamiokande data.

The behaviours of the $P_{\alpha\beta}$'s as a function of L/E for various values of ΔM^2 are shown in Figure 2. The curves are averaged over a Gaussian L/E distribution with 30% width.

For the smallest value of ΔM^2 (0.25 eV^2), the similarity of $P_{e\mu}$ and $P_{e\tau}$ implies the expectation of a vanishing up/down ν_e asymmetry A_e in the Superkamiokande experiment, in complete agreement with the measured value [5, 6].

On the other hand, for the two larger values of ΔM^2 (1.8 and 3.0 eV^2), the dominance of $P_{e\mu}$ over $P_{e\tau}$ for $L/E > 10^3$ m/MeV corresponds to positive values of A_e. This is still an open possibility as the over-all fit to the Superkamiokande data of the

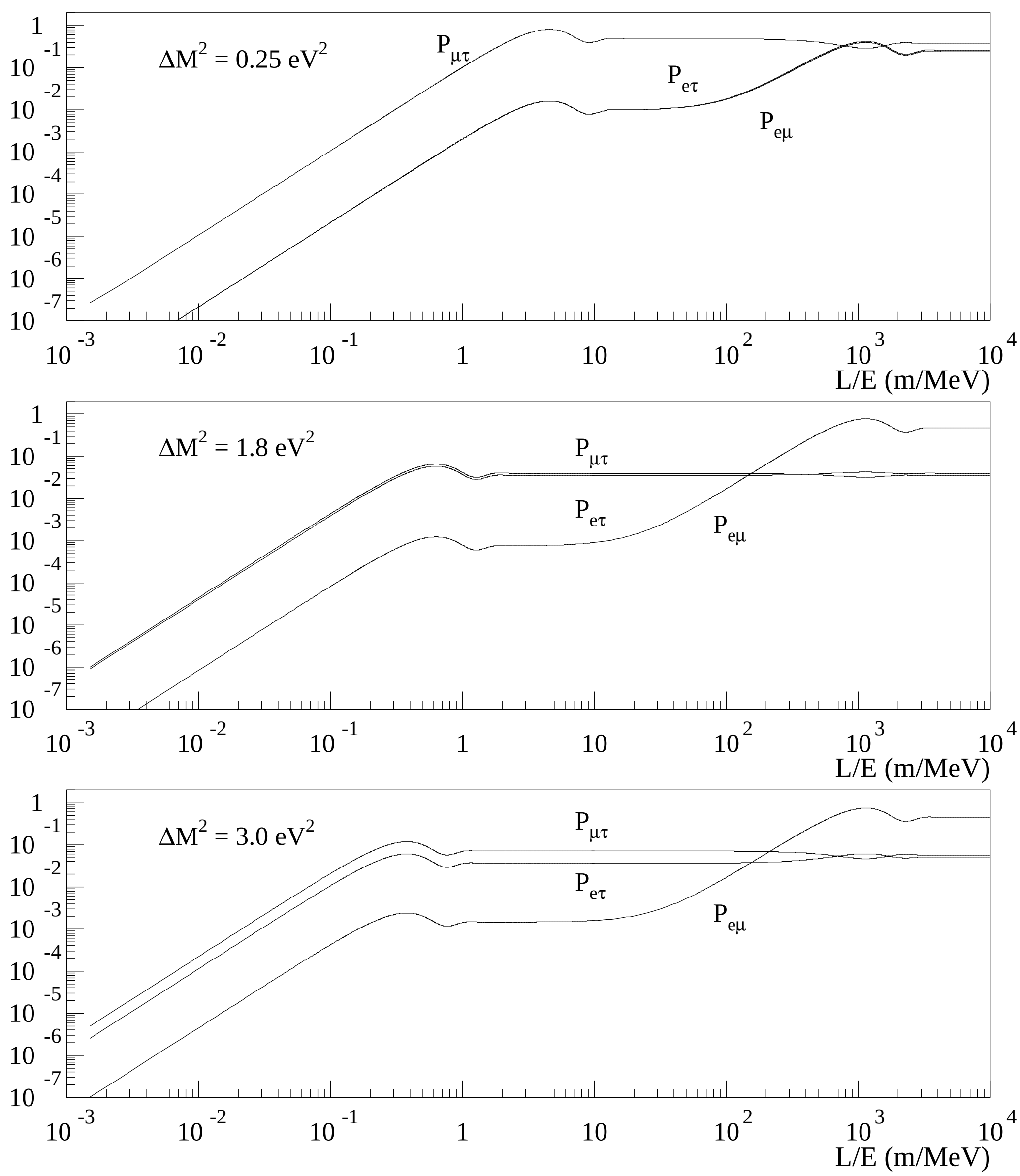

Figure 2: The transition probabilities $P_{\alpha\beta}$ calculated as a function of L/E for the three typical values of ΔM^2. The curves are averaged over a Gaussian L/E distribution with 30% width.

two-flavour hypothesis of ν_e - ν_μ oscillations alone is quite acceptable (C. L. = 4.4%) and yields a large $P_{e\mu}$ for $L/E > 10^3$ m/MeV ($\sin^2(2\theta) = 0.93$) [5, 6].

Future better data may be able to clarify this issue and possibly further restrict the acceptable range of ΔM^2.

4 Conclusions

Standard three-flavour neutrino-mixing phenomenology, supplemented by the hypotheses of natural mass hierarchy ($m_1 << m_2 << m_3$) and strong correspondence between flavour and mass eigenstates ($s_{ij} < c_{ij}$), is quite adequate to interpret all neutrino phenomena observed so far.

Together with the oscillation analysis of the atmospheric neutrino data, the LSND result implies a Δm^2 of about 10^{-3} eV^2.

For $L/E < 10^3$ m/MeV all phenomena depend only on the two angles θ_{13} and θ_{23}. The available upper limits on the quantities 1 - $(P_{ee})_1$ and $(P_{\mu\tau})_1$ provide enough information to calculate the upper limit on $(A_{e\mu})_1 = 4s_{23}^2 s_{13}^2 c_{13}^2$ as a function of ΔM^2 shown in Figure 1. From the compatibility with the LSND result, the allowed range of ΔM^2 is

$$2.5 \times 10^{-1} < \Delta M^2 < 3.0 \text{ eV}^2,$$

implying the immediate cosmological consequence that the Universe cannot be closed by neutrinos alone.

Experiments studying the range $L/E \sim 1$ m/MeV [22, 23] have clearly optimal chances to detect the characteristic feature of oscillations, namely a modulation as a function of L/E. On the basis of the observed ν_μ - ν_e transition probability, conservatively taken as $(P_{e\mu})_{\text{LSND}} \sim 1 \times 10^{-3}$, a $(P_{\mu\tau})_{\text{LSND}} \sim 5 \times 10^{-2}$ is expected.

It should be noted that $(P_{e\mu})_1$ and $(P_{\mu\tau})_1$ are linked by the relation

$$(P_{e\mu})_1/(P_{\mu\tau})_1 = s_{13}^2/(c_{13}^2 c_{23}^2)$$

so that an increase in $(P_{e\mu})_1$ implies necessarily a larger $(P_{\mu\tau})_1$.

The calculated values of r are in good agreement with the so far not very accurate measurements reported by the Kamiokande experiment. However, the comparison between calculations and experimental results is really meaningful only in the two extremes of the L/E range, the region in-between depending on many experimental features. It should be emphasized that a value of $r(\Theta = \text{small})$ measured to be significantly below 1, together with the smallness of $(P_{e\mu})_{\text{LSND}}$ and of $1 - \langle P_{ee}\rangle_1$ would constitute an experimental evidence for a sizeable $\langle P_{\mu\tau}\rangle_1$.

The large ν_e - ν_μ transition expected for $L/E > 10^3$ m/MeV is compatible with the results of the CHOOZ experiment [19] only for $\Delta m^2 \sim 10^{-3}$ eV^2 or smaller.

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