LABORATORI NAZIONALI DI FRASCATI
SIS-Pubblicazioni

LNF-98/020(P)
24 Giugno 1998

Supersymmetric hadronic bound state detection at e^+e^- colliders

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Abstract

We review the possibility of formation for a bound state made out of a stop quark and its antiparticle. The detection of a signal from its decay has been investigated for the case of a e^+e^- collider.

PACS:13.60.Le;14.80.-j;14.80.Ly

Submitted to Physics Letters B

1 Introduction

In the Standard Model it has been verified that there is creation of bound states for every quark but the top [1,2]. The latter possibility is ruled out due to the high value of the top quark mass, which is responsible for its short lifetime. The natural step forward would be to consider the possibility of bound states creation outside the Standard Model. In this case we focus our attention to the supersymmetric extensions of the Standard Model [3], in particular to the detection of a bound state (supermeson) created from a stop and an anti–stop ("stoponium") at e^+e^- colliders.

2 Bound States

In this Section we will review the bound states creation. For the SUSY case, our assumption will be that the bound state creation does not differ from the SM case, as the relevant interaction is again driven by QCD, and is regulated by the mass of the constituent (s)quarks.

A formation criterion states that [2] the formation of a hadron can occur only if the level splitting between the lying levels of the bound states, which depend upon the strength of the strong force between the (s)quarks and their relative distance [1], is larger than the natural width of the state. It means that, if

$$\Delta E_{2S-1S} \geq \Gamma \tag{1}$$

where $\Delta E_{2S-1S} = E_{2S} - E_{1S}$, Γ is the width of the would–be bound state, then the bound state exists.

For the case of a scalar bound state $\tilde{t}\bar{\tilde{t}}$, without referencing to a particular supersymmetric model, we should consider the Coulombic two–body interaction

$$V(r) = -\frac{4}{3}\,\frac{\alpha_s}{r} \tag{2}$$

with the two–loop expression for α_s [4]

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \log\left[Q^2/\Lambda^2_{\overline{MS}}\right]} \left\{1 - \frac{2\beta_1}{\beta_0^2}\frac{\log\left[\log\left[Q^2/\Lambda^2_{\overline{MS}}\right]\right]}{\log\left[Q^2/\Lambda^2_{\overline{MS}}\right]}\right\} \tag{3}$$

with $\beta_0 = 11 - \frac{2}{3}n_f$, $\beta_1 = 51 - \frac{19}{3}n_f$. Due to the present limits on the stop mass [5,6] and due to the fact that in our assumptions the stop is lighter than the top quark, we should set $n_f = 5$. The α_s expression (3) has to be evaluated at a fixed scale $Q^2 = 1/r_B^2$, where r_B is the Bohr radius

$$r_B = \frac{3}{4\mu\alpha_s} \tag{4}$$

and μ is the reduced mass of the system. It has been shown in [2,1] that in the case of high quark mass values, the predictions of the Coulombic potential evaluated at this scale do not differ from the other potential model predictions.

In figures 1 and 2 we show a plot of the energy splitting for the first two levels for the stoponium bound state with respect to the stop mass, for the LHC and the NLC case respectively. As from (1), those figures have to be compared to the width of the stoponium. The width of the stoponium, $\Gamma_{\tilde{t}\bar{\tilde{t}}}$, is twice the width of the single stop squark, as each should decay in a manner independent from the other.

There are several ways a stop should decay [7], depending on the assumptions made for the other superpartners. In the most interesting cases, the highest width value for a range of the stop squark mass of 60 to 100 GeV, relevant to LEP, and up to 500 GeV for NLC, will not exceed the value of a few KeV. Those are to be compared to the ΔE_{2S-1S} values, which from figures 1 and 2 are of the order of the GeV , thus larger than the width of the bound state for three orders of magnitude, fulfilling eventually the requirement of (1).

A different formation criterion states that the bound state exists if the revolution time, $t_R = 2\pi r/v$, is larger than the lifetime of the rotating quarks, $\tau = 1/\Gamma$ [8], that is

$$t_R < \tau \ . \tag{5}$$

This criterion has been proven to be stronger than (1) by about a factor of two on the upper mass limit [1]. In any case the choice of either formation criterion does not change the results obtained so far, and we shall conclude that the stoponium could be formed.

3 Cross Section and Decay Width

The next natural step would be to see whether the stoponium could be detected on an e^+e^- collider with LEP or future NLC characteristics. For this purpose we shall calculate its cross section and decay modes; basing our predictions on [9], and updating their results.

We should look for the production and decay of the P wave state, since we are interested in the search of the bound state at a e^+e^- collider, conserving thus quantum numbers.

We use the Breit–Wigner formula to evaluate the total cross section [5]:

$$\sigma = \frac{3\pi}{M^2} \times \frac{\Gamma_e \Gamma_{tot}}{(E-M)^2 + \Gamma_{tot}^2/4} \tag{6}$$

where M is the mass of the resonance, E is the centre–of–mass energy, Γ_{tot} is the total width, and Γ_e is the decay width to electrons.

The first decay we will investigate is the leptonic one, which is given by the Van Royen–Weisskopf formula [10]

$$\Gamma(2P \to e^+e^-) = 24\alpha^2 Q^2 \frac{|R'(0)|^2}{M^4} \tag{7}$$

$R'(0)$ is the derivative of the radial wavefunction calculated at the origin, M the mass of the bound state, α the QED constant, Q the (s)quark charge.

For this and following cases, we shall make use of the radial wavefunctions of the Coulombic model, as presented in Section (1). Those are, for the $1S$ state

$$R_{1S}(r) = \left(\frac{2}{r_B}\right)^{3/2} \exp\left(-\frac{r}{r_B}\right) \tag{8}$$

and for the $2P$

$$R_{2P}(r) = \frac{1}{\sqrt{3}} \left(\frac{1}{2r_B}\right)^{3/2} \frac{r}{r_B} \exp\left(-\frac{r}{2r_B}\right) \tag{9}$$

r_B is the Bohr radius defined in (4) .

For the hadronic width decay we have the following expression

$$\Gamma(2P \to 3g) = \frac{64}{9} \alpha_s^2 \frac{|R'(0)|^2}{M^4} \log(m_{\tilde{t}} r_B) \tag{10}$$

where the Bohr radius acts as an infrared cutoff [9].

The $2P$ state could also decay into a $1S$ state and emit a photon. The width decay in this case is given by

$$\Gamma(2P \to 1S + \gamma) = \frac{4}{9} \alpha Q^2 (\Delta E_{2S-1S})^3 D_{2,1} \tag{11}$$

where ΔE_{2S-1S} is the energy of the emitted photon, and $D_{2,1} = \langle 2P|r|1S\rangle$ is the dipole moment [11]. In figures 3 and 4 we present the decays of the $2P$ state into hadrons and into a $1S$ state plus a photon as a function of the stop mass, as predicted by the Coulombic model. One observes that there is not a strong variation of the decay widths with respect to the stop mass, and that both have similar values of the order of some KeV. We could notice also a small threshold effect due to the inclusion of the top flavour.

For a light stop – i.e. lighter than its Standard Model partner – the analysed modes so far are the dominant widths [7].

Figures 5 and 6 shows the peak cross section obtained from (6) as function of the stop mass. While the peak cross section is in the nb range, the resonance is practically undetectable at the present colliders because its width is much smaller than the typical beam energy spread (of the order of 200 MeV at LEP2 [5]). The effect of a growth of the total width – due to e.g. other squarks or R–parity violating terms [12] – does not change the result, as the net effect will be a decrease of the peak cross section. This is clearly illustrated in figure 7 where the Breit–Wigner formula (6) is folded with the typical energy spread of the beam of 200 MeV, and in figure 8, where the beam energy spread of the NLC is taken to be of the order of 2.8% [13].

4 Conclusions

We have shown that because of the high energy binding and of the narrow decay width the formation of a $\tilde{t}\bar{\tilde{t}}$ bound state is possible. Our result shows that this supersymmetric bound state cannot be detected at the present and even future e^+e^- collider, and this result holds true even for bound states made out of sqarks different from the $\tilde{t}$. The latter fact proves also that it gives a negligible contribution to the $\tilde{q}\bar{\tilde{q}}$ production cross section.

Acknowledgments
We Thank G. Pancheri for useful discussion and suggestions. One of us (N.F.) wishes to thank A. Masiero for useful discussion.

References

[1] N. Fabiano, *Eur. Phys. J.* **C2** 345 (1998).

[2] N. Fabiano, A. Grau and G. Pancheri, *Phys. Rev.* **D50** (1994) 3173;
"Toponium from different potential models", Nuovo Cimento A, Vol 107, 2789,(1994).

[3] For a review see:
Ed. M. Jacob, *Supersymmetry and Supergravity*.

[4] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, *Phys. Rev.* **D18** (1978) 3998;
W.J. Marciano, *Phys. Rev.* **D29** (1984) 580.

[5] Review of Particle Physics, *Phys. Rev.* **D54** (1996) 1,
http://pdg.lbl.gov/

[6] ALEPH Collaboration, *Search for Scalar Top and Scalar Bottom Quarks at LEP2, Phys. Lett.* **B 413** (1997) 431;
Search for Light Top Squarks in $p\bar{p}$ Collisions at $\sqrt{s} = 1.8$ TeV, Phys. Rev. Lett. **76** (1996) 2222;
OPAL Collaboration, *Search for Scalar Top and Scalar Bottom Quarks at $\sqrt{s}$= 170 GeV - 172 GeV in e^+ e^- Collisions, Z. Phys.* **C 75** (1997) 409.

[7] K. Hikasa and M. Kobayashi, *Phys. Rev.* **D36** (1987) 724.

[8] I. Bigi, Y. Dokshitzer, V. Khoze, J. Kuhn and P. Zerwas, *Phys. Lett.* **B181** (1986) 157.

[9] C. Nappi, *Phys. Rev.* **D25** (1982) 84.

[10] R.Van Royen and V.Weisskopf, *Nuovo Cimento* **50A** (1967) 617.

[11] J. Kühn and P.M. Zerwas, *Phys. Rev.* **167** (1988) 321.

[12] A. Bartl et al., *Phys. Lett.* **B384** (1996) 151; *Z. Phys.* **C73** (1997) 469.

[13] Ed. R. Brinkmann, G. Materlik, J. Rossbach, A. Wagner, *Conceptual Design of a 500 GeV* e^+e^- *Linear Collider with Integrated X–Ray Laser Facility*, Vol. 1 (1997).

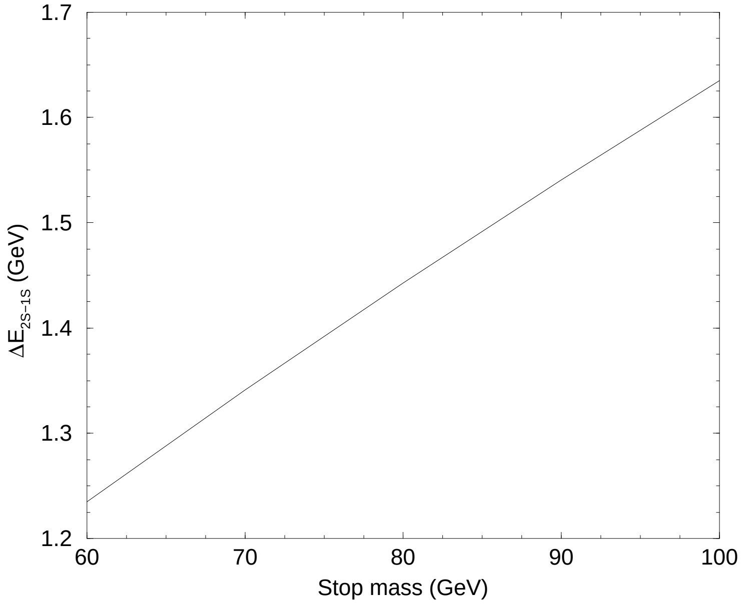

Figure 1: ΔE_{2S-1S} *as a function of the stop mass up to 100 GeV for the Coulombic model.*

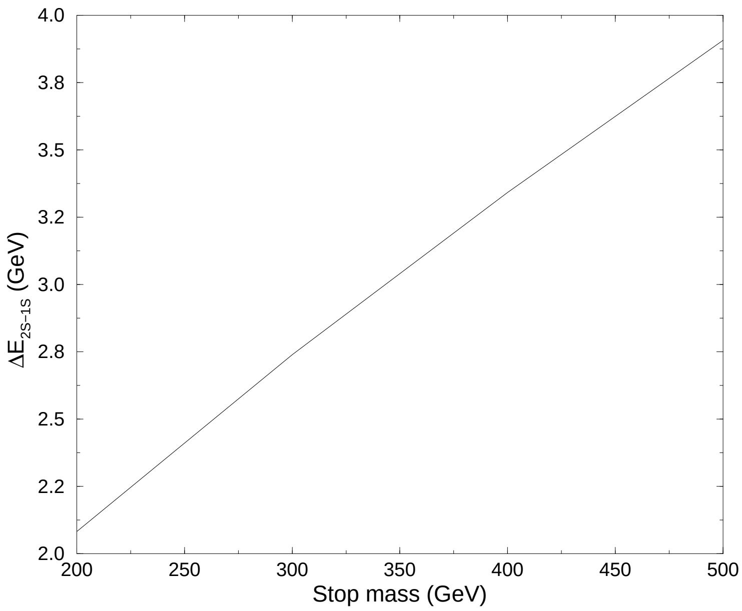

Figure 2: ΔE_{2S-1S} *as a function of the stop mass up to 500 GeV for the Coulombic model.*

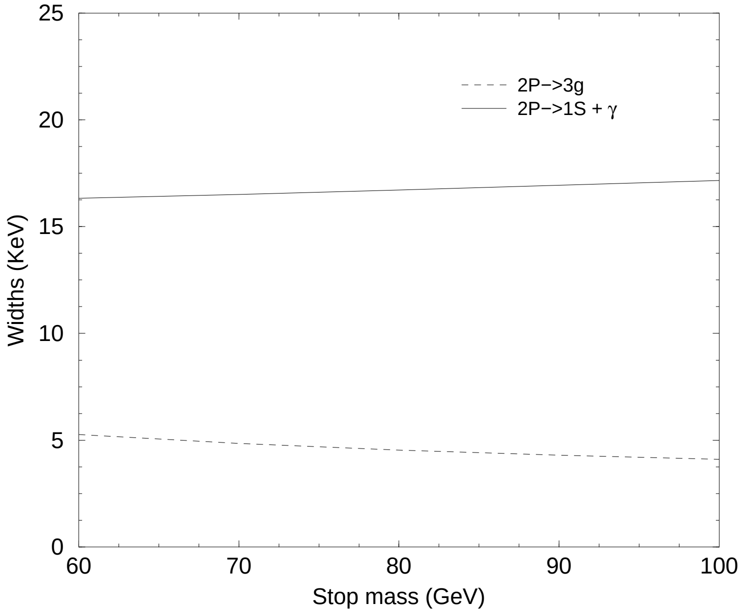

Figure 3: *Decay widths for the 2P state with respect to the stop mass for the Coulombic model. The dashed line represents the decay into hadrons, the continuos line the decay into the* $1S$ *state and an emitted photon.*

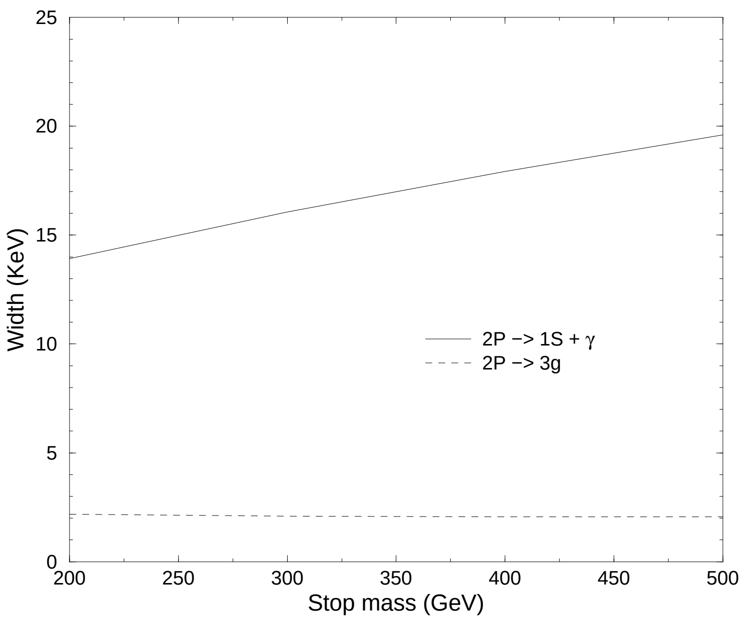

Figure 4: *Like Fig. 3, for a mass range of up to 500 GeV, for NLC.*

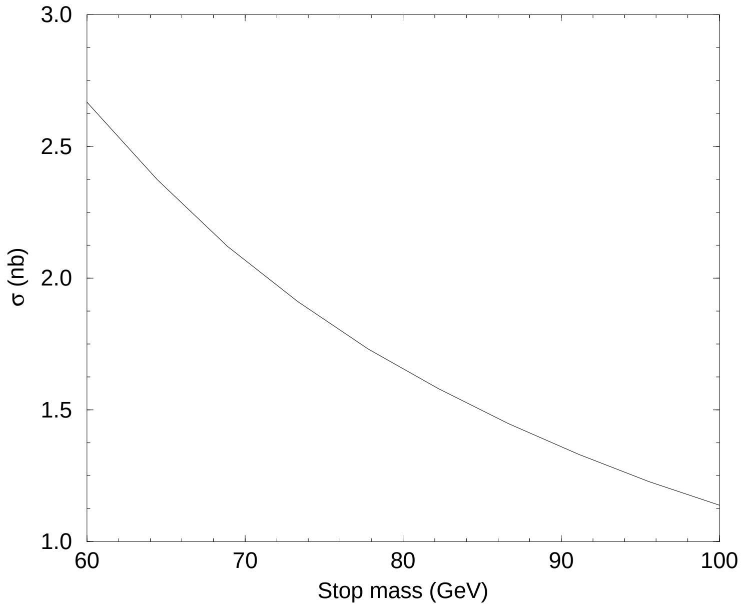

Figure 5: *Peak cross section as a function of the stop mass, for the LEP case.*

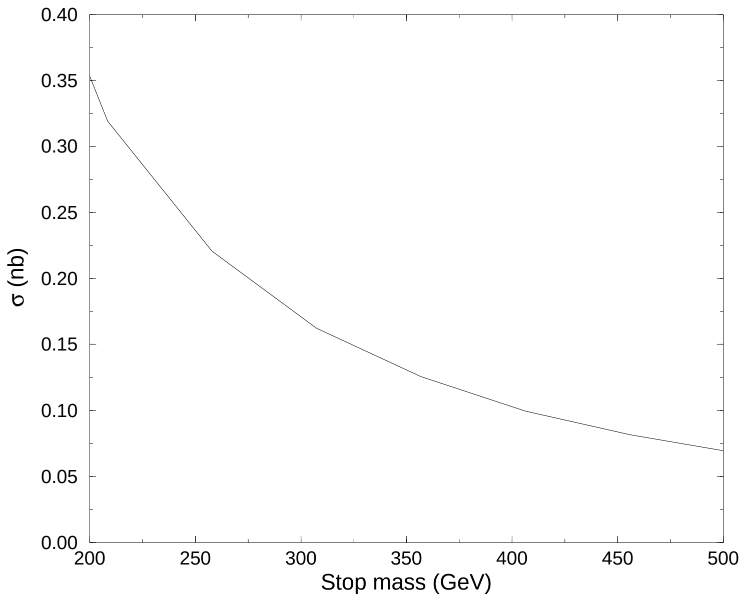

Figure 6: *Peak cross section as a function of the stop mass, for the NLC case.*

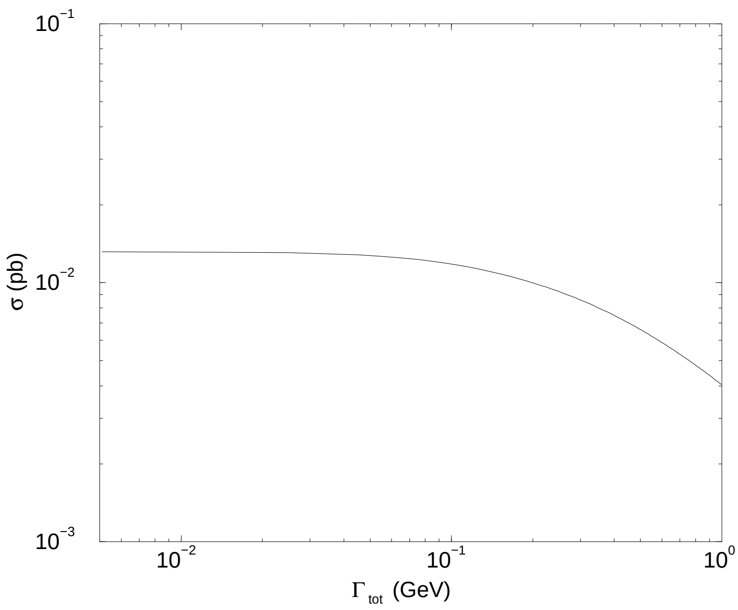

Figure 7: *Total cross section folded with a beam energy spread of 200 MeV as a function of the total width of the stop. The plot has been obtained for a stop mass of 100 GeV.*

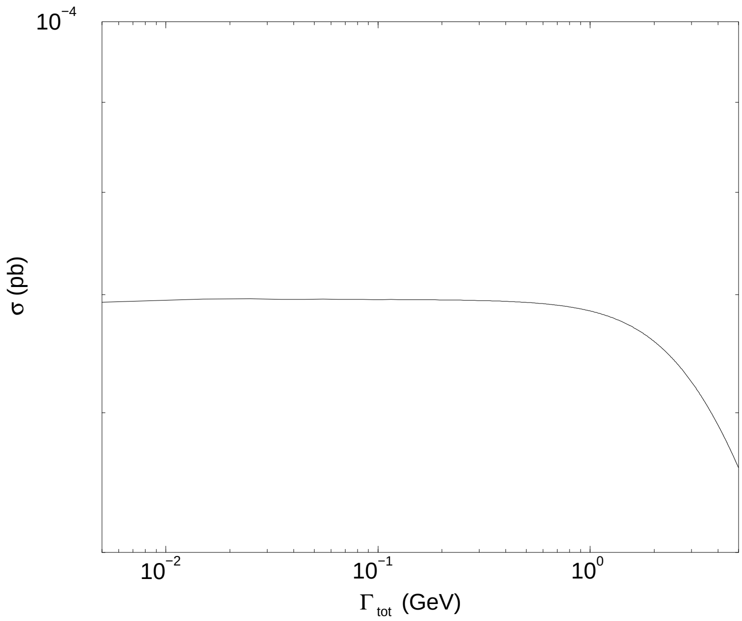

Figure 8: *Like Fig. 7, for a beam energy spread of 6 GeV (NLC). The plot has been obtained for a stop mass of 200 GeV.*